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Fréchet Distribution Applied to Salary Incomes in Spain from 1999 to 2014. An Engineering Approach to Changes in Salaries' Distribution

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Abstract: The official data in relation to salaries paid in Spain from 1999 to 2014 has been analyzed. The inadequate data format does not reflect the whole salary distribution. Fréchet distributions have been fitted to the data. This simple distribution has similar accuracy in relation to the data when compared to other distributions (Log-Normal, Gamma, Dagum, GB2). Analysis of the data through the fitted Fréchet distributions reveals a tendency towards more balanced (i.e., less skewed) salary distributions from 2002 to 2014 in Spain.

Keywords: Spanish economy; salary distribution; parametric modelling; Fréchet distribution; minimum wage

JEL Classification: C22; J31

1. Introduction

In November 2015, the salary distribution data in Spain was updated in the statistics database of the Spanish Tax Agency.[1] This data is organized by taking into account several aspects of the Spanish society (gender, age, economic sectors, etc.). According to the best-selling newspaper in Spain,[2] the most significant information was the salaries earned in relation to the salary brackets (see in Tables 1 and 2 the number of salaried people and its weight in terms of total amount of paid euros, in relation to the salary, expressed in times the minimum wage), which demonstrates that the average annual salary in Spain fell in 2014 to the levels of 2007. Other key information from the statistics reflected in the media was the imbalance between genders.[3]

[1] Agencia Tributaria de España. Mercado de Trabajo y Pensiones en las Fuentes Tributarias. http://www.agenciatributaria.es/AEAT.internet/datosabiertos/catalogo/hacienda/Mercado_de_Trabajo_y_Pensiones_en_las_Fuentes_Tributarias.shtml.

[2] Average annual Spanish salary falls to lowest level since 2007 (Jiménez 2015a).

[3] Men account for 82% of highest salaries in Spain, says new report (Jiménez 2015b).

Table 1. Salary earners per income bracket in Spain from 1999 to 2014 (expressed in million people). Source: *Agencia Tributaria de España*.

Income	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
0 to 0.5 min. wage	2.784	2.774	2.795	2.833	2.911	2.851	3.232	3.286	2.988	3.090	3.405	3.420	3.466	3.494	3.642	3.695
0.5 to 1 min. wage	1.688	1.739	1.760	1.817	1.829	1.887	2.145	2.187	2.214	2.284	2.251	2.207	2.211	2.122	2.110	2.197
1 to 1.5 min. wage	1.743	1.819	1.839	1.870	1.872	1.970	2.211	2.359	2.492	2.481	2.284	2.215	2.175	2.051	1.982	2.047
1.5 to 2 min. wage	2.182	2.341	2.388	2.440	2.418	2.583	2.824	3.010	3.169	2.966	2.678	2.592	2.500	2.417	2.224	2.220
2 to 2.5 min. wage	1.595	1.760	1.926	2.024	2.144	2.174	2.188	2.281	2.318	2.281	2.038	1.979	1.956	1.896	1.762	1.769
2.5 to 3 min. wage	1.033	1.114	1.215	1.282	1.374	1.399	1.411	1.467	1.528	1.548	1.430	1.389	1.373	1.344	1.251	1.244
3 to 3.5 min. wage	0.799	0.848	0.899	0.950	0.982	1.013	1.035	1.081	1.109	1.122	1.048	1.059	1.073	1.017	0.984	0.990
3.5 to 4 min. wage	0.642	0.689	0.727	0.746	0.785	0.793	0.809	0.847	0.875	0.879	0.831	0.823	0.807	0.768	0.761	0.756
4 to 4.5 min. wage	0.524	0.548	0.579	0.605	0.641	0.643	0.639	0.657	0.677	0.689	0.654	0.641	0.630	0.553	0.567	0.569
4.5 to 5 min. wage	0.374	0.417	0.454	0.484	0.521	0.522	0.480	0.491	0.511	0.521	0.498	0.456	0.419	0.342	0.357	0.361
5 to 7.5 min. wage	0.718	0.791	0.866	0.935	1.030	1.003	0.936	0.953	0.958	0.979	0.916	0.851	0.803	0.727	0.722	0.726
7.5 to 10 min. wage	0.205	0.224	0.247	0.265	0.290	0.282	0.263	0.267	0.276	0.277	0.250	0.236	0.225	0.201	0.195	0.197
More than 10 min. wage	0.143	0.156	0.177	0.187	0.204	0.199	0.187	0.185	0.194	0.194	0.168	0.156	0.149	0.133	0.125	0.128
Total amount	14.431	15.220	15.871	16.438	17.001	17.321	18.360	19.070	19.309	19.311	18.452	18.025	17.788	17.063	16.682	16.899

Table 2. Salaries payed per income bracket in Spain from 1999 to 2014 (expressed in billion euros). Source: *Agencia Tributaria de España*.

Income	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
0 to 0.5 min. wage	3.428	3.499	3.615	3.733	3.856	4.122	4.991	5.303	5.221	5.632	6.371	6.392	6.513	6.407	6.586	6.797
0.5 to 1 min. wage	7.363	7.746	8.002	8.424	8.659	9.420	11.542	12.440	13.320	14.420	14.684	14.571	14.789	14.203	14.216	14.791
1 to 1.5 min. wage	12.778	13.601	14.031	14.534	14.831	16.453	19.914	22.471	25.085	26.241	25.107	24.716	24.544	23.128	22.464	23.172
1.5 to 2 min. wage	22.269	24.417	25.470	26.584	26.922	30.288	35.597	39.996	44.331	43.662	40.927	40.203	39.278	37.991	35.191	35.111
2 to 2.5 min. wage	20.731	23.326	26.059	27.944	30.219	32.280	34.993	38.500	41.291	42.758	39.756	39.181	39.223	38.002	35.558	35.704
2.5 to 3 min. wage	16.480	18.110	20.168	21.698	23.731	25.460	27.674	30.349	33.338	35.536	34.118	33.635	33.707	33.073	30.925	30.757
3 to 3.5 min. wage	15.097	16.348	17.707	19.081	20.113	21.888	24.105	26.549	28.721	30.570	29.697	30.445	31.226	29.571	28.807	28.978
3.5 to 4 min. wage	14.007	15.329	16.516	17.279	18.533	19.749	21.738	24.008	26.151	27.623	27.164	27.292	27.099	25.768	25.710	25.547
4 to 4.5 min. wage	12.931	13.828	14.899	15.871	17.152	18.140	19.458	21.106	22.943	24.528	24.203	24.110	23.975	20.996	21.671	21.755
4.5 to 5 min. wage	10.305	11.732	13.052	14.202	15.600	16.467	16.299	17.573	19.310	20.718	20.581	19.133	17.779	14.550	15.258	15.415
5 to 7.5 min. wage	24.948	28.028	31.311	34.474	38.690	39.767	40.058	42.975	45.575	48.938	47.509	44.870	42.974	38.942	38.911	39.099
7.5 to 10 min. wage	10.173	11.350	12.779	13.986	15.623	16.000	16.109	17.227	18.786	19.842	18.658	17.795	17.218	15.364	14.955	15.148
More than 10 min. wage	13.049	14.792	17.513	18.397	20.460	21.172	21.600	22.819	25.165	26.353	23.370	22.163	21.463	19.402	18.443	19.006
Total amount	183.56	202.11	221.12	236.21	254.39	271.21	294.08	321.32	349.24	366.82	352.15	344.51	339.79	317.40	308.70	311.28

The aforementioned statistical information (included in Tables 1 and 2) is organized in brackets of half minimum wage size until five times the minimum wage. This wage limit is followed by a bracket from 5 to 7.5 times the minimum wage and then by a bracket from 7.5 to 10 times the minimum wage. Finally, the statistics end with a more than 10 times the minimum wage bracket. In Figure 1, the distribution of the salaries paid is shown as a function of the salary bracket in 2014. A significant concern arises from this figure, as the different size of the brackets does not allow for fitting a mathematical distribution in order to obtain more information from the data.

