# WHAT IS THE OPTIMAL STITCHING ORIENTATION FROM AN AERIAL SURVEY PERSPECTIVE?

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**Abstract.** Mosaicing is a technique that allows obtaining a large high resolution image by stitching several images together. These base images are usually acquired from an elevated point of view.
Until recently, low-altitude image acquisition has been performed typically by using using airplanes, as well as other manned platforms. However, mini unmanned aerial vehicles (MUAV) endowed with a camera have lately made this task more available for small for cicil applications, for example for small farmers in order to obtain accurate agronomic information about their crop fields.
The stitching orientation, or the image acquisition orientation usually coincides with the aircraft heading assuming a downwards orientation of the camera. In this paper, the effect of the image orientation in the efficiency of the aerial coverage path planning is studied. Moreover, an algorithm to compute an optimal stitching orientation angle is proposed and results are numerically compared with classical approaches.

**Keywords:** Mini Unmanned Aerial Vehicles, Image mosaicing, Optimal heading, Computational geometry

## 1 Introduction

Nowadays, the mini unmanned aerial vehicles (MUAV) market makes available to small farmers a high variety of aircraft with an affordable cost. Most of them allow, among other features, way-point programming, which is ideal for aerial imaging.

Nevertheless, farmers require tools that support them in order to establish all the parameters that are essential to fully define a flight. The flight program typically is made of a series of waypoints (i.e. points where pictures have to be taken). A correct definition of these waypoints should assure a specific spatial resolution as well as an important overlapping between consecutive images. This overlapping is required in order to guarantee an optimal mosaicing despite small errors in position and orientation during the flight.

The desired spatial resolution (i.e. the pixels/cm required in the resulting image) determines the flight altitude. The camera resolution (number of pixels), the sensor size and shape as well as the optical parameters (i.e., focal length) have to be also considered.

There are many references (Valente et al, 2013) where farmers or drone operators can find the equations required for determining the altitude and distance among waypoints to fulfill these requirements. Moreover, some drones already rely on software that allows computing the mentioned parameters, or even programming a complete flight considering rectangular shape fields.

Unfortunately, many fields do not rely on rectangular shapes or perfectly north-aligned boundaries. In these cases, farmers are forced to define enormous limits in order to obtain flight plans that cover the entire field, which turns out to be non-optimal.

Apart from the optimization in the number of images required for coverage, if a general case is considered, (i.e., unshaped field boundaries), the decision of the angle used as main direction acquires special relevance. Moreover, depending on the type of aircraft used for the flight, this angle can have influence in all the mission definition. Thus, when a fixed wing aircraft (i.e. a plane) is used, this angle will determine the flight direction and therefore will affect the entire flight plan so as to minimize the number of turns. On the other hand, if a rotary wing (e.g., helicopter or multi-rotor) with hovering capabilities is used, the influence for the flight is lower considering downwards camera orientation.

This work is therefore focused on determining the best orientation for aerial pictures in a coverage mission considering a further mosaicing process. The section 2 is dedicated to formally define the problem considered. The section 3 introduces the smallest area enclosing rectangle (SAER) concept, which is required by the algorithms. Section 4 illustrates the optimal heading approach. Section 5 provides with results, including several numerical algorithm validations and finally, section 6 summarizes the conclusions.

## 2 Problem definition

The problem addressed in this paper is to compute the minimum number of rectangular polygons - finite and homogeneous set - that stitched together are capable of covering a convex or concave polygon.

The problem could also be described in a less computational geometry manner: given a robot workspace with any polygonal shape and configuration, compute the minimum number of images needed to map it, subject to a specific spatial resolution and overlapping.

The overall sampling algorithm proposed is shown in Figure 1. However, in this paper the problem tackled is the one related with founding the minimum effective area of coverage. Herein, is proved that the minimum area of effective coverage is obtained by finding the SAER of the workspace in a pre-sampling iteration. The SAER orientation is then the MUAV heading in each point during the coverage task considering a fixed link between the camera and the vehicle frame, which is usual in order to reduce the weight of the payload to the maximum.

