Coherent Detection

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Coherent Detection

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1- Transmitted Signal

- The complex electric field of the transmitted signal is:

$$E_s(t) = A_s(t)e^{j\omega_s t} = \left|A_s(t)\right| e^{j(\omega_s t + \theta_s(t))}$$

$A_s(t)$ is the complex amplitude $\rightarrow A_s(t) = \left|A_s(t)\right| e^{j\theta_s(t)}$

ω_s is the angular frequency

$\theta_s(t)$ is the phase

- The real electric field is:

$$\xi_s(t) = \mathrm{Re}\{E_s(t)\}$$

- The <u>signal power</u> is:

$$P_s(t) = \frac{C}{2}\left|E_s(t)\right|^2 = \frac{C}{2}\left(E_s(t)\right)\left(E_s(t)\right)^* = \frac{C}{2}\left|A_s(t)\right|^2$$

$$C = \frac{S_{eff}}{Z_0}$$

S_{eff} is the effective beam area
Z_0 the impedance of the free space

2- Local Oscillator

- The complex electric field of the local oscillator is:

$$E_{LO}(t) = A_{LO} e^{j\omega_{OL} t} = \left| A_{LO} \right| e^{j(\omega_{LO} t + \theta_{LO}(t))}$$

A_{LO} is the complex amplitude $\rightarrow A_{LO}(t) = \left| A_{OL} \right| e^{j\theta_{LO}(t)}$

ω_{LO} is the angular frequency

$\theta_{LO}(t)$ is the phase

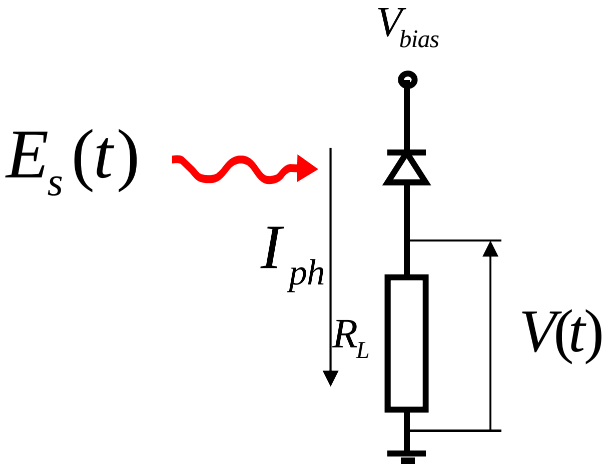
- The real electric field is:

$$\xi_{LO}(t) = \mathrm{Re}\{E_{LO}(t)\}$$

- The local oscillator power is:

$$P_{LO}(t) = \frac{C}{2}\left(E_{LO}(t)\right)\left(E_{LO}(t)\right)^{*} = C\,\frac{\left|A_{LO}\right|^{2}}{2}$$