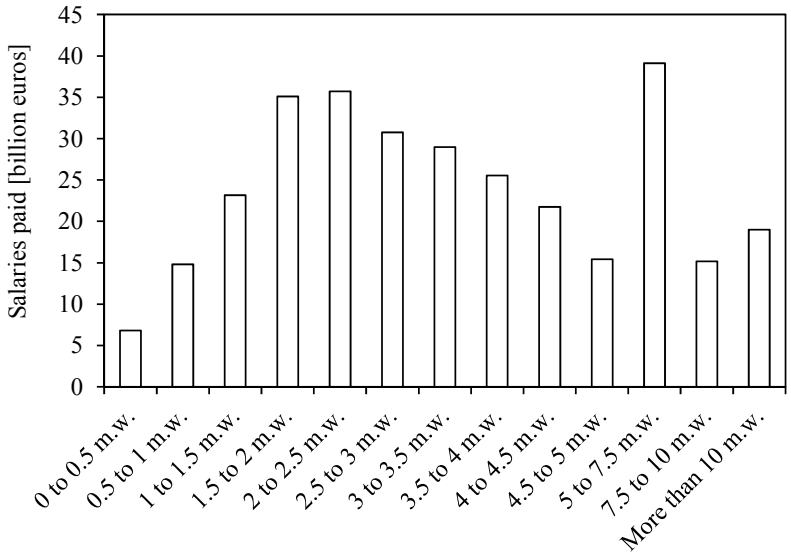

Figure 1. Distribution of salaries paid in Spain (2014) as a function of the minimum wage (m.w.) brackets established by the Spanish Tax Agency (*Agencia Tributaria de España*).

The aim of the present work is to draw some conclusions from this data based on the empirical analysis normally used by engineers and scientists when studying a problem, which requires the use of mathematical models that can be parametrized in terms of a limited number of non-dimensional variables. This approach facilitates working with distributional functions that retain the most significant information reflected by the data. In addition, the possibility of defining the aforementioned functions from a limited amount of data (e.g., surveys) should be also mentioned as one of the main advantages of this methodology. To perform the present study:

(1) The data from Tables 1 and 2 was post-processed in order to obtain an estimation of the salaries earned in brackets of half minimum wage from 5 to 7.5 times the minimum wage and from 7.5 to 10 times the minimum wage. To do this, the evolution of the minimum wage in Spain from 1999 to 2014 is required (see Table 3).

(2) The data (salaries paid) was made non-dimensional in order to directly compare the distributions from different years.

(3) A corrected Fréchet distribution was fitted to the data (once post-processed), the evolution from 1999 to 2014 of this distribution being analyzed in order to detect changes in their main parameters and study the data through them.

Table 3. Minimum wage (paid yearly) in Spain from 1998 to 2014.

Year	Minumum Wage [€]	Year	Minumum Wage [€]
1998	5725.02	2007	7988.4
1999	5828.48	2008	8400
2000	5947.2	2009	8736
2001	6068.3	2010	8866.2
2002	6190.8	2011	8979.6
2003	6316.8	2012	8979.6
2004	6659.1	2013	9034.2
2005	7182	2014	9034.2
2006	7572.6		

The three-parameter Weibull distribution (Weibull 1951) was initially proposed to model the salary distribution in the present work. This is a simple but versatile distribution widely used in wind engineering and wind energy assessment (Justus et al. 1978; Rehman et al. 1994; Lun and Lam 2000; Seguro and Lambert 2000; Dorvlo 2002; Ulgen and Hepbasli 2002; Azad et al. 2014; Shittu and Adepoju 2014). The corrected Weibull probability density function (PDF) is expressed as:

$$f(\phi) = \lambda\left(\frac{k}{c}\right)\left(\frac{\phi - d}{c}\right)^{k-1} \exp\left[-\left(\frac{\phi - d}{c}\right)^{k}\right] \quad (1)$$

where c is the scale parameter, k is the shape parameter, and d is the location parameter (Cran 1988). The correction parameter, λ, is used in order to obtain a better fit to the data, bearing in mind that the salaries interval is half minimum wage, and, therefore, the value of this parameter should be approximately $\lambda = 0.5$. In the present study, the function f stands for the density of salaries earned, whereas the variable ϕ stands for the salary (expressed in terms of times the minimum salary). However, the results of the fittings performed in the present work showed negative values of the shape and correction parameters. For this reason, the Fréchet distribution, which is closely related to the Weibull distribution (Abbas and Tang 2012; Mann 1984; De Gusmão and Ortega 2011; Khan et al. 2008), was finally selected.[4] The Fréchet distribution is expressed as:

$$f(\phi) = \lambda\left(\frac{\gamma}{c}\right)\left(\frac{c}{\phi - d}\right)^{\gamma+1} \exp\left[-\left(\frac{c}{\phi - d}\right)^{\gamma}\right] \quad (2)$$

where, as aforementioned, c is the scale parameter, γ is the shape parameter, and d is the location parameter. Obviously, once the scale parameter is proven to be $c > 0$, it is easy to derive Equation (2) from Equation (1) by assuming $\gamma = -k$. Both aforementioned distributions, Weibull and Fréchet, are commonly applied to analyze the extreme values from other probability distributions (Goda et al. 2010; Carmona 2014) and represent the Fisher-Tippett FT-III and FT-II distributions, respectively (Fisher and Tippett 1928).

After a review of the available literature, it seems that the Weibull or Fréchet distributions are not commonly used in economics, compared to other distributions. Nevertheless, some examples of the use of these distributions have been found; for example, to describe the income distribution in a society (Chotikapanich et al. 2007; Jagielski and Kutner 2013; Atkinson and Bourguignon 2015) or the inequality associated with that distribution (Krause 2014; McDonald and Ransom 2008; Lubrano 2016). In the present work, the analysis is focused on wages, leaving aside other revenue types normally included in the analysis of incomes or wealth.

[4] According to Kleiber and Kotz (2003), the Weibull distribution is surrounded by some controversy as the "French would argue that this is nothing else but Fréchet distribution".

Just focusing on wages, the recent work by García et al. (2014) is worthy of mention. These authors analyze different PDFs to study some specific aspects of the salary distributions is Spain, suggesting the use of the four-parameter GB2 distribution as the best choice when fitting to the data. However, in that work, the reference data is obtained from the National Statistics Institute of Spain (*Instituto Nacional de Estadística*) annual survey, whereas in the present analysis the raw data is based on the entire database from the Spanish Tax Agency. In addition, the PDFs analyzed seem to be quite complicated when compared to the Fréchet distribution equation. Therefore, in the present work the authors have followed the suggestion by Banerjee et al. (2006), who stated 'that a useful description of the data is the one that has the minimal number of parameters, yet reasonably (but not necessarily perfectly) agrees with the data'.

