Thickness dimensioning of water flow glazing facades

F. Javier Escoto
Department of Applied Mathematics, School of Aeronautics and Space Engineering. Technical University of Madrid (UPM),
javier.escoto.lopez@alumnos.upm.es
ORCID: 0000-0002-5315-7277

Juan A. Hernández
Department of Applied Mathematics, School of Aeronautics and Space Engineering. Technical University of Madrid (UPM),
juanantonio.hernandez@upm.es
ORCID: 0000-0002-5188-1456

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Abstract: The fluid inside water fluid glazing facades creates a linear distribution pressure along the vertical direction. Hence, the glazings must be designed to with stand pressure loads satisfying deflection and stresses requirements given by the norms and standards. The straightforward solution is to increase the glass thickness until the deflection is below the required limit. This implies an increase in weight and amount of material rising the price of the glazing, which is undesirable. Therefore, pillars or stripes can be considered as an alternative solution to limit glass deflection.

In order to ensure that the structural behavior requirements are fulfilled, a proper mathematical model and simulation must be carried out. The elastic plate model must be sufficiently detailed to retain the most relevant physical aspects of the process but simple enough not to over dimension the problem. To solve the mathematical model, it was necessary to apply a numerical method to the partial differential equations resultant. For this task a High Order Finite Difference Method (HOFM) was implemented for both linear and non-linear models, which led to satisfactory results from the viewpoint of the accuracy of the solution.

From the development of the structural model and simulation for Water Flow Glazings thickness dimensioning two conclusions were extracted. First, the plate model was sufficient to dimension the glazings and second, the presence of pillars or stripes permits to reduce significantly the thickness dimension. Hence, introducing pillars or stripes is considered an effective solution to solve the hydrostatic pressure problem and lead to a more efficient design of Water Flow Glazings.

***Keywords**:* *Water-Flow Glazing (WFG), pillars, stripes, hydrostatic pressure, mathematical models*

1. INTRODUCTION

In order to design long life Water-Flow Glazings it is crucial to guarantee that the sealing of the water chamber maintains its tightness during all service. Is in this process in which the filling up of the chamber plays an important role on the stresses to which the silicon adhesive which fixes both frames of the glasses that comprises the glazing. To reduce the traction loads it is desirable to maintain the hydrostatic pressure below the atmospheric pressure level on the maximum possible surface of the glass. However, this can lead to a deflection that brings together the glasses of the glazing putting them too close or even in contact which is not assumable. To avoid this situation without increasing the thickness of the glass, a solution can be to define a configuration of transversal elements that restrict the deformation in certain regions of the glass surface. The design of this configuration and thickness selection must be done through the simulation of the structural behavior of the glazing with the considered configurations.

In the following pages, a mathematical model for the structural behavior of the glazing and an algorithm for High Order Finite Difference Method are presented [1]. As well, a simulation with an example of the application of the algorithm and model is presented.

2. MATHEMATICAL MODEL

The geometry of each glass of the glazing can be considered independently as the domain Ω:

$$\Omega = \{(x, y, z) \in \mathbb{R}^3 | 0 \leq x \leq L_x, 0 \leq y \leq L_y, -t/2 \leq z \leq t/2\}, \qquad (1)$$

where L_x is the width of the glass, L_y its height and t its thickness.

As the glazings satisfy the relational order $t \ll L_x \sim L_y$, dependence on z of the magnitudes can be decoupled using the elastic plate theory [2]. Thus, the transversal deflection and membrane stresses are assumed to remain constant through the thickness and all the other magnitudes vary linearly with z in each point (x, y) of the plane $z = 0$.

First of all, it is necessary to define the hydrostatic pressure q that acts on the inner walls of the glazing. The hydrostatic assumption over the fluid gives the linear distribution of pressure $p(y)$ expressed in Eq. [2]:

$$q(y) = p(y) - p_0 = -\rho g (y - h_0), \qquad (2)$$

in which ρ is the fluid density, g is the gravitational constant, p_0 is the atmospheric pressure and h_0 is the height of the zero-pressure line. Hence, for the region $y > h_0$ the pressure inside the glazing is less than the atmospheric and the glasses deflect towards each other.

