ANALYTICAL MODELS FOR TEMPERATURE PREDICTION ON WELDING

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Abstract

Temperature field has an important influence on the phase changes and, finally, on the microstructure and mechanical properties, residual stresses and strains development in the welded joints. During the welding, temperature field shape and its profile depend on many factors but, assuming some assumptions regarding homogeneity and materials isotropy, the mathematical relations for different practical cases can be obtained. Besides, welded bodies can be infinite or semi-infinite as the plates, bars and massive bodies are considered. The instantaneous or permanent thermal sources, fixed or mobile have an important influence on the temperatures distribution. Therefore, the paper presents some analytical solutions for the temperatures prediction in the welded joints in case of the mobile sources with 2D and 3D Gauss distribution.

1. Equations of the temperature field

The general equation of the energy which is the start point for the temperature field analysis has the following expression:

$$\rho \cdot c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) - \rho \cdot c\left(v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z}\right) + Q_v \tag{1}$$

where v_x, v_y, v_z are the speed components on the three directions.

Considering the thermal source moving on the direction x (v_x=v, v_y=v_z=0) and the coordinates system mobile solidary with the thermal source, the energy balance equation can be written as:

$$\frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) - \rho \cdot c \cdot v \frac{\partial T}{\partial x} + Q_v = 0 \tag{2}$$

Due to the existence of the solid (s) and liquid (l) phases in the weld pool, the energy balance equations, in the quasi-stationary period, can be expressed as:

- in case of solid phase:

$$\frac{\partial}{\partial x}\left(\lambda_s \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_s \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_s \frac{\partial T}{\partial z}\right) - \rho \cdot c_s \cdot v \frac{\partial T}{\partial x} + Q_v = 0, \tag{3}$$

- in case of liquid phase:

$$\frac{\partial}{\partial x}\left(\lambda_l \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_l \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda_l \frac{\partial T}{\partial z}\right) - \rho \cdot c_l \cdot v \frac{\partial T}{\partial x} + Q_v = 0, \tag{4}$$

at solid-liquid interface:

$$(n \cdot v) \cdot \rho \cdot \Delta H = \lambda_s \left(\frac{\partial T}{\partial n}\right)_s - \lambda_l \left(\frac{\partial T}{\partial n}\right)_l; \qquad T = T_{top}. \tag{5}$$

The upper surface of the piece is under the influence of the thermal source and also of the heat lost in the environment. Therefore, the contour condition can be written as:

$$-\lambda \left.\frac{\partial T}{\partial z}\right|_{z=0} = q \tag{6}$$

2. Mobile thermal source of 2D Gauss distribution

2.1. Temperature field analysis in the butt welded joints

Considering the mobile thermal source of Gauss distribution, Eagar and Tsai changed the Rosenthal's theory [2]. Their solution represents an important step for the temperatures' approximation in the vicinity of the thermal source. Assuming an exponential repartition of the heat flow, its mathematical expression can be written as:

$$q = q_{\max} e^{-k \cdot r^2} , \qquad (7)$$

where k is the concentration factor which depends on the thermal source type.

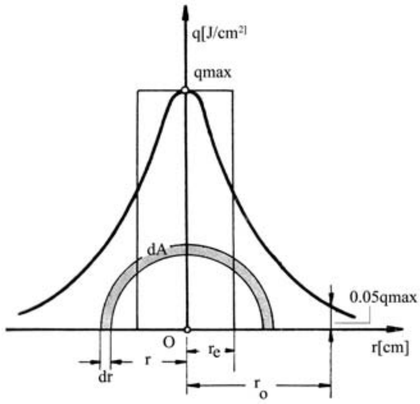

Figure 1. Gauss distribution of the heat flow.

Taking account of Gauss distribution of the heat flow, Pavelic et al. [1], developed the calculus expressions of the heat flow for the contact area and outside of the contact area:

$$q = \frac{3Q}{\pi \cdot r^2} \exp \frac{x^2 + y^2}{-r^2/3} \qquad \text{for: } \left(x^2 + y^2\right)^{1/2} \le r , \qquad (8)$$

where Q=η·U·I and $r=r_0$ is the effective radius of the thermal source when the heat flow decreases at 5% of maximum value 3Q/πr^2.
Outside of the area, which is under the influence of the thermal source, the heat flow's expression is:

$$q = -\left[\alpha \cdot (T - T_0) + \varepsilon \cdot C \cdot \left(T^4 - T_0^4\right)\right]$$

for: $\left(x^2 + y^2\right)^{1/2} \succ r$. (9)

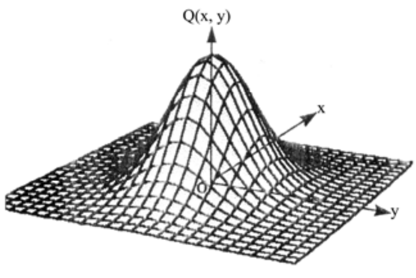

Figure 2. 2D Gauss distribution of heat flow.

