

EXPLORATION OF THE JUPITER PLASMA TORUS WITH A SELF-POWERED ELECTRODYNAMIC TETHER

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The dynamics and power generation of an electrodynamic tether (EDT) placed in the three body system formed by Jupiter, Io and the spacecraft are analyzed. In the region surrounding Io's orbital path, a region of increased electron density called the plasma torus offers a suitable location to operate an EDT. The electrodynamic interaction between the conducting cable of the EDT and the strong magnetic field of the planet leads to non-negligible electrodynamic force, that perturbs the natural three body motion. New equilibrium positions are found in the synodic frame, which coincide with the classical triangular Lagrangian points only when the electrodynamic force vanishes. The locations of equilibrium positions are computed as a function of tether length, width and spacecraft mass. While in this equilibrium position, the tethered system can generate kilowatts of electrical power without de-orbiting the system, the energy coming from the super-rotating plasma sphere of Jupiter. The motion around the new equilibrium positions is evaluated, for both small linear motion confined to a neighborhood of the equilibrium point, and for large amplitude non-linear motions. As an application of this study, a mission profile capable to explore the whole plasma torus is presented. This plasma torus explorer can perform an internal "scan" of the torus itself while generating electrical power useful for loads on board the spacecraft.

INTRODUCTION

The exploration of the Jupiter plasma torus with an electrodynamic tether offers several attractive advantages, both from the scientific and technological point of views. The plasma torus of Jupiter is a structure which exhibits unique features in the whole Solar System; it acts as an outstanding place where to study the interaction between a moon and a plasmasphere. Because of its volcanic activity, Io acts as a source of neutral material that is then ionized and taken to the state of a cold plasma, forming the plasma torus; the interaction between this plasma and the Jupiter's magnetosphere causes several phenomena which are not fully investigated.

An electrodynamic tether placed at Jupiter cannot operate at all locations, because the small environmental electron density implies a small current flowing along the cable. Beside the region close

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to Jupiter, the plasma torus offers a region with an increased electron density N_e , that is a suitable location to operate an EDT. When placed in the plasma torus, the electrodynamic interaction of the EDT with the magnetosphere and the plasmasphere allows to generate kW of electrical power, that can be used on board the spacecraft. The power generation occurs at the expense of the thermal energy of the plasmasphere. An electrodynamic force occurs simultaneously with the power generation process; such a force can be used in a controlled manner to obtain stable orbits around equilibrium positions. Using the electrodynamic force developed by an EDT, new equilibrium positions within the perturbed circular restricted three body problem (CRTBP) can be found. When placed in such equilibrium positions, the EDT is capable of extracting power from the environment –without deorbiting the system. Power generation happens at the expense of the energy of the plasma co-rotating with Jupiter.

Here we will show that the use of an EDT for the exploration of inner jovian system is made feasible by the high-power level that such a system can provide together with the propellantless propulsion provided by the system. High power level means more scientific instrumentation on board of the spacecraft and possibility of high-data-rate link with Earth.

EARLIER STUDIES

Jovian exploration using electrodynamic tethers was proposed during the 80s, as an application of the innovative concept proposed by Hannes Alfvén based on the utilization of the energy contained in the cosmic plasmas for spacecraft propulsion.¹ In 1987 Gabriel, Jones and Garrett² firstly studied the dynamical behavior of an EDT placed at Jupiter, and realized that “*hundreds of kW of power can be generated at distances less than about $5 R_j$ for a 10 km tether*”, and that “*outside $2.24 R_j$ an accelerating force and electric power generation occur simultaneously, the energy being derived from the co-rotating plasma*” . In 1991 Hammond et al.³ proposed a jovian EDT experiment, confirming the capability of the system, but identifying in the hollow cathode plasma contactor the most critical device. Two years later in 1993 Sanmartin et al.⁴ proposed the innovative concept of a *bare* EDT, which is capable to offer several attractive advantages with respect to the standard insulated tethers. In 1998 NASA published the results of a study of an insulated EDT for propulsion and power generation at Jupiter, confirming previous preliminar results: “*The environment of the Jovian system has properties which are particularly favorable for utilization of an electrodynamic tether. Specifically, the planet has a strong magnetic field and the mass of the planet dictates high orbital velocities which, combined with the planet’s rapid rotation rate, can produce very large relative velocities between the magnetic field and the spacecraft*”.⁵ Talley et al.⁶ considered a rotating EDT at Jupiter. The first application of the new concept of using a bare EDT at Jupiter was published in 2005, in which Sanmartin and Lorenzini⁷ studied an entire Jovian tour using an EDT made with a thin conductive tape tens of kilometers long and a few centimeters wide. They demonstrated the feasibility of capturing, lowering and raising apojove/perijove, and reaching escape velocity by using elliptical orbits with periapsis and apoapsis conveniently positioned with respect to Jupiter’s stationary orbit. Their study was subsequently extended in the 2006 Ariadna study for ESA.⁸ Recently Peláez and Scheeres,^{9,10} proposed a permanent tethered observatory at Jupiter, placed in the proximity of one of the first moonlets of Jupiter, and developed an in-depth analysis on the dynamics and the stability of the EDT. The analysis has been developed in a three-body gravitational environment, and they characterized the dynamical behaviour of the EDT in proximity of Lagrangian points of Jupiter-moon system.

Here we study the dynamics of a jovian observatory capable to explore and scan the inner zone of

the jovian system, comprising of Io and its plasma torus. We will show that the EDT observatory can act as a permanent high-power station placed at Jupiter, where it can be used for example as a high data-rate communication link with the Earth to transmit science data from smaller auxiliary probes navigating in the Jovian system. The EDT observatory placed in the plasma torus can explore the inner Jovian system, and the volcanic moon Io. Relevant scientific aspects concerning the physics of the torus and the moon can be investigated; the Jovian plasmasphere is strongly influenced by the mass ejection of Io’s volcanic plumes. The moon acts as a source of about 10^3 kg/s (an estimation of this value can be found, for example, in Hill *et al.*¹¹) of sulphur compounds that are injected in the jovian magnetosphere, where they are subsequently ionized and taken to a state of cold plasma.

In-situ exploration of the plasma torus happened when the first NASA probes executed their fly-by at Jupiter. As Voyager 1 made its swing-by of Io, it passed through and behind the satellite’s plasma torus, obtaining the first *in situ* data of the torus. In 1981 Bagenal¹² made one of the first models, and described the torus morphology in the following manner: “A sharp gradient in plasma temperature of $\approx 7 \times 10^5$ K/ R_j at $5.7 R_j$ divides the torus into two parts, a cold inner region, where the ions are closely confined to the centrifugal equator, and a warm outer region, which includes the orbit of Io and has a thickness scale height of $1 R_j$. The outer edge of the warm torus is defined by a drop in plasma density near $7.5 R_j$ ”. Several models of the electron distribution of the torus of Io have been developed (Warwick *et al.* 1979; Birmingham *et al.* 1981; Bagenal *et al.*, 1981), and were calibrated using the Doppler signature received in the signals of *Voyager-1*, at NASA/JPL stations. The standard reference model of the charged particles environment at Jupiter was published in 1983 by Divine and Garrett.¹³ An accurate description of the plasma torus structure, the Io’s plasma interaction, and its dynamical interaction with the Jovian magnetosphere can be found in the monograph of Bagenal *et al.*¹⁴

DYNAMIC MODEL OF THE EDT

Assumptions

The motion of a spacecraft inside the Plasma Torus is influenced by two main attractors, Jupiter and Io. Furthermore, all the Lagrangian points of the Jupiter-Io-spacecraft system are contained inside the torus. The dynamical scheme of the circular restricted three body problem has been selected to describe the motion, with appropriate perturbation to take into account the electrodynamic interaction of the EDT with the environment.