In the case in which the transversal deflection of the plate u is much lesser than its thickness$t$ the resultant model is linear. The differential equation that controls the deformation of the glazing is given by Eqs. [3,4]:

$$\nabla^4 u + \frac{\rho g}{D}(y - h_0) = 0, \qquad (3)$$

$$D = \frac{E t^3}{12(1 - \nu^2)}, \qquad (4)$$

where$\nabla^4 = \nabla^2 \nabla^2$ is the biharmonic operator,D is the stiffness of the plate, E is the Young modulus of the glass and ν its Poisson's coefficient. The maximum stresses that appear as consequence of the bending of the glasses are given by Eqs. [5-7]

$$\sigma_{xx} = -\frac{6D}{t^2}\left(\frac{\partial^2 u}{\partial x^2} + \nu \frac{\partial^2 u}{\partial y^2}\right), \qquad (5)$$

$$\sigma_{yy} = -\frac{6D}{t^2}\left(\frac{\partial^2 u}{\partial y^2} + \nu\frac{\partial^2 u}{\partial x^2}\right), \tag{6}$$

$$\sigma_{xy} = -\frac{6D}{t^2}(1-\nu)\frac{\partial^2 u}{\partial x \partial y}. \tag{7}$$

These stresses appear at the outer and inner surfaces of the glass with opposite directions. For example, if one point (x, y, z) is submitted to a traction normal stress its opposite point $(x, y, -z)$ is submitted to a compression normal stress.

The effect of the silicon sealing is included as simply supported boundary conditions:

$$u(x,0) = \frac{\partial^2 u}{\partial x^2}(x,0) = \frac{\partial^2 u}{\partial y^2}(x,0) = 0, \qquad u(x,L_y) = \frac{\partial^2 u}{\partial x^2}(x,L_y) = \frac{\partial^2 u}{\partial y^2}(x,L_y) = 0, \tag{8}$$

$$u(0,y) = \frac{\partial^2 u}{\partial x^2}(0,y) = \frac{\partial^2 u}{\partial y^2}(0,y) = 0, \qquad u(L_x,y) = \frac{\partial^2 u}{\partial x^2}(L_x,y) = \frac{\partial^2 u}{\partial y^2}(L_x,y) = 0. \tag{9}$$

Whenever geometrical non linearity is considered, that is, the deflection u is of the same order or greater than the thickness, the effect of the membrane stresses σ_{xx}^m, σ_{yy}^m and σ_{xy}^m must be taken in account [3]. In these conditions, the Eq. [3] must be substituted by Eqs. [10,11]:

$$\nabla^4 u + \frac{t}{D}\mathcal{L}(u,\phi) + \frac{\rho g}{D}(y - h_0) = 0, \tag{10}$$

$$\nabla^4 \phi + \frac{E}{2}\mathcal{L}(u,u) = 0. \tag{11}$$

where ϕ is the Airy stress function and $\mathcal{L}$ is a non linear differential operator which verify respectively Eqs. [12,13]

$$\sigma_{xx}^m = \frac{\partial^2 \phi}{\partial y^2}, \qquad \sigma_{yy}^m = \frac{\partial^2 \phi}{\partial x^2}, \qquad \sigma_{xy}^m = \frac{\partial^2 \phi}{\partial x \partial y}, \tag{12}$$

$$\mathcal{L}(u,\phi) = \frac{\partial^2 u}{\partial x^2}\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 u}{\partial y^2}\frac{\partial^2 \phi}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y}\frac{\partial^2 \phi}{\partial x \partial y}. \tag{13}$$

