THE FIRST THERMAL ANALYSIS OF AN ARCH BRIDGE: THOMAS YOUNG 1817

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Abstract

The engineers of the 18th century were well aware of the movements suffered by arch bridges due to changes of temperature. In 1801 this problem caused concern to the experts reporting on Telford's design for a 600 feet iron arch. Vicat reported in 1824 "un mouvement périodique" in the arches of the bridge of Souillac and George Rennie published in 1842 the movements observed in Southwark Bridge after its completion in 1818. However the analysis of these perturbations was studied much later, within the frame of elastic theory, by Bresse in 1854 and Rankine in 1862, followed by others like Winkler or Clapeyron, in the last quarter of the 19th century. This is the information that can be found in the general books on the History of the Theory of Structures. In fact, Thomas Young included in his article "Bridge" written for the *Supplement to the fourth edition of the Encyclopaedia Britannica*, and published anonymously in 1817, an analysis of the thermal effects in an iron arch bridge. He deduced first the general expression for shallow segmental arches and then applied it to the arch of Southwark Bridge. Thomas Young's analysis preceded, then, by ca. 40 years the work of Bresse or Rankine. However he received no recognition for this discovery, and it appears that no contemporary English engineer was able to understand his derivations.

Keywords: Arches, iron, temperature effects, history of bridge analysis

1.0 Introduction

Architects and engineers have been aware of the movements experimented by buildings due to changes of temperature. For example, the cracking of the dome of Saint Peter in Rome was attributed ca. 1740 by several experts to thermal effects. The different expansion coefficients of stone and iron were also (at least qualitatively) known, and when Poleni recommended in 1743 to put iron rings outside the dome he observed that they should be placed in summer [1]. Movements in stone bridges were also reported and, for example, in 1824 the French engineer Vicat registered the movements of the Pont de Souillac, which he attributed to changes of temperature. He made, also, a crucial observation. The actual state of the bridge was always changing, and suggested that this should be taken into account: ". . . les grandes voûtes exposées à toutes les intempéries ne sont jamais en équilibre. Je laisse aux savans qui sont particulièrement occupés des conditions de cet équilibre, à discuter l'influence perturbatrice des mouvemens dont je viens de constater la realité" [2]. (The complete answer to the question of the "actual" state of a structure was only possible within the frame of Limit Analysis and the Safe Theorem [3].) In Britain, the collapse of several iron bridges ca. 1800 led to some engineers to the conclusion that the failures were due to the expansion of iron [4] and in the reports written about the feasibility of Telford's grand design for an iron arch of 600 feet over the Thames in London, one of the arguments against the proposal was the thermal effects [5]. It appears that the first systematic study of the movements of an iron bridge due to changes in temperature was made by George Rennie in 1818 during the construction of the arches of Southwark Bridge [6], [7]. However, no structural analysis was made to explain the results. In the classical books on the history of the theory of structures by Todhunter [8], Timoshenko [9] and Charlton [10], the first analysis of the thermal effects in an arch is attributed to Bresse in 1854 [11] and, indeed, the book by Bresse constitutes an exhaustive study of the elastic arches, though he only considered thermal effects in two-hinged arches. Rankine, in his *Manual of civil engineering* of 1862, studied in an approximate way iron arches with fixed ends [12]. In the last quarter of the 19th century the study of the thermal effects in elastic arches appears in many texts on structural analysis. The book by Castigliano [13], published in 1879, is particularly useful for the clarity of exposition and the calculation examples. However, Thomas Young (1773B1829) in the article "Bridge" written in 1817 for the *Supplement* to the 4th edition of the Encyclopaedia Britannica [14], included a correct analysis of the thermal effects on a shallow segmental arch with encastré ends. In fact, the article addresses the whole theory of arch bridge design and Young's fundamental contributions to arch theory have been discussed elsewhere [15]. In what follows a detailed study will be made of Young's pioneering contribution on the thermal analysis of arches.

2.0 "Resistance of materials": the calculation of strains

The article "Bridge" is divided in six parts. We are concerned here mainly with the first part, "Resistance of materials". In it Young extended his theory of "passive strength" (elasticity) already expounded in his *Lectures* of 1807 [16] with a view to its application to arches. Young makes an effort to explain the theory in rigorous terms. The method used by Young is the Aclassical@ method of stating a proposition (named alphabetically from A to Z) and then demonstrating it. This way of exposition makes difficult to follow the general line of reasoning and results particularly exasperating to a modern reader. The propositions though formulated in a general manner are directed to study the arch problem. In this part Young is treating two problems: a) the eccentric compression of a block and the calculation of the resulting stresses; b) the stresses due to thermal effects. We will follow Young's order of exposition.