The tether has been assumed to be a rigid dumbbell of variable length L , with two lumped masses m_{T1} and m_{T2} at the tips. The total mass of the spacecraft is $m = m_{T1} + m_{T2}$. The length L can be time-varying according to a control-law $L(t)$. Its inertia tensor $[\mathbf{I}_C]$ when expressed in the body-frame is a diagonal matrix with a zero component:

$$[\mathbf{I}_C] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I_s & 0 \\ 0 & 0 & I_s \end{bmatrix} \quad (1)$$

where the first component vanishes because the EDT is assumed to be a long one-dimensional structure. The principal inertia moment I_s is a function of the total length L and of the reduce mass: $I_s = \frac{m_{T1}m_{T2}}{m_{T1}+m_{T2}}L^2 = m_r L^2$.

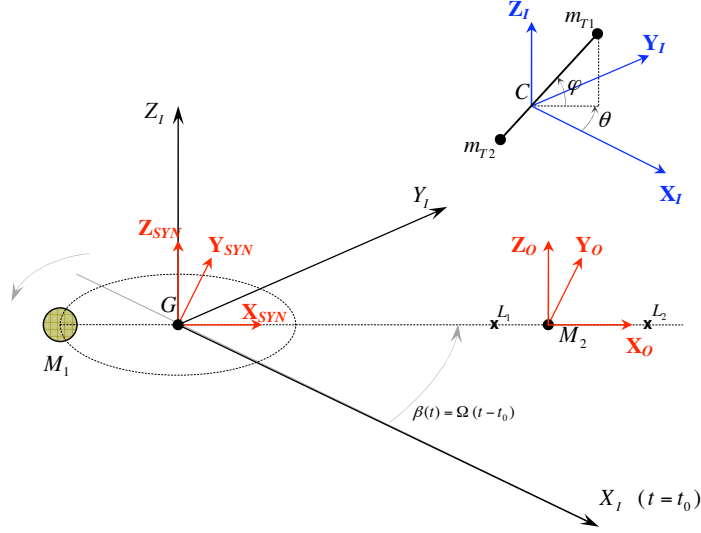


Figure 1. Reference Frames.

Equations of motion

Figure 1 shows the reference frames used for the analysis. The reference frame of navigation (synodic frame) is attached to the center of mass G of the planetary system, with X-axis pointing from Jupiter to Io and corotating with it, (XY)-plane coincident with the plane of revolution of Io around Jupiter, and Z-axis to complete a right-handed cartesian frame. Within this non-inertial frames, the resulting Newton equations are the classical CRTBP equations plus the perturbation due to the EDT electrodynamic force. For the attitude motion of the tether we have used the two angles θ, ϕ (in-plane, out-of-plane), describing the orientation of the dumbbell with respect to the inertial frame (X_I, Y_I, Z_I) . The resulting equation of motion (navigation and attitude) of the tether in the limiting case when the ratio (L/R) is small (ratio between tether length L and distance R from the gravitational attractor) are:

$$\ddot{\vec{R}} + 2\vec{\Omega} \times \dot{\vec{R}} + \vec{\Omega} \times (\vec{\Omega} \times \vec{R}) = \vec{f}_{gr1} + \vec{f}_{gr2} + \vec{f}_{el} \quad (2)$$

$$[\dot{I}_C]\vec{\omega} + [I_C]\dot{\vec{\omega}} + \vec{\omega} \times [I_C]\vec{\omega} = \vec{M}_{gr1} + \vec{M}_{gr2} + \vec{M}_{el} \quad (3)$$

where \vec{R} is the position of the center of mass of the EDT, $\vec{\Omega} = (0, 0, \Omega)$ is the angular velocity of revolution of the two primaries, $\Omega = \sqrt{G(M_1 + M_2)/d^3}$ is evaluated from the third Kepler law, $\vec{\omega}$ is the angular velocity of the body, d the distance Jupiter-Io, $[I_C]$ the inertia tensor of the body, \vec{f}_{gr} the specific gravitational force (the subscript 1 stands for Jupiter, 2 for Io), and \vec{f}_{el} the specific electrodynamic force due to the interaction of the EDT with the environment. Gravitational forces of Jupiter \vec{f}_{gr1} and Io \vec{f}_{gr2} have been modeled using only the first harmonic of the gravitational field, leading to: $\vec{f}_{gr} = -(\mu/R^3)\vec{R}$. Second-order effects due to deviation from sphericity (*i.e.*: Jupiter flattening, non-sphericity of Io, *etc.*) have been neglected. Gravitational torques are given by the classical first-order formulation: $\vec{M}_{gr} = 3\mu/R^5\vec{R} \times [I_C]\vec{R}$. Electrodynamic torques \vec{M}_{el} have been neglected for simplicity. The resulting motion equations of the EDT are:

$$\ddot{\vec{R}} + 2\vec{\Omega} \times \dot{\vec{R}} + \vec{\Omega} \times \vec{\Omega} \times \vec{R} = -\frac{\mu_1}{R_1^3} \vec{R}_1 - \frac{\mu_2}{R_2^3} \vec{R}_2 + \vec{f}_{el} \quad (4)$$

$$[I_C] \dot{\vec{\omega}} + [I_C] \vec{\omega} + \vec{\omega} \times [I_C] \vec{\omega} = \frac{3\mu_1}{R_1^5} \vec{R}_1 \times [I_C] \vec{R}_1 + \frac{3\mu_2}{R_2^5} \vec{R}_2 \times [I_C] \vec{R}_2 \quad (5)$$

where $\vec{R}_1 = \vec{R} - \vec{d}_1 = (x + d\nu_2, y, z)$ is the distance vector of the EDT from Jupiter and $\vec{R}_2 = \vec{R} - \vec{d}_2 = (x - d\nu_1, y, z)$ is the distance vector from Io, $\nu = \nu_2 = M_2/(M_1 + M_2)$ and $\nu_1 = M_1/(M_1 + M_2)$ are the mass parameters, d_1 and d_2 the distances of the two primaries from the barycenter G measured along the Jupiter-Io line. Eq. (4) and Eq. (5) constitute a system of 5 independent ODEs in the 5 unknowns of position vector (3 unknowns) and orientation angles (2 unknowns). Because of their strong non-linear nature, numerical integration with an Adams-Bashforth numerical integrator has been used to solve the equations.