Together with the aforementioned work by García et al. (2014), a few examples in the available literature related to distributions applied to analysis of wages have been found. Shatnawi et al. (2013) studied gender discrimination in relation to wages.[5] These authors stated that the wage density distribution related to a specific group of individuals (in this case male workers) corresponds to the log-normal distribution. In this sense, Benoit Mandelbrot in his work 'Paretian distributions and income maximization',[6] stated some decades before that different distributions of offers (i.e., wages) made to a single individual are conditioned by how each sector weights the attributes of this individual (Mandelbrot 1962). Therefore, in a whole economic system there is a coexistence of distributions that affect different social groups. An example of this mixture of density functions has been recently proposed to model wage distributions in the Czech Republic (Marek and Vrabec 2013). In addition, works by Rigby and Stasinopoulos (2015), Machado and Mata (2005), and Sohn et al. (2014) can also be mentioned.[7] In these works, a procedure to fit a mixture of distributional functions is developed, the works by Machado and Mata (2005), and Sohn et al. (2014) being respectively applied to wage distribution in Portugal from 1986 to 1995 and in Germany (years 1992 and 2010). These procedures have helped to analyze how inequality affects different subgroups of the population studied.

The Spanish economy has undergone drastic changes in the last 15 years. The evolution of the salaried people from 1999 to 2014 is included in Figure 2, together with the evolution of the total amount of salaries earned. After the crisis from 1994 to 1999, the Spanish economy boomed and unemployment fell. From 2006, the Spanish economy suffered the longest crisis within the period of democracy (i.e., from 1977) (Galindo 2015). This has affected the salary distribution in relation to the salary earned, thereby widening this distribution as a result of the demand for highly skilled workers, which increases the effect of the college premium for the high salaries (Carrasco et al. 2015). This conclusion had already arisen with regard to the 1994–1999 crisis (Febrer and López 2004). In contrast, inequality had fallen in the previous period, from 1985 to 1992, as a result of the unemployment reduction and the decrease in the college premium earnings for the high-skilled jobs (Pijoan-Mas and Sánchez-Marcos 2010). Inequality as a consequence of the present crisis has been the focus of many researchers' attention, the most noteworthy example in terms of interest generated in all establishments around the world (political, economic, academic ...) being the work by Thomas Piketty *'El Capital en el Siglo XXI'* (Capital in the Twenty-First Century) (Piketty 2014).[8] Regarding the Spanish economy, it seems that the adjustment is far from being fair in terms of society wealth (Orsini 2014).

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[5] Selezneva and Van Kerm, published another interesting work on gender discrimination in wage distribution in Germany, showing a larger gender gap at the bottom of the distribution (Selezneva and Van Kerm 2016).

[6] This work was dedicated by B. Mandelbrot to Maurice Fréchet, who proposed in 1927 the distribution selected in the present work to study the wages distribution in Spain.

[7] This work by Sohn et al. is the 2014 working version of the 2015 paper 'A Semiparametric Analysis of Conditional Income Distributions' (Sohn et al. 2015).

[8] This work, published initially in 2013 in French (and in 2014 in English), was cited more than 4700 times by the end of 2016, according to Google Scholar.

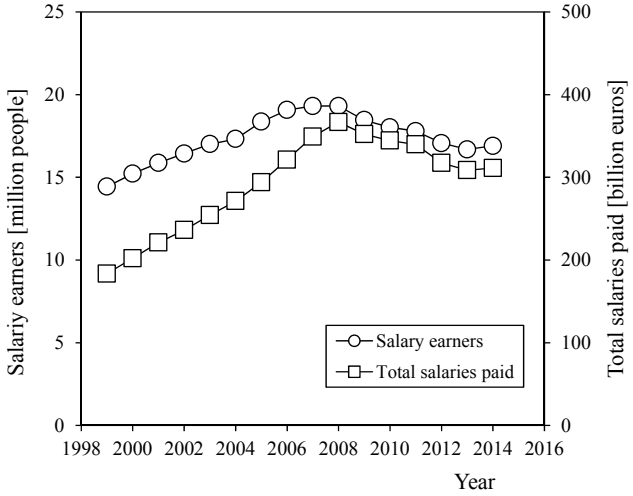

Figure 2. Evolution from 1999 to 2014 of salary earners (left y-axis) and total salaries paid (right y-axis) in Spain.

As mentioned, the purpose of the authors is to analyze the salary distributions in Spain from their point of view as engineers and researchers (Cubas et al. 2014a; Mendaña and Pindado 2013; Pindado et al. 2015; Pindado et al. 2013; Cubas et al. 2014b), relying only on mathematical modeling of the data and using the Fréchet distribution. In addition, the authors have tried to work as far as possible from the constant echoes coming from different ideologies that try to explain the origin of the present economic crisis in Spain and its possible solution. Wages and distribution of wealth or incomes are not completely dependent on one another, other factors such as education or household economic structure also being important for wealth sharing in a modern society (Espejo and Pascual 2007; Asplund and Barth 2005). In terms of the intellectual challenge involved in post-processing and understanding the data, the authors find the present moment of the economy fascinating. However, let us also say that, focusing only on cold mathematical modeling, we are well aware of the difficulties many Spanish people have in finding anything *fascinating* about the present situation of the economy.

The present work is organized as follows: in Section 2, the proposed methodology to fit the proposed Fréchet distribution to the data is included. In Section 3, the results are included. Finally, conclusions are summarized in Section 4.

2. Methodology

To estimate the total earned salary corresponding to sub-brackets of half minimum wage from 5 to 7.5 times the minimum wage and from 7.5 to 10 times the minimum wage, two assumptions are made:

- the number of salaried people depends linearly on the salary, and
- the average salary corresponding to each new sub-bracket of half minimum wage is centered in relation to the aforementioned bracket.

The linear distribution in each bracket is characterized by the number of salaried people in the first sub-bracket, s_1, and the reduction in the number of earners from one sub-bracket to the following one, Δ. Therefore, if S is the population (salaried people) within the bracket to be split into five new sub-brackets, M is the total amount of salaries paid in the brackets to be split, Φ is the annual minimum wage, and ϕ_1 is the center of the first sub-bracket (ϕ_1 = 5.25 times the minimum wage in the case of the 5 to 7.5 times the minimum wage salaries bracket and ϕ_1 = 7.75 times the minimum wage in the case of the 7.5 to 10 times the minimum wage salaries bracket, see Figure 3), the following expressions can be derived for Δ and s_1:

