Experimental analysis of stability limits of capillary liquid bridges

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This paper deals with the influence of axial microgravity on the stability limits of axisymmetric, cylindrical liquid columns held by capillary forces between two circular, concentric, solid disks. A fair number of experiments have been performed and both the maximum and the minimum volume of liquid that a capillary liquid bridge can withstand have been obtained as a function of the geometry of the liquid bridge and of the value of the axial microgravity acting on it. Experimental results are compared with published theoretical predictions made by other investigators and discrepancies between those results criticized.

I. INTRODUCTION

The fluid configuration analyzed in this paper consists of an isothermal, axisymmetric mass of liquid held by surface tension forces between two parallel, coaxial, solid disks of the same diameter, as sketched in Fig. 1. The equilibrium shapes of such capillary liquid bridges, when subjected to a gravity field acting parallel to the liquid bridge axis, is determined, assuming the liquid interface is anchored to the edges of the disks, by the following dimensionless parameters: the slenderness, defined as the ratio of gap separation to disk diameter, $\Lambda = L/(2R)$, the dimensionless volume of liquid, $V = \mathrm{V}/(\pi R^2 L)$, defined as the ratio of the physical volume V to the volume of a cylinder of the same L and R, and the Bond number, $B = \rho g R^2/\sigma$, where ρ is the difference between the liquid density and the surrounding medium density, g the axial acceleration, and σ the surface tension (both ρ and σ are assumed to be constant).

Equilibrium shapes and stability limits of capillary liquid bridges have been the subject of many studies, both theoretical and experimental, during the last decades. Early studies concern liquid bridges between equal disks under gravitationless conditions,[1–5] whereas the influence of Bond number is the subject of more recent publications.[6–13] The influence on stability limits of disks of different diameters has also been considered.[13–16] Although the behavior at stability limit is qualitatively similar no matter if the diameters are equal or unequal, only liquid bridges between equal disks are considered in the following.

It is well known that, for each value of Bond number, the stability diagram of capillary liquid bridges can be represented by a single closed piecewise curve on the slenderness/volume plane (Fig. 2). Liquid bridge configurations represented by points inside this stability limit curve are stable, whereas those lying outside are unstable. According to the above results, in each stability limit curve it is possible to distinguish three different parts. If both the liquid bridge volume and the slenderness are small enough, the stability limit is governed by the detachment of the interface (the three-phase contact line) from the edges of the disks (curve *AB*, Fig. 2). The other part of the stability limit curve, corresponding to a minimum in volume, is characterized by the axisymmetric breakage of the liquid bridge (curve *BC*), such a part of the stability curve is known in the literature as minimum volume stability limit. Finally, there is another limit of maximum volume (curve *CA*), which means an upper bound to the stability region (the transition between both limits, minimum and maximum volume, is not clearly defined, at least from an experimental point of view, and to point out this fact this part of the curve in Fig. 2 has been represented using a dashed line instead of a solid one).

Stability limits of minimum volume are well documented both theoretically and experimentally, and there are no relevant discrepancies in the results reported by different authors. This agreement is no longer valid in the case of stability limits of maximum volume, where the differences can be remarkable. In an early paper published by Da Riva and Martínez[4] concerning liquid bridges between equal disks in gravitationless conditions ($B = 0$), the maximum volume stability limit is calculated under the assumption that such a limit is reached when the contact angle at the edges of the disks becomes greater than some specified value (they assumed this maximum value of the contact angle was π), and that if this value is exceeded, the liquid spreads over the lateral surfaces of the supporting disks. On the other hand, as far as we know, there are two works dealing with the same problem, liquid bridges between equal disks in gravitationless conditions, in which a more conservative stability limit of maximum volume was independently found (in one of them, first published in Russian[8] and later in English,[9] the problem is studied from a theoretical point of view, whereas in the other[11] the stability limit of maximum volume is analyzed both theoretically and experimentally). According to these papers, the maximum volume of a liquid bridge is limited by a nonaxisymmetric instability that occurs before the above-quoted

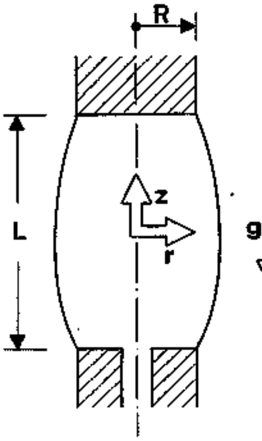

FIG. 1. Geometry and coordinate system for the liquid bridge problem.