Electrodynamic force

The specific force \vec{f}_{el} [N/kg] takes into account the electrodynamical interaction of the EDT with the planetary magnetic field and the environmental electron density. The interaction between the tether and the environment is driven by the presence of a motional electric field $\vec{E} = \vec{v}_{rel} \times \vec{B}$, which is due to the relative motion with velocity \vec{v}_{rel} of the body with respect to the planetary magnetic field \vec{B} . The motional electric field \vec{E} drives a current i along the conductive cable of the tether (after assuming the presence of plasma contactors at the tips), which in turns is responsible for the electrodynamic force \vec{F}_{el} [N]:

$$\vec{F}_{el} = \int_0^L i(l) d\hat{l} \times \vec{B} = I_{avg} L (\hat{u} \times \hat{B}) \quad (6)$$

The current depends of the capability of the electrodynamic tether to collect electrons from the plasma sphere at the anodic end, and to reject them at the cathodic end. Electron collection occurs in the OML regime for a bare tether, because the tether width is smaller than the Debye length λ_D and Larmor radius r_L . In fact, assuming the electron population has a temperature $36 < kT_e < 360$ [eV], the Debye length at the level of Io's orbit results: $0.99 < \lambda_D < 3.2$ [m], and the Larmor radius $9.6 < r_L < 30.5$ [m] (with magnetic field $B = 2 \times 10^{-6}$ [T], and electron density $N_e = 2 \times 10^9$ [m⁻³]). For a bare tether with small ohmic resistance, the length-averaged current is given by the following formula, valid for the OML regime:⁴

$$I_{avg} = \frac{2}{5} \left(\frac{2wL}{\pi} \right) q_e N_e \sqrt{\frac{2q_e E_t L}{m_e}} \quad (7)$$

where w and L are tether width and length, q_e and m_e the electron charge and mass, N_e is the environmental electron density, and $E_t = \hat{u} \cdot \vec{E}$ is the component of the motional electric field \vec{E} along the tether. For the evaluation of the motional electric field \vec{E} the knowledge of the local magnetic field \vec{B} is required. The local magnetic field is given by the superimposition of three contributions: Jupiter's field, Io's field and other external fields (*i.e.*: interplanetary magnetic field, *etc.*). Since the most important contribution is due to Jupiter, a simple dipolar model is a good approximation for the inner region ($r < 20R_j$). Furthermore, "*Io does not possess an appreciable*

internal magnetic field”, was stated by Khurana et al. from the analysis of *Galileo* fly-by near the volcanic moon.¹⁵ A classical dipolar model has been used to describe the Jupiter magnetic field:

$$\vec{B}(\vec{r}) = \frac{m}{r^3} [3(\hat{m} \cdot \hat{r})\hat{r} - \hat{m}] \quad (8)$$

where $\vec{m} = \mu_m R_j^3 \hat{m}$ is the magnetic dipole moment vector of the planet, \hat{m} is its unit vector, μ_m is the intensity of the dipole [Tesla], R_j is the planet equatorial radius, \vec{r} is the position vector of the spacecraft in the magnetic reference frame.

The electron density N_e has been evaluated using the Divine and Garrett model.¹³ The model provides the densities of protons, electrons and six positive ion species. Here we only consider the electron density N_e , because it is the plasma parameter affecting the EDT behavior. The model allows to obtain the electron density as a function of the position vector of the spacecraft; Figure 2 shows the electron density profile as a function of the radial distance from Jupiter. The locations of Io and of the four moonlets is evidenced with vertical lines in the figure. The presence of the plasma torus can be seen from the electron density increase around Io location ($\approx 5.9R_j$).

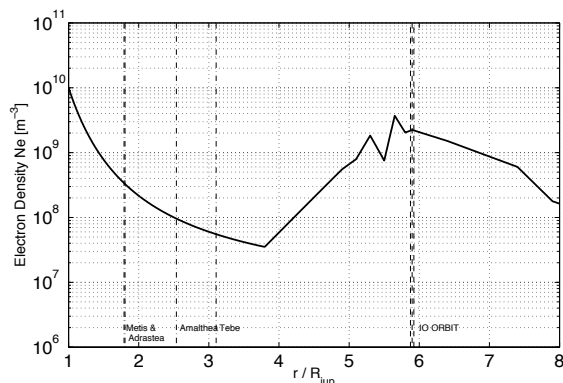


Figure 2. Electron density at Jupiter with Divine and Garrett model.

The Jupiter magnetic field rotates with the planet at the same angular velocity, with velocity $v_B = \Omega_j R$, where Ω_j is the Jupiter angular rotation rate and R is the distance in the equatorial plane. Due to the high rotation rate of Jupiter, v_B results to be of the order of ≈ 74 km/s at $5.9 R_j$ (Io’s orbit). As a consequence a spacecraft co-orbiting with Io ($v_s \approx 17.3$ km/s) has a relative velocity of $v_{rel} \approx 57$ km/s, and a consequent motional electric field $E \approx 0.1$ V/m (with a local magnetic field $B \approx 2 \times 10^{-6}$ T).

Adimensionalization

In order to reduce the number of parameters involved in the equations of motion it is convenient to switch to non-dimensional equations. Length, mass and time have been respectively adimensionalized with: the distance between primaries $d = 1$, the total mass of the planetary system $M_1 + M_2 = 1$, and the inverse of the mean angular velocity of the primaries $1/\Omega = 1$. The resulting equations of motion are:

$$\ddot{\xi} - 2\dot{\eta} = \xi - \frac{\nu_1}{\rho_1^3}(\xi + \nu_2) - \frac{\nu_2}{\rho_2^3}(\xi - \nu_1) + \frac{F_{el,x}}{md\Omega^2} \quad (9)$$

$$\ddot{\eta} + 2\dot{\xi} = \eta - \frac{\nu_1}{\rho_1^3}\eta - \frac{\nu_2}{\rho_2^3}\eta + \frac{F_{el,y}}{md\Omega^2} \quad (10)$$

$$\ddot{\zeta} = -\frac{\nu_1}{\rho_1^3}\zeta - \frac{\nu_2}{\rho_2^3}\zeta + \frac{F_{el,z}}{md\Omega^2} \quad (11)$$

$$\ddot{\theta} + 2\frac{\dot{L}}{L}\dot{\theta} - 2\dot{\theta}\dot{\varphi}\tan\varphi = \frac{1}{\cos\varphi} \left[\frac{3\nu_1}{\rho_1^5}(\vec{\rho}_1 \cdot \hat{u})(\vec{\rho}_1 \cdot \hat{v}) + \frac{3\nu_2}{\rho_2^5}(\vec{\rho}_2 \cdot \hat{u})(\vec{\rho}_2 \cdot \hat{v}) \right] \quad (12)$$

$$\ddot{\varphi} + 2\frac{\dot{L}}{L}\dot{\varphi} + \dot{\theta}^2 \cos\varphi \sin\varphi = -\frac{3\nu_1}{\rho_1^5}(\vec{\rho}_1 \cdot \hat{u})(\vec{\rho}_1 \cdot \hat{w}) - \frac{3\nu_2}{\rho_2^5}(\vec{\rho}_2 \cdot \hat{u})(\vec{\rho}_2 \cdot \hat{w}) \quad (13)$$

where (ξ, η, ζ) are the adimensional coordinates of orbital position, $\nu_1 = 1 - \nu_2 = \frac{M_1}{M_1+M_2}$ and $\nu_2 = \frac{M_2}{M_1+M_2}$ are the mass parameters of Jupiter and Io, and:

$$\rho_1 = \sqrt{(\xi + \nu_2)^2 + \eta^2 + \zeta^2} \quad (14)$$

$$\rho_2 = \sqrt{(\xi - \nu_1)^2 + \eta^2 + \zeta^2} \quad (15)$$

are the adimensional distances of the center of mass of the spacecraft from the two primaries.