$$\Delta = \frac{1}{5}\left[S(1+\phi_1) - \frac{M}{\Phi}\right] \quad (3)$$

$$s_1 = 2\Delta + \frac{1}{5}S \tag{4}$$

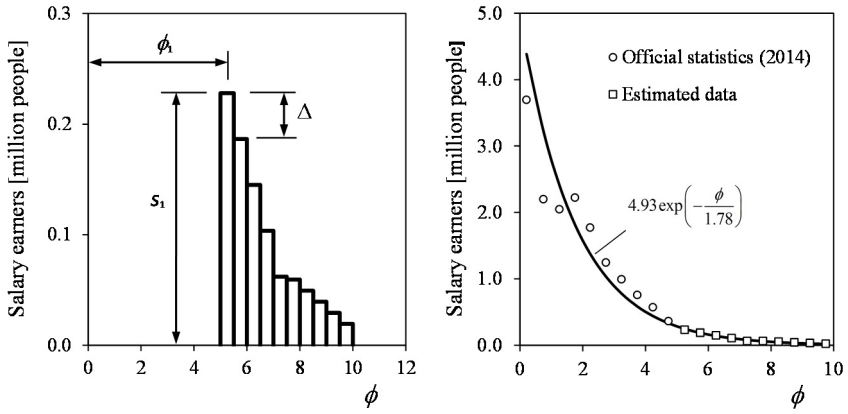

Figure 3. (**Left**) Estimated number of salary earners as a function of the non-dimensional salary paid, ϕ (expressed in multiples of the minimum wage), within the 0.5 minimum wage sub-brackets, which divide the official 5–7.5 and 7.5–10 times the minimum wage brackets, in Spain (2014). The variables corresponding to Equations (3) and (4) in relation to the official 5–7.5 bracket are indicated in the graph. (**Right**) Salary earners in Spain (2014) as a function of the non-dimensional salary paid, ϕ. (Official Statistics 2014, see footnote 1)

Therefore, the salaried people within each sub-bracket j can be expressed as:

$$s_{j=}\, s_1 - \Delta(j-1) \tag{5}$$

The salary earners in each half-minimum wage sub-brackets from 0 to 10 times the minimum wage are also shown for 2014 in Figure 3. Once the number of earners in each sub-bracket has been estimated, the amount of salaries paid can be calculated by multiplying this quantity by the salary corresponding to the center of the sub-bracket. In Figure 4, the salary distribution corresponding to 2014 (made non-dimensional by dividing by the total amount of salaries paid that year), is shown. In this graph the open circles correspond to the data from Tables 1 and 2, whereas open squares correspond to the calculated data with Equations (3) to (5). It should also be mentioned that other more complex approaches to the data within the studied brackets (quadratic or cubic distributions, instead of the linear ones) are possible.

Furthermore, it can be observed in the right graph of Figure 3 that an exponential equation:

$$s = k_1 \exp\left(-\frac{\phi}{k_2}\right) \tag{6}$$

fits the distribution of salary earners, s, as a function of the salary earned expressed in number of times the minimum salary, ϕ, well. From this expression, it is possible to estimate the salary earners in the bracket between ϕ_1 and ϕ_2 minimum salaries:

$$S(\phi_1;\, \phi_2) = 2\int_{\phi_1}^{\phi_2} k_1 \exp\left(-\frac{\phi}{k_2}\right) \mathrm{d}\phi = 2k_1k_2\left[\exp\left(-\frac{\phi_1}{k_2}\right) - \exp\left(-\frac{\phi_2}{k_2}\right)\right] \tag{7}$$

It should be mentioned that the factor of 2 included in the expression above is necessary, as the data is discretized in terms of half minimum wage. Equation (7) gives the total amount of salaried people in 2014 with a 3.9% error rate.

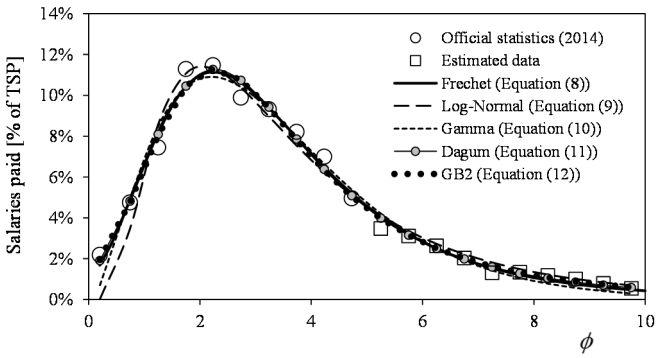

Figure 4. Salaries paid (in terms of percentage of the total amount, TSP) in 2014 as a function of the non-dimensional salary paid, ϕ (expressed in times the minimum wage). The Fréchet distribution has been fitted to the data (Equation (8); $\lambda = 0.479$; $\gamma = 8.55$; $c = 13.6$; $d = -11.18$), together with the Log-Normal (Equation (9); $\lambda = 0.478$; $a = 1.15$; $b = 0.658$), the Gamma (Equation (10); $\lambda = 0.463$; $a = 2.856$; $b = 1.188$), the Dagum (Equation (11); $\lambda = 0.490$; $a = 3.740$; $b = 4.160$; $p = 2.287$; $d = -2.445$), and the GB 2 (Equation (12); $\lambda = 0.493$; $a = 5.218$; $b = 5.046$; $\xi = 2.112$; $\eta = 0.740$; $d = -3.532$) distributions.

The Fréchet Distribution Fitted to the Data

The proposed Fréchet distribution (i.e., Equation (2)), reproduced here for convenience:

$$f(\phi) = \lambda\left(\frac{\gamma}{c}\right)\left(\frac{c}{\phi - d}\right)^{\gamma+1} \exp\left[-\left(\frac{c}{\phi - d}\right)^{\gamma}\right] \tag{8}$$

has been fitted to the normalized distribution of 2014 salaries earned in relation to the non-dimensional minimum wage, ϕ, see Figure 4. Fitting was performed with the least squares method using MATLAB. As can be observed in the figure, the correlation seems to be accurate. Other PDFs used in income analysis (Atkinson and Bourguignon 2015; Kleiber and Kotz 2003; McDonald and Ransom 1979; Klein et al. 2015) such as the log-normal distribution:

$$f(\phi) = \lambda\frac{1}{\phi b\sqrt{2\pi}} \exp\left(-\frac{[\ln(\phi) - a]^2}{2b^2}\right) \tag{9}$$

the gamma distribution:

$$f(\phi) = \lambda\frac{1}{b^a\Gamma(a)}\phi^{a-1} \exp\left(-\frac{\phi}{b}\right) \tag{10}$$

and the Dagum distribution (Kleiber 2008):

$$f(\phi) = \lambda\frac{ap}{b}\frac{\left(\frac{\phi-d}{b}\right)^{ap-1}}{\left[1 + \left(\frac{\phi-d}{b}\right)^{a}\right]^{p+1}} \tag{11}$$

have been also fitted to the data with good results. Furthermore, the GB2 distribution mentioned in the first section (García et al. 2014):

$$f(\phi) = \lambda\frac{a(\phi - d)^{a\xi-1}\Gamma(\xi + \eta)}{b^{a\xi}\Gamma(\xi)\Gamma(\eta)[1 + \left(\frac{\phi-d}{b}\right)^{a}]^{\xi+\eta}} \tag{12}$$

was also included in the study in order to compare the suggested approach with a more complex (and better)[9] distribution. The above PDFs have been compared, for the 2014 data, with the proposed Fréchet distribution in terms of Root Mean Square Error (RMSE):

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(y_n - f(\phi_n)\right)^2} \tag{13}$$

where y_n is the percentage of the salaries paid at the income bracket corresponding to the salary ϕ_n and $f\ (\phi_n)$ is the figure from the selected PDF at ϕ_n. The results show similar values of this error (RMSE = 4.423×10^{-2} [Fréchet]; RMSE = 4.410×10^{-2} [log-normal]; RMSE = 4.443×10^{-2} [gamma]; RMSE = 4.415×10^{-2} [Dagum]; and RMSE = 4.403×10^{-2} [GB2]) for the five distributions.