First he states the proportionality between tensions and deformations and to justify this he expounds a theory of cohesive and repulsive molecular forces and states that even if the law of this forces is not linear (*fig.* 1 in Figure 5)., the effect will be proportional for a small Achange of dimensions@.

He then treats the eccentrical compression of a block and states and begins considering the limit position of an eccentric force so that all the section remains in compression and the corresponding increase in the stresses. However, the way he expressed the problem is as follows: AThe strength of block or beam must be reduced to one half, before its cohesive and repulsive forces can both be called into action@. A modern engineer may have no difficulty in interpreting this: Young is obviously referring to the Amiddle third@ concept and the maximum stress is double as the mean stress. To demonstrate this Young assumes explicitly that plane sections remain plane after the deformation. It follows that the deformations vary linearly and Aconsequently the forces may always be represented, like the pressure of a fluid, at different depths, by the ordinates of a triangle; and their result may be considered as concentrated in the centre of gravity of the triangle, or of such of its portions as are contained within the depth of the substance.@ Here Young is struggling with the concept of stress and he uses the analogy of the pressure of a fluid. However he tries always to speak in terms of deformations, the Aforces@ or Apressures@ being always proportional to them, as stated in the first proposition (*fig.* 2 in Fig.1).

The next proposition states that the compression or the extension of the axis of the block or beam is always proportional to the force, reduced to the direction of the axis, at whatever distance it may be applied@. The deformation of the axis is always equal to the mean deformation, produced by the normal component of the force applied in the middle of the section. The transverse component of the force will be resisted by Alateral adhesion@ (shear) and if the force is normal to the axis Athe length of the axis will remain unaltered@.

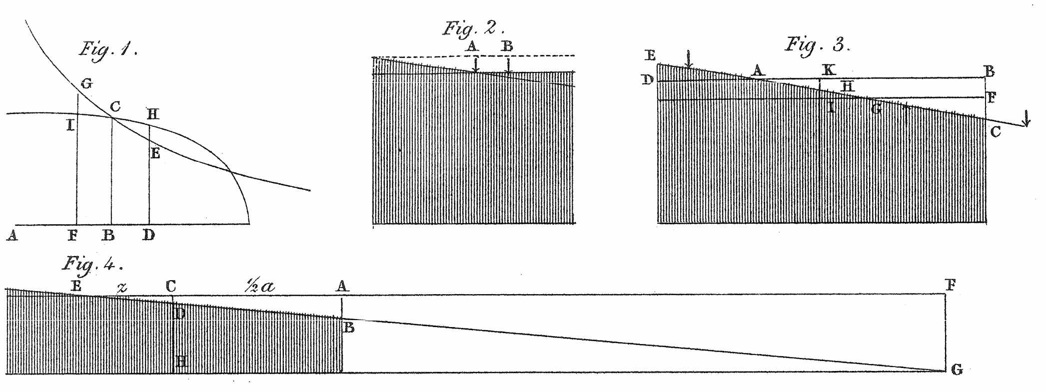
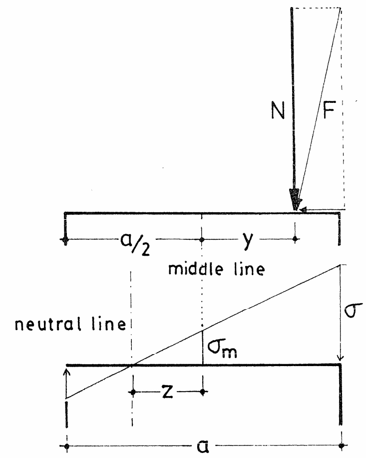


Fig. 1. Drawings on AResistance of materials@. Young is concerned with the Acompression@ or Aextension@ of a joint which remains plane after deformation [14].

Then Young proceeds to locate the neutral point for this general force placed at any distance: AThe distance of the neutral point from the axis is to the depth, as the depth to twelve times the distance of the force, measured in the transverse section@. In an algebraical form

$$z = \frac{a^2}{12y} \tag{1}$$

where z is the distance of the neutral point from the axis, a is de depth of the section and y is the distance of the point of application of the force to the axis (Fig. 2). Young=s demonstration is based in the proportionality of the stress resultants and the triangular form of the stress blocks.



