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Islamic domes of crossed-arches: Origin, geometry and structural behavior

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ABSTRACT: Crossed-arch domes are one of the earliest types of ribbed vaults. In them the ribs are intertwined forming polygons. The earliest known vaults of this type are found in the Great Mosque of Córdoba in Spain built in the mid 10th century, though the type appeared later in places as far as Armenia or Persia. This has generated a debate on their possible origin; a historical outline is given and the different hypotheses are discussed. Geometry is a fundamental part and the different patterns are examined. Though geometry has been thoroughly studied in Hispanic-Muslim decoration, the geometry of domes has very rarely been considered. The geometrical patterns in plan will be examined and afterwards, the geometric problems of passing from the plan to the three-dimensional space will be considered. Finally, a discussion about the possible structural behaviour of these domes is sketched.

Crossed-arch domes are a singular type of ribbed vaults. Their characteristic feature is that the ribs that form the vault are intertwined, forming polygons or stars, leaving an empty space in the centre. The fact that the earliest known vaults of this type are found in the Great Mosque of Córdoba, built in the mid 10th century, has generated a debate on their possible origin. The thesis that appears to have most support is that of the eastern origin. This article describes the different hypothesis, to then proceed with a discussion of the geometry. Geometry is a topic that has been thoroughly studied in Hispanic-Muslim decoration; however, the geometry of domes has very rarely been considered. Also, the function of the ribs has never been discussed, in contrast with the abundant literature on gothic ribs. In what follows, we will give first a historical overview of the evolution of this structural type. Then the geometrical patterns in plan will be examined. Afterwards, the geometric problems of passing from the plan to the three-dimensional space will be considered. Finally, a discussion about the possible structural behavior of these domes will be given.

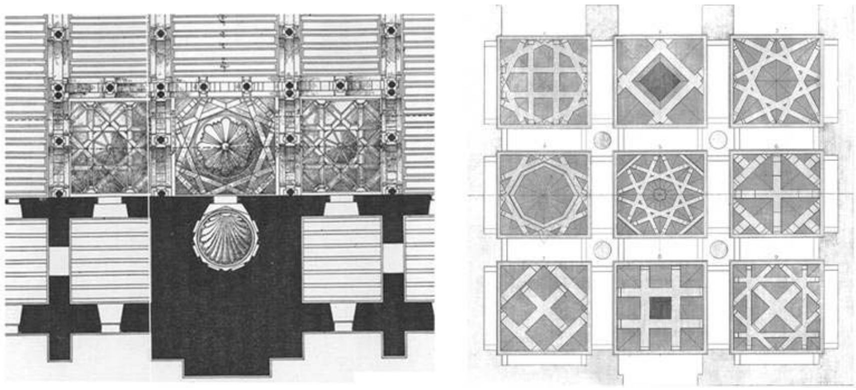
1 HISTORICAL DEVELOPMENT

The first examples known appear in Al-hakam II's extension of the Great Mosque of Córdoba, between the years 961 and 965. Four domes were constructed during these years (Fig.1.a); three of them close to the mihrab, while the forth was built in the place where the old mihrab of Abd al-Rahman II's mosque used to be.

The mosque of Bib al-Mardum or Cristo de la Luz, in Toledo, appears ca. 50 years later. A full catalogue of possible solutions for crossed-arch domes is offered in this building (Fig.1.b). Each of the nine bays has a different layout, some repeating what had already been seen in Córdoba, while others will have an influence in later buildings, as we will see.

Throughout the 11th century, other crossed-arch domes are built, such as the one in the Tornerías Mosque, or that in the Belén Chapel, in the convent of Santa Fe, both in Toledo. Another example is the small dome covering the niche located over the central column of the church of San Baudelio de Berlanga, in the province of Soria.

From the end of the 10th century, ribbed domes appear in Armenia and Persia, featuring, however, important differences between them. Muslim architecture employs a network of thin ribs, made of brick, and multiplies intertwining between them, creating polygons or stars. Geometry acquires a key role in these domes. In Armenia, nonetheless, large arches are built in good quality stone voussoirs. Armenian builders divide the vault in parts, while Arab builders make use of the ribs to organize the decorative brick bond in the webs, but always constructing a homogeneous intrados. The scale is also different: in Armenia, these vaults are elements applied to the whole of the building, while in Muslim architecture the vault only covers part of the room (Baltrusaitis J., 1939).