Once Equation (8) is fitted to the data, it is possible to estimate the salaries paid in the bracket between ϕ_1 and ϕ_2 minimum salaries:

$$\begin{aligned} M = (\phi_1;\phi_2) \quad &= \mathrm{TSP} \times 2\int_{\phi_1}^{\phi_2} \lambda\left(\tfrac{\gamma}{c}\right)\left(\tfrac{c}{\phi-d}\right)^{\gamma+1} \exp\left[-\left(\tfrac{c}{\phi-d}\right)^{\gamma}\right] \mathrm{d}\phi \\ &= \mathrm{TSP} \times 2\lambda\left(\exp\left[-\left(\tfrac{c}{\phi_2-d}\right)^{\gamma}\right] - \exp\left[-\left(\tfrac{c}{\phi_1-d}\right)^{\gamma}\right]\right) \end{aligned} \tag{14}$$

where TSP is the total amount of salaries paid (TSP = 311.28 billion euro in 2014, see Table 2). As aforementioned, the factor of 2 included in the expression above is necessary because the data is discretized in terms of half minimum wage. Equation (14) gives the total amount of salaries paid in 2014 with a 4.6% error rate (calculated from $\phi = 0$ to $\phi \to \infty$, that is, leaving aside the contribution within the bracket $[d, 0]$ [the location parameter, d, has a negative value in all studied cases]).

3. Results and Discussion

In Table 4, the coefficients resulting from the salary earners and the salaries paid distributions (Equations (6) and (8), respectively) from 1999 to 2014 are included. These results are shown in the graphs of Figure 5. As expected, the lower graph of the figure reveals the same progressive change regarding the salaries paid from 2008 as the one reflected in Figure 2. The coefficients have a local variation in 2003. The RMSE evolution of the proposed Fréchet distribution calculated in relation to the data (Equation (13)) is plotted in Figure 6 (left). The results indicate that this distribution comes closer to the statistical data, as it shows decreasing values of the RMSE from 1999 to 2014. A higher value of this error is locally reached in 2003, reproducing the local variation shown in Figure 5. This might be explained by the removal of the restriction in relation to the hiring of civil servants by both the central and regional authorities in Spain (Argimón and Gómez 2006) (it should be mentioned that the importance of the public servants in the statistics is significant). As an example, the number of civil servants in Spain represented 14.1% of total salary earners and 19.4% of the salaries paid in 2004 (Botella et al. 2009). Moreover, the total weight of the public sector on the Gross Domestic Product in Spain has been estimated to be approximately 45%).[10] In addition, two distinct patterns are shown by the evolution of the RMSE in the aforementioned graph. From 1999 to 2007, a slightly decreasing pattern is shown, whereas from 2008 the negative slope of that pattern is accentuated. This effect could be the result of the changes in the labor market law implemented in 2010 and 2012 in Spain (without leaving aside the changes faced by the Spanish economy since the beginning of the present crisis in 2007).[11]

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[9] MacDonald and Ransom claim that the GB2 distribution fits the income distributions better than other simpler distributions such as log-normal or Weibull (McDonald and Ransom 2008).

[10] Mario Alonso. President of the Institute of Auditors and the Spanish Accounting (*Instituto de Censores Jurados de Cuentas*) (Alonso 2016).

[11] The change of the labor market law in 2010 started to be studied in 2008. During two years the government of President Rodríguez Zapatero tried to reach a wide agreement that could include both the employers' association and the trade unions.

Table 4. Coefficients of the mathematical expressions (Equations (6) and (8)) fitted to the salary earners and the salaries paid (Tables 1 and 2, respectively) from 1999 to 2014. Coefficient k_1 is expressed in millions of people; k_2, γ, c, d, λ are dimensionless.

Year	k_1	k_2	γ	c	d	λ
1999	3.93	1.89	3.78	6.40	−3.86	0.488
2000	4.11	1.92	3.42	5.83	−3.25	0.489
2001	4.21	1.95	3.37	5.78	−3.15	0.486
2002	4.32	1.97	3.32	5.76	−3.10	0.488
2003	4.38	2.01	6.39	10.15	−7.68	0.480
2004	4.55	1.98	3.25	5.62	−2.95	0.487
2005	5.01	1.90	3.51	5.78	−3.24	0.484
2006	5.24	1.89	3.35	5.44	−2.92	0.485
2007	5.27	1.90	3.13	5.06	−2.54	0.486
2008	5.29	1.90	3.68	6.04	−3.51	0.485
2009	5.16	1.87	4.63	7.71	−5.18	0.486
2010	5.10	1.84	5.41	8.78	−6.29	0.483
2011	5.09	1.82	6.39	10.15	−7.68	0.480
2012	4.95	1.79	6.93	10.49	−8.08	0.475
2013	4.87	1.78	8.62	13.61	−11.18	0.479
2014	4.93	1.78	8.55	13.60	−11.18	0.479

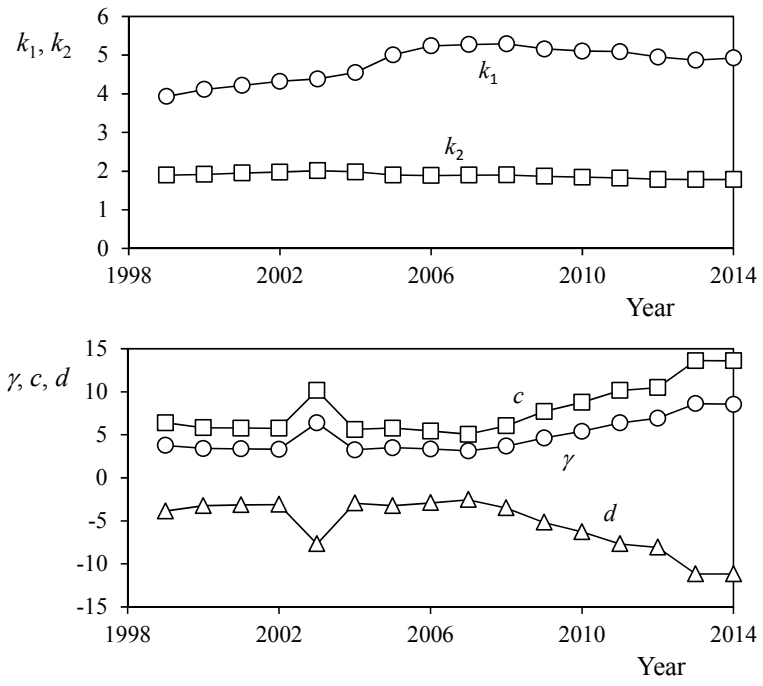

Figure 5. Evolution of coefficients from Equations (6) and (8), once fitted to the official statistics data (Tables 1 and 2).