EQUILIBRIUM POSITIONS IN THE COROTATING FRAME

When the electrodynamic tether is active (*i.e.* when the current is flowing along the cable), new equilibrium positions appear in the corotating synodic frame, this fixed to the rotation of Io around Jupiter. These positions are the equilibrium solutions ($\vec{R} = \vec{R} = \vec{0}$, and $[\dot{I}_C] = \vec{\omega} = \vec{0}$) of Eq. (2) and Eq. (3):

$$\vec{\Omega} \times (\vec{\Omega} \times \vec{R}) = -\frac{\mu_1}{R_1^3}\vec{R}_1 - \frac{\mu_2}{R_2^3}\vec{R}_2 + \vec{f}_{el} \quad (16)$$

$$\vec{\omega} \times [I_C]\vec{\omega} = \frac{3\mu_1}{R_1^5}\vec{R}_1 \times [I_C]\vec{R}_1 + \frac{3\mu_2}{R_2^5}\vec{R}_2 \times [I_C]\vec{R}_2 \quad (17)$$

The equilibrium positions in the synodic frame are given by the zeros of Eq. (16);. When written in adimensional form, ($F_{el}/(md\Omega^2) = f_{el}^*$) the equilibrium electrodynamic force components are:

$$f_{el,\xi}^* = -\xi + \frac{\nu_1}{\rho_1^3}(\xi + \nu_2) + \frac{\nu_2}{\rho_2^3}(\xi - \nu_1) \quad (18)$$

$$f_{el,\eta}^* = -\eta + \frac{\nu_1}{\rho_1^3}\eta + \frac{\nu_2}{\rho_2^3}\eta \quad (19)$$

$$f_{el,\zeta}^* = \frac{\nu_1}{\rho_1^3}\zeta + \frac{\nu_2}{\rho_2^3}\zeta \quad (20)$$

The location of new equilibrium positions vary as a function of the perturbing electrodynamic force \vec{f}_{el}^* ; these positions coincide with the classical five Lagrangian points when \vec{f}_{el}^* vanishes.

Equilibrium positions along Io's orbital path

We are interested in obtaining expressions for the equilibrium positions that lies inside the plane of revolution of the two primaries, and in particular positions along the orbital path of the secondary (Io). It is thus convenient to express the position vector in appropriate polar coordinates:

$$\begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix} = r_a \begin{bmatrix} \cos \alpha \\ \sin \alpha \\ 0 \end{bmatrix} + \begin{bmatrix} -\nu_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_a \cos \alpha - \nu_2 \\ r_a \sin \alpha \\ 0 \end{bmatrix} \quad (21)$$

where $r_a = r/d$ is the adimensional radial position measured from Jupiter and α is the *phase angle*. The distance from the primaries take the form: $\rho_1 = r_a$, and $\rho_2 = \sqrt{r_a^2 + 1 - 2r_a \cos \alpha}$. Figure 3 shows the geometry involved in the analysis.

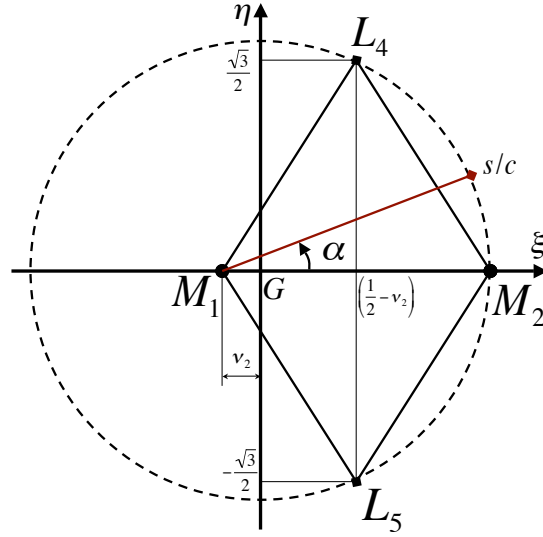


Figure 3. Geometry of equilibrium positions analysis.

Substituting Eq. (21) in Eq. (18)-(20) and imposing the condition $r_a = 1$ (positions on the orbital path of Io), the equation simplifies to the following form:

$$f_{el,\xi}^* = \nu \left(\frac{1}{\rho_2^3} - 1 \right) (\cos \alpha - 1) \quad (22)$$

$$f_{el,\eta}^* = \nu \left(\frac{1}{\rho_2^3} - 1 \right) \sin \alpha \quad (23)$$

$$f_{el,\zeta}^* = 0 \quad (24)$$

where $\nu = M_2/(M_1 + M_2)$, $\rho_2 = \sqrt{2 - 2 \cos \alpha}$ (and $\rho_1 = 1$). Eqs. (22)-(24) define the electrodynamic force that the tether (or any other thrust-device) must supply to remain stationary with respect to the synodic reference frame at a given angular position α when the tether is located in the orbital path of the secondary ($r_a = 1$). As expected from the model, Eq. (24) shows that no

out-of-plane force components must occur for equilibrium positions within the plane of revolution of the two primaries. In dimensional units the magnitude of the force F_{el} [N] can easily be computed as:

$$F_{el} = md\Omega^2 \sqrt{(f_{el,\xi}^*)^2 + (f_{el,\eta}^*)^2 + (f_{el,\zeta}^*)^2} = md\Omega^2 \nu \rho_2 \left| \frac{1}{\rho_2^3} - 1 \right| \quad (25)$$

For a given angular location α along the orbital path of the secondary, the external specific force required to maintain the spacecraft in equilibrium with the synodic frame is a function of the mass parameter ν and the distance between the two primaries.

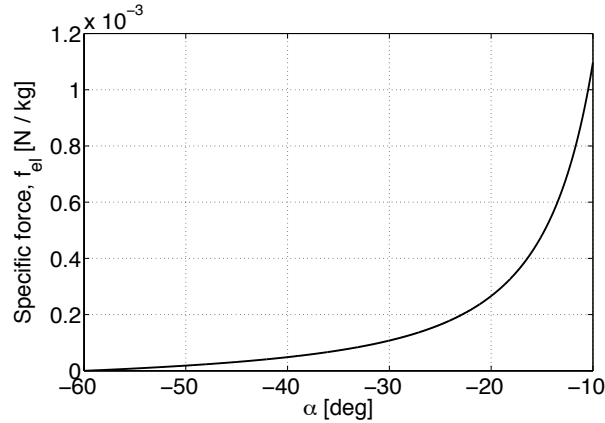


Figure 4. Specific force required for equilibrium in the Jupiter-Io corotating frame.

Figure 4 shows the specific force required for equilibrium in the Jupiter-Io corotating frame. No external force is required to maintain an object at $|\alpha| = 60$ deg, that is at a triangular Lagrangian point because of stable nature of those points.