(a) (b)

Figure 1 : (a) Plan of the domes of the maqsura in the Great Mosque of Córdoba, (b) Domes of Cristo de la Luz

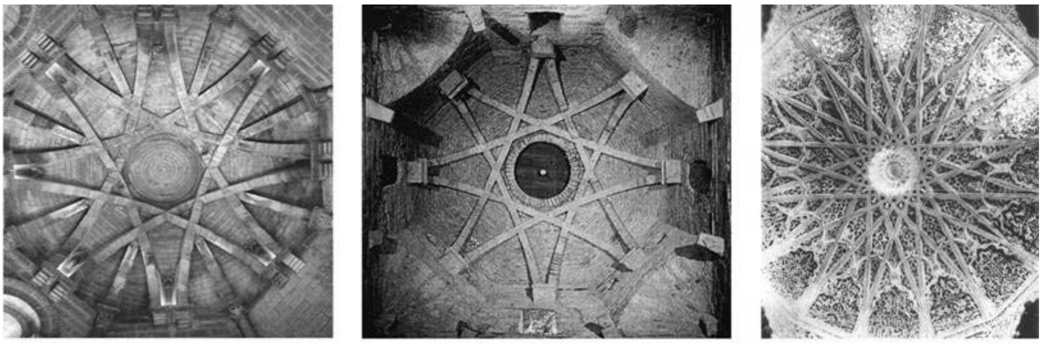
Towards the end of the 12th century, two types of vaults can be distinguished. On one hand, in Christian architecture, the vaults feature thick ribs, usually creating drawings based on the square or the octagon. Various examples can be found in Spain and the South of France. In Islamic architecture, on the contrary, the number of ribs increases while their thickness is reduced, and the domes often feature stalactite ornament and fretwork webs. Some of the most important Christian examples are the churches of Torres del Río (Fig.2.a) and San Miguel de Almazán in Spain, and Santa Cruz de Olorón and San Blas Hospital in France (Fig.2.b). As far as the Islamic type is concerned, various examples are found in Spain and the North of Africa. It is worth mentioning those of the Toro-Buiza house, in the Alcazar of Seville, the vaults in the Torre del Homenaje of the Villena Castle in Alicante and the vault covering the San Sebastian Chapel in Granada. Further examples can be seen in the North of Africa, such as the vault over the top floor of the Koutoubiya minaret. Also worthy of mention are the tracery vaults of Tremecen, Taza and Fez (Fig.2.c).

At the beginning of the 16th century, this type of vault is revived in Spain to be used in some of the most important ciboria, such as the ones in Zaragoza, Teruel and Tarazona (Fig.3). The vaults of these ciboria reproduce one of the designs found in Córdoba, combining the Hispanic-Muslim influences with the contemporary Gothic style. On the outside, these constructions feature a superposition of prismatic elements.

Spanish 16th Century ciboria are not the last examples of this type of domes. In the Italian Baroque, Guarino Guarini designed several churches inspired in the earlier examples and built the dome of San Lorenzo in Turin using crossed-arches. Again in Italy, in the 18th century Vittone included these vaults in some of his buildings, such as the Chapel of San Luigi Gonzaga. Even in the 20th century some examples of crossed-arch vaults have been produced. In Spain, Luis Moya roofs some of the most important constructions of his architectural production with these domes, including the Chapel of the Universidad Laboral in Zamora or the parish church of Torrelavega, in Cantabria.