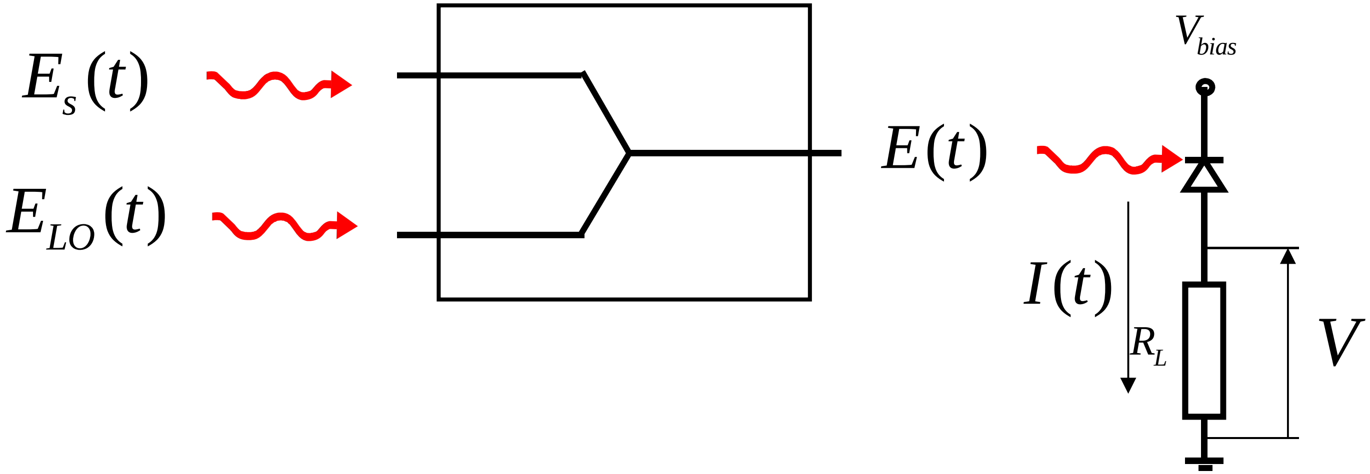
3- Direct detection



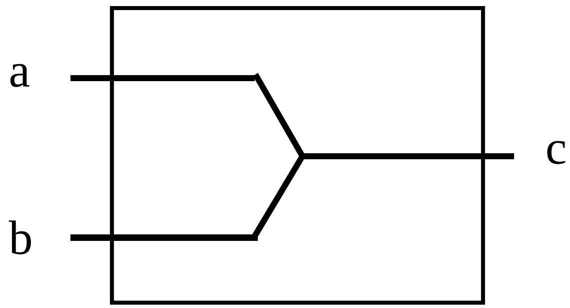
$$I_{ph}(t) = \Re P_s(t) = \frac{R_L \Re C}{2} |A_s(t)|^2$$

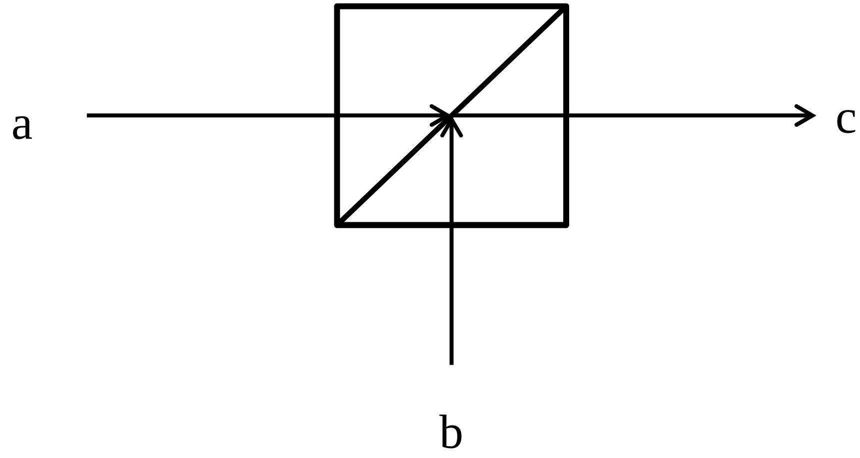
$$V(t) = R_L I_{ph}(t) = R_L \Re P_s(t) = \frac{R_L \Re C}{2} |A_s(t)|^2$$

4- Coherent Detection (3 dB coupler/ Beam Splitter)



3 dB Coupler / Beam splitter





$$E_c = \frac{1}{\sqrt{2}}(E_a + E_b)$$

$$E(t)=\frac{1}{\sqrt{2}}\left(E_s(t)+E_{LO}(t)\right)=\frac{1}{\sqrt{2}}\left(\left|A_s(t)\right|e^{j(\omega_s t+\theta_s(t))}+\left|A_{LO}\right|e^{j(\omega_{OL} t+\theta_{LO}(t))}\right)$$

$$P(t)=\frac{C}{2}\left(E_1(t)\right)\left(E_1(t)\right)^{*}=$$

$$=\frac{C}{2}\left(\left|A_s(t)\right|^2+\left|A_{LO}\right|^2+2\left|A_s(t)\right|\left|A_{LO}\right|\cos\left[(\omega_s-\omega_{OL})t+(\theta_s(t)-\theta_{LO}(t))\right]\right.$$

$$=P_s(t)+P_{LO}+2\sqrt{P_s(t)P_{LO}}\cos\left[\omega_{IF}t+(\theta_s(t)-\theta_{LO}(t))\right]$$

$$\omega_{IF}=\omega_s-\omega_{LO}\rightarrow\begin{cases}\text{Heterodyne}\rightarrow\omega_{IF}\neq 0\\ \text{Homodyne}\rightarrow\omega_{IF}=0\end{cases}$$