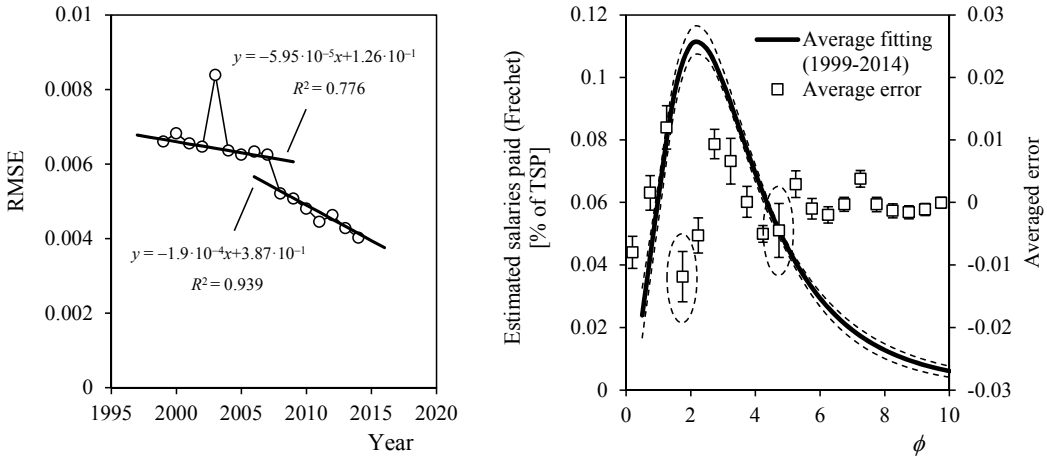

Figure 6. (**Left**) RMSE related to Equation (8) fittings to the official statistics data, see Table 2. (**Right**) Average values of the aforementioned fittings (dashed lines represent the higher and the lower values of that fittings) in relation to the perceived salary, ϕ (expressed in times the minimum salary). On the right y-axis: averaged error when comparing the fittings and the official statistics data (the standard deviation bars have also been included; the highest levels of the standard deviation have been indicated by dashed ellipses).

In addition, the curve formed by the averaged values (from 1999 to 2014) of the percentage salaries paid, estimated through the fitted Fréchet distributions, is shown in Figure 6 (right), together with the curves that represent the maximum and minimum values, plotted with dashed lines. Also, the averaged differences (averaged error) between the Fréchet distribution and the official statistical data at all 0.5 minimum wage brackets are plotted in this graph from 1999 to 2014. It can be said that, although based on the results (Figure 4), the Fréchet distribution fits the data well, some degree of error is locally shown. However, the fact that the proposed wages distribution, once fitted to the data, represents several different groups of individuals (with different skills and different evaluation of them), should be taken into account when trying to explain these differences. Finally, one more effect can be observed in this graph. A higher deviation, related to the differences between the fitted distribution and the official data, is shown for $\phi = 2$ and $\phi = 5$, suggesting a boundary between groups of salaried people and indicating two possible changes from one year to the next; in money (i.e., the salary payed to the people at one side of this boundary has changed) or in people (i.e., some people have moved, increasing or decreasing their salary, to the other side of the boundary).[12]

In order to analyze the data from Tables 1 and 2 and estimate the accuracy of the proposed analytical approach, the number of salaried people responsible for the amount of salaries paid are placed in three different brackets: 0%–30%, 30%–70%, and more than 70%. From Equation (14), the following expressions can be derived for the salaries that represent the boundaries between the three brackets, $\phi_{30\%}$ and $\phi_{70\%}$:

$$\phi_{30\%} = d + \frac{c}{\left[\ln\left(\frac{2\lambda}{0.3}\right)\right]^{\frac{1}{\gamma}}} \tag{15}$$

$$\phi_{70\%} = d + \frac{c}{\left[\ln\left(\frac{2\lambda}{0.7}\right)\right]^{\frac{1}{\gamma}}} \tag{16}$$

[12] This fact agrees with what Paul Krugman said, quoting a work by (Burkhauser and Couch 2009); 'The majority of economic mobility occurs over fairly small spans of the distribution'. On Invincible Ignorance (Krugman 2016a). Also: *La irreductible ignorancia* (Krugman 2016b).

Once the aforementioned boundary salaries, $\phi_{30\%}$ and $\phi_{70\%}$, are calculated, the salaried people in the brackets mentioned in Equation (7) can be estimated:

$$S_{0\%-30\%} = S(0;\phi_{30\%}) = 2k_1k_2\left[1-\exp\left(-\frac{\phi_{30\%}}{k_2}\right)\right] \tag{17}$$

$$S_{30\%-70\%} = S(\phi_{30\%};\phi_{70\%}) = 2k_1k_2\left[\exp\left(-\frac{\phi_{30\%}}{k_2}\right)-\exp\left(-\frac{\phi_{70\%}}{k_2}\right)\right] \tag{18}$$

$$S_{70\%-\infty} = S(\phi_{70\%};\infty) = 2k_1k_2\exp\left(-\frac{\phi_{70\%}}{k_2}\right) \tag{19}$$

The results (in terms of percentage in relation to the total amount of salary earners each year) are included in Table 5 and Figure 7. It can be observed that the results based on the analytical approximation follow the general trend shown by the results calculated with direct interpolations on the statistical data from Tables 1 and 2. However, a noteworthy deviation of the analytical approximation can also be observed in the graph.

Another interesting result is observed when the annual increases on the salaried people within the 0%–30% and 30%–70% brackets (respectively, $\Delta_{0\%–30\%}$ and $\Delta_{30\%–70\%}$) are compared. The data shows a very good correlation indicating that each increase (decrease) in the number of earners within the 0%–30% salaries paid bracket is well correlated with a proportional decrease (increase) in the number of earners within the 30%–70% salaries paid bracket (see right graph of Figure 7). As it can be observed in Table 5, the boundary salary $\phi_{30\%}$ has a value of approximately $\phi_{30\%} = 2$, which was identified as one of the points where a higher transfer of salaried people from one side to the other of this salary level was produced from 1999 to 2014. The estimated results (based on the Fréchet distribution fitting), however, do not reflect the aforementioned correlation. In addition, it should also be said that, according to the graph in Figure 7, the percentage of earners within the more than 70% salaries paid bracket remains constant throughout the studied period (1999–2014).

Table 5. Wage levels, $\varphi_{30\%}$ and $\varphi_{70\%}$, that indicate 30% and 70% of the total amount of salaries paid and the number of salaries paid (salaried people) within the 0%–30%, 30%–70%, and more than 70% salaries paid brackets from 1999 to 2014. Two different figures are included: 'calc.' stands for the figures calculated by interpolating on the official statistical data, whereas 'est.' stands for the figures estimated with the fittings of Equations (6) and (8).

