Alpha decay perturbations by atomic effects at extreme conditions

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ARTICLE INFO

ABSTRACT

The alpha tunneling effect in the presence of electron screening is calculated within the Debye model. Calculations show that very small effects are predicted by cooling the metal to low temperatures. However the alpha lifetime decay may be reduced by about 15% if solid samples of the alpha emitters are cooled and compressed to relatively high densities. These conditions can be achieved at high pressures by using existing diamond anvil cells (DACs). Even so, practical consequences for speeding-up the decay of actinides (from the nuclear waste) seem to be negligible.

Keywords:
Alpha decay
Lifetime reduction
Extreme atomic conditions

1. Introduction and background

Nuclear reactions and decays are influenced by the electron screening in a condensed matter environment [1–12]. The electron cloud surrounding the nuclear interaction is effectively a screening potential causing the projectile under consideration to see a modified Coulomb potential barrier. In particular, for alpha decays, the barrier is reduced in size and extension implying a higher cross section in comparison with the bare nuclear interaction [9]. This effect can be calculated from the Debye plasma model for quasi-free electrons [1]. The Debye model has been also used successfully to calculate the effects of electron screening on fusion reactions in stellar plasmas [3].

In a recent publication [1] it was suggested that Debye plasma model applied to the quasi-free metallic electrons predicts a significant reduction in the alpha decay lifetime if the metal is cooled to low temperatures. In particular, in a metal cooled to a temperature $T = 4$ K, the lifetime of the alpha decay of $^{210}\mathrm{Po} \rightarrow {}^{206}\mathrm{Pb} + \alpha$ is reduced from 138 days to 0.5 days and the transuranium waste $^{226}\mathrm{Ra} \rightarrow {}^{222}\mathrm{Rn} + \alpha$ is shortened from 1600 years to 1.3 years. If this is verified to be correct then it should be of enormous importance.

In this Letter it is shown that this prediction is not justified in the context of the Debye model. For this purpose the alpha tunneling effect in the presence of electron screening is calculated. The calculations show that no visible effect is predicted by cooling the metal to low temperatures. It is suggested that the alpha decay lifetime may be reduced (about 15%) by compressing the solid sample to high pressures as a result of using a diamond anvil cell (DAC).

Findings presented in this Letter are in agreement with recent publications [8,12] where it is stated that alpha decay rate enhancement in a metallic environment is unlikely from the viewpoint of standard screening models [8] and experimental results show that the decay lifetime in metal embeddings is not significantly shorter than in insulator embeddings [12]. However, our Letter presents additional analysis and calculations, particularly related to the modifications caused by a higher density environment, where stronger effects are predicted, although not large enough to be useful as a tool for nuclear waste management.

2. Calculation model

The lifetime τ of an alpha emitter nucleus, with an atomic mass A and charge Z, is given by

$$\frac{1}{\tau} = \left(\frac{1}{\tau_{00}}\right) e^{-G}, \tag{1}$$

$$G = \frac{2}{\hbar}\int_{R}^{b} dr \sqrt{2\mu\big(U(r) - E_\alpha\big)}. \tag{2}$$

U is the potential describing the α decay [12], $\hbar$ is Planck constant divided by 2π, $\mu = m_\alpha m_{A-4}/(m_\alpha + m_{A-4}) \approx m_\alpha$ is the α reduced mass, R is the nucleus radius and E_α is the kinetic energy of the free α particle and $1/\tau_{00}$ is the rate that the α particle is hitting the barrier and trying to tunnel outside the potential $U(r)$ and it is practically fixed from experiment for the given U and E_α. The limit b is defined by

$$E_\alpha = \frac{1}{2}\mu v^2 = \frac{2(Z-2)e^2}{b} = U(b) \tag{3}$$

where e is the value of the electron charge, v is final velocity of the α particle. e^{-G} is the transparency factor for the tunneling problem and is calculated from the integral in Eq. (2)

$$G = \frac{1}{\hbar}\big[8\mu 2(Z-2)e^2 b\big]^{1/2}\left\{\cos^{-1}\left(\sqrt{\frac{R}{b}}\right) - \sqrt{\left(\frac{R}{b}\right) - \left(\frac{R}{b}\right)^2}\right\} \approx \frac{2\pi}{\hbar}\big[\mu(Z-2)e^2 b\big]^{1/2} = \frac{4\pi(Z-2)e^2}{\hbar v} \quad \text{for } \frac{R}{b} \ll 1. \tag{4}$$