In addition to Eqs. [8,9], boundary conditions to the Airy function must be imposed:

$$\frac{\partial^2 \phi}{\partial x^2}(x,0) = \frac{\partial^2 \phi}{\partial y^2}(x,0) = 0, \qquad \frac{\partial^2 \phi}{\partial x^2}(x,L_y) = \frac{\partial^2 \phi}{\partial y^2}(x,L_y) = 0, \tag{14}$$

$$\frac{\partial^2 \phi}{\partial x^2}(0,y) = \frac{\partial^2 \phi}{\partial y^2}(0,y) = 0, \qquad \frac{\partial^2 \phi}{\partial x^2}(L_x,y) = \frac{\partial^2 \phi}{\partial y^2}(L_x,y) = 0. \tag{15}$$

The presence of pillars or stripes is introduced as a zero displacement condition for the points which are in the inner region of the element. These conditions can be imposed on both pillars' region Ω_p and the stripes region Ω_s, substituting the differential equations for the linear or nonlinear plate by Eqs. [16,17] in this region:

$$u|_{\Omega_\mathrm{p}} = 0, \tag{16}$$

$$u|_{\Omega_\mathrm{s}} = 0. \tag{17}$$

3. HIGH ORDER FINITE DIFFERENCE METHOD

In order to solve the differential equations involved the domain$[0, L_x] \times [0, L_y]$ is discretized in $(N_x + 1)(N_y + 1)$ nodal points. In this manner the solution is also discretized in a vector $U \in \mathbb{R}^{N_v(N_x+1)(N_y+1)}$, where N_v is the number of variables (1 in the linear case, 2 in the nonlinear case). In the same manner the derivatives involved in the differential equation are approximated by arbitrary order finite differences. Therefore, the differential operator turns into a difference operator $F: \mathbb{R}^{N_v(N_x+1)(N_y+1)} \longrightarrow \mathbb{R}^{N_v(N_x-1)(N_y-1)}$ for U which satisfies:

$$F(U) = 0. \tag{18}$$

Furthermore, the boundary conditions merge as a difference operator as well.$G: \mathbb{R}^{N_v(N_x+1)(N_y+1)} \longrightarrow \mathbb{R}^{2N_v(N_x+N_y)}$

$$G(U) = 0. \tag{19}$$

Both systems of equations can be agglutinated into a single system $H: \mathbb{R}^{N_v(N_x+1)(N_y+1)} \longrightarrow \mathbb{R}^{N_v(N_x+1)(N_y+1)}$

$$H(U) = \begin{bmatrix} F(U) \\ G(U) \end{bmatrix} = 0. \tag{20}$$

Solving the system of Eq. [15] will provide the desired solution. In the case in which the problem is linear, the system is solved by LU decomposition and when it is nonlinear is required an iterative method such as Newton's.

4. SIMULATION

To illustrate the effect that pillars and stripes have on the structural behavior of the glazing four cases will be considered in this document. The dimensions of the glazing will be: $L_x = 1.3$ m, $L_y = 3$ m,$t = 10$ mm. Its elastic properties are $E = 72$ GPa and $\nu = 0.22$. The four distinct considered cases are:

a) A linear glazing with no pillars or stripes and zero pressure line at h_0=0.5 m.
b) A linear glazing with one pillar at $x = 0.65$ m, $y = 0.86$ m and zero pressure line at h_0=0.5 m.
c) A linear glazing with one stripe at $x = 0.65$ m and zero pressure line at h_0=0.5 m.
d) A linear glazing with two stripes at $x = 0.43$ m, and zero pressure line at h_0=0.5 m.

As it is shown in Figure 1, the plate with no stripes suffers a deflection that can be superior to 5 cm, which is reduced to approximately 1 cm when a pillar is present. The effect of stripes is more effective leading to deflections of 2.5 mm for one stripe and 0.4 mm for two stripes. It is also remarkable that the presence of transversal elements divides the surface area in different regions that in the case of stripes act as independent plates with proper boundary conditions.