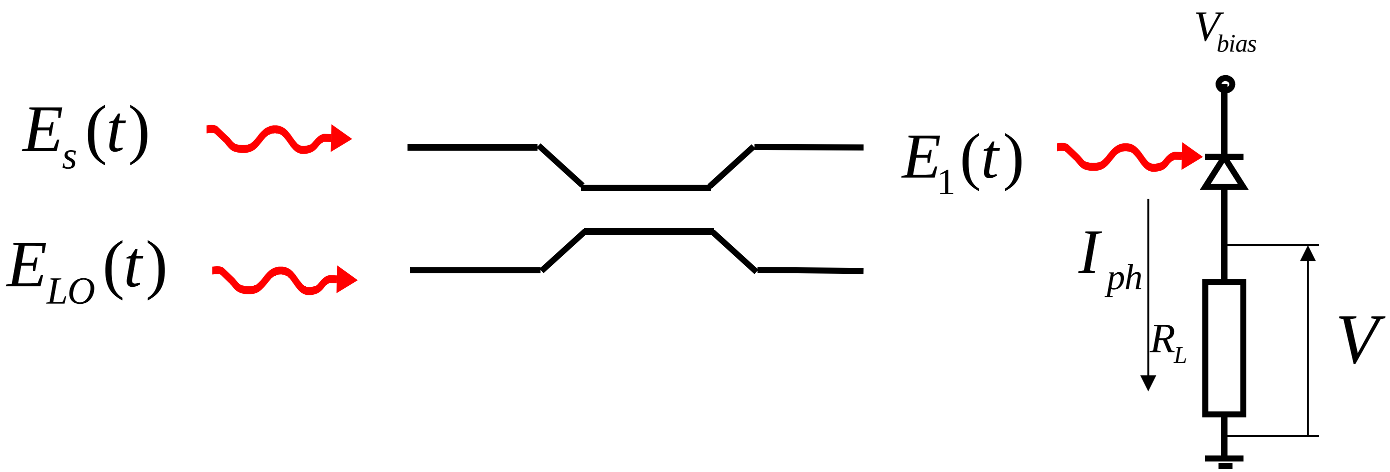
$$\boxed{\Delta\phi(t) = \omega_{IF} t + (\theta_s(t) - \theta_{LO}(t))}$$

$$I(t) = \frac{\Re}{2}\left[P_s(t) + P_{LO} + 2\sqrt{P_s(t) P_{LO}} \cos\left[\omega_{IF} t + (\theta_s(t) - \theta_{LO}(t))\right]\right]$$

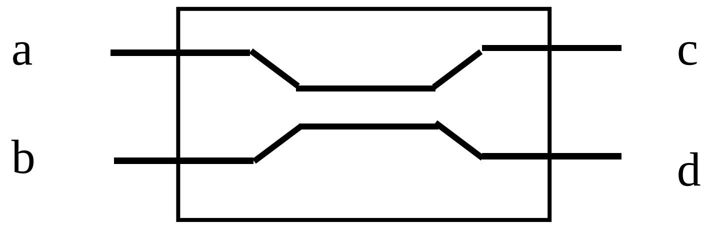
$$= \frac{\Re}{2}\left[P_s(t) + P_{LO} + 2\sqrt{P_s(t) P_{LO}} \cos\left(\Delta\phi(t)\right)\right]$$

$$\begin{cases} \bar{I} = \frac{\Re}{2}\left[P_s(t) + P_{LO} \right] \\ I_{peak} = \frac{\Re}{2}\left[P_s(t) + P_{LO} + 2\sqrt{P_s(t) P_{LO}} \right] \end{cases}$$

5- Coherent Detection (Directional coupler)