Influence of tether size on locations of equilibrium positions

Eq. (22)-(24) have been used to obtain the size of an electrodynamic tether capable to provide the electrodynamic force necessary to maintain the spacecraft in equilibrium in the corotating frame. The angle identifying the equilibrium position α_{eq} has been numerically determined for a set of cases as a function of size parameters L, w, m , that are the tether length, tether width and spacecraft mass, respectively.

As expected, it was found that the EDT parameters influence the magnitude of the electrodynamic force and, consequently, the location of the equilibrium point. A set of numerical simulations was carried out with several values of mass, length and width, in order to identify the equilibrium position for each configuration. Figure 5 shows the results of a subset of these simulations, using a 5 cm width tether. Spacecraft masses from 200 to 1200 kg have been considered, and four tether lengths equal to $L=10,20,30,40$ km. The figure depicts the equilibrium position, defined by the angle α from the Jupiter-Io line vs. the spacecraft mass, for different values of the tether length L . Table 1 shows the numerical values of the graph. All positions lie on the Io's orbital path.

Figure 6 shows simulated results for an electrodynamic tether in equilibrium in the co-rotating frame of Jupiter and Io. The position of the spacecraft is marked by the arrow-tail sign; Jupiter and

Table 1. Equilibrium positions α_{eq} [deg] for Width = 5 cm.

α_{eq} [deg], Width = 5 cm				
m [kg]	L = 10 km	L = 20 km	L = 30 km	L = 40 km
200	-34.36	-16.69	-10.14	-7.51
400	-41.85	-22.77	-14.37	-10.05
600	-43.96	-27.03	-17.41	-12.31
800	-48.65	-29.99	-19.93	-14.19
1000	-50.38	-32.56	-21.82	-15.85
1200	-51.63	-34.61	-23.52	-17.21

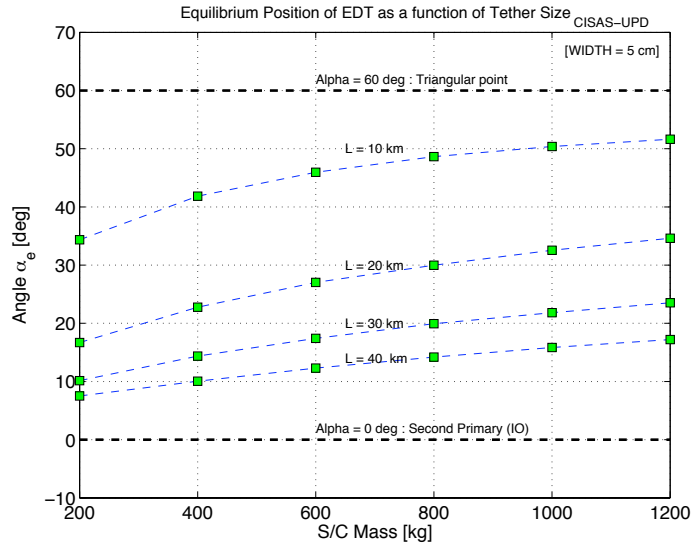


Figure 5 Equilibrium angle α_{eq} at L_5 as a function of spacecraft mass, for 4 values of tether length $L = 10, 20, 30, 40$ km, and for a bare tape tether of width $w = 5$ cm. The angle α_{eq} is the angle from the Jupiter-Io line.

Io are to scale, and their triangular points have also been evidenced. The data for the simulation are: tether length $L = 20$ km, width $w = 5$ cm, spacecraft mass $m = 600$ kg; simulation time $t = 40T_{rev} \approx 70.76$ days.

From the simulations summarized in Figure 5 the following considerations can be derived:

1. A “short” tether is sufficient to stay in equilibrium near the triangular points;
2. The more the position moves away from triangular point in the direction of the second primary (Io), the longer the tether required or the smaller the satellite mass; this implies that a greater electrodynamic force must be exerted to stay near Io;
3. For example, a tether with fixed length $L=10$ km, and the suitable spacecraft mass can allow equilibrium position within the range $\alpha = 34 - 52$ degrees;
4. Using a controlled variable-length tether and a constant-mass spacecraft, the equilibrium positions can be moved along Io’s orbital path.

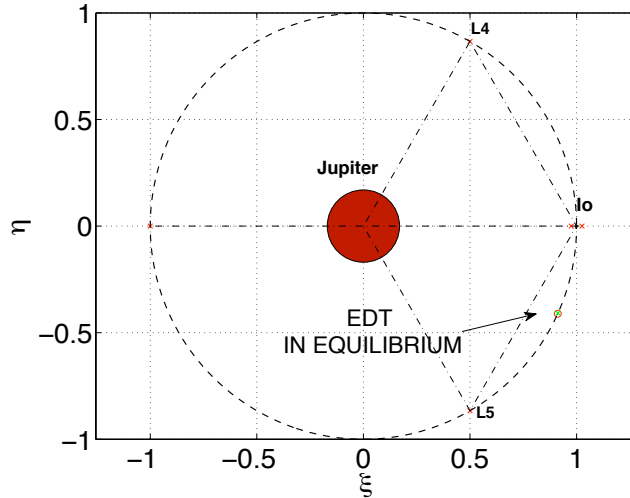


Figure 6 Electrodynamic tether in equilibrium in the corotating frame of Jupiter and Io. The position of the spacecraft is marked with the arrow-tail sign; Jupiter and Io are to scale, and their triangular points have also been evidenced. (Data: length $L = 20$ km, width $w = 5$ cm, spacecraft mass $m = 600$ kg; simulation time $t = 40T_{rev} \approx 70.76$ days.)

The new equilibrium positions coincide with the classical triangular points at $\alpha = \pm 60$ deg only when the electrodynamic force is equal to zero.

Power Generated at Equilibrium

Electrical power P is extracted at the expense of the corotating plasma. The mechanical ideal power is the scalar product of the electrodynamic force and the velocity relative to the plasma, $P_{id} = \vec{F}_{el} \cdot \vec{v}_{rel}$. The useful power is a fraction of P_{id} and is equal to $P_u \approx 0.31P_{id}$ for a vertical librating tether, and reduces to $P_u \approx 0.22P_{id}$ for a rotating tether. Power scales with a power law of the length, $P \propto L^{5/2}$, meaning that longer tethers produce much more electrical power. Tables 2 and 3 report the ideal and useful power obtained from numerical simulations. All simulations considered a vertical tether librating around its natural attitude equilibrium position; the natural attitude positions have been found by Robinson,¹⁶ who showed that a dumbbell placed at triangular Lagrangian points orients itself toward the center of mass of the planetary system. The values of ideal power are shown in Fig. 7. The figure shows the scaling of power with the length. An ideal power of about 5 kW can be extracted from the plasmasphere by using a 20 km-length tether. The useful power P_u results to be reduced due to the non-ideal electron collection. As can be seen from Table 3, the useful power that can be extracted with an EDT of moderate length between (10 and 20 km), is in the range of 300 - 1700 W. Higher power levels can be easily obtained increasing the length of the tether.