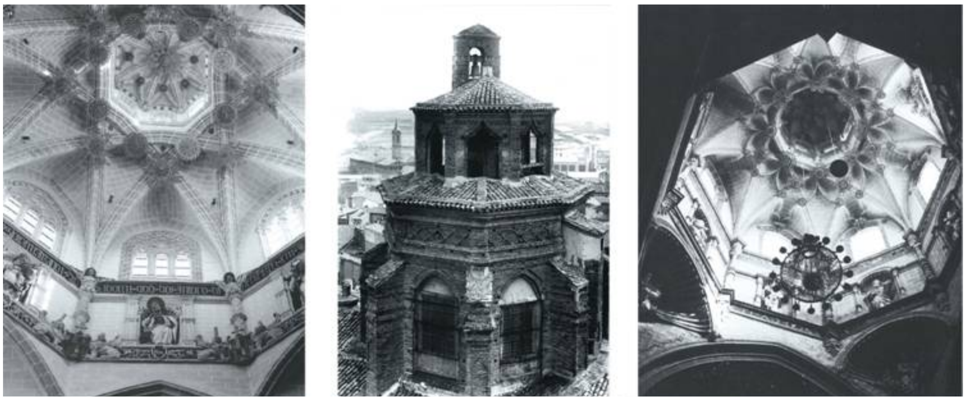
2 ORIGIN

The theories on the origin of these domes are diverse. The first known domes are those of the Great Mosque of Cordoba, but the level of complexity and perfection achieved in these domes is sufficient to consider that earlier examples must have existed.



(a) (b) (c)

Figure 2: (a) Torres del Río, (b) Santa Cruz de Olorón, (c) Mosque of Taza (Terrasse 1943)



(a) (b) (c)

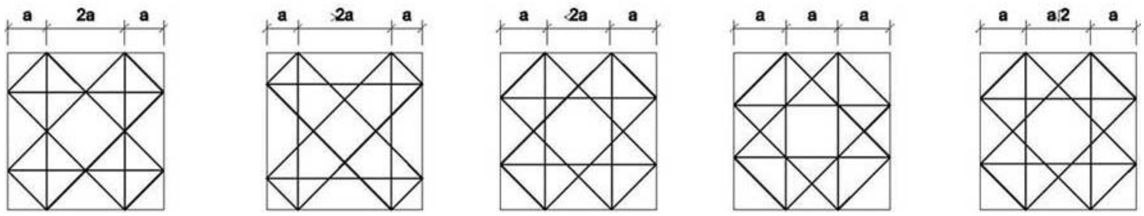
Figure 3 : Spanish ciboria: (a) and (b) Zaragoza interior and exterior views, (b) Tarazona

The 'official' theory maintains that crossed-arch domes are of Eastern origin. However, this theory has not yet been proved. Marçais suggests the possibility that earlier plane crossed arch examples may have resulted in the use of crossed arches in domes: "Le même architecte était, à la rigueur, capable d´imaginer ces deux applications si curieuses du même principe". However, in the Spanish edition of 1983 he supports the theory that these vaults come from Persia. Velázquez Bosco (1894) suggests the possibility that these domes are the masonry interpretation of timber domes constructed in some parts of Asia Minor, which features a hole in the centre for fume extraction. Lampérez supports the theory of Velázquez Bosco. Terrasse is of the opinion that the origin is in the arches that originally framed the squinches and believes that these vaults must have originated in Mesopotamia or Persia. After Godard's work, Terrasse supports the theory of the Persian origin. Baltrusaitis, in *Le probléme de l'ogive et l'Arménie*, presents the differences and similarities between Armenian and Arab domes. He believes that ribs only carry a structural function in Armenian domes, while in those of Arab origin, ribs are merely decorative. He compares these ribs with the geometric patterns that appear in ornaments. Despite the oldest examples of this ribbed system are found in the west, he believes that it did not originate in the west, but in the east. He rules out the hypothesis of eastern examples originating from the western ones, and considers them to be two independent events developed in parallel: "Les ogives arméniennes et les coupoles d´Islam seraient deux formations indépendantes et parallèles inspirées par un modèle unique, l´une interprétée par des géomètres, l´autre par des architectes" (Baltrusaitis J., 1936). Lambert points at the domes of the great mosques of Kairouan (836) and Tunis (864) as possible precedents of the domes in Cordoba, for they are chronologically located between the extension of the mosque by Abd ar-Rahman III and Al-Hakam II. He emphasizes the evolution between the pumpkin dome of Kairouan, and the dome of Tunis, featuring arches with rectangular section between the domes webs. Gómez-Moreno proposes a possible Mesopotamian origin (Gómez Moreno M., 1951). Galdieri supports this theory. Pavón Maldonado writes about a possible origin in Roman antiquity. The last book on the subject by Giese-Vogeli doesn't give new evidence or theories. It appears that the matter is still not settled.