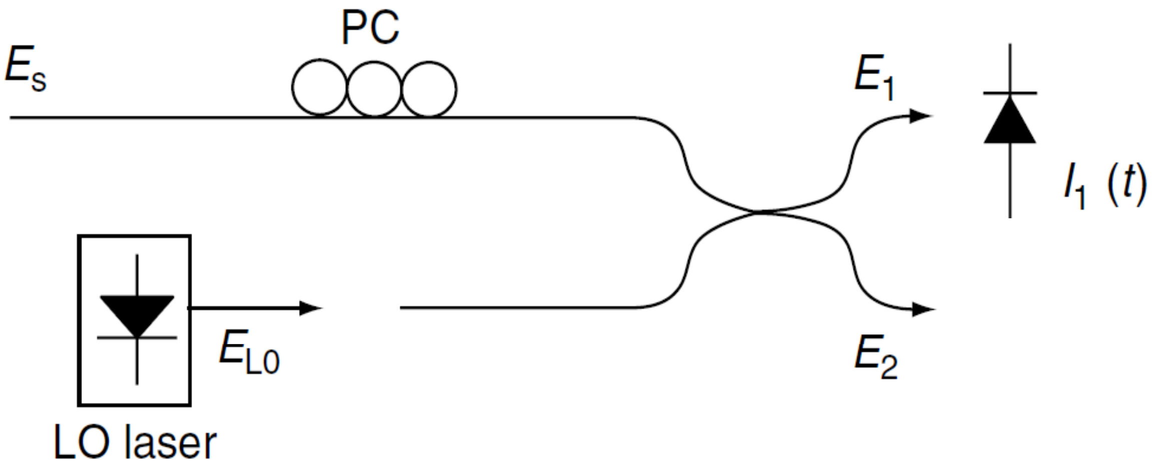


Directional Coupler



$$\begin{pmatrix} E_c \\ E_d \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -j \\ -j & 1 \end{pmatrix} \begin{pmatrix} E_a \\ E_b \end{pmatrix}$$

5-1 Single-Branch Coherent Receiver



$$E_1(t) = \frac{1}{\sqrt{2}}\left(E_s(t) - jE_{LO}(t)\right) = \frac{1}{\sqrt{2}}\left(\left|A_s(t)\right| e^{j(\omega_s t + \theta_s(t))} - j\left|A_{LO}\right| e^{j(\omega_{OL} t + \theta_{LO}(t))}\right)$$

$$P_1(t) = \frac{C}{2}\left(E_1(t)\right)\left(E_1(t)\right)^* =$$

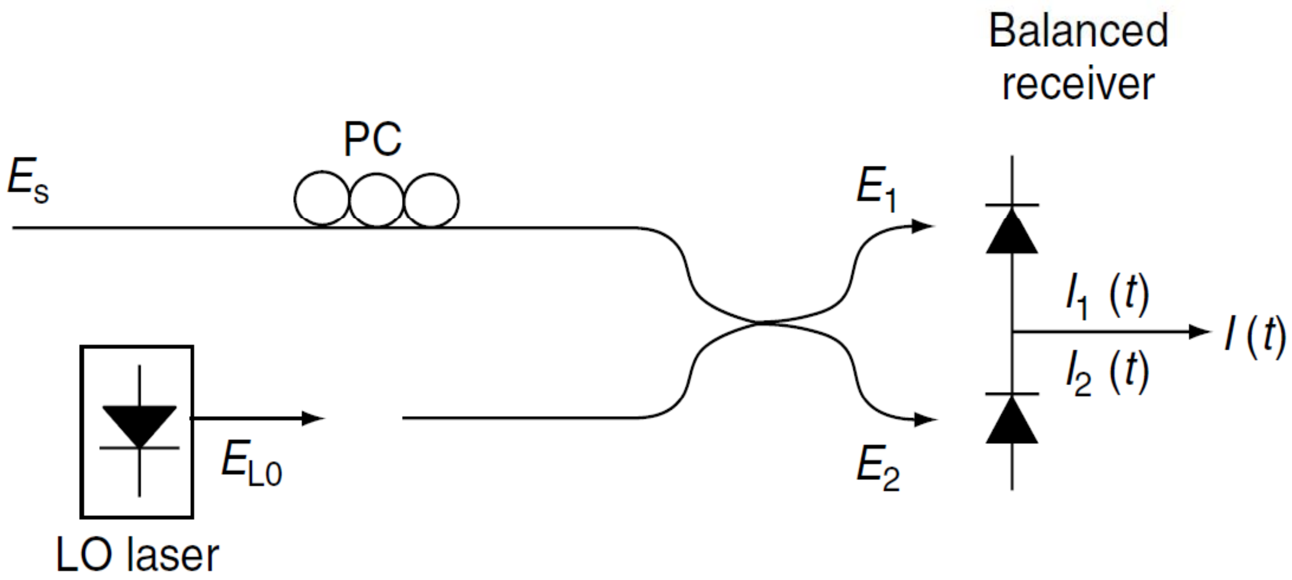
$$= \frac{C}{2}\left(\left|A_s(t)\right|^2 + \left|A_{LO}\right|^2 - 2\left|A_s(t)\right|\left|A_{LO}\right|\sin\left[(\omega_s - \omega_{OL})t + (\theta_s(t) - \theta_{LO}(t))\right]\right.$$