2.2. Temperature field analysis in the fillet welds

Fillet welds represent 80% of the total naval production. Therefore, taking account of two-dimensional distribution of heat flow, Jeong and Cho [3] analysed temperature field in fillet welds and found the analytical solutions for the temperatures calculus. (Figure 2).

Since the geometrical shape and the electric arc distribution are complex, the authors replaced the real system with an equivalent one (Figure 3).

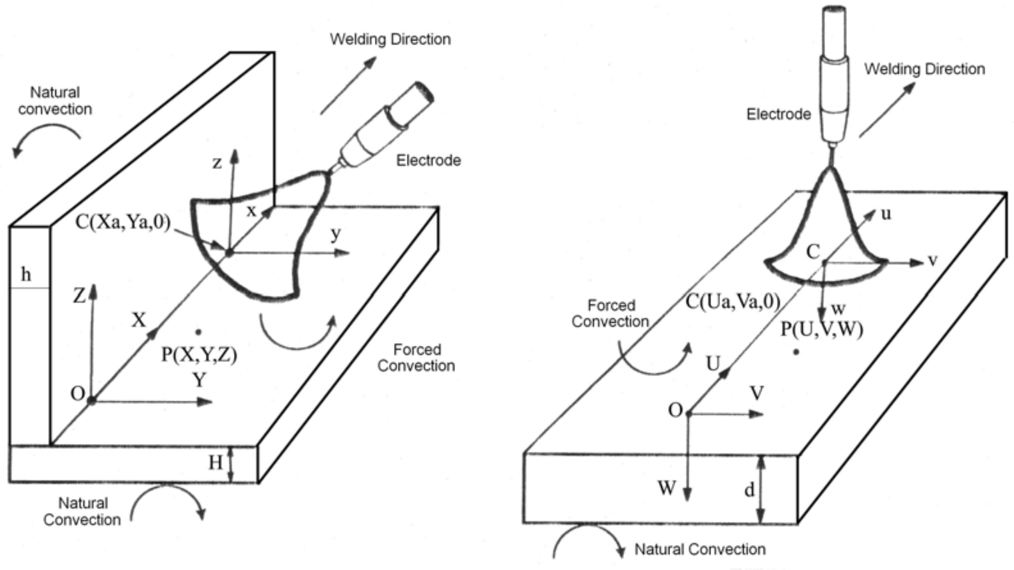

Figure 3. The real and equivalent systems in case of fillet welds [3].

Finally, the mathematical model of the heat transfer in the changed coordinates system is given by the following equations (10)...(16).

- Conduction equation in case of solids:

$$\frac{\partial T_u}{\partial t} = a\left(\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} + \frac{\partial^2}{\partial w^2}\right)T_u \tag{10}$$

- Contour condition on the upper surface:

$$-\lambda \frac{\partial T_u(U,V,0)}{\partial W} + \alpha_1\left(T_u(U,V,0) - T_0\right) = 0 \tag{11}$$

- Contour condition on the lower surface:

$$\lambda \frac{\partial T_u(U,V,d)}{\partial W} + \alpha_2\left(T_u(U,V,d) - T_0\right) = 0 \tag{12}$$

- Boundary condition at the infinite distance from the thermal source:

$$\lim_{r\to\infty} T_u(U,V,W,t) = T_0, \qquad r = \sqrt{(U-U')^2 + (V-V')^2 + W^2}, \tag{13}$$

Where r is the distance from the thermal source located in the coordinates point (U', V', 0).