3 GEOMETRY

Geometry is a key feature of crossed-arch vaults. There is an enormous range of possibilities for the design of the plan projection of these vaults. However, spatial geometry must be simultaneously considered, as some designs might present incompatibilities in their geometry when implemented in three dimensions. Different layouts are used that adapt to the requirements of each particular case. As we have seen, three different layouts can be found in the four domes of Córdoba. Fifty years later, with the construction of the nine domes of Cristo de la Luz, almost all the possible layouts are finally defined, and most of what was built afterwards was based in these designs.

The octagon is possibly the most used figure in the layout design of these vaults. Octagons can be obtained in different ways. One of the most common methods consists in dividing the sides of a square in three parts, and placing eight ribs spanning between the internal divisions, so that two ribs arrive at each point, one rib parallel to the side of the square, and the other in the diagonal direction. According to the ratio between the divisions, the design in plan is different.



(a) (b) (c) (d) (e)

Figure 4 : Design variations obtained from the same initial diagram

Two squares are always present in the centre, rotated by 45º. When the side of the square is divided so that the middle section is smaller than twice the lateral sections (Figs.4.c and 4.e), an octagon is created in the middle. Furthermore, if the proportion of Fig.4.e is met, the octagons, both the external and the internal one, will be regular polygons, and all ribs will be equal, making construction a lot easier. The latter two are among the most common cases. They appear in the lateral domes of the maqsura in the Great Mosque of Cordoba, in the Belén Chapel and in the Spanish ciboria in Aragon. These designs can adapt to real plans, such as the ciboria of Zaragoza and Teruel; where the plans are not square, but the same scheme is adopted. These changes in plan have a large impact on the final spatial arrangement. While ribs for square layouts could be semi-circular, in the case of a rectangular layout they can't be so, for they would then not cross each other.

One other common layout, especially in the 12th and 13th centuries in Spain, is that found for the first time in the dome of Cristo de la Luz. From an octagonal layout, ribs spring in pairs from the middle points of each side, defining another octagon in the centre. All ribs are equal in this case. The octagon must be regular in order that the ribs meet in space. This pattern is also found in Torres del Río and Almazán in Spain; and Olorón and the Hospital of San Blas in the South of France. In San Miguel de Almazán the layout of the transept is a skew square, and thus the ribs cannot all be equal, but those that are parallel will be. Further ahead we will talk about what happens at a spatial level in these cases, Fig.5.

Another scheme found in Cristo de la Luz is the one shown in Fig.6.a. Starting from a square plan, ribs are drawn from the centre of the square to a corner. All ribs are equal in this case. The scheme can also be found at the Hospital of Santa Cruz in Toledo (Fig.6.b), where four additional ribs are added spanning from a corner of the square to a corner of the octagon, and at the Purificación chapel in the cathedral of Tarazona (Fig.6.c), where the scheme becomes more complex with the introduction of curved ribs.

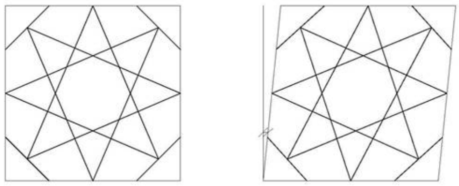
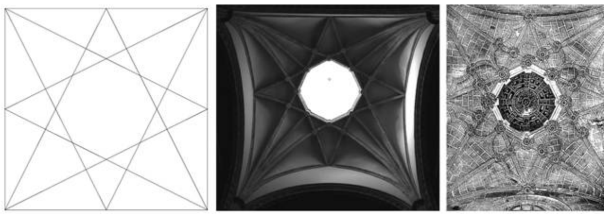


Figure 5 : Plans of Torres del Río (left) and Almazán (note the slight skew of the plan)



(a) (b) (c)

Figure 6 : (a) Schematic plan, (b) Hospital de la Santa Cruz, (c) Capilla de la Purificación