$$= P_s(t) + P_{LO} - 2\sqrt{P_s(t)P_{LO}}\,\sin\left[\omega_{IF} t + (\theta_s(t) - \theta_{LO}(t))\right]$$

$$I_1(t) = \frac{\Re}{2}\left[P_s(t) + P_{LO} - 2\sqrt{P_s(t) P_{LO}} \sin\left(\Delta\phi(t)\right) \right]$$

$$\begin{cases} \overline{I} = \dfrac{\Re}{2}\left[P_s(t) + P_{LO} \right] \\ I_{peak} = \dfrac{\Re}{2}\left[P_s(t) + P_{LO} + 2\sqrt{P_s(t) P_{LO}} \right] \end{cases}$$

5-2 Balanced Coherent Receiver



Removes the dc components and maximizes the beating term

- $E_1(t)$ is the same as the Single-Branch Coherent Receiver

- $E_2(t) = \frac{1}{\sqrt{2}}\left(-jE_s(t) + E_{LO}(t)\right) = \frac{1}{\sqrt{2}}(-j\left|A_s(t)\right|e^{j(\omega_s t+\theta_s(t))} + \left|A_{OL}\right|e^{j(\omega_{LO} t+\theta_{LO}(t))})$

$$P_2(t) = \frac{C}{2}\left(E_2(t)\right)\left(E_2(t)\right)^* =$$

$$= \frac{C}{2}(\left|A_s(t)\right|^2 + \left|A_{LO}\right|^2 + 2\left|A_s(t)\right|\left|A_{LO}\right|\sin\left[(\omega_s - \omega_{LO})t + (\theta_s(t) - \theta_{LO}(t))\right]$$

$$= P_s(t) + P_{LO} + 2\sqrt{P_s(t)P_{LO}}\sin\left[\omega_{IF}t + (\theta_s(t) - \theta_{LO}(t))\right]$$

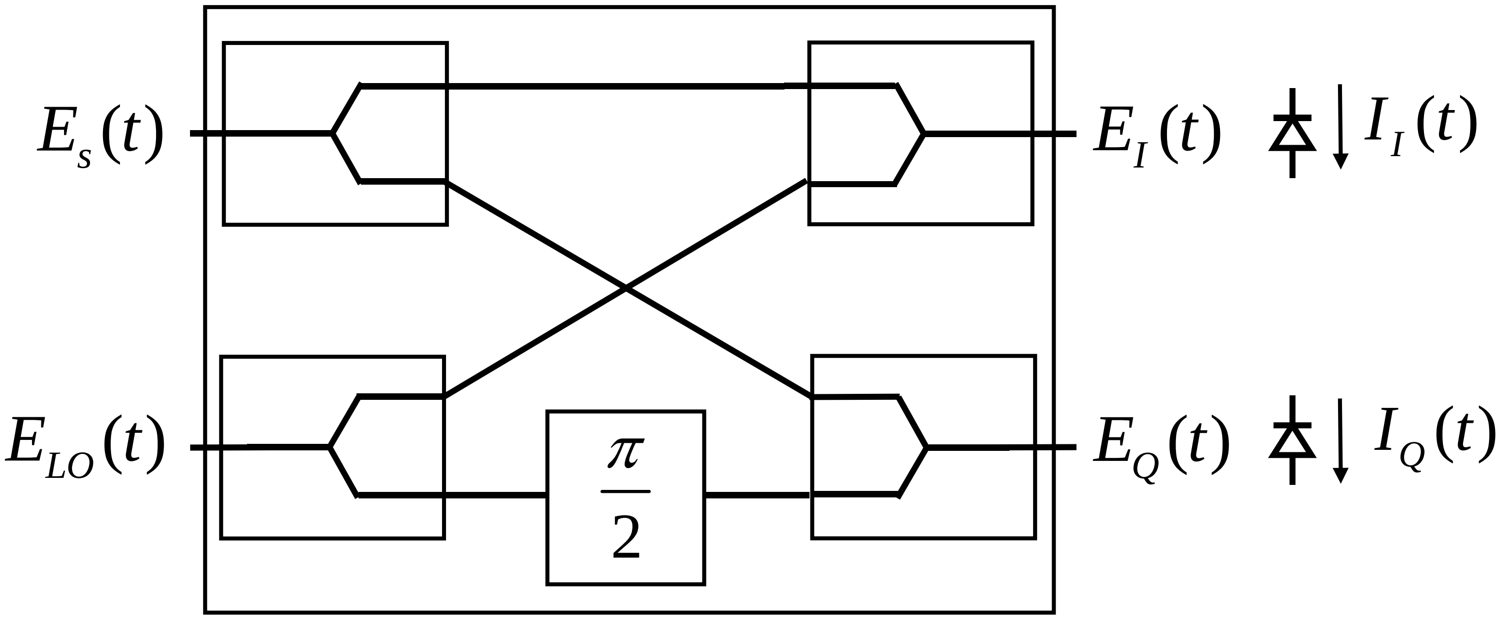
$$I_1(t) = \frac{\Re}{2}\left[P_s(t) + P_{LO} - 2\sqrt{P_s(t)P_{LO}}\,\sin\left(\Delta\phi(t)\right) \right]$$