- Temperatures distribution at time $t=t_0$:

$$T_u(U,V,W,0) = T_0 \tag{14}$$

- Heat flow two-dimensional distribution is given by the equation:

$$Q(u,v,t) = \frac{q(t)}{2\pi\sigma_u\sigma_v}\exp\left[-\left(\frac{u^2}{2\sigma_u{}^2} + \frac{v^2}{2\sigma_v{}^2}\right)\right], \tag{15}$$

if $\sigma^2 = \sigma_u \times \sigma_v$, the equation (15) can be written as:

$$Q(u,v,t) = \frac{q(t)}{2\pi\sigma^2}\exp\left(-\frac{\sigma_v{}^2u^2 + \sigma_u{}^2v^2}{2\sigma^4}\right). \tag{16}$$

In the equation (16) σ_u, σ_v are the distribution parameters and $q(t)=\eta UI(t)$ is the useful power of the electric arc. Solving the equations 10)...(16), the analytical solutions of the temperatures in the changed coordinates system can be obtained for three analyzed cases:

1. Temperature at t moment, in the point P of (U,V,W) coordinates, in the infinite plate which is under the influence of the thermal source q_i, located in the point of (U', V', 0) coordinates at t_1 moment

$$T_u(U,V,W,t)=\frac{q_i}{2\pi\cdot\lambda\cdot d\cdot(t-t_1)}\cdot\exp\left(-\frac{(U-U')^2+(V-V')^2}{4a\cdot(t-t_1)}\right)\cdot\sum_{n=0}^{\infty}A_n\exp\left(-\mu_n^2(t-t_1)\right)\cdot$$
$$\cdot\left[\cos\left(\frac{\mu_n}{\sqrt{a}}W\right)+\frac{\beta_1\sqrt{a}}{\mu_n}\sin\left(\frac{\mu_n}{\sqrt{a}}W\right)\right] \tag{17}$$

where: $A_n=\dfrac{\mu_n^2}{\mu_n^2+a\beta_1^2+\dfrac{2a\beta_1}{d}}$, $\beta_1=\dfrac{\alpha_1}{\lambda}$, $\beta_2=\dfrac{\alpha_2}{\lambda}$, $\lambda=a\cdot c\cdot\rho$

and μ_n is a positive value which satisfies the equality: $tg\left(\dfrac{\mu_n d}{\sqrt{a}}\right)=\dfrac{\sqrt{a}\mu_n(\beta_1+\beta_2)}{\mu_n^2-\beta_1\cdot\beta_2\cdot a}$.

2. Temperature in the infinite plate which is under the influence of the two-dimensional Gauss thermal source located in a fixed point at t_1 moment and obtained by overlapped instantaneous thermal sources

$$dT_u(t_1)=\frac{q(t_1)\cdot d(t_1)}{\pi\cdot\rho\cdot c\cdot d}\cdot\frac{\sigma^2}{\sqrt{\sigma^4+2a\cdot[t-t_1]\sigma_v^2}\cdot\sqrt{\sigma^4+2a\cdot[t-t_1]\sigma_u^2}}\cdot$$
$$\cdot\exp\left[-\frac{\left(\sigma_v^2+2a\cdot(t-t_1)\cdot U^2\right)+\left(\sigma_u^2+2a\cdot(t-t_1)\cdot V^2\right)}{2\left(\sigma_u^2+2a\cdot(t-t_1)\right)\cdot\left(\sigma_v^2+2a\cdot(t-t_1)\right)}\right]\cdot \tag{18}$$
$$\cdot\sum_{n=0}^{\infty}A_n\exp\left(-\mu_n^2(t-t_1)\right)\cdot\left[\cos\left(\frac{\mu_n}{\sqrt{a}}W\right)+\frac{\beta_1\sqrt{a}}{\mu_n}\sin\left(\frac{\mu_n}{\sqrt{a}}W\right)\right]$$

3. Temperature in the infinite plate which is under the influence of the two-dimensional Gauss thermal source, located in the point of (U_a, V_a, 0) coordinates at t_1 moment and obtained by overlapped instantaneous thermal sources

$$dT_u(t_1)=\frac{q(t_1)\cdot d(t_1)}{\pi\cdot\rho\cdot c\cdot d}\cdot\frac{\sigma^2}{\sqrt{\sigma^4+2a\cdot[t-t_1]\sigma_v^2}\cdot\sqrt{\sigma^4+2a\cdot[t-t_1]\sigma_u^2}}\cdot$$
$$\cdot\exp\left[-\frac{\left(\sigma_v^2+2a\cdot(t-t_1)\cdot(U-U_a)^2\right)+\left(\sigma_u^2+2a(t-t_1)\cdot(V-V_a)^2\right)}{2\left(\sigma_u^2+2a(t-t_1)\right)\cdot\left(\sigma_v^2+2a(t-t_1)\right)}\right]\cdot \tag{19}$$
$$\cdot\sum_{n=0}^{\infty}A_n\exp\left(-\mu_n^2(t-t_1)\right)\cdot\left[\cos\left(\frac{\mu_n}{\sqrt{a}}W\right)+\frac{\beta_1\sqrt{a}}{\mu_n}\sin\left(\frac{\mu_n}{\sqrt{a}}W\right)\right]$$