The electron screening changes the potential U in the Debye model according to the following equation

$$U = \frac{2(Z-2)e^2}{r} - \frac{2Z^* e^2 e^{-r/\lambda_D}}{r} \approx \frac{2(Z-2)e^2}{r}\left[1 - \frac{Z^*}{Z-2} + \left(\frac{Z^*}{Z-2}\right)\left(\frac{r}{\lambda_D}\right)\right] \quad \text{for } \frac{r}{\lambda_D} \ll 1. \tag{5}$$

The Debye length, λ_D (in c.g.s. units), is

$$\lambda_D = \left(\frac{k_B T}{4\pi e^2 n_e}\right)^{1/2} \approx \left(\frac{k_B m_p A}{4\pi e^2 Z^*}\right)^{1/2}\left(\frac{T}{\rho}\right)^{1/2} = 8.9250\times 10^{-12}\left(\frac{A}{Z^*}\right)^{1/2}\left(\frac{T(\mathrm{K})}{\rho(\mathrm{g/cm^3})}\right)^{1/2} \text{[cm]} \tag{6}$$

where m_p is the proton mass, Z^* is the number of free electrons per nucleus, T is the temperature (in kelvins) and ρ is the matter density (in g/cm^3).

The relation between b and E_α is given by $U(r=b) = E_\alpha$, yielding

$$b = 2(Z-2)e^2\left(1 - \frac{Z^*}{Z-2}\right)\left(E_\alpha - \frac{2Z^* e^2}{\lambda_D}\right)^{-1}. \tag{7}$$

Substituting (7) into (4) for $R/b \ll 1$ and using (6) one gets from Eq. (1) the lifetime for α decay

$$\tau \approx \tau_0 \exp\left(-\frac{U_D}{2E_\alpha}\right)\exp\left[-\frac{Z^*}{2(Z-2)}\right], \tag{8}$$

$$U_D = \frac{2Z^* e^2}{\lambda_D} \approx 3.23\times 10^{-2}\left(\frac{Z^{*3}\rho}{AT}\right)^{1/2} \text{ MeV}. \tag{9}$$

τ_0 in this equation is taken as the experimental alpha decay half life time of a bare nucleus. Z^* is the number of free or quasi-free electrons in the solid plasma model.

In order to increase the number of free electrons in a solid we suggest to cool the sample to low temperatures and to compress it to high pressures. There are two ways of compressing: dynamic compression by shock waves and static compression by diamond anvil cell (DAC). The largest pressures derived in a diamond anvil cell are of the order of four mega-bars [13] while the maximum dynamic pressure achieved is about one giga-bar [14]. The maximum pressures derived by shock waves are larger than those obtained by DAC, but the time duration of the dynamic experiments last about one micro-second or less (shorter than 1 nanosecond with laser driven compression). Furthermore in a shock wave experiment the target is heated and therefore U_D is decreased according to Eq. (9). Therefore the static experiments seem to be favorable for checking the α decay lifetime.

The relation between pressure and density is described by the equation of state [15]. Examples for lead ($Z = 82$, $A = 207$; [16]) and thorium ($Z = 90$, $A = 232$; [17]) are given in Table 1 for DAC (diamond anvil cell) experiments; the results are given for the pressure P in Mbars (1 Mbar = 100 GPa) versus the compression ρ/ρ_0.

Table 1
DAC (diamond anvil cell) experimental results for lead (Pb: $Z = 82$, $A = 207$, density $\rho_0 = 11.35$ g/cm^3) and thorium (Th: $Z = 90$, $A = 232$, density $\rho_0 = 11.72$ g/cm^3), lifetime $\tau_\alpha = 1.4\times 10^{10}$ years; under strong compression. P is the pressure in Mbars (1 Mbar = 100 GPa) and ρ/ρ_0 is the compression.

P [Mbars]	0.2	1	2	3
ρ/ρ_0 [Pb]	1.25	1.72	2.10	2.31
ρ/ρ_0 [Th]	1.26	1.89	2.23	2.47

3. The pressure ionization model

In order to estimate the dependence of Z^* (in Eqs. (8) and (9)) with density we use the pressure ionization model [18–20]. When atoms are pushed together by compressing the matter, some of the extended wave functions of the bound electrons form bands which propagate between atoms. The pressure ionization occurs when electron wave functions of neighbor atoms overlap and these atomic states create energy bands. The energy width of these bands grows with density and at high compression the bands merge into the free-electron continuum states. The number of these free electrons is denoted by Z^*.