Influence of the position of the thrust:

location of the neutral line:

$$z = \frac{a^2}{12y}$$

increase of the stress:

$$\sigma = \sigma_m \left(\frac{a + 6y}{a} \right)$$

where

$$\sigma_m = \frac{N}{a} \quad \text{(per unit breadth)}$$

Fig. 2. Young=s propositions on AResistance of materials@ expressed in modern terms (stresses).

The next proposition tries to relate the increase of the normal stresses in terms of the distance of the force from the axis (Fig. 2): AThe power of a given force to crush a block, is increased by its removal from the axis, supposing its direction unaltered, in the same proportion as the depth of the block is increased by the addition of six times the distance of the point of application of the force, measured in the transverse section@. Young is referring to the increase of the strains (stresses) due to the eccentricity of the load. In modern terms, if we call σ_m the mean compressive stress produced by the force applied in the centre of the section, the removal of the force at a distance y will produce a stress σ given by

$$\sigma = \sigma_m \left(\frac{a + 6y}{a} \right) \tag{2}$$

Young demonstrates the assertion, again, for similar triangles (*fig*.3 in Fig. 1). Therefore, now we are in the situation to ascertain the Astrength@ of any section acted by any force located at any distance, comparing the maximum deformation (strain) of the section with the corresponding fracture value for the material. This was the objective of the first four propositions.

Now Young turns to the study of the flexural deformation of the blocks or sections of the arch. He begins studying the curvature produced in the neutral line by any given force: AThe curvature of the neutral line of a beam at any point, produced by a given force, is proportional to the distance of the line of the direction of the force from the given point of the axis, whatever that direction may be@. He is stating that the curvature is proportional to the bending moment produced by the force. He has already shown that the distance z of the neutral point to the axis is inversely proportional to the distance of the force y, equation (1). Now, with reference to *fig.* 4 in Fig. 1, is evident that z/CD = r/(CH) or the curvature $\kappa = 1/r = (1/z)$(CH/CD) = y (CH/CD). As (CH/CD) is proportional to the force f, it follows that

$$\kappa = \text{ConstantH } (f\,y) \tag{3}$$

i.e. is proportional to the bending moment. Young do not use the term ‘bending moment’, but remarks that if the force f is inclined the distance y should be measured through the perpendicular to the direction of the force. Finally, Young states that Athe radius of curvature of the axis will always be to that of the neutral line as the acquired to the original length of the axis@, as is evident with reference to the cited figure.

Then, Young defines the constant which relates the curvature κ with the bending moment ($f\,y$): AThe radius of curvature of the neutral line is to the distance of the neutral point as the original length of the axis to the alteration of that length; or as a given certain quantity to the external force: and this quantity has been termed the Modulus of elasticity@. For this he makes use of the concept of AModulus of elasticity@, which, in modern terms relates the stresses and strains. He considers that the reader is already familiar with the concept. In fact, Young=s definition of the modulus of elasticity, given in his *Lectures* [16] is anything but clear: AThe modulus of elasticity of any substance is a column of the same substance, capable of producing a pressure on its base which is to the weight of causing a certain degree of compression, as the length of this substance is to the diminution of its length.@ Young sometimes speaks of the Aweight of the modulus@ (which is EA, being E the modern definition of the modulus and A the area of the cross section) or of the Aheight of the modulus@ (which is E/γ, being γ the specific weight of the material; this last definition is independent of the cross section).

Using the same equation as in the preceding proposition $r/z = M/f$, being M ($=EA$) the weight of the modulus. Now, as $z = a^2/(12y)$, $r = Mz/f = M\,a^2/(12fy)$. In modern terms

$$(fy) = (1/r)(Ma^2/12) = \kappa\,(EI), \tag{4}$$

being I the second moment of area of a rectangle of height a and breadth unity.

The next proposition establish the relationship between the strain of the axis and the strain in the line of the direction of the force: AThe flexibility, referred to the direction of the force, is expressed by unity, increased twelve times the square of the distance, divided by that of the depth@. Now Young is looking after the relationship between the strain of the middle axis ε_m and the strain in the line of action of the force ε. It is evident in *fig*.4 of Fig. 1, that CD/z = FG/($z + y$), then (FG/CD) = ($z + y$)/z = 1 + (y/z), that is

$$\frac{\varepsilon}{\varepsilon_m} = 1 + 12\frac{y^2}{a^2}\,. \tag{5}$$

If the direction of the force is oblique the relationship remains the same if we consider the strain of the axis in a direction parallel to that of the force, as will be obvious from.