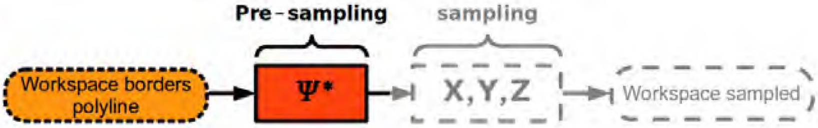

**Fig. 1.** Map-based algorithm abstract procedure.

## 3 The Smallest-Area enclosing rectangle problem

The SAER (aka minimum bounding box (MBB)) problem refers to find the smallest rectangle (i.e. minimum area or volume) that encloses a polygon. It should be notice that MBB is different from the minimum bounding rectangle (MBR). The MBR addresses a rectangle with the minimum and maximum coordinates of a polygon within its coordinate system.

This problem can be solved in $O(n^2)$ running time by calculating for each edge of the polygon a rectangle with at least one side collinear. After some time, this approach was improved to $O(n)$ with a method denoted as rotating calipers (Toussaint, 1983). This algorithm can be applied in different fields, e.g, pattern recognition, computer vision or robotics. Figure 2 illustrates the problem addressed.

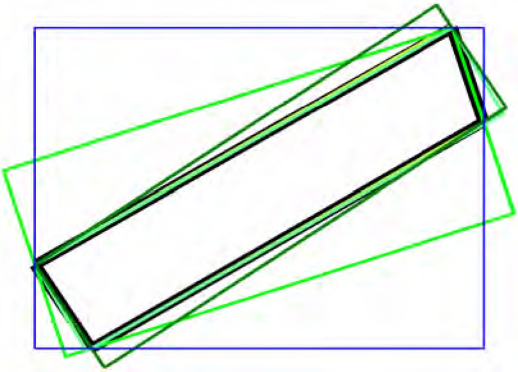

**Fig. 2.** SAER candidates (green) of rectangular polygon (black). MBR solution drawn in blue.

## 4 Optimal heading $\Psi^*$ approach

The heading computation plays an important role in aerial missions with both holonomic (a holonomic robot is one which is able to move instantaneously in any direction in the space of its degrees of freedom) and non-holonomic MUAVs. For instance, in case of non-holonomic MUAV, the heading has to be known in order to perform turn around maneuvers whilst holonomic MUAV heading could be oriented to the geodetic north (aka, true north) or any other fixed angle.

Although this clearly simplifies the algorithm and its execution time, it does not ensure a minimum number of samples over the workspace. When the workspace is sampled for computing coverage trajectories with non-holonomic MUAV with fixed sensor frame - such as, for example a digital camera, with a determinate sensor coverage - might be important to determine the MUAV orientation over the trajectory if we wish to obtain image data with a concrete constant orientation and optimize the coverage samples in according to it. Herein the optimal heading computation is addressed employing the combination of computation geometry algorithms, as well as deterministic methods.

**Lemma 1.** *The optimal heading angle is equal to the SAER orientation of a given field.*

*Proof.* The minimum area of effective coverage is proportional to the SAER of a determinate field. Therefore, a minimum area coverage is obtained if the MUAV heading angle is the same of the SAER orientation.

**Lemma 2.** *The optimal number of samples (i.e. lower bound) to cover a field is obtained by finding the number of samples that cover the SAER of that field*

*Proof.* Each sample is a photograph. A photograph is a rectangular projection of the surface. The set of images stitched together will have the appearance of a rectangle. It is assumed that the resulting rectangle bounds the workspace. How much effective is that bounding rectangle is the problem of finding the SAER.

**Lemma 3.** *The number of points to sample the SAER is less than or equal to the number of points to sample the MBR*

*Proof.* The MBR is the maximum rectangular area enclosing a polygon. Consequently, any candidate SAER area will be less than or equal to the MBR area of that polygon.

In this study, two approaches have been analyzed in order to compute the optimal heading. The first one is derived from 3. An alternative solution based on typical farmers criteria (i.e. consider the longest side of the field crop boundary) has been also considered.