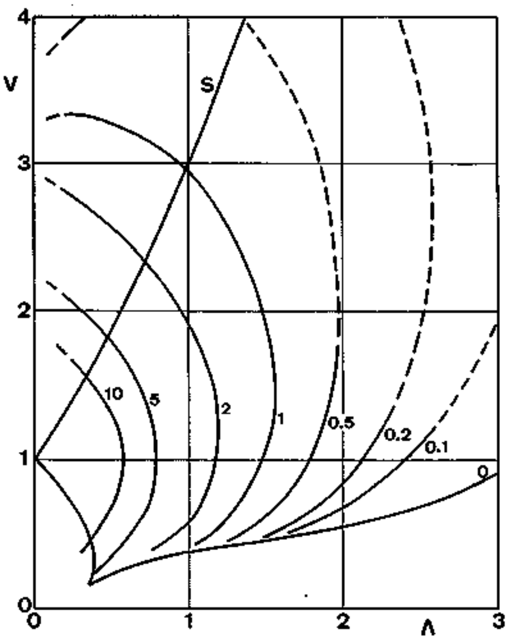

FIG. 3. Influence of the Bond number on the stability limits of liquid bridges between equal disks according to Martínez *et al.*[12] (curves drawn in dashed lines indicate extrapolated results). Numbers on the curves indicate the value of the Bond number. The curve labeled S corresponds to the maximum volume stability limit corresponding to $B=0$ reported by Myshkis *et al.*[9] and Russo and Steen.[11]

limit based on a maximum value of the contact angle, this nonaxisymmetric instability being characterized by the appearance of a single azimuthal bulge at the interface, which causes a nonaxisymmetric spreading of the liquid (this instability is clearly illustrated by Russo and Steen[11] in their Fig. 5). To our knowledge, there is only one publication on the influence of Bond number on stability limits of maximum volume.[12] In Fig. 7 of this paper,[12] the stability regions for different values of the Bond number are shown. Unfortunately, no indication is given on how such stability curves have been calculated, although it seems that some similar criterion to that of the maximum contact angle has been used. To point out the differences between both criteria, in Fig. 3 the stability limits reported by Martínez *et al.*[12] are compared with those corresponding to $B=0$ of Myshkis *et al.*[9] and Russo and Steen.[11] It can be observed that the differences are not appreciable in the case of minimum volume limit, but become unacceptable in the case of stability limits of maximum volume.

Aiming to clarify this situation, the influence of Bond number on the stability limits of capillary liquid bridges

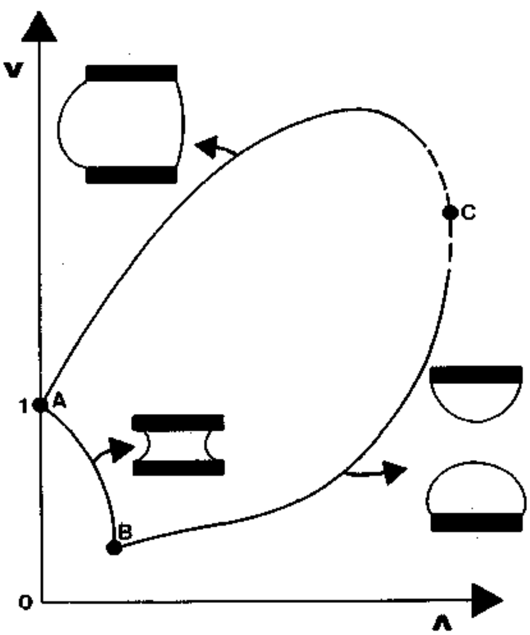

FIG. 2. Typical stability diagram of liquid bridges between equal disks subjected to an axial acceleration. The sketches indicate the different types of instability appearing on the different parts of the stability curve.

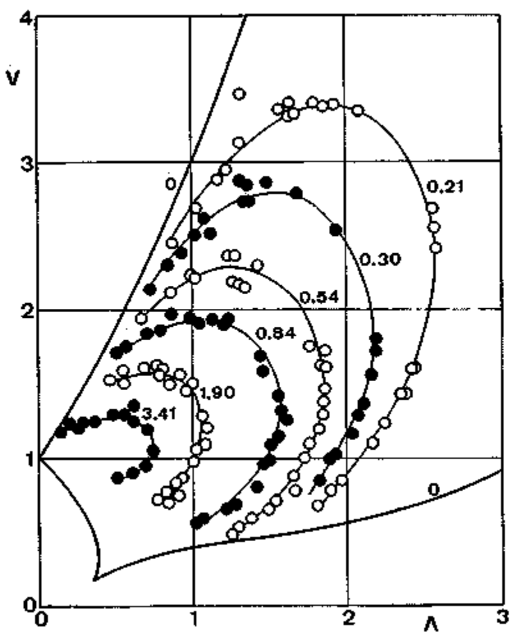

FIG. 4. Stability limits of liquid bridges between equal disks. The symbols indicate experimental results. Numbers on the curves indicate the value of Bond number B. The curves labeled as $B=0$ are from Myshkis *et al.*[9]