$$I_2(t) = \frac{\Re}{2}\left[P_s(t) + P_{LO} + 2\sqrt{P_s(t)P_{LO}}\,\sin\left(\Delta\phi(t)\right) \right]$$

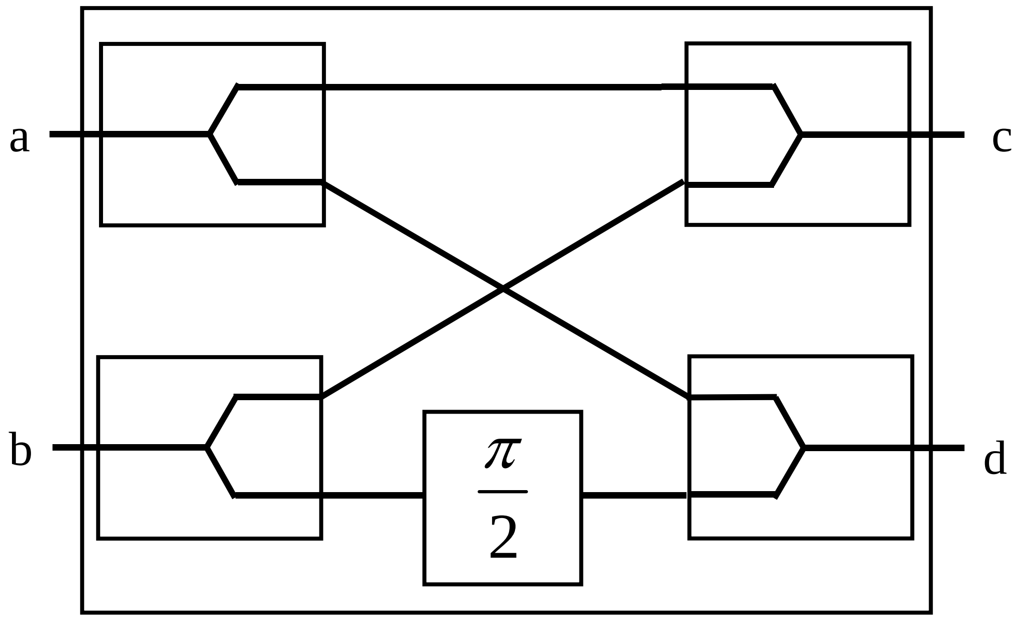
$$I(t) = I_1(t) - I_2(t) = -2\Re\sqrt{P_s(t)P_{LO}}\sin\left(\Delta\phi(t)\right)$$

$$\begin{cases} \overline{I} = 0 \\ I_{peak} = 2\Re\sqrt{P_s(t)P_{LO}} \end{cases}$$