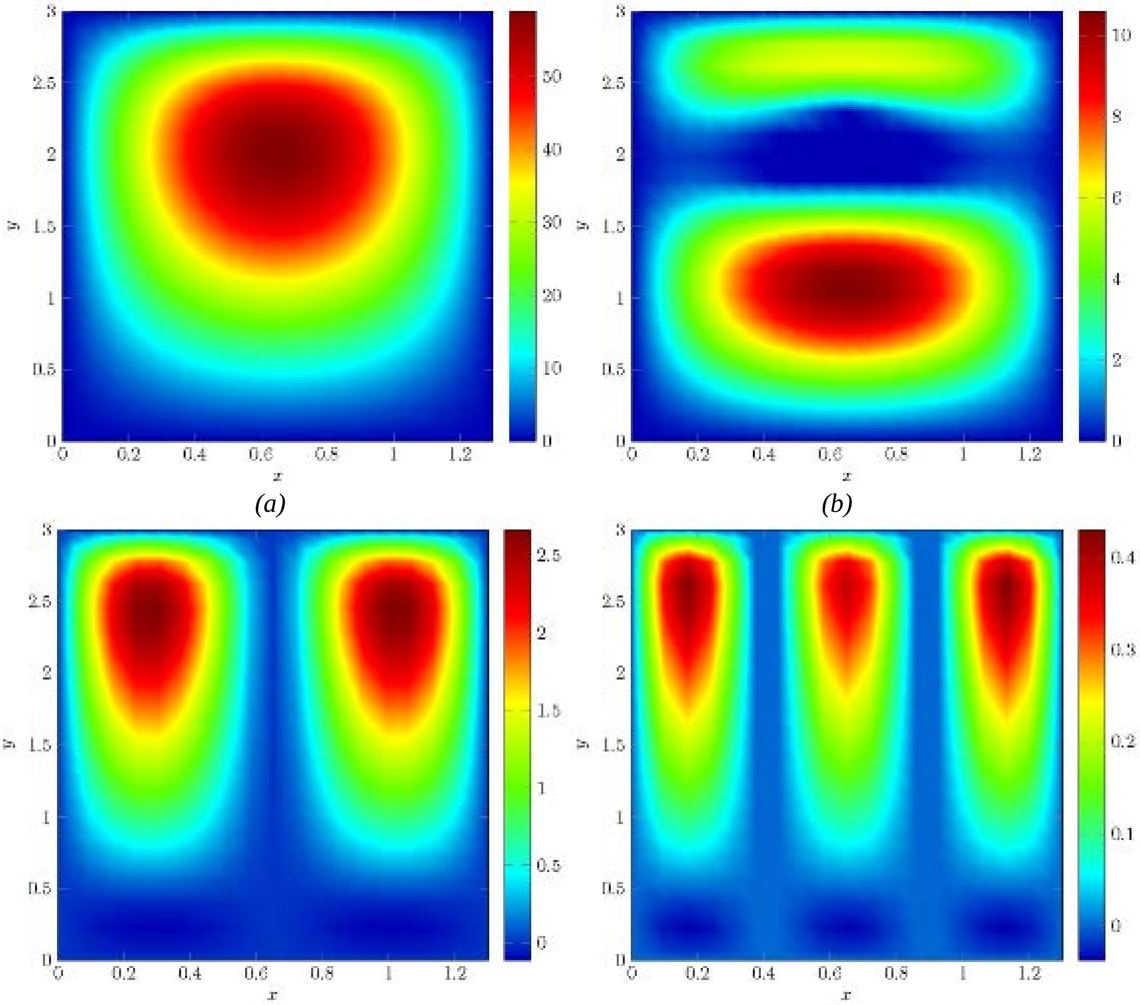


Figure 1. Deflection distribution $u(x, y)$ of the glass expressed on mm for the four cases. (a) Deflection with no pillars or stripes, (b) Deflection with one pillar, (c) Deflection with one stripe, (d) Deflection with two stripes.

Comparing the maximum bending stresses produced in each case, Figure 2 shows that the effect of pillars and stripes is similar to the one caused for deflection. When the glass is free of transversal elements, the stress can be greater than 120 MPa. With one pillar, the maximum value is reduced to 80 MPa but the stress concentration is higher. One middle stripe reduces both the value and the stress concentration obtained with one pillar considering a maximum stress of around 50 MPa. When a second stripe is added the result is even better obtaining lower stress concentration and a maximum value of approximately 15 MPa.