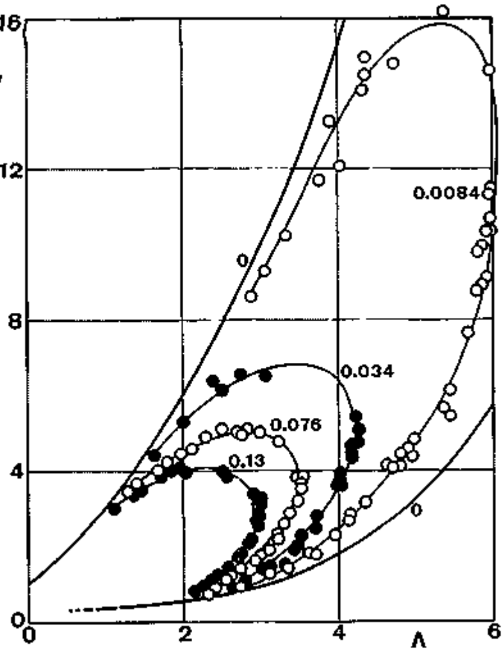

FIG. 5. Stability limits of liquid bridges between equal disks. The symbols indicate experimental results. Numbers on the curves indicate the value of Bond number B. The curves labeled as $B = 0$ are from Myshkis *et al.*[9]

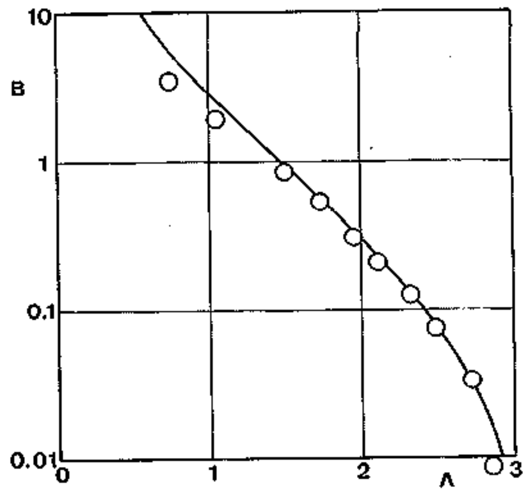

FIG. 6. Variation with the Bond number B of the maximum stable slenderness Λ of liquid bridges between equal disks having cylindrical volume ($V = 1$). The symbols represent experimental results deduced from Figs. 4 and 5 whereas the curve corresponds to theoretical results calculated by Coriell *et al.*[6]

has been experimentally analyzed. Experiments have been performed by using millimetric liquid bridges, using water as working fluid, and both minimum and maximum volume stability limits have been measured for a large number of liquid bridge configurations. Experimental results concerning stability limits of minimum volume are in agreement with those published by other investigators, and in the case of stability limits of maximum volume experimental results, corroborate those obtained by Slobozhanin and Russo and Steen: In all of the configurations tested, a nonaxisymmetric deformation of the interface has been detected before the spreading of the liquid over the lateral surfaces of the disks and, if the slenderness was large enough, the breaking of the liquid column.

II. EXPERIMENTAL APPARATUS AND RESULTS

Experiments have been carried out in a millimetric liquid bridge facility already described by Bezdenejnykh and Meseguer.[16] The liquid bridge, as sketched in Fig. 1, is formed between two parallel, coaxial, equal-diameter disks. Working fluid (water) is added or withdrawn from the capillary bridge through a small hole made at the center of each one of the bottom disks, which is connected to a calibrated syringe by flexible tubing. The length of the liquid bridge is adjusted by raising and lowering the upper disk.

A typical experiment is as follows: Once a stable liquid bridge of the desired slenderness is formed, water is added or withdrawn until the corresponding stability limit is reached. The volume of the liquid bridge is slowly increased or decreased in very small steps until the breaking of the liquid bridge takes place or an asymmetry in the interface appears. From time to time, photographs of the liquid bridge are taken, these pictures being used later to calculate the volume of the liquid bridge at stability limits as a redundancy measure of the readings of the displacement of the syringe piston.

Ten different sets of disks were used, their diameters ranging from 0.5 to 10 mm, which give, taking $\rho = 10^3$ kg m^{-3}, $g = 9.8$ m sec^{-2}, and $\sigma = 0.072$ N m^{-1}, values of Bond number ranging from 0.0084 to 3.41. Experimental results are shown in Fig. 4 ($0.21 \leqslant B \leqslant 3.41$) and in Fig. 5 ($0.0084 \leqslant B \leqslant 0.13$). Note that experimental results concerning maximum volume stability limits are more disperse than those corresponding to minimum volume stability limits. The reason for this difference resides in the nature of the instability to be detected; while in the case of stability limits of minimum volume, there is no doubt to when such a limit has been reached (the liquid bridge breaks); in the case of maximum volume, the instability only causes, initially, the asymmetry of the interface, which in some cases is difficult to observe. Despite this spreading of the experimental results, the agreement with theoretical predictions of Slobozhanin and theoretical and experimental results of Russo and Steen corresponding to gravitationless conditions must be pointed out; note that all experimental points in Figs. 4 and 5 lie under the curve labeled as $B = 0$. The inaccuracy of the theoretical predictions presented by Martínez *et al.*[12] must also be pointed out. (Compare experimental results shown in Fig. 4 with theoretical curves plotted in Fig. 3. In both plots the same scales have been used.)

Finally, the influence of Bond number on the maximum stable slenderness of a liquid bridge having cylindrical volume ($V = 1$) is shown in Fig. 6. In this plot, the symbols represent experimental results deduced from Figs. 4 and 5, whereas the curve corresponds to the theoretical limit calculated by Coriell *et al.*[6]

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