In case of mobile system, solidary with the electrode, the temperature $T_u(u,v,w)$, at t moment, can be computed with the following mathematical relation:

$$T_u(u,v,w,t)=\int_0^t \frac{q(t_1)}{\pi\cdot\rho\cdot c\cdot d}\cdot\frac{\sigma^2}{\sqrt{\sigma^4+2a[t-t_1]\cdot\sigma_v^2}\cdot\sqrt{\sigma^4+2a[t-t_1]\cdot\sigma_u^2}}\cdot$$
$$\cdot\exp\left[-\frac{\left(\sigma_v^2+2a\cdot(t-t_1)\cdot(u+U_a(t)-U_a(t_1))^2\right)+\left(\sigma_u^2+2a(t-t_1)\cdot(v+V_a(t)-V_a(t_1))^2\right)}{2\left(\sigma_u^2+2a(t-t_1)\right)\cdot\left(\sigma_v^2+2a(t-t_1)\right)}\right]\cdot \tag{20}$$
$$\cdot\sum_{n=0}^{\infty}A_n\exp\left(-\mu_n^2(t-t_1)\right)\cdot\left[\cos\left(\frac{\mu_n}{\sqrt{a}}W\right)+\frac{\beta_1\sqrt{a}}{\mu_n}\sin\left(\frac{\mu_n}{\sqrt{a}}W\right)\right]\cdot dt_1$$

According to Jeong and Cho [3], the analytical solution for the temperatures calculus in the fillet joints is:

$$T(x,y,z,t)=T_u(u,v,w,t)\cdot\frac{d}{H}\sqrt{\frac{\sqrt{\left(e^A\cdot\cos B+a\right)^2+\left(e^A\cdot\sin B\right)^2}}{\sqrt{\left(e^A\cdot\cos B-1\right)^2+\left(e^A\cdot\sin B\right)^2}^{\,2}}},\quad A=\frac{\pi\cdot v}{d},\quad B=\frac{\pi\cdot w}{d} \tag{21}$$

3. Mobile thermal source of 3D Gauss distribution

Due to the important penetration in case of the semi-infinite welded bodies, Goldak, Chakravarti and Bibby proposed the three-dimensional mobile source to predict the temperatures.

3.1. Semi-ellipsoidal thermal source

Firstly, Goldak et al. proposed a semi-ellipsoidal thermal source of the heat flow in any point of (x, y, z) which can be computed with the equation [4]:

$$Q(x,y,z,)=\frac{6\sqrt{3}\eta\cdot U\cdot I}{a_h\cdot b_h\cdot c_h\cdot\pi\sqrt{\pi}}\cdot\exp\left(-\frac{3x^2}{c_h^2}-\frac{3y^2}{a_h^2}-\frac{3z^2}{b_h^2}\right), \tag{22}$$

where a_h, b_h, c_h are the parameters of the semi-ellipsoidal thermal source (Fig. 4) and x, y, z are the coordinates of the thermal source.

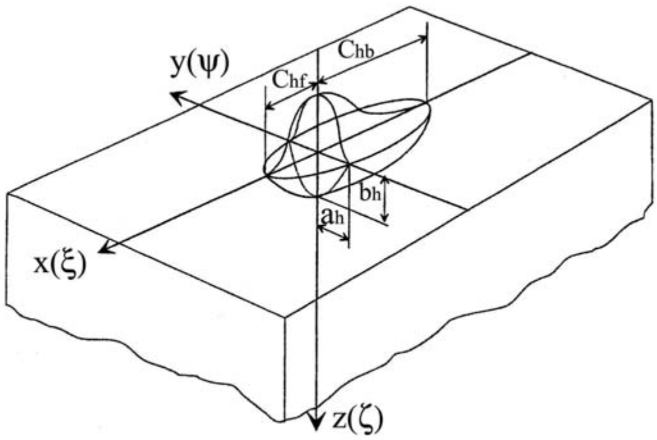

Figure 4. 3D Gauss distribution of the heat flow (c_{hf}=c_{hb}=c_h).