MOTION AROUND EQUILIBRIUM POSITIONS

The new equilibrium points found within the perturbed three-body problem are stable points, and stable orbits can be obtained around them. In this section we show some examples of motion around equilibrium positions, which are useful to understand the behavior of the orbital motion of the EDT in the synodic frame. We are here interested in the position of the EDT center of mass as a function of time, when the system is placed in proximity of its equilibrium position. Three sets

Table 2. Ideal Power.

P_{id} [W], Width = 5 cm				
m [kg]	L = 10 km	L = 20 km	L = 30 km	L = 40 km
200	1019	4979	12908	25497
400	1059	5195	13090	26728
600	1071	5491	13629	26295
800	1083	5543	13946	26894
1000	1087	5648	14184	27096
1200	1090	5723	14449	27922

Table 3. Useful Power.

P_u [W], Width = 5 cm				
m [kg]	L = 10 km	L = 20 km	L = 30 km	L = 40 km
200	316	1543	4001	7904
400	328	1611	4058	8286
600	332	1702	4225	8151
800	335	1718	4323	8337
1000	337	1751	4397	8400
1200	338	1774	4479	8656

of simulations have been considered to summarize the behavior: (1) EDT placed with the center of mass coincident with the equilibrium point; (2) EDT placed with a *small* deviation of angle α with respect to the equilibrium point; and (3) an EDT placed with a *large* deviation of the angle α with respect to the equilibrium point. In the following three subsections we describe each simulation to outline the main features of the dynamics.

Center of mass coincident or very closely to the equilibrium point

We adopted the following EDT configuration: a tape tether of moderate size with $L = 10$ km, width $w = 5$ cm, and a total spacecraft mass $m = 600$ kg. Using the graph of Figure 5 (or equivalently Table 1), we obtain the equilibrium angle $\alpha_{eq} = -43.96$ deg, that identifies the location of equilibrium point within the orbital path of Io. When we simulate the motion of the tether starting at α_{eq} , the system stays at the equilibrium position for the entire simulation time, confirming the stable nature of the equilibrium point.

When the initial orbital position is displaced by a small quantity δ from the equilibrium point ($\delta/d < 10^{-3}$, where d is the distance between the primaries), the motion exhibits a behavior very similar to the classical linear motion at triangular points. The motion is characterized by two eigenfrequencies, a long-time frequency and a short-time one, with different amplitudes. To a first approximation, these two eigenfrequencies can be estimated using the classical Szebehely formulas for the eigenfrequencies at triangular points.¹⁷

Longer tethers have their equilibrium position at locations closer to the second primary. For example, a 20-km tether ($w = 5$ cm, $m_{S/C} = 600$ kg) has its position at $\alpha_{eq} = -27.03$ deg, and

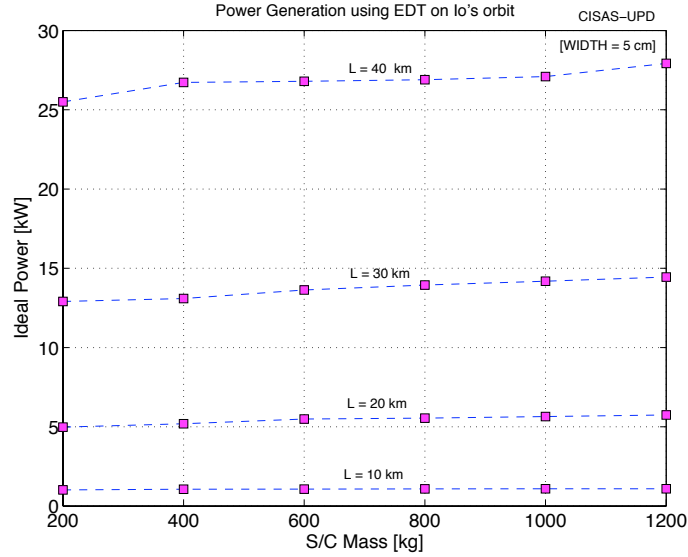


Figure 7 Ideal electrical Power generated using a bare EDT with tape section of width = 5 cm, as a function of tether mass (on abscissa axis) and tether length (the four parametric curves are for L=10,20,30,40 km). Useful power is approx $0.31P_{id}$ for a librating tether, and $0.22P_{id}$ for a rotating tether.

the power generated is $P_{id} = 5888$ W and $P_{ut} = 1825$ W.

Small oscillations

For small initial deviations of the angle α from the equilibrium point, small oscillations around the point occur. The center of mass of the EDT spacecraft oscillates around its equilibrium point, as can be seen in Figure 9. The same tether as before (length = 10 km, width = 5 cm and spacecraft mass = 600 kg) was placed at an initial angle displaced by $\Delta\alpha = 10$ degrees with respect to the equilibrium position $\alpha_{eq} = -43.96$ deg. The starting point is marked with a red circle, and the final point with a green x. The trajectory oscillates around a center placed exactly at the equilibrium point $\alpha = \alpha_{eq}$.

Large oscillations

When the EDT is placed at a large initial angle displacement $\Delta\alpha$ with respect to the equilibrium point, the type of motion becomes similar to the well known zero-velocity orbits around the triangular points of the CRTBP.¹⁷ In fact, when the amplitude of the angular deviation is increased, an interesting horseshoe-shaped orbit takes place, and non-linear effects became evident. Figure 10 depicts a very large amplitude orbit. The same tether as before (length = 10 km, width = 5 cm and spacecraft mass = 600 kg) was initially placed at $\alpha_0 = -8.96$ deg ($\Delta\alpha = 35$ deg). The trajectory is a modified horseshoe orbit. Classical horseshoes are symmetrical with respect the Jupiter-Io line (ξ -axis), while the new orbit is rotated by an angle $\Delta\alpha$ counterclockwise.

Out-of-plane orbital motion

If the trajectory has an out-of-plane component, it oscillates harmonically around the plane of revolution of the two primaries, due to the symmetry of the system. Figure 11 shows an example

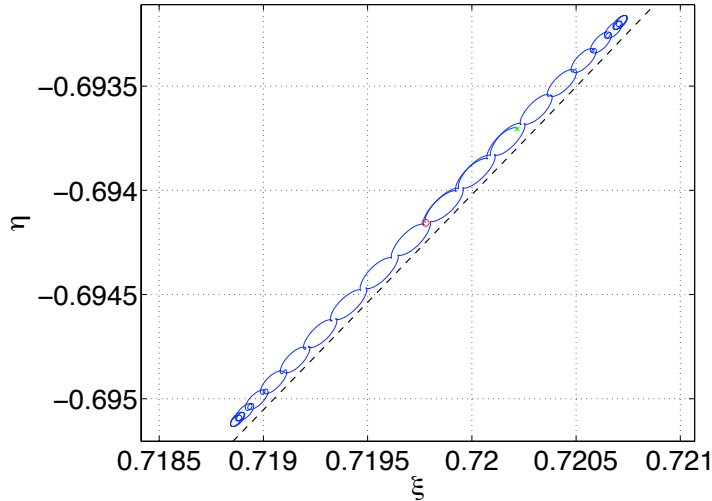


Figure 8 Small orbit around the equilibrium position; the two characteristic eigenfrequencies can be easily recognized.

of out-of-plane orbital motion. The initial conditions are the same of the small oscillation case, but with the addition of an initial out-of-plane component.