3.0 Thermal effects

Now Young has all what he need to attack the problem of the effect of a temperature change. An increment of temperature θ will produce a strain ($\theta\ v$) where v is the coefficient of thermal expansion. In an arch of fixed abutments this strain will induce a set of internal forces which will be superposed to the internal forces due to the load. The calculation of the strain of a straight bar is easy: Young set himself a difficult problem to calculate the relationship between the strain of the cord and the maximum strain in a shallow circular arch with encastré ends. He resumed the result of his investigations in the following proposition:

AIf a solid bar have its axis curved a little into a circular form, and an external force be then applied in the direction of the chord, while the extremities retain their angular position, the greatest compression or extension of the substance will ultimately be to the mean compression or extension which takes place in the direction of the cord as

$$\left(1+\frac{4h}{a}\right) \text{ to } \left(1+\frac{16h^2}{15a^2}\right)$$

a being the depth of the bar, and h the actual versed sine, or the height of the arch@.

The solution of the problem implies the elastic analysis of a segmental circular arch with fixed ends. Young applied the Aprinciple of superposition@ and ideas of the compatibility of deformation: AWe must here separate the actions of the forces retaining the ends of the bar into two parts, the one simply urging the bar in the direction of the chord, and the other, which is of a more complicated nature, keeping the angular direction unaltered; and we must first calculate the variation of the angular situation of the ends, in consequence of the bending of the bar by the first portion, and then the strain required to obviate that change, by means of a force acting in the direction of the middle of the bar, while the ends are supposed to be fixed@.

There is no drawing in the article to explain the reader this application of the principle of superposition. The approach has been summarized in Fig. 3. Young knows that the deformation must be symmetrical. First, then, he calculates the angular variation of the extreme of a semi-arch fixed at the keystone acted by a horizontal force p at the lower end, Fig. 3 (a). Calling the angle which determines the position of a section x, r the radius of curvature of the middle line of the arch and y the vertical distance of the section to the chord of the arc, then the bending moment will be p ($r$cos$x$B b), where b is the cosine of the whole semi arc c (b = $r$cos$c$). If we call the inclination of the bar i the curvature κ = di/ds = p (r cos$x$ Bb)/EI; but ds = rdx, and di = (r/EI) p(r cos$x$ Bb)dx. The angular variation will be the integral for the whole semi arc, that is, it will be proportional to p (r sin c B bc), and this is the result which Young correctly gives (he drops the constant 1/EI).

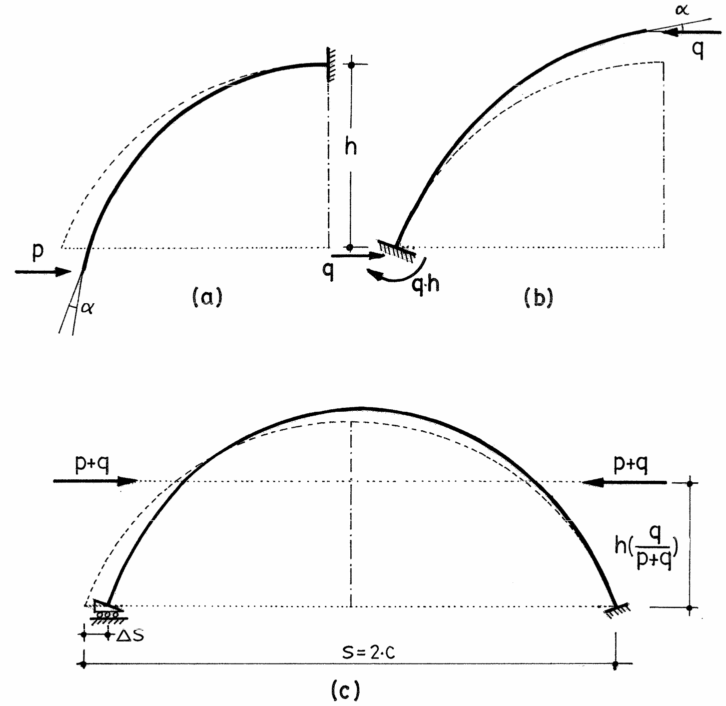


Fig. 3. Study of the stresses in an arch encastré in both abutments due to a horizontal displacement of the abutments, maintaining the angular inclination constant.