In general, matter at high compression can be successfully described by the Thomas–Fermi (TF) model [15]. In the TF model the electrons are considered as a degenerate fluid placed in a self-consistent electrostatic field described by Poisson equation. The electrons are degenerate when the temperature energy ($\sim k_B T$, where T is the temperature and k_B is Boltzmann constant) is smaller than the Fermi energy (ε_F). Since $\varepsilon_F \sim 10^4$ K for a solid density and scales as $(\rho/\rho_0)^{2/3}$ (ρ/ρ_0 is the compression) it is appropriate in our case to use TF model at $T = 0$.

In the TF model the Z^* is the number of electrons per cell (the volume of one atom) whose energy is positive relative to the ion sphere boundary potential [20]. The following useful fit for Z^* in TF model was derived by More [20],

$$\begin{cases} \dfrac{Z^*}{Z} = \dfrac{x}{1+x+\sqrt{1+2x}}, \\ x = \begin{cases} 14.3139\left(\dfrac{\rho}{ZA}\right)^{0.6624} \\ 0.1129\left(\dfrac{\rho}{\rho_0}\right)^{0.6624} \end{cases} \quad \text{for Pb: } Z = 82,\ A = 207,\ \rho_0 = 11.35\ \text{g/cm}^3. \end{cases} \tag{10}$$

From this equation one can see that for $x \ll 1$ one gets $Z^* \sim (\rho/\rho_0)^{2/3}$.

4. Results

In order to show that the Debye model does not predict the significant reduction in the alpha decay lifetime as suggested in some papers (e.g., [1]) we present our estimations in Table 2 consistent with the recent literature [8,12]. In particular, cooling to liquid helium temperature, $T \approx 4$ K, the Debye energy U_D/E_a (Eq. (9)) and τ/τ_0 (Eq. (8)) are estimated for ^{210}Po and ^{226}Ra. The densities ρ_0 and ρ (for

Table 2
E_α, U_D/E_α and τ/τ_0 are given for $^{210}\text{Po} \to {}^{206}\text{Pb} + \alpha$ (part (a)) and $^{236}\text{Ra} \to {}^{232}\text{Rn} + \alpha$ (part (b)), at a compression of 2.5 with an initial matrix-Pb density ρ_0 of 11.35 g/cm^3 and a temperature of 4 K (the first 2 columns) and $T = 300$ K the last column. (τ_0 stands for the bare nucleus lifetime.)

(a) $^{210}_{84}\text{Po} \to {}^{206}_{82}\text{Pb} + \alpha$, $E_\alpha = 5.30$ MeV	$Z^* = 6$ ($T = 4$ K)	$Z^* = 8$ ($T = 4$ K)	$Z^* = 6$ ($T = 300$ K)
U_D/E_α	1.86×10^{-2}	2.9×10^{-2}	2.15×10^{-3}
τ/τ_0	0.99	0.98	1.00

(b) $^{236}_{88}\text{Ra} \to {}^{232}_{86}\text{Rn} + \alpha$, $E_\alpha = 4.78$ MeV	$Z^* = 2$ ($T = 4$ K)	$Z^* = 2.7$ ($T = 4$ K)	$Z_0^* = 2$ ($T = 300$ K)
U_D/E_α	3.37×10^{-3}	5.28×10^{-3}	3.9×10^{-4}
τ/τ_0	1.00	0.99	1.00

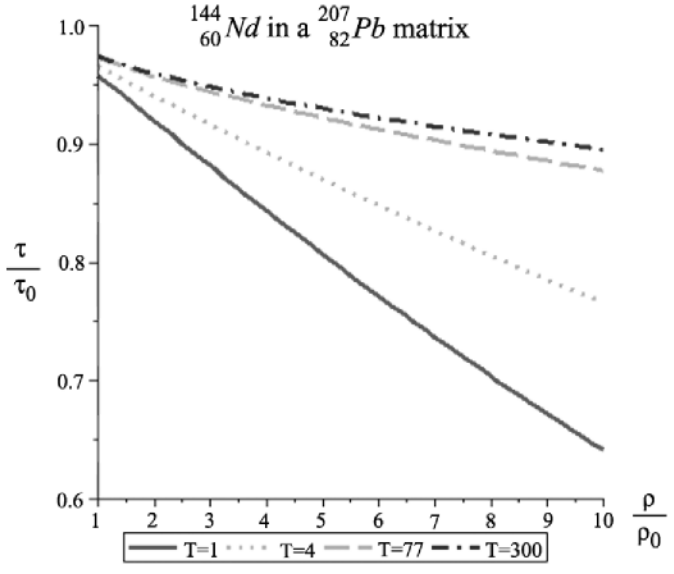

Fig. 1. The alpha decay lifetime reduction during compression of a lead (Pb: $Z = 82$, $A = 207$, $\rho_0 = 11.35$ g/cm^3) matrix containing the ^{144}Nd isotope ($E_\alpha = 1.85$ MeV, $\tau_0 = 2.3 \times 10^{15}$ years). The lifetime reduction ratio τ/τ_0 is described as a function of the compression ρ/ρ_0 for room temperature (300 K), liquid nitrogen temperature (77 K) and liquid helium temperatures (4 K and 1 K). (τ_0 stands for the bare nucleus lifetime.)