PLASMA TORUS EXPLORATION AND POWER GENERATION

Scanning the Plasma Torus

The dynamical behavior of the EDT around the new equilibrium points can be conveniently used to construct a mission profile for the exploration of the Jovian plasma torus and for the extraction of power from Jovian plasmasphere.

The mission has been constructed with the aim of exploring the plasma torus at several latitudes and longitudes, by performing an internal “scan” of the torus itself. The trajectory consists in a large-amplitude oscillation around the new equilibrium position. An out-of-plane motion component allows the oscillation along the ζ synodical axis, in order to scan the torus at several latitudes (see Figure 12).

Several spacecraft configurations allow this kind of trajectories. One of the most interesting configuration utilizes a 14-kilometers EDT, made with a conductive tape of width $w = 5$ cm, thickness $t = 50\mu m$, mounted on a spacecraft with a total mass $m_{S/C} = 600$ kg. Features of the plasma torus EDT-Explorer are reported in Table 4.

Table 4. Configuration of the Plasma Torus EDT Explorer

Plasma Torus EDT Explorer	
Length	14 [km]
Width	5 [cm]
Thick	50 [μm]
S/C mass	600 [kg]

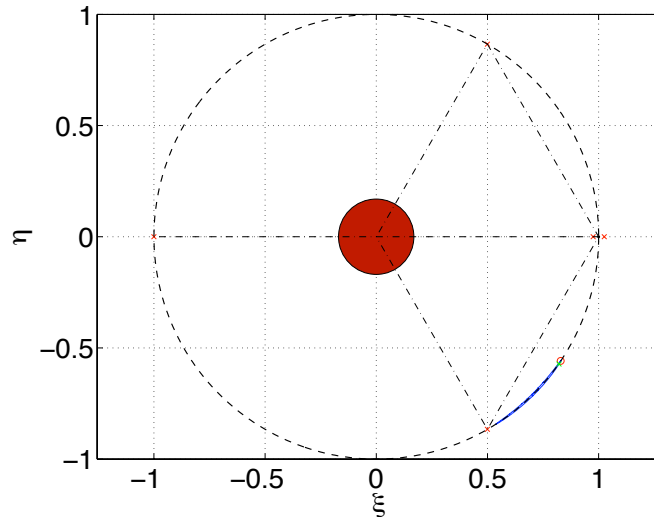


Figure 9 Small oscillations around the equilibrium position with initial position displaced by 10 degrees.

Results of a numerical simulation showing the mission profile are shown in Figure 12. The spacecraft oscillates between the two triangular Lagrangian points L_4 and L_5 , crossing all longitudes of the Plasma Torus, except for a forbidden zone containing the second primary (Io).

Power generation

One of the very attractive capabilities of the Plasma Torus EDT Explorer is the high-level power extraction from the plasmasphere. The torus is a high electron density region within the Jovian system, in which it is possible to generate high electrical power on board the spacecraft. When placed in the Plasma Torus, even a moderate-length EDT like the one we considered here (Table 4) acts as a device capable of converting the energy contained in the high-speed plasmasphere into electrical power that can be used on board the spacecraft.

The power profile for our case shows (Figure 13) shows an ideal power $P_{id} = 856$ W, and a useful power of $P_u = 265$ W. Such power profile can be increased by using other tether configurations. The power is proportional to the tether length, with a power law $P \propto L^{5/2}$. Power at kiloWatt levels could be reached and they are surely of great interest for Jovian missions, that are always handcuffed by a scarcity of power. An electrodynamic tether extracts energy directly from the corotating plasmasphere, and converts it into electrical power useful for loads on board of the satellite.

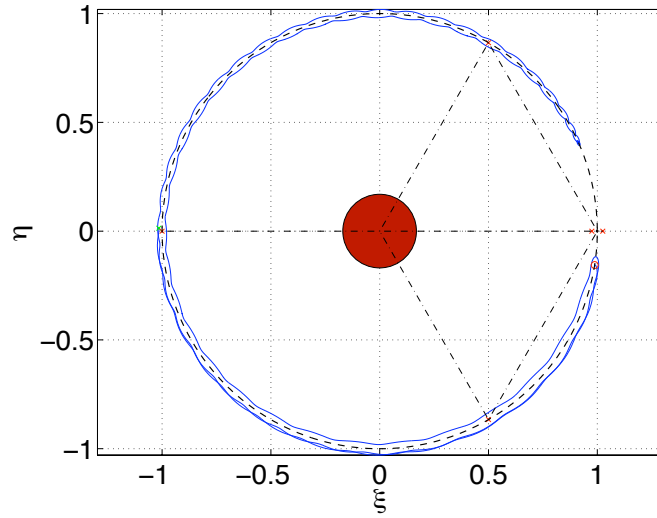


Figure 10 Large oscillations around the equilibrium position (modified horseshoe orbit), starting from $\alpha = -8.96$ deg.

Table 5. Useful Power of a Rotating tether inside the plasma torus.

P_u [W], Width = 5 cm				
$L/m_{S/C}$	10 km	20 km	30 km	40 km
400 [kg]	51.1	272.3	709.5	1406.5
600 [kg]	51.6	283.7	737.1	1430.6
800 [kg]	52.3	295.9	754.0	1451.1
1000 [kg]	51.9	295.2	761.9	1486.3
1200 [kg]	52.9	298.5	770.1	1512.5

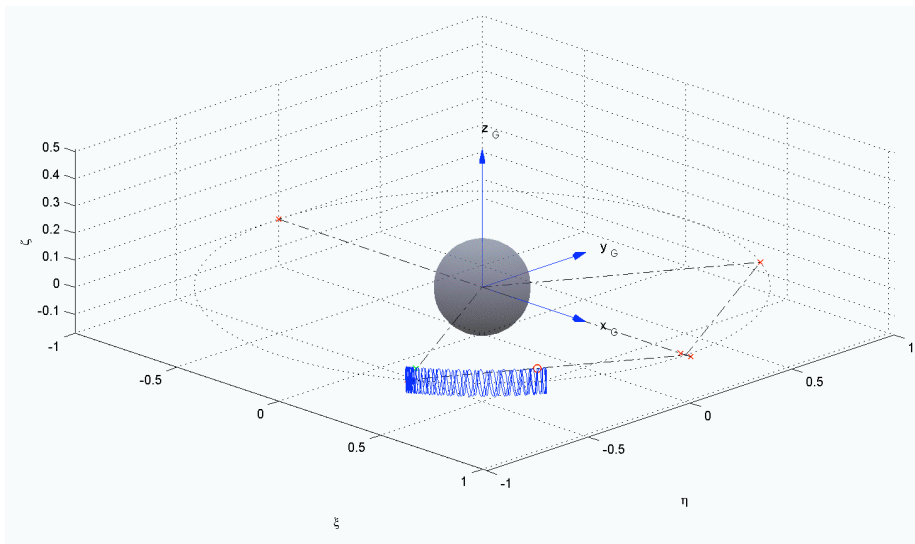


Figure 11. Out-of-plane harmonic motion.