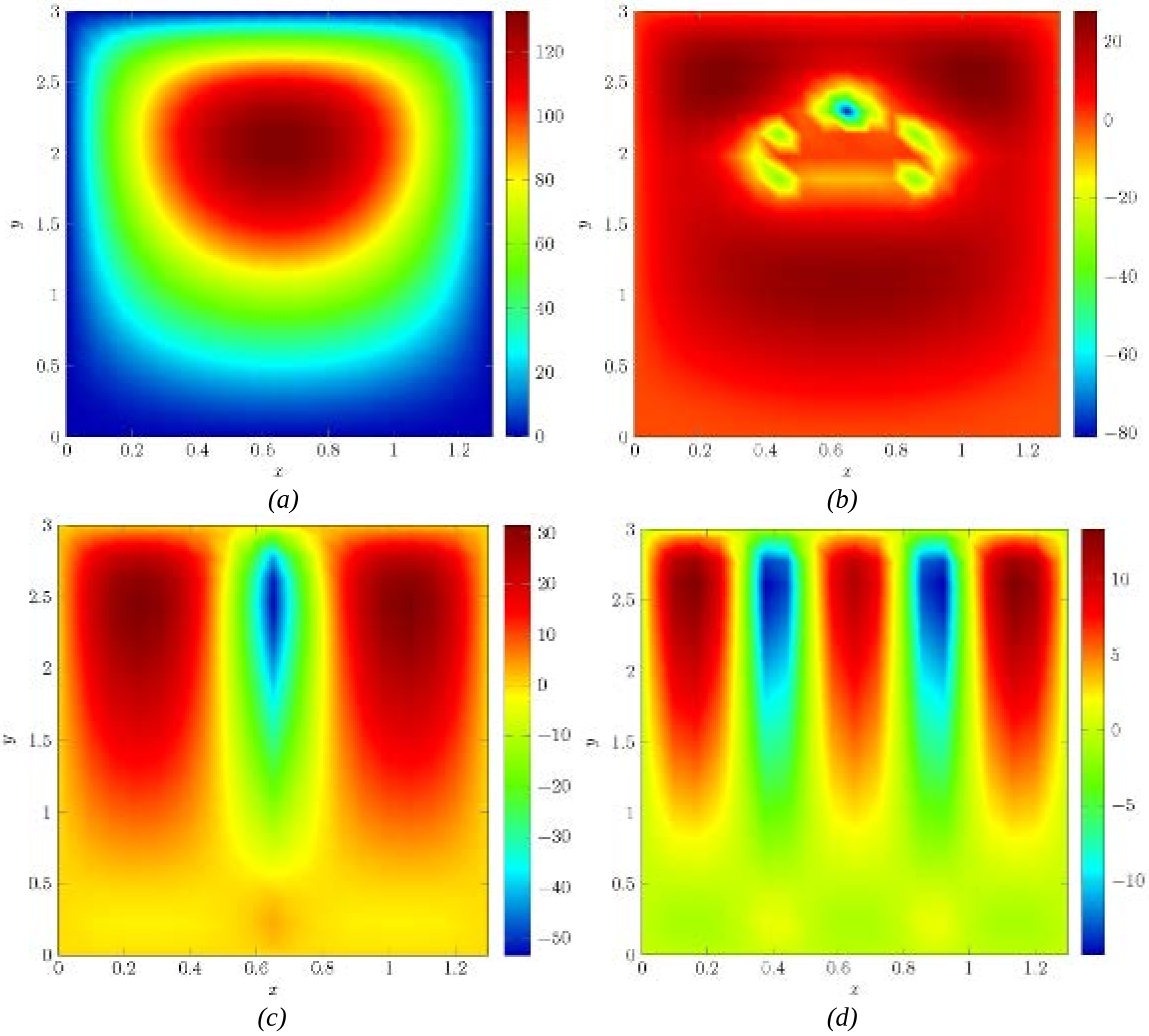