In order to apply the second criterion, lets consider the following hypothesis:

**Hypothesis 1** *The SAER of a polygon can be directly obtained by finding its longest edge and compute the rectangle colinear to that edge.*

In this way, the goal is to find the longest edge of the polygon and compute the rectangle collinear to it. Although the longest-edge approach computing time is also linear (i.e. $O(n)$), it can be slightly moderated for a smaller $n$ since the bounding rectangle is just computed once. The rectangle boundaries are just computed when the longest edge is found. The longest edge is computed in linear time through basic arithmetic operations. However, the running time has not really a significant influence in the overall system since it works offline. In addition to this, one possible point in favor of the later approach is that is a fairly easy and intuitive from an implementation point of view.

## 5 Numeric simulations

In this section the algorithm reliability is demonstrated by trial within several workspaces that have been chosen in order to deal with a wide spectrum of cases.

### 5.1 Shapes set based in real data

In order to develop and test the methodologies explained in the previous sections a set of fields (or parcels) from the Community of Madrid (Spain) and Cundinamarca Department (Colombia) were carefully selected. The fields data set is shown in Figure 3.

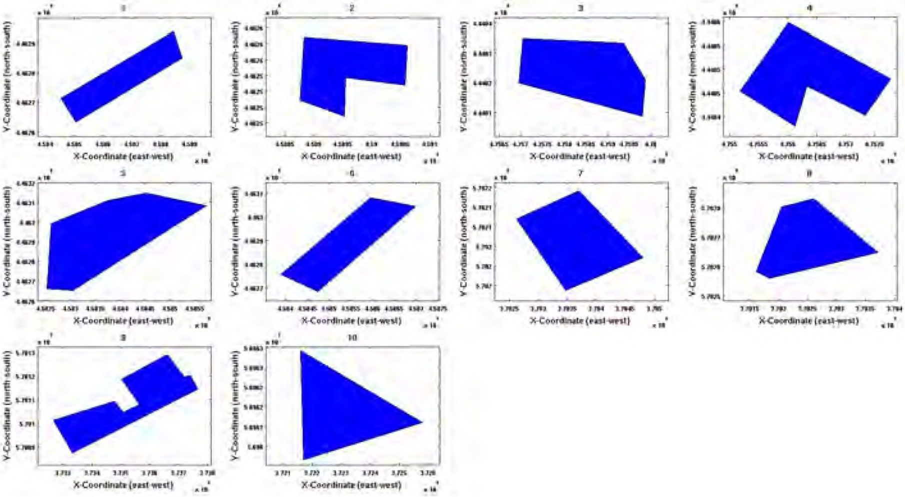

**Fig. 3.** Fields data set.

The fields were chosen to meet different characteristics, such as be convex, concave, and have different geometric shapes. Therefore, several shape indexes were computed to identify the heterogeneous magnitude of the fields set. Field shapes analysis have been used before in agricultural studies for several purposes, e.g., classification and studying the relationship with field operations (Oksanen, 2013). The shape indexes measured are: *Convexity*, the convexity is the ratio between the shape area and its convex hull area. It indicates how convex is the shape of the field; *Rectangularity*, the rectagularity is the ratio between the shape area and it MBR area. It gives the field shape order of rectagularity; *Triangularity*, it can be obtained by several ways, however for the sake of simplicity the same idea of rectangularity is followed as stated in (Rosin, 2003). Therefore, the triangularity is the ratio between the shape area and the Minimum Bounding Triangle (MBT) area. It indicates how triangular is the shape of the field. *Elipsity*, it gives the field shape order of elipsity. The moment invariant method was used to compute this magnitude.

Table 1 shows both individual and average shape indexes used in the algorithm deployment.

| Shape | Area ($m^2$) | Convexity | Rectangularity | Triangularity | Compactness | Ellipsity |
|---|---|---|---|---|---|---|
| 1 | 41.41K | 1 | 0.32 | 0 | 0.25 | 0.09 |
| 2 | 11.71K | 0.82 | 0.63 | 0 | 0.38 | 0.05 |
| 3 | 53.98K | 1 | 0.70 | 0 | 0.47 | 0.35 |
| 4 | 27.75K | 0.85 | 0.49 | 0 | 0.51 | 0.13 |
| 5 | 82.02K | 1 | 0.51 | 0 | 0.37 | 0.39 |
| 6 | 43.96K | 1 | 0.36 | 0 | 0.30 | 0.12 |
| 7 | 28.68K | 1 | 0.52 | 0.56 | 0.57 | 0.17 |
| 8 | 31.38K | 1 | 0.55 | 0 | 0.50 | 0.17 |
| 9 | 69.52K | 0.81 | 0.33 | 0 | 0.31 | 0.21 |
| 10 | 57.59K | 1 | 0.49 | 1 | 0.34 | 0.14 |