6- Single-Branch IQ Coherent Receiver



2x2 90º Hybrid



$$\begin{pmatrix} E_c \\ E_d \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -j \end{pmatrix} \begin{pmatrix} E_a \\ E_b \end{pmatrix}$$

$$E_I(t) = \frac{1}{\sqrt{2}}\left(E_s(t) + E_{LO}(t)\right)$$

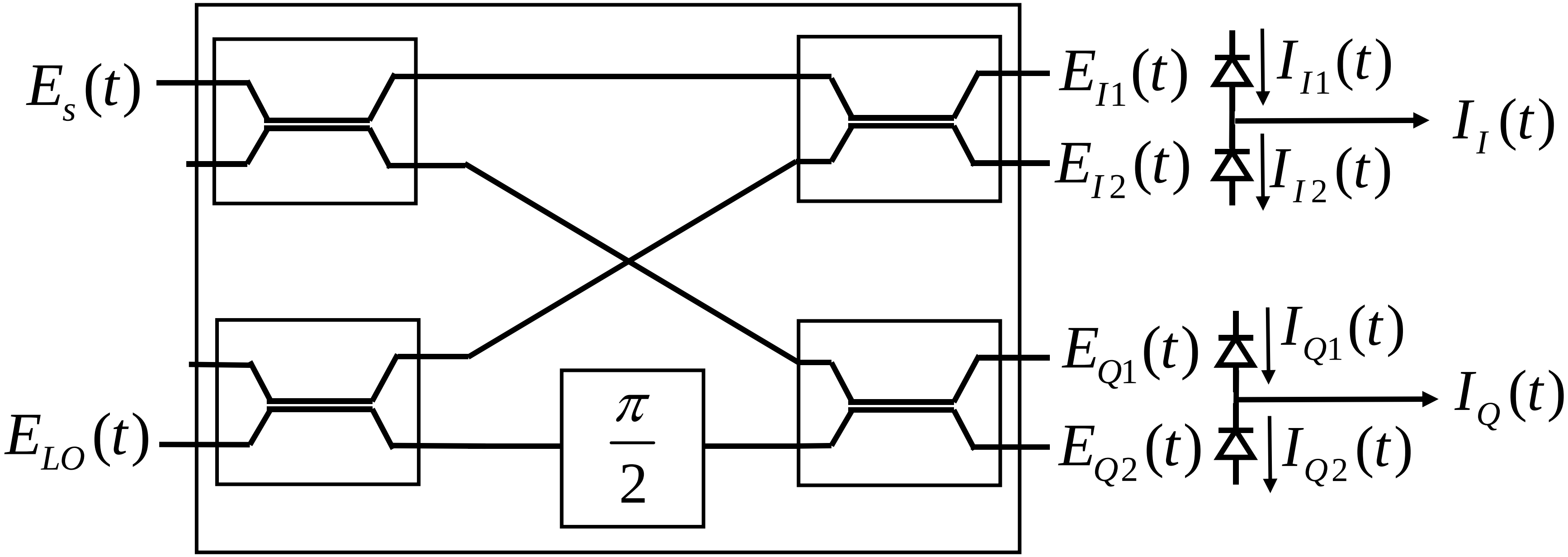
$$I_I(t) = \frac{\Re}{2}\left[P_s(t) + P_{LO} + 2\sqrt{P_s(t)P_{LO}}\cos\left(\Delta\phi(t)\right)\right]$$