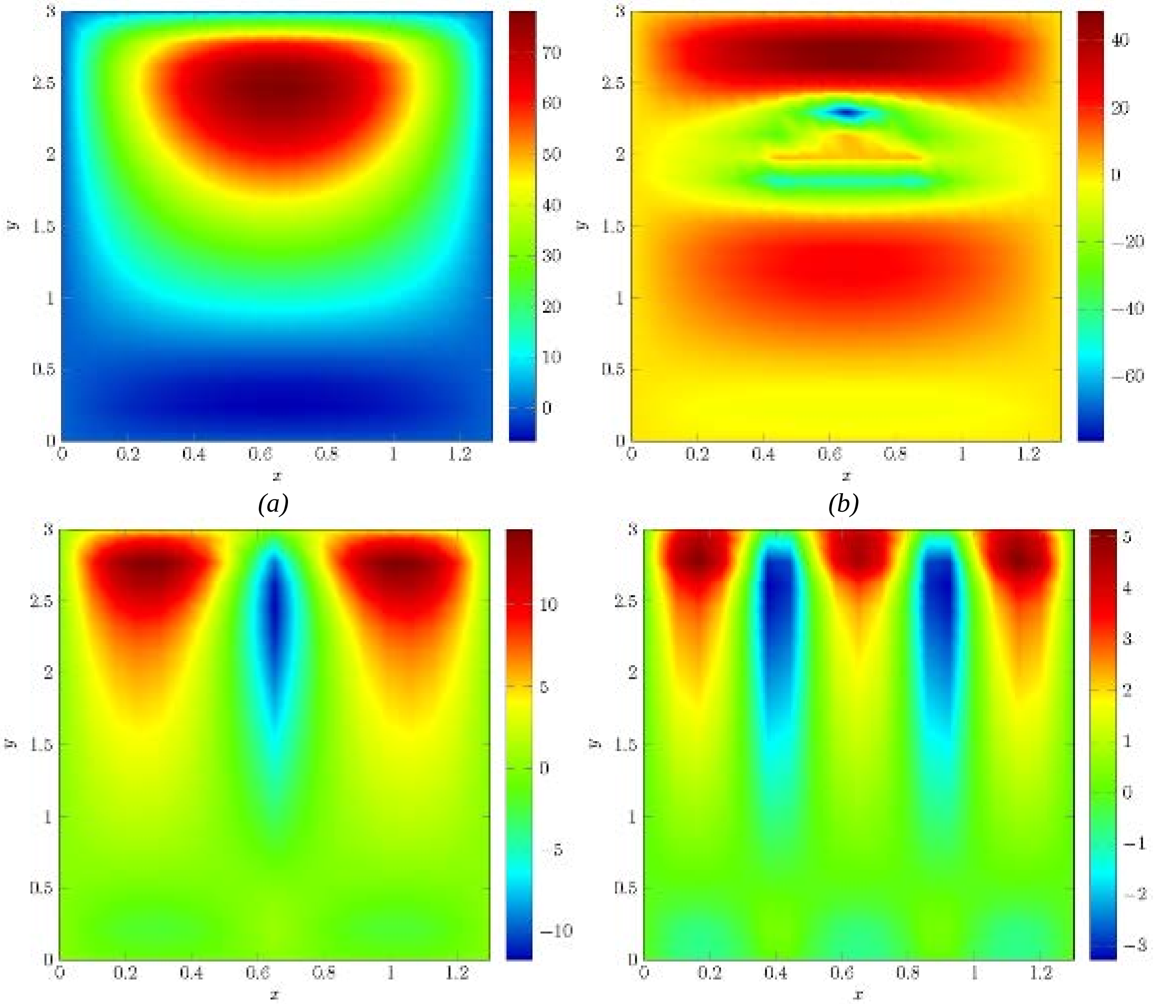