Year	$\phi_{30\%}$		$\phi_{70\%}$		$S_{0\%–30\%}$ [%]		$S_{30\%–70\%}$ [%]		$S_{70\%-\infty}$ [%]	
	calc.	**est.**	**calc.**	**est.**	**calc.**	**est.**	**calc.**	**est.**	**calc.**	**est.**
1999	2.04	2.27	4.50	4.72	63.1	72.1	27.8	22.5	9.1	8.5
2000	2.07	2.30	4.56	4.79	62.6	72.4	28.2	22.6	9.2	8.5
2001	2.12	2.36	4.67	4.90	62.4	72.7	28.4	22.5	9.2	8.4
2002	2.14	2.38	4.71	4.93	62.3	72.8	28.5	22.5	9.3	8.5
2003	2.01	2.23	4.29	4.48	62.3	69.5	28.4	23.1	9.3	11.2
2004	2.15	2.40	4.72	4.95	61.8	73.1	28.8	22.5	9.4	8.6
2005	2.05	2.29	4.50	4.73	62.2	72.5	28.5	22.5	9.2	8.6
2006	2.03	2.27	4.45	4.69	61.9	72.6	28.8	22.4	9.3	8.7
2007	2.02	2.27	4.45	4.69	61.2	72.3	29.4	22.6	9.5	8.8
2008	2.04	2.28	4.46	4.69	61.6	72.8	29.0	22.5	9.5	8.8
2009	2.04	2.26	4.42	4.62	62.7	73.3	28.0	22.3	9.3	8.8
2010	2.02	2.25	4.35	4.54	62.8	73.5	27.9	22.0	9.3	8.9
2011	2.01	2.23	4.29	4.48	62.9	73.6	27.7	21.7	9.3	9.0
2012	1.97	2.20	4.18	4.37	63.0	73.4	27.7	21.4	9.3	9.0
2013	1.99	2.20	4.24	4.40	63.9	73.8	26.9	21.5	9.2	8.9
2014	1.98	2.19	4.24	4.40	64.1	73.4	26.8	21.7	9.1	8.8

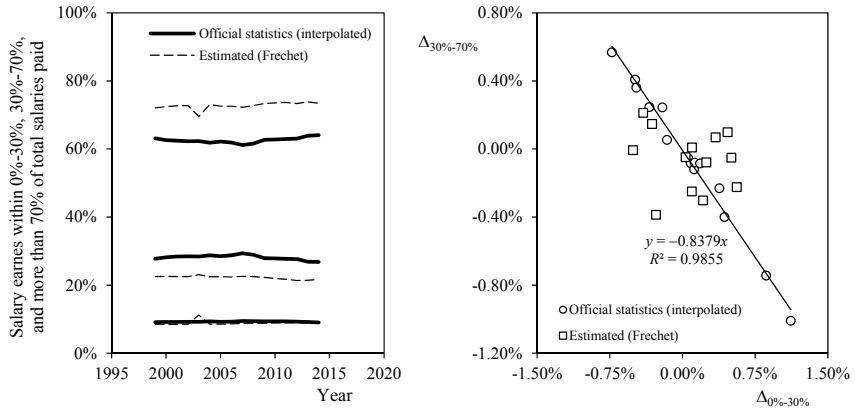

Figure 7. (**Left**) Evolution from 1999 to 2014 of the of the percentage salary earners within the 0%–30%, 30%–70%, and more than 70% of the total salaries paid brackets; (**Right**) Yearly variations of the aforementioned the percentage salary earners within the 30%–70% bracket in relation to the variations of the the percentage salary earners within the 0%–30% bracket.

Finally, the distribution of the salaries paid can be studied in terms of skewness; that is, the asymmetry of the distribution (or the difference between both tails weight). The skewness, s_{kw}, of the Fréchet distribution is calculated using the gamma function (De Gusmão and Ortega 2011; Khan et al. 2008):

$$S_{kw} = \frac{\Gamma\left(1-\frac{3}{\gamma}\right) - 3\Gamma\left(1-\frac{2}{\gamma}\right)\Gamma\left(1-\frac{1}{\gamma}\right) + 2\Gamma^3\left(1-\frac{1}{\gamma}\right)}{\left[\Gamma\left(1-\frac{2}{\gamma}\right) - \Gamma^2\left(1-\frac{1}{\gamma}\right)\right]^{\frac{3}{2}}} \tag{20}$$

However, to analyze the asymmetry of the fittings and its evolution within the studied period, a simpler parameter was selected. This asymmetry parameter, ψ, is defined as the difference between the median:

$$\phi_{50\%} = d + \frac{c}{\left[\ln\left(\frac{2\lambda}{0.5}\right)\right]^{\frac{1}{\gamma}}} \tag{21}$$

and the mode:

$$\phi_m = d + c\left(\frac{\gamma}{1+\gamma}\right)^{\frac{1}{\gamma}} \tag{22}$$

of the distribution. Therefore:

$$\Psi = \phi_{50\%} - \phi_m = c\left(\left[\ln\left(\frac{2\lambda}{0.5}\right)\right]^{\frac{1}{\gamma}} - \left(\frac{\gamma}{1+\gamma}\right)^{\frac{1}{\gamma}}\right) \tag{23}$$

With this definition of the asymmetry, we have defined a parameter that would help to measure how much the salaries paid distribution detach from a normal (or Gaussian) distribution, which is characterized by zero skewness (and equal values of the mode and the median). A normal distribution of the salaries would be more heavily weighted on the middle wage levels, with equally weighted tails (i.e., the weight of the large number of low wages counterbalances the weight of the low number of high wages). In Figure 8, the results are shown. Leaving aside the result from 2003, produced by a larger error of the fitting from the official statistics data that year, it can be said that after a quite constant level of asymmetry from 1999 to 2007, ψ = 1.11–1.18, the asymmetry falls to 0.87 in 2013–2014. Hence, the results seem to indicate that a more balanced salary distribution was reached in 2013–2014, after the booming years of the Spanish economy (from 1999–2007). Finally, it should be underlined that this transition towards a less unequal salary distribution sharply contrasts with the latest reports on inequality in Spain regarding incomes and wealth.

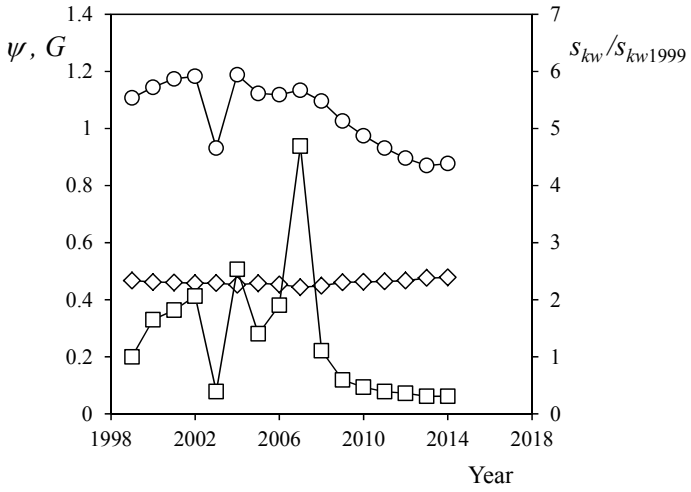

Figure 8. Evolution from 1999 to 2014 of the Fréchet distribution fittings asymmetry parameter, ψ, (open circles; defined with equation (23)). The calculated skewness of the Fréchet distribution fittings, s_{kw}, (open squares), and the evolution of the Gini coefficient (open rhombi) have been also included in the graph.