$$E_Q(t) = \frac{1}{\sqrt{2}}\left(E_s(t) - jE_{LO}(t)\right)$$

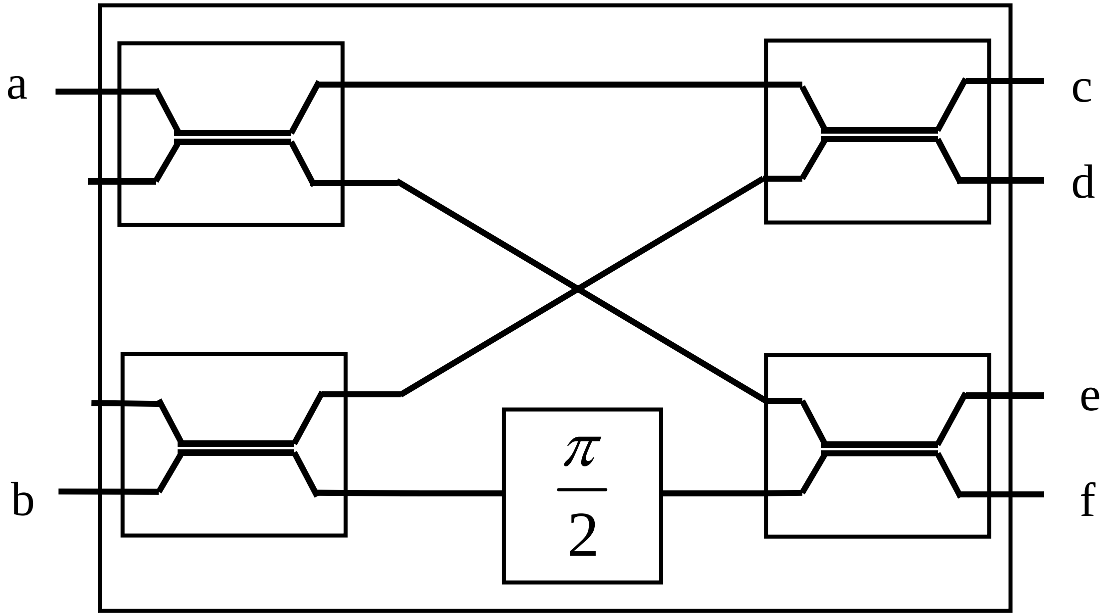
$$I_Q(t) = \frac{\Re}{2}\left[P_s(t) + P_{LO} - 2\sqrt{P_s(t)P_{LO}}\sin\left(\Delta\phi(t)\right)\right]$$

$$\underline{I(t)} = \underbrace{I_I(t) - jI_Q(t)}_{\text{Electronic processing}} = \Re\left[\sqrt{P_{LO}}\,e^{j(\omega_{IF}t-\theta_{LO}(t))}\right]\underline{E_s(t)}$$

7- Balanced IQ Coherent Receiver



2x4 90º Hybrid



$$\begin{pmatrix} E_c \\ E_d \\ E_e \\ E_f \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -j & -j \\ -j & -1 \\ -1 & -j \end{pmatrix} \begin{pmatrix} E_a \\ E_b \end{pmatrix}$$

$$\begin{cases} E_{I1}(t) = \frac{1}{2}\left(E_s(t) - E_{LO}(t)\right) \\ I_{I1}(t) = \frac{\Re}{4}\left[P_s(t) + P_{LO} - 2\sqrt{P_s(t)P_{LO}}\cos\left(\Delta\phi(t)\right)\right] \end{cases}$$

$$\begin{cases} E_{I2}(t) = -j\frac{1}{2}\left(E_s(t) + E_{LO}(t)\right) \\ I_{I2}(t) = \frac{\Re}{4}\left[P_s(t) + P_{LO} + 2\sqrt{P_s(t)P_{LO}}\cos\left(\Delta\phi(t)\right)\right] \end{cases}$$

$$I_{I1}(t) = I_{I1}(t) - I_{I2}(t) = -\frac{\Re}{2}\left[\sqrt{P_s(t)P_{LO}}\cos\left(\Delta\phi(t)\right)\right]$$

$$\begin{cases} E_{Q1}(t) = -j\dfrac{1}{2}\left(E_s(t) - jE_{LO}(t)\right) \\ I_{Q1}(t) = \dfrac{\Re}{4}\left[P_s(t) + P_{LO} - 2\sqrt{P_s(t)P_{LO}}\sin\left(\Delta\phi(t)\right)\right] \end{cases}$$

$$\begin{cases} E_{Q2}(t) = -\dfrac{1}{2}\left(E_s(t) + jE_{LO}(t)\right) \\ I_{Q2}(t) = \dfrac{\Re}{4}\left[P_s(t) + P_{LO} + 2\sqrt{P_s(t)P_{LO}}\sin\left(\Delta\phi(t)\right)\right] \end{cases}$$

$$I_{Q1}(t) = I_{Q1}(t) - I_{Q2}(t) = -\frac{\Re}{2}\left[\sqrt{P_s(t)P_{LO}}\sin\left(\Delta\phi(t)\right)\right]$$

$$\underline{I(t)} = \underbrace{-\left(I_I(t) + jI_Q(t)\right)}_{\text{Electronic processing}} = \frac{\Re}{2}\left[\sqrt{P_{LO}}e^{j(\omega_{IF}t-\theta_{LO}(t))}\right]\underline{E_s(t)}$$