Figure 2. Normal stress distribution $\sigma_{xx}(x, y)$ of the glasses expressed on MPa for the four cases. (a) Stress with no pillars or stripes, (b) Stress with one pillar, (c) Stress with one stripe, (d) Stress with two stripes.

The qualitative behavior of the normal stress along y is different. In Figure 3 it can be observed that for the case of free surface, the maximum value is less than 80 MPa. When a pillar is introduced, a stress concentration is produced leading to a maximum value of around 80 MPa that is, no significant improvement from this viewpoint. On the contrary when one stripe is introduced, the stress concentration is reduced compared to the pillar case and the maximum stress is below 15 MPa, reducing greatly the maximum stress value. The effect is more notorious in the presence of two stripes, leading to a maximum stress value of 5 MPa.



(a) (b) (c) (d)

Figure 3. Normal stress distribution $\sigma_{yy}(x,y)$of the glasses expressed on MPa for the four cases. (a) Stress with no pillars or stripes, (b) Stress with one pillar, (c) Stress with one stripe, (d) Stress with two stripes.

Finally, the shearing stress is represented in Figure 4, obtaining as expected, better results for the presence of pillars and stripes than for the free surface. For the free surface the maximum value obtained is produced on the top corners and its value is around 80 MPa. When the pillar is introduced this value reduces to 30 MPa. In the presence of one stripe this value goes down to less than 20 MPa and around 8 MPa when two stripes are introduced.