In order to check the robustness of the present analysis, some additional calculations were carried out. Firstly, the skewness, s_{kw}, was calculated in order to compare it with the suggested symmetry parameter, ψ. The results, made non dimensional with the value of the skewness in 1999, are also included in Figure 8. A similar trend between the skewness and parameter ψ can be observed. In addition, it seems that parameter ψ can filter some noise shown by the skewness in those points where the scale parameter, γ, is close to $\gamma = 3$, as the skewness of the Fréchet distribution presents a singularity at this point.

Secondly, the evolution of the wages distribution was analyzed using the Lorenz curves. This approach is normally applied to income inequality (Krause 2014; Lubrano 2016). According to Lubrano (2016), the Lorenz curve is 'a graphical representation of the cumulative income distribution. It shows for the bottom p_1% of households, what percentage p_2% of the total income they have'. In Figure 9, the Lorenz curves corresponding to 1999, 2005, and 2013 are shown. In this graph, the percentage of the salaried people, p, is plotted on the x-axis, whereas the percentage of the wages received by this number of salaried people, $L(p)$, is plotted on the y-axis. The Lorenz curves were calculated, from 1999 to 2014, with the data from Tables 1 and 2. As can be observed in Figure 9, all curves have the same pattern. Nevertheless, some additional information can be derived in order to establish an evolution pattern followed by the wages distribution. If the difference between the Lorenz curve and the theoretical values that represent the maximum equality (Krause 2014), $p - L(p)$, is plotted as a function of the percentage of the salaried people, p, it is possible to analyze the position of the maximum of this curve (see Figure 10). If this maximum is displaced towards $p = 1$, the existence of an elite within the salaried group is revealed, whereas if it is displaced towards $p = 0$ it could be said that a group with extreme low wages exists. The evolution of this maximum position, $p_{\max|p-L(p)|}$, from 1999 to 2014 is shown in Figure 11. It can be appreciated that this maximum detaches its position from $p = 1$, which indicates a reduction in the importance of the higher wages (the elite) in relation to the whole group. This result agrees with the pattern shown by the symmetry parameter ψ in Figure 8; that is, the salary distribution in Spain has changed towards a more balanced situation since 2007.

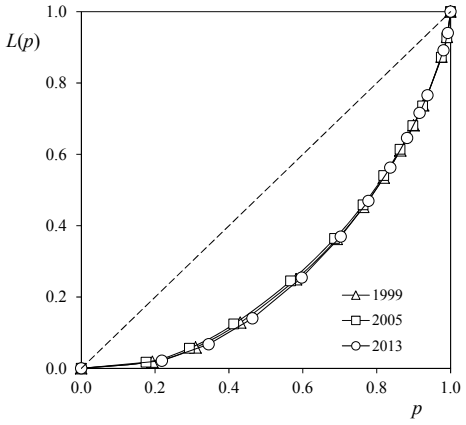

Figure 9. Lorenz curves representing the wage distributions in 1999, 2005, and 2013. Percentage of the wages, $L(p)$, perceived by a percentage p of the salaried people in relation to the percentage of the salaried people, p. The dashed straight line corresponds to the theoretical maximum equality level.

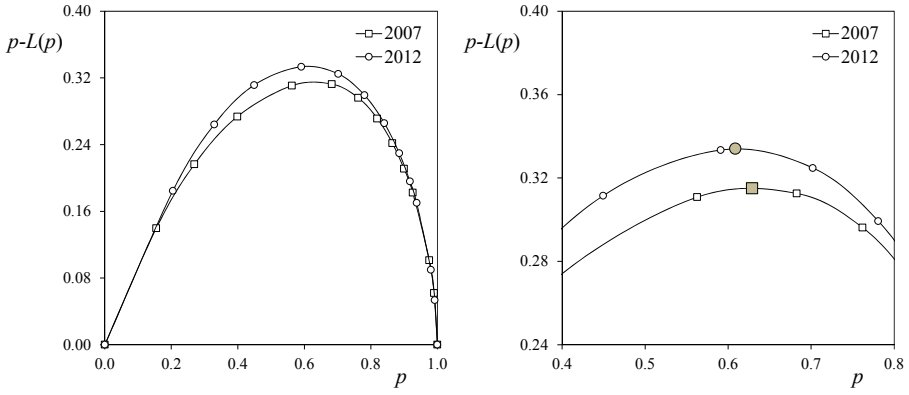

Figure 10. Difference between the Lorenz curves and the theoretical maximum equality level, $p - L(p)$, in relation to the percentage of salaried people, p, in 2007 and 2012 (left). Location of the maximum point on these curves (right).

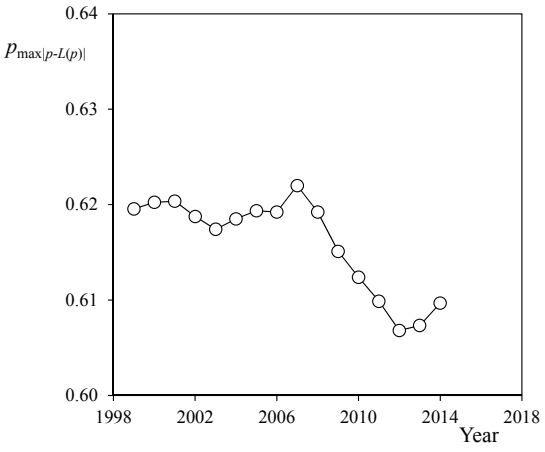

Figure 11. Evolution from 1999 to 2014 of the salaried people percentage corresponding to the maximum difference between the Lorentz curve applied to the salaries distribution and the theoretical maximum equality level.

Finally, we have performed a further analysis of the data, studying the evolution of the Gini coefficient, G, defined as (Krause 2014; Lubrano 2016):

$$G = 1 - 2\int_0^1 L(p)\mathrm{d}p \tag{24}$$

The evolution of this coefficient, G, between 1999 and 2014 has been plotted in Figure 8 in order to compare it with the evolution of the defined asymmetry parameter, ψ, and the skewness, s_{kw}. It seems that the value of this coefficient remains quite constant, with a very slight increase starting in 2007, indicating an increase of inequality. This result contrasts with the previous results, based on the asymmetry of the wage distributions, and indicates a margin for further research.

4. Conclusions

In the present work, the official data in relation to salaries paid in Spain from 1999 to 2014 has been analyzed. The Fréchet distribution was used as a way of studying the data using an analytic expression that depends on a limited number of parameters. The most significant conclusions resulting from this work are:

- The Fréchet distribution has proven to fit the studied salaries' distributions well, having a similar accuracy in relation to the data when compared to other distributions (Log-Normal, Gamma, Dagum, GB2) that can be considered more complex.
- The analysis of the results showed that changes in the salary distribution as a result of the economy evolution are reflected in the 0%–30% and 30%–70% brackets of the total salaries paid, the top 30% (affecting 9% of the salaried people) being quite resilient to those changes from 2002 to 2014 in Spain.
- Finally, the results based on the distributions' asymmetry (skewness) indicate an increasingly more balanced salaries distribution (i.e., less skewed) starting in 2007. However, this seems to be in contrast with the evolution of the Gini coefficient.

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Conflicts of Interest: The authors declare no conflict of interest.

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