**Table 1.** Fields data set characteristics.

The shape data set used as trial are from 10 different workspaces with an average area of approximately 4ha. Table 1 shows the shape indexes measured for each workspace. Its clear-cut that the convexity is predominant in the data set, i.e., less concave fields than convex. Moreover, the round and elliptic shapes are less present than rectangular and triangular ones.

### 5.2 Results

The bounding rectangles areas have been computed with three different methods: MBR (TN), rotating calipers (RC), and longest-edge (LE). The orientation of each field has been computed relative to the true north and in CCW. Table 2 presents the respective values computed for each field.

The values from table 2 show that the effective area of coverage by the MUAV significantly improve if an SAER is computed before mapping a field. This can be stated by using the comparison from the first and second column. The area magnitudes of the second column are far lower than those of the first column.

| Field | TN Area ($m^2$) | RC Area ($m^2$) | LE Area ($m^2$) | Angle ($rad$) |
|---|---|---|---|---|
| 1 | 128763 | 43190 | 43190 | 0.52592 |
| 2 | 18608 | 17396 | 17396 | -0.058933 |
| 3 | 76865 | 76144 | 76144 | 2.7614 |
| 4 | 56633 | 38314 | 38314 | -5.59136 |
| 5 | 159411 | 114822 | 114822 | 1.013 |
| 6 | 122958 | 47179 | 47179 | 1.0136 |
| 7 | 55093 | 31851 | 31851 | 0.55579 |
| 8 | 56777 | 45833 | **45894** | -2.6766 |
| 9 | 209378 | 94032 | 94032 | 0.55721 |
| 10 | 116773 | 115185 | 115185 | 2.7304 |

**Table 2.** Bounding rectangle areas and angles computed in the field set.

Therefore, employing the SAER algorithm to the contour of the workspace an effective coverage improvement from 1.4% – 66% is obtained.

On the other hand, by comparing the rotating calipers and longest-edge results it can be noticed that the SAER computed is the same for all the fields, less for one.

In Figure 4 is shown the result of the three approaches after being applied to the contour of field 8.

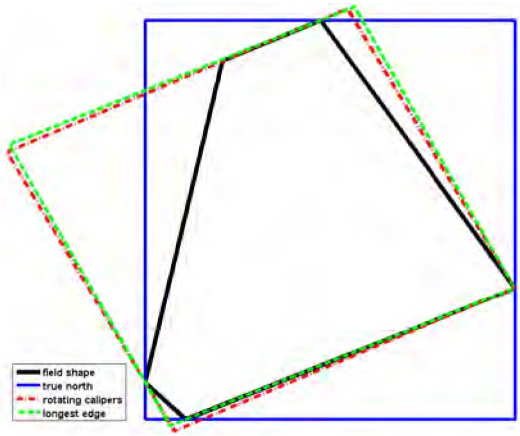

**Fig. 4.** Bounding rectangles computed with the different approaches (field 8).

The result of the preliminary study with 10 fields showed that a simple procedure as the Longest-edge method provides good results. Only one in 10 cases, a more complex method (i.e. rotating calipers) achieved slightly better results that the traditional approach of considering the longest edge of the parcel as guide for agricultural tasks.

Nevertheless, it is worth noting that agricultural fields usually rely on quite regular shapes. When strange shapes are in use, rotating calipers method is recommended so as to guarantee better results.

## 6 Conclusions

This study is focused on solving a common problem that happens in aerial imaging when mosaicing process is required, which is to determine the optimal heading angle to align the pictures. This heading angle affects to the flight plans, specially when non-holonomic aircrafts are used.

A formal analysis based on well-known computational techniques has been performed and numerical simulations considering several representative field shapes is provided.

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