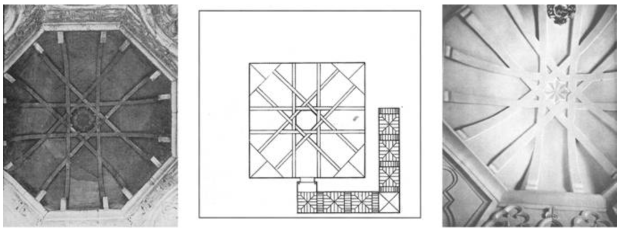
In the central dome of the Great Mosque of Cordoba we find a scheme originated from two squares rotated in plan, and inscribed inside a third square (Figs.7.a and 7.b). Thus, an octagon is again drawn both inside and outside. In order that these two octagons are regular, the relationships given in Fig.7 must be met, or, equivalently, the initial two squares must be rotated by a 22.5° angle with respect to the external square. This scheme is also found at the Almoravid quba in Marrakech (Fig.7.c)

(a) (b) (c)

Figure 7 : (a) Schematic plan, (b) Central dome in the maqsura of the mosque of Córdoba, (c) Quba of Marrakech (Meunié and Terrasse 1957).

In all the designs described above, the ribs spring in pairs. However, there are other domes in which ribs spring individually; Torres Balbás attributes these constructions to the Almohads or their influences. There are various examples of this kind of vaults: the vault in the tower at the prison of Alcalá la Real, the chapel of la Asunción in the monastery of las Huelgas, or the vault over the first floor of the Tower of the Homenaje at the castle of Villena (Fig.8). Similar to these three, but incorporating some particularities, is the Chapel of Talavera in the old cathedral of Salamanca (Fig.9, left). In this case, each rib spans from a corner of the octagon to the mid-point of a side, skipping four corners. Eight equal ribs thus appear, each one parallel to another one, but not parallel to the sides of the octagon.

A further singular design is found at the Chapel of Villaviciosa Fig.9, right), in the Great Mosque of Cordoba. The plan is rectangular, measuring 7.35 x 8.35 m (Gómez Moreno, 1951). Four arches, parallel to the sides of the rectangle, cross forming a square. A further four arches form a diamond inscribed in the plan. The latter four arches cross the earlier four at the corners of the square, so that three arches cross at one point.



(a) (b) (c)

Figure 8 : (a) Minaret of the Koutoubitya (Basset and Terrase 1926), (b) Alcalá la Real (Pavón Maldonado, 2009), (c) Chapel of la Asunción (Torres Balbás, 1949).

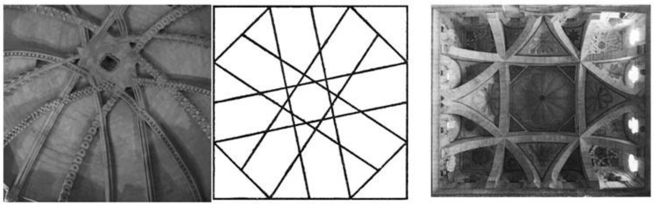


Figure 9 : Chapel of Talavera: Photo and plan layout (left) Chapel of Villaviciosa

When transferring these schemes on plan to three dimensional designs, we come across different cases. For the simplest one, ribs are contained in a spherical surface, as is the case of the Hospital of Santa Cruz, or the vaults of the maqsura of the Great Mosque of Cordoba. Furthermore, all arches can be equal. In the first case, the square layout is defined by four arches, so that the mid points of the square are higher than the corners. The ribs, therefore, are rampant arches (Fig. 10).

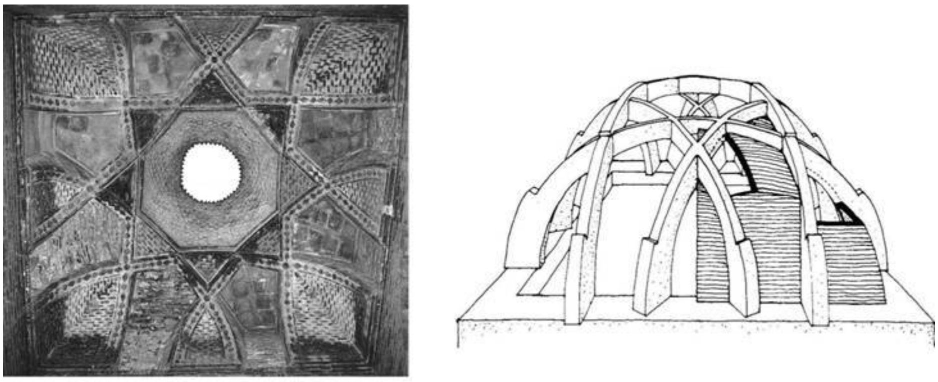
(a) (b) (c)