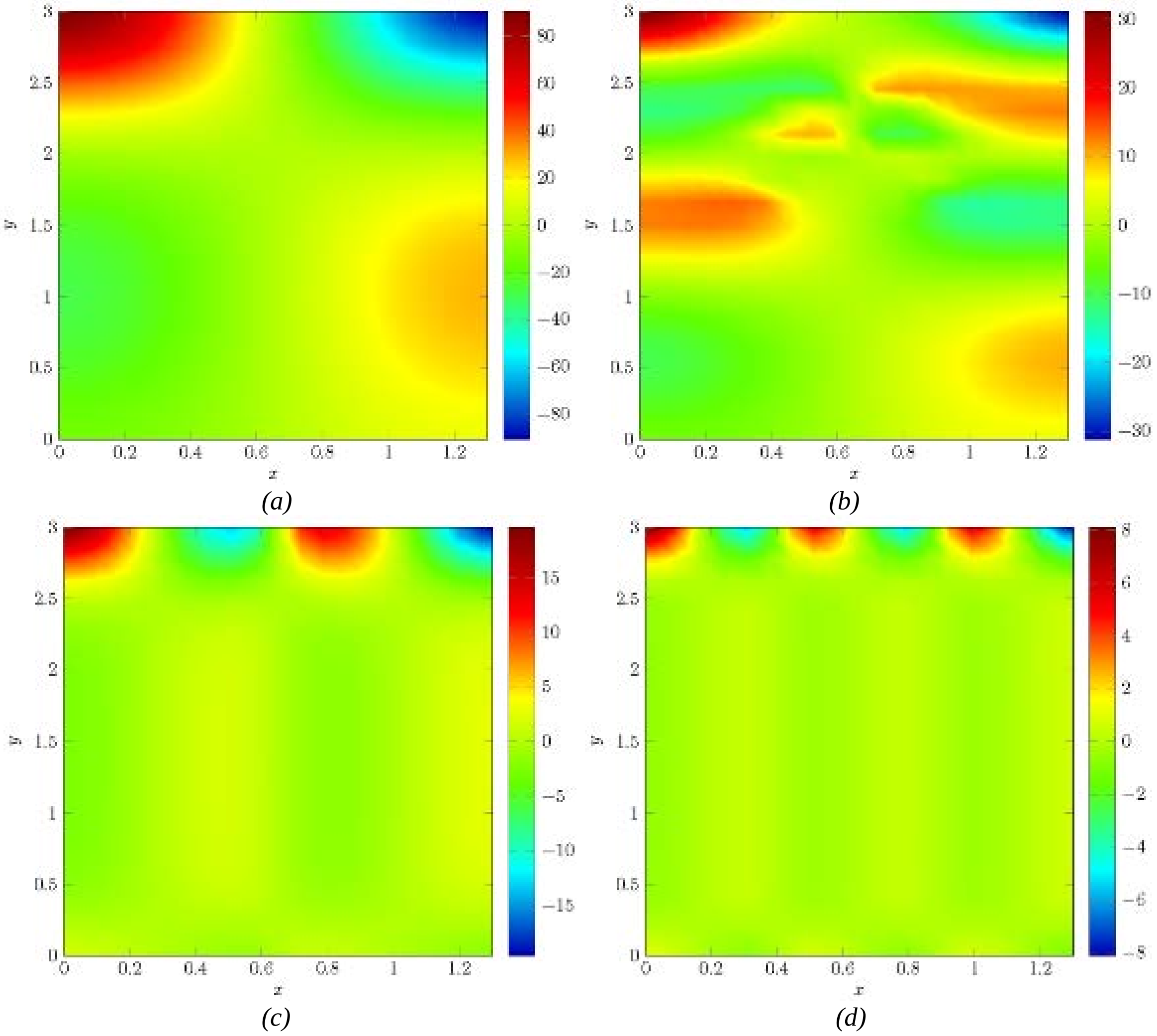


Figure 4. Shear stress distribution $\sigma_{xy}(x,y)$of the glasses expressed on MPa for the four cases. (a) Stress with no pillars or stripes, (b) Stress with one pillar, (c) Stress with one stripe, (d) Stress with two stripes.

4. RESULTS AND DISCUSSION

To sum up the results obtained, the maximum values of each case are listed on Table 1. In this example it can be appreciated a way of selecting the configuration of transversal elements. For example, in the case in which the water chamber thickness is set to be 16 mm, the only viable configuration if the thickness is not increased or the zero-line pressure raised, is the two stripes configuration. This is so, as in other situation the glasses will come to close to each other. In other cases, it will be necessary to perform a trade-off between the thickness, zero-line pressure and the configuration of transversal elements.

Table 1. Maximum values for the different configurations of transversal elements

Case	u (mm)	σ_{xx}(MPa)	σ_{yy}(MPa)	σ_{xy}(MPa)
a)	58.92	132.55	78.16	90.45
b)	10.60	80.89	79.11	31.12
c)	2.66	53.27	14.59	19.59
d)	0.43	14.86	5.14	8.10

It is interesting to remark, that these simulations carried out using the nonlinear plate model lead to a results with the same accuracy than in the linear case. In particular, for case a) the maximum deflection for a nonlinear plate of the same dimensions and hydrostatic pressure distribution lead to a maximum deflection of 59.41 mm and maximum bending stresses σ_{xx} =133.93 MPa, σ_{yy} =78.89 MPa, σ_{xy} =90.79 MPa. The membrane stresses obtained are $\sigma_{xx}^{m} = 0.08$ MPa, $\sigma_{yy}^{m} = 0.15$ MPa, $\sigma_{xy}^{m} = 0.09$ MPa, which are negligible compared to the bending stresses. Hence, in a first approximation the linear plate model suffices to estimate the structural behavior of the glazing.

5. CONCLUSIONS

From the process of modeling the structural behavior of water flow glazings for dimensioning purposes, several conclusions are extracted.

1. The linear plate theory is sufficient model to retain in a first predesign phase the most relevant physical aspects involved in the structural behavior of the glazing. Even though the plate theory conditions are not valid in a region nearby the transversal element, the stresses and deflections predicted by the model are conservative with respect to a three dimensional elastic solid model.

2. The presence of pillars and specially stripes permits to reduce the maximum deflection of each glass for a fixed thickness. Conversely, this will permit to reduce the thickness of the glasses given a maximum deflection value.

3. The presence of transversal elements will prevent the silicon which fixes the frame of the glazing to be submitted to traction stresses which will reduce the lifetime of the glazing.

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