3.2. Temperature field developed by the semi-ellipsoidal thermal source

Taking account of the solution obtained by Carslaw and Jaeger in case of the instantaneous source, the researchers [2] established the temperature field produced by the semi-ellipsoidal thermal source:

$$dT_{t'}=\frac{\delta Q\cdot dt'}{\rho\cdot c\cdot[4\pi\cdot a\cdot(t-t')]^{3/2}}\cdot\exp\left(-\frac{(x-x')^2+(y-y')^2+(z-z')^2}{4a\cdot(t-t')}\right). \tag{23}$$

Overlapping more instantaneous thermal sources and taking account of semi-ellipsoidal thermal source distribution, it can be written the equation (24) [2]:

$$dT_{t'} = \frac{1}{2}\int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' \int_{-\infty}^{\infty} dz' \frac{dt'}{\rho \cdot c \cdot [4\pi \cdot a \cdot (t-t')]^{3/2}} \cdot \frac{6\sqrt{3}\eta \cdot U \cdot I}{a_h \cdot b_h \cdot c_h \cdot \pi\sqrt{\pi}} \cdot \exp\left(-\frac{3x'^2}{c_h^2} - \frac{3y'^2}{a_h^2} - \frac{3z'^2}{b_h^2}\right) \cdot \exp\left(-\frac{(x-x')^2 + (y-y')^2 + (z-z')^2}{4a \cdot (t-t')}\right) \tag{24}$$

and rewritten as:

$$dT_{t'} = \frac{3\sqrt{3}\eta \cdot U \cdot I \cdot dt'}{\rho \cdot c \cdot \pi\sqrt{\pi}} \cdot \frac{1}{\sqrt{12a \cdot (t-t') + a_h^2}} \cdot \frac{1}{\sqrt{12a \cdot (t-t') + b_h^2}} \cdot \frac{1}{\sqrt{12a \cdot (t-t') + c_h^2}} \cdot \exp\left(-\frac{3x^2}{12a \cdot (t-t') + c_h^2} - \frac{3y^2}{12a \cdot (t-t') + a_h^2} - \frac{3z^2}{12a \cdot (t-t') + b_h^2}\right) \tag{25}$$

Considering that the thermal source moves with constant speed from initial moment $t'=0$ to the moment $t'=t$, the increase of temperature in this range time can be computed as [2]:

$$T - T_0 = \frac{3\sqrt{3}\eta \cdot U \cdot I}{\rho \cdot c \cdot \pi\sqrt{\pi}} \cdot \int_0^t \frac{dt'}{\sqrt{12a \cdot (t-t') + a_h^2}\sqrt{12a \cdot (t-t') + b_h^2} \cdot \sqrt{12a \cdot (t-t') + c_h^2}} \cdot \exp\left(-\frac{3(x-vt')^2}{12a \cdot (t-t') + c_h^2} - \frac{3y^2}{12a \cdot (t-t') + a_h^2} - \frac{3z^2}{12a \cdot (t-t') + b_h^2}\right) \tag{26}$$

Appling the equation from above, the analytical solutions can be obtained for three specific cases as [2]:

1. If $a_h=b_h=c_h=r_h$, the thermal source becomes a semi-sphere with radius of r_h, and the relation (26) has the expression:

$$T - T_0 = \frac{3\sqrt{3}\eta \cdot U \cdot I}{\rho \cdot c \cdot \pi\sqrt{\pi}} \cdot \int_0^t \frac{dt'}{\left[12a \cdot (t-t') + r_h^2\right]^{3/2}} \cdot \exp\left(-\frac{3(x-vt')^2 + 3y^2 + 3z^2}{12a \cdot (t-t') + r_h^2}\right). \tag{27}$$

When $r_h = \sqrt{3}\sigma$ the equation (2.40) becomes:

$$T - T_0 = \frac{\eta \cdot U \cdot I}{\rho \cdot c \cdot \pi\sqrt{\pi}} \cdot \int_0^t \frac{dt'}{\left[4a \cdot (t-t') + \sigma^2\right]^{3/2}} \cdot \exp\left(-\frac{(x-vt')^2 + y^2 + z^2}{4a \cdot (t-t') + \sigma^2}\right). \tag{28}$$