Figure 10 : Spatial layouts: (a) Hospital of Santa Cruz and (b), (c) San Miguel de Almazán (Martínez de Aguirre and Gil Cornet, 2004)

In the case of using pointed arches, as in Torres del Río, the arches cannot be contained in a sphere. Nonetheless, if the octagon is regular, all arches will be equal and will cross in space. Problems appear when the octagon is not regular, as is the case of San Miguel de Almazán. As seen above, the arches cannot all be equal and their axes don't cross. A possible solution would be not to consider this problem, making all arches instead with the same curvature and with a vertical springing. Accepting this hypothesis, if the distances between the axes of the arches are measured at the points where they would theoretically cross, these distances are found to lie between 11 and 91 mm, that is, between 0.2 and 1.5% of the total span of the vault. Carefully observing the intersections between ribs, it is indeed possible to see small differences in level between the crossing arches (Fig.10.b, 10.c). In any case, this is only a hypothesis and a detailed survey of the vault's geometry would be necessary to either corroborate or dismiss it.

4 STRUCTURAL BEHAVIOR

A crossed-arch dome is a ribbed vault. The question arises as to the actual function of the ribs in supporting the dome. This question has been debated for more than two centuries in relation to the gothic rib, but it has been rarely, if ever, posed in relation to crossed-arch domes (Frankl, 1960). One reason may be the emphasis on decoration of the studies on Muslim architecture. Ribs are considered mostly as decorative. However the size of the arches in the vaults of the Maqsura of Córdoba, in the Armenian domes and in the Spanish ciboria does not fit with this hypothesis. Besides, recent constructive studies (Galdieri, 1981) made on ribbed Persian domes has demonstrated that the thinness of the arches is only apparent, as the ribs project to the outside of the extrados, as may be easily seen in Fig.11.



(a) (b)

Figure 11 : Vault 60 of the mosque of Isfahan, ca. 1100. (Galdieri, 1981)

In Gothic architecture, there is wide agreement that the ribs serve to define the geometry of the vault and form a permanent centring during construction. Besides, they cover the groin hiding the difficult encounter of the masonry of the webs. What was at the heart of the debate was the structural function of the rib. Are the ribs the active structure and the webs only a passive load, or, on the contrary, the ribs have merely decorative? (The first theory corresponds to Viollet-le-Duc and Choisy and was considered right until the 1930's. The second originated in the work of V. Sabouret and, above all, P. Abraham (Huerta, 2009)). If this question is pertinent for gothic vaults it will be also pertinent in the case of the Muslim ribbed vaults and domes. However, the whole matter of the structural behavior of Islamic vaults and domes has been so far neglected. This is curious as some scholars have (Lambert, 1928, Torres Balbás, 1935, Pope, 1963) seen the very origin of the gothic rib in the earlier crossed-arch domes.

The recent book by Giese-Vogeli tackles briefly the matter of the structural behavior of crossed-arch domes, mainly with reference to the gothic debate. Giese-Vogeli, after a brief review of modern literature, takes side with Abraham: "We know today that the ribs do not carry the vault shell". However, she doesn´t seem consider the modern theory of Limit Analysis of Masonry Structures, developed mainly by Jacques Heyman since 1960's (Heyman, 1995). It is only within the correct frame of Limit Analysis, which leads to the approach of equilibrium, that masonry structures of any kind can be understood. The whole debate of the function of the rib has been obscured by the false supposition that there is an "actual" observable state of the structure (Heyman, 1968, Huerta, 2009). In fact, in a hyperstatic structure there are infinite possible states of equilibrium and the actual state cannot be find, depending mainly on small variations of the boundary conditions (Heyman, 2008). Then, depending on the specific situation the rib may or may not carry, the shell may be supported or not, or, perhaps, a certain indeterminate amount is supported by the ribs and the rest by the shell, and the proportion may vary with time.

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