Then he calculates the angular variation of the extreme of the semi arch fixed at the lower end due to the action of another horizontal force q acting at the superior end, Fig. 3 (b). Now the bending moment is $q(r\mathrm{B}r\cos x)$ and, as before, $di = (1/EI)\ q(r\ \mathrm{B}\ r\ \cos x)dx$. The angular variation for the whole arc, after the integration, will be proportional to $q(rc\mathrm{B}\ r\sin c)$, the constant the same as before $1/EI$. For compatibility of deformation the two angular variations must be equal, therefore

$$\frac{p}{q} = \frac{rc - r\sin c}{r\sin c - bc} \tag{6}$$

The deduction is completely general for any arch with an opening angle $2c$. Now Young introduces the simplification for a shallow arch. When the arc is small

$$\sin c = c - \frac{1}{6}c^3 + \frac{1}{120}c^5 \ldots$$

considering only the first two terms and supposing that ($r\ \mathrm{B}\ b$), the height of the arch, is very nearly $(2)rc^2$ (i.e. approximating the arc to a parabola), and now c representing the length of the semi chord. Substituting these values, we obtain $p/q = 1/2$.

Now, in the whole arch the two actions must be summed. At the lower ends of the arch act a force $(p + q)$ and a moment qh. We can reduce this to a single force $(p + q)$ acting at a distance $(p + q)/q = (1 + p/q)$ from the chord, Fig. 3 (c). For a shallow arch ($p/q = ½$) this distance becomes 2/3 of the height of the arch or, as Young expresses it Awhen the force is considered as single, the distance *d* of the line of its direction from the summit must ultimately be one-third of the versed sine or height@.

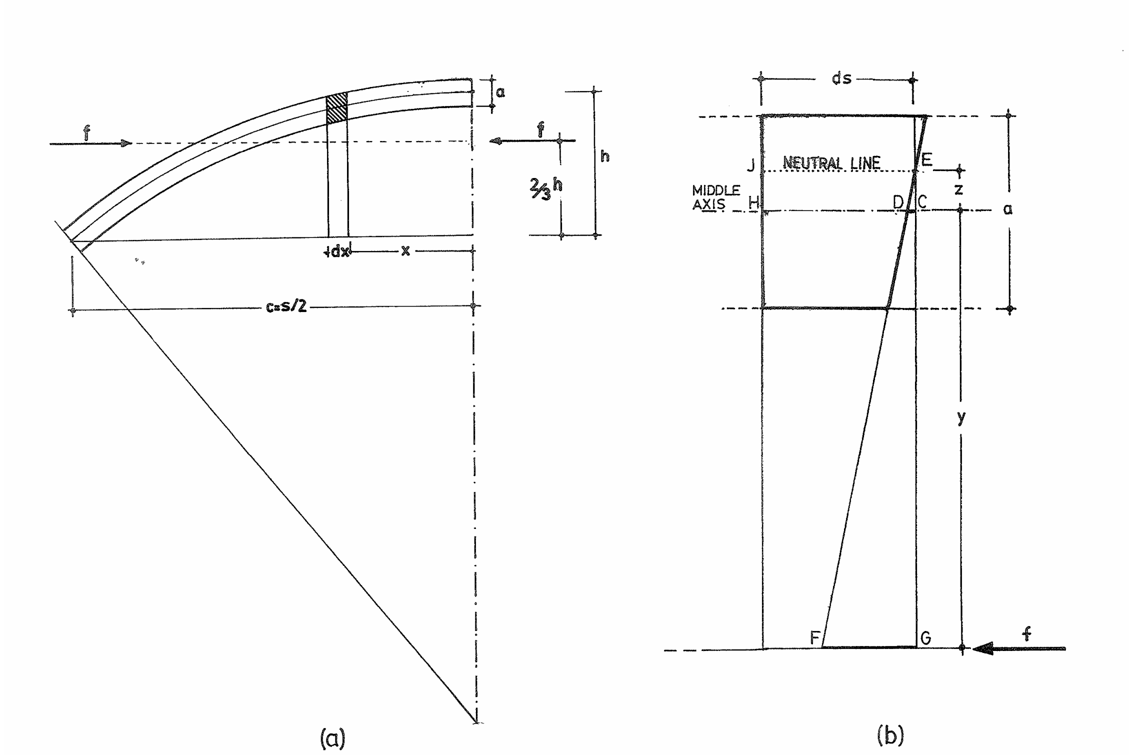


Fig.4. Deformation of an arch due to the normal strains.