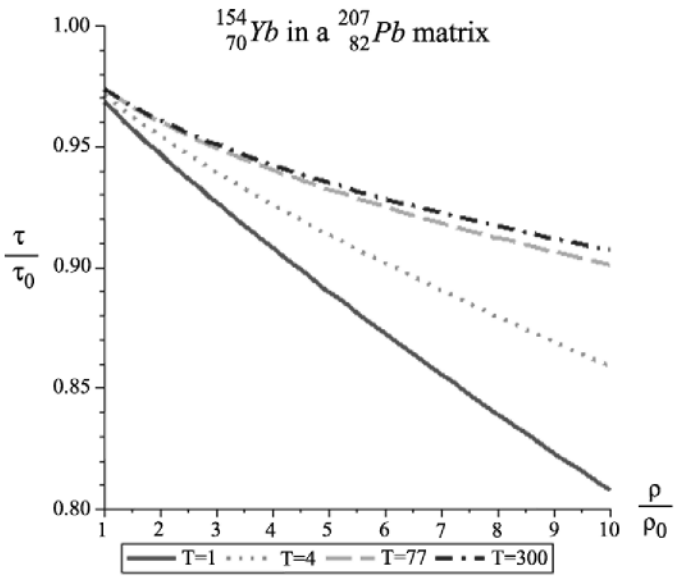

Fig. 2. The alpha decay lifetime reduction during compression of a lead (Pb: $Z = 82$, $A = 207$, $\rho_0 = 11.35$ g/cm^3) matrix containing ^{154}Yb ($E_\alpha = 5.31$ MeV, $\tau_0 = 0.4$ s). The lifetime reduction ratio τ/τ_0 is described as a function of the compression ρ/ρ_0 for room temperature (300 K), liquid nitrogen temperature (77 K) and liquid helium temperatures (4 K and 1 K). (τ_0 stands for the bare nucleus lifetime.)

pressure 3 Mbars) are taken for the matrix material of lead from Table 1. The results E_α, U_D/E_α and τ/τ_0 are given in Table 2. As one can see from Table 2 the alpha decay lifetimes decreases by about 1% or less.

It is important to point out that the temperature T cannot go to zero since it is implicitly assumed that the Debye length is larger than the b value in Eq. (7) (which in turn is much larger than the nucleus radius) yielding

$$T[\mathrm{K}] > \rho\left[\mathrm{g/cm^3}\right] \cdot \left(\frac{Z^*}{A}\right) \approx 1. \tag{11}$$

For the transition metals some of the d-band electrons can be ionized while for actinides some of the f-band electrons might also be ionized. We estimate the alpha decay reduction during diamond anvil cell (DAC) compression of a lead (Pb: $Z = 82$, $A = 207$, $\rho_0 = 11.35$ g/cm^3) matrix containing two Lanthanides [21]: the ^{144}Nd with $E_\alpha = 1.85$ MeV, $\tau_0 = 2.3 \times 10^{15}$ years and ^{154}Yb with $E_\alpha = 5.31$ MeV, $\tau_0 = 0.4$ seconds. The results are summarized in Figs. 1 and 2. In these figures the lifetime reduction τ/τ_0 is described as a function of the compression ρ/ρ_0 for room temperature (300 K), liquid nitrogen temperature (77 K) and liquid helium temperatures (4 K and 1 K). As one can see the cooling is important in the Debye model, however for significant reduction in the lifetime the compression plays the major role. At liquid helium temperatures and a compression of 5 a reduction of about 20% in the alpha decay lifetime is predicted.

Even so, these values are very far from the lifetime reduction by several orders of magnitude claimed in recent publications [1], and would not be useful for speeding-up the decay of the actinides generated in the spent nuclear fuel. However, findings of this Letter point out that experimental tests using DCA at low temperatures seem to be necessary to assess the actual significance of these perturbations in the decay mechanism of alpha emitters.

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