2. If the distribution parameter is σ=0, the thermal source becomes a point one, the analytical solution confirming the solution of Carslaw and Jaeger:

$$T - T_0 = \frac{\eta \cdot U \cdot I}{\rho \cdot c} \cdot \int_0^t \frac{dt'}{[4\pi a \cdot (t-t')]^{3/2}} \cdot \exp\left(-\frac{(x-vt')^2 + y^2 + z^2}{4a \cdot (t-t')}\right). \tag{29}$$

3. If $b_h=0$, then the semi-ellipsoidal thermal source becomes a surface semi-elliptical thermal source and the equation (26) is the following:

$$T - T_0 = \frac{3\eta \cdot U \cdot I}{\pi \cdot \rho \cdot c} \cdot \int_0^t \frac{dt'}{\sqrt{4\pi \cdot a \cdot (t-t')}\sqrt{12a \cdot (t-t') + a_h^2}\sqrt{12a \cdot (t-t') + c_h^2}} \cdot \exp\left(-\frac{3(x-vt')^2}{12a \cdot (t-t') + c_h^2} - \frac{3y^2}{12a \cdot (t-t') + a_h^2} - \frac{z^2}{4a \cdot (t-t')}\right) \tag{30}$$

When $a_h = c_h = \sqrt{6} \cdot \sigma$, the thermal source changes in a circular one. In this case, the equation (28) is similar to the solution of Eagar and Tsai:

$$T - T_0 = \frac{\eta \cdot U \cdot I}{\pi \cdot \rho \cdot c} \cdot \int_0^t \frac{dt'}{\sqrt{4\pi \cdot a \cdot (t - t')} \cdot \left[4a \cdot (t - t') + 2\sigma^2\right]} \cdot \exp\left(- \frac{(x - vt')^2 + y^2}{4a \cdot (t - t') + 2\sigma^2} - \frac{z^2}{4a \cdot (t - t')} \right) \quad (31)$$

3.3. Double ellipsoidal thermal source

Combining two semi-ellipses, Goldak, Chakravarti and Bibby proposed the double ellipsoidal thermal source (Figure 4) [4]. For the semi-ellipse located in front of the welding arc, the heat flow can be computed in each point of (x, y, z) coordinates using the following mathematical relation:

$$Q(x, y, z,) = \frac{6\sqrt{3} \cdot r_f \cdot \eta \cdot U \cdot I}{a_h \cdot b_h \cdot c_{hf} \cdot \pi\sqrt{\pi}} \cdot \exp\left(- \frac{3x^2}{c_{hf}^2} - \frac{3y^2}{a_h^2} - \frac{3z^2}{b_h^2} \right). \quad (32)$$

For the second semi-ellipse located behind of the welding arc, the heat flow can be computed in each point of (x, y, z) coordinates using the following mathematical relation:

$$Q(x, y, z,) = \frac{6\sqrt{3} \cdot r_b \cdot \eta \cdot U \cdot I}{a_h \cdot b_h \cdot c_{hb} \cdot \pi\sqrt{\pi}} \cdot \exp\left(- \frac{3x^2}{c_{hb}^2} - \frac{3y^2}{a_h^2} - \frac{3z^2}{b_h^2} \right). \quad (33)$$

In the equations (32) and (33) r_f and r_b represent the repartition coefficients of the heat in front and behind of the thermal source and their mathematical relations are the following [4]:

$$r_f = 2c_{hf} / (c_{hf} + c_{hb}), \qquad r_b = 2c_{hb} / (c_{hf} + c_{hb}) \quad , \quad (34)$$

where the thermal source dimensions are similar to the weld pool geometrical characteristics [4].

3.4. Temperature field developed by the double ellipsoidal thermal source

In case of the double ellipsoidal distribution of the thermal source, for the volume corresponding to the areas located in front and behind of the thermal source, the following equation can be written:

$$dT_{t'} = \frac{1}{4} \cdot \frac{6\sqrt{3} \cdot \eta \cdot U \cdot I \cdot dt'}{\rho \cdot c \cdot a_h \cdot b_h \cdot \pi\sqrt{\pi}\left[4\pi \cdot a \cdot (t - t')\right]^{3/2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(- \frac{(x - x')^2 + (y - y')^2 + (z - z')^2}{4a \cdot (t - t')} \right) \cdot$$
$$\cdot \left(\frac{r_f}{c_{hf}} \cdot \exp\left(- \frac{3x'^2}{c_{hf}^2} - \frac{3y'^2}{a_h^2} - \frac{3z'^2}{b_h^2} \right) + \frac{r_b}{c_{hb}} \cdot \exp\left(- \frac{3x'^2}{c_{hb}^2} - \frac{3y'^2}{a_h^2} - \frac{3z'^2}{b_h^2} \right) \right) \cdot dx' \cdot dy' \cdot dz' \quad (35)$$