Now Young turns to calculate the reduction of length of the arch chord. The differential variation of the chord of the arch due to the action of a certain force *f*, for a cross section *A* and a modulus of elasticity *E*, is

$$d\delta = \varepsilon_m \left(1 + 12\frac{y^2}{a^2}\right)dx = \left[\frac{f}{EA}\right]\left(1 + 12\frac{y^2}{a^2}\right)dx \qquad (7)$$

where *y* is the vertical distance of the middle line to the chord, i. e. the line of action of the force *f*.

The variation of length will be the integral through the whole span $s = 2c$. As the arch is shallow, Young approximates the circular arc to a parabola, $y = (h/3)$B$(x^2/(2r))$, where *h* is the height of the arc and *r* is the radius of curvature at the crown. Another property of the parabola is that $h = c^2/(2r)$, where *c* is half the chord. Substituting these values in eqn. (7), and integrating, we obtain:

$$\delta = \Delta s = 2\left[\frac{f}{EA}\right]\left(c + \frac{16h^2c}{15a^2}\right) = s\left[\frac{f}{EA}\right]\left(1 + \frac{16h^2}{15a^2}\right) \tag{8}$$

and the right bracket the relationship between the variation of length of the chord of the arch and that of the straight bar of the length of the chord. For a given deformation the force *f* at the abutments of the arch will be reduced by this factor in relation with the force applied to the straight bar for the same deformation, and the mean stress at the crown will be reduced by the same factor.

Now this force *f* is located at (2/3)*h*, and the maximum stress will be at the springings and the relationship to the mean stress at the crown will be given by eqn. (2). Therefore the stress at the springings will be (1+4(*h*/*a*)) times the mean stress (disregarding the effect of the inclination of the section, which is very small for shallow arches). Therefore the maximum stress or strain at the springing of the arch, due to a certain increment or decrement of the span of the arch, maintaining the inclination of the springings, will be

$$\sigma = \sigma_m \left[\frac{1 + \frac{4h}{a}}{1 + \frac{16h^2}{15a^2}}\right]. \tag{9}$$

2.1 Example of application

Immediately after the preceding demonstration, Young gives an example of the application of his proposition to the calculation of the maximum stress due to a certain increase (or decrease) of temperature. He considers an arch with a constant depth a = 10 ft. and a height h = 20 ft.; as we have seen the span *s* does not enter in the calculations. Then the term in brackets, which gives the relationship between the maximum stress in the springing and the mean stress at the crown, will have a value of (9/5.267) or nearly 17/10

If the arch suffers a change of temperature of θ=32° Fahrenheit (17.8° C), this will lead to a variation of length (a 'strain' ε in modern terms) of $\theta\upsilon$ = 1/5000. (In his *Lectures* he gives for cast iron υ = 6.18 10^{-6} °F, referring to the experiences of Lavoisier; so it appears he is rounding the value of 1/5056.) The whole arch will try to expand or contract 1/5000, but the length of the chord, between the abutments must remain the same, and therefore an external force must act which will reduce its length by this amount. The maximum strain at the abutments will be 10/17 of this quantity, that is nearly 1/3000, "which is the equivalent of the pressure of a column of the metal of about 3300 feet in height, since *M*, the height of the modulus of elasticity is found, for iron and steel, to be about 10,000,000 feet".

Young obtains the stress multiplying the strain by the modulus of elasticity, but he is obtaining the "stress" not as force divided by area, but as the stress at the base of a column of constant section of certain height. It is not difficult to see that both methods lead to the same result. As we have seen, $M = E/\gamma$, then $\varepsilon M = \varepsilon\, E/\gamma = \sigma/\gamma$ which is the height of a column of uniform section build of a material of specific weight γ which presents a stress σ at its base. (This form of measuring the stresses in buildings was first proposed by Gautier in 1716 and was employed by Perronet to compare the stresses in different buildings. The breaking strength, then, was the limit height which can be build with a certain material and this parameter was still present in many engineering handbooks of the 19^{th} [17].)