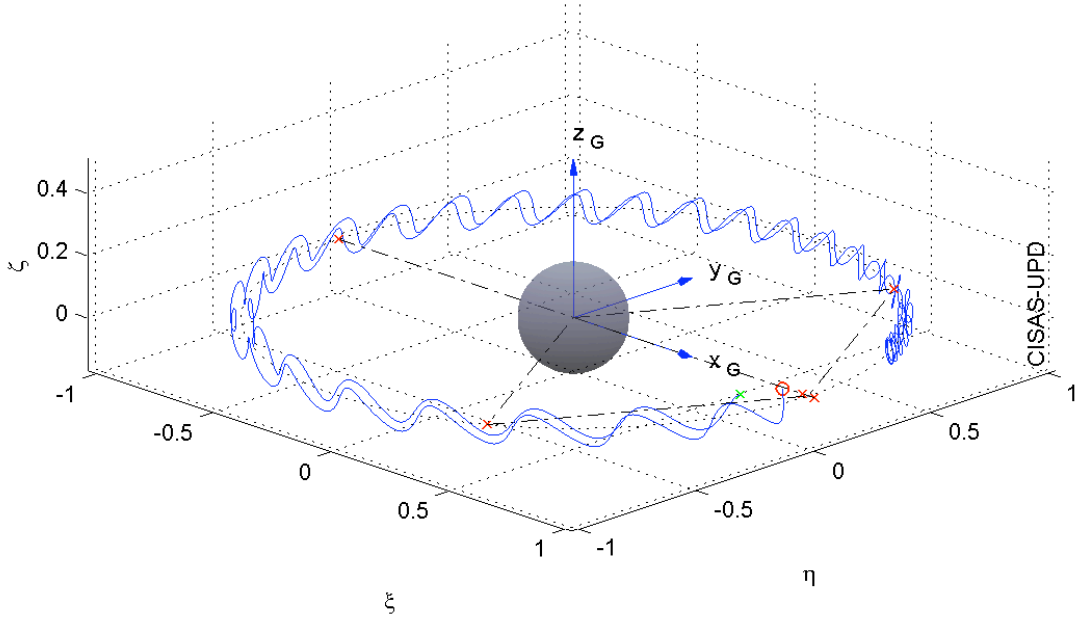


Figure 12 Plasma Torus EDT Explorer; $L = 14$ km, Width = 5 cm, $m = 600$ kg, simulation time: $T = 70T_{rev} \approx 123.8$ days. Rendering of the trajectory in 3D, in the Jupiter-Io system. Tether initial position is marked with a red circle.

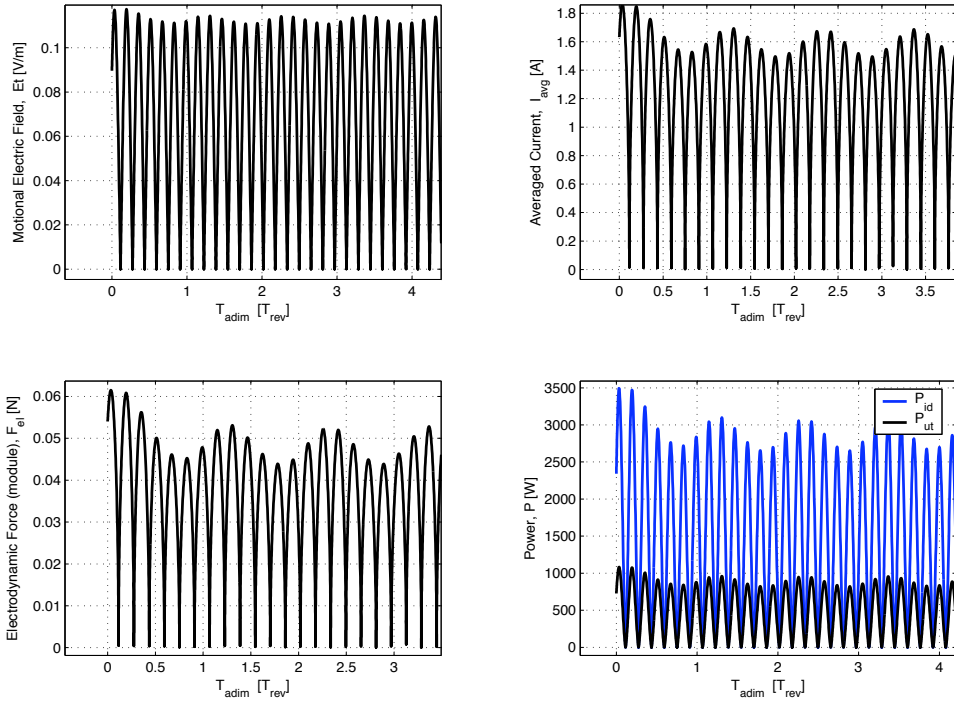


Figure 13 Motional electric field, Length-averaged current, electrodynamic force and Electrical power produced by an electrodynamic tether orbiting inside the Plasma Torus.

CONCLUSION

The EDT concept is a unique power device capable to extract kW of power directly from the jovian environment, that is more effective than solar photovoltaic systems and providing power levels comparable with RTGs power plants. The two (stable) triangular Lagrangian points L-4 and L-5 of the Io-Jupiter system lie inside the plasma torus and they act as attractors for bodies that co-orbit along the Io's orbit. We have used these two points to demonstrate that an EDT can be placed at a fixed distance from Io (and co-orbiting with it) away from a Lagrangian point (for example ahead of L-5) by using a thrust electrodynamic force. The force, produced by the conductive tether through the interaction with Jupiter's magnetic field, counteracts the gravitational and inertial forces that push the system towards L-5. While in this new equilibrium position, the tethered system generates a few kWatts of power (with a tether length of order 20 km) that can be used on board the spacecraft (S/C). From another viewpoint, in this case the EDT extracts energy from the plasma sphere that co-rotates with Jupiter at an orbital rate greater than the orbital rate of the S/C itself, while the S/C remains at a constant distance from Io along its orbit. In another version of the same concept, the spacecraft with the attached EDT can be made to explore the entire Io torus without the need for chemical propellant but rather by utilizing the electrodynamic force produced through the interaction with the super-rotating plasma sphere and magnetic field of Jupiter, while generating electrical power at the same time. The research conducted can open the door for a number of new studies that will enrich our knowledge of the Jupiter moon system through the use of electrodynamic propulsion and power generation that exploits the local environment.

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NOTATION

EDT	Electrodynamic Tether
CRTBP	Circular Restricted Three Body Problem
G	Center of mass of the two primaries (Jupiter and Io)
R_j	Jupiter radius, $1 R_j \approx 71400 km$
\vec{R}	Position vector of the center of mass of the EDT [km]
$\vec{\Omega}$	angular velocity of revolution of Jupiter-Io [rad/s]
μ	Gravitational parameter [km^3/s^2]
\vec{R}	Position vector of the center of mass of the EDT
\vec{R}_1	Distance vector of the EDT from the primary (<i>i.e.</i> from Jupiter)
\vec{R}_2	Distance vector of the EDT from the secondary (<i>i.e.</i> from Io)

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