The equation (35) can be adapted as:

$$dT_{t'} = \frac{3\sqrt{3} \cdot \eta \cdot U \cdot I \cdot dt'}{2\pi \cdot \rho \cdot c \cdot \sqrt{\pi \cdot \left(12a \cdot (t - t') + a_h^2\right) \cdot \left(12a \cdot (t - t') + b_h^2\right)}} \cdot \left(\frac{A}{\sqrt{12a \cdot (t - t') + c_{hf}^2}} + \frac{B}{\sqrt{12a \cdot (t - t') + c_{hb}^2}} \right) \quad (36)$$

$$\textit{where} \quad A = r_f \cdot \exp\left(- \frac{3x^2}{12a \cdot (t - t') + c_{hf}^2} - \frac{3y^2}{12a \cdot (t - t') + a_h^2} - \frac{3z^2}{12a \cdot (t - t') + b_h^2} \right).$$

$$B = r_b \cdot \exp\left(- \frac{3x^2}{12a \cdot (t - t') + c_{hb}^2} - \frac{3y^2}{12a \cdot (t - t') + a_h^2} - \frac{3z^2}{12a \cdot (t - t') + b_h^2} \right)$$

Similarly, considering that the thermal source moves with constant speed from initial moment $t'=0$ to the moment $t'=t$, the increase of temperature in this range time can be computed as [2]:

$$T-T_0=\frac{3\sqrt{3}\cdot\eta\cdot U\cdot I}{2\pi\cdot\rho\cdot c\cdot\sqrt{\pi}}\cdot\int_0^t\frac{dt'}{\sqrt{\left(12a\cdot(t-t')+a_h^2\right)\cdot\left(12a\cdot(t-t')+b_h^2\right)}}\left(\frac{A}{\sqrt{12a\cdot(t-t')+c_{hf}^2}}+\frac{B}{\sqrt{12a\cdot(t-t')+c_{hb}^2}}\right)$$

$$where\ \ A=r_f\cdot\exp\left(-\frac{3(x-vt')^2}{12a\cdot(t-t')+c_{hf}^2}-\frac{3y^2}{12a\cdot(t-t')+a_h^2}-\frac{3z^2}{12a\cdot(t-t')+b_h^2}\right) \qquad (37)$$

$$B=r_b\cdot\exp\left(-\frac{3(x-vt')^2}{12a\cdot(t-t')+c_{hb}^2}-\frac{3y^2}{12a\cdot(t-t')+a_h^2}-\frac{3z^2}{12a\cdot(t-t')+b_h^2}\right)$$

The equation from above represents the analytical solution of temperature calculus in the transitory regime in case of the semi-infinite body which is under the influence of the double ellipsoidal thermal source. If $c_{hf}=c_{hb}=c_h$, the equations (36) and (37) can be written as the equations (25) and (26) which describe the temperature variation of the semi-infinite body under the influence of the semi-ellipsoidal thermal source.

4. Conclusion

- Investigations concerning the analytical solutions of the thermal transfer in the welded joints in case of 2D and 3D thermal sources are presented in this paper. The analytical models don't take account neither of thermo-physical properties dependent of temperature nor of heat lost by convection and radiation.
- Goldak [4] proposes the different distribution of heat flow in front and behind the thermal source and its real dimensions in the analytical modelling of the thermal transfer in the welded joints.

Notations

a – thermal diffusivity;
a_h, b_h, c_{hf}, c_{hb} – ellipsoidal thermal source;
c – specific heat at ambient temperature;
c_S – specific heat for the solid phase;
c_L – specific heat for the liquid phase;
ΔH – fusion heat per mass unit;
λ - material thermal conductivity;
λ_x, λ_y, λ_z – material thermal conductivities on x, y, z directions;
r_f, r_b – repartition coefficients of the heat in front and behind of the thermal source;
σ – distribution parameter specific for the thermal source;
V_x, V_y, V_z – speed components on x, y, z directions.

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