A height of 3300 feet leads to a stress of 5 tons/sqin. Or 77 N/mm². Young finds this stress quite high and affirms that in this case "[it] would certainly require particular precaution, to prevent the destruction of the stones forming the abutment by a force so much greater than they are capable of withstanding without assistance".

2.2 Southwark bridge

The last part of the article Bridge is dedicated to the "description of some of the most remarkable bridges which have been erected in modern times". It begins with a brief, but excellent, history of cast iron bridges, but most of the space is dedicated to a detailed analysis, employing his theory of arches and passive strength, of two bridges, the Southwark and Waterloo bridges. The first is of cast iron and the second of stone, and it is evident that Young wants to show that his theory can be applied to either material.

In the case of Southwark Bridge a thermal analysis is made, following his method, explained above. The span of the central arch was 240 feet and the engineer John Rennie was worried about the possible effects of thermal expansion. The Wearmouth Bridge, of similar dimensions, has presented many problems and some other cast iron bridges have collapsed. In his autobiography, he says, referring to the arches of Southwark bridge: "A those arches were the largest of the kind ever constructed, considerable doubts as to their stability occurred to many, and the subject was discussed amongst scientific men with considerable energy; and amongst others, the celebrated Dr. Young undertook to investigate Mr. Rennies calculations, and came to the conclusion that the bridge was well designed, and would be a perfectly safe structure"[7]. It appears that Rennie is referring to the article of the Encyclopaedia Britannica, as Napier, the editor, wrote to Young just after publication: "I have been amused with two or three conjectures as to the write of this remarkable article. Rennie has been sadly puzzled by the signature O.R. which, when separated, stands for two other contributors in the list, but whose combined strength, as he rightly says, could not have produced Bridge" [18]. (Young contributed anonymously to the Encyclopaedia, and signed his articles by two consecutive letters of the Latin phrase "Fortunam ex aliis"; only in 1824, five years before his death, he gave permission to include his name. He was afraid that his work in other fields could damage his profession of physician.) It is surprising that George Rennie does not even mention Young's contribution in his paper on the expansion of arches [6]. No doubt he was unable to understand it.

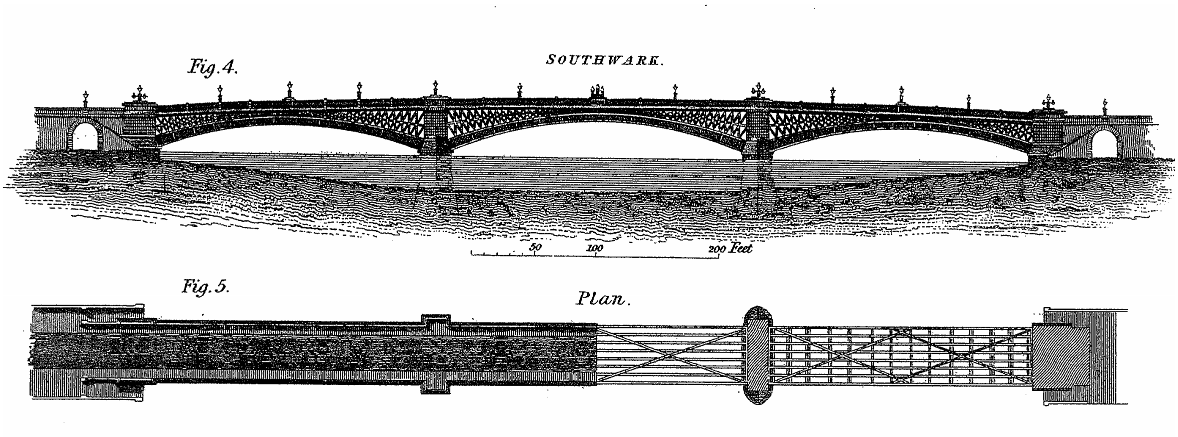


Fig. 5. Southwark Bridge

Southwark arches have varying section, so Young took a mean depth of *a* = 7 feet. The height of the arch *h* = 23 feet. Then, $1+4(h/a) = 14.14$ and $1+(16h^2/15a^2)=12.52$. Then the relationship between the maximum and mean stress is 1.129. "If, in a long and sever frost, the temperature varied from 52° to 20°, since the general dimensions will contract 1/5000, the extreme parts of the blocks near the abutments would vary 1.129/5000 of their length". Now, as the height of the modulus is 10.000.000 feet, the stress is represented by a column of 2258 feet height, that is 3 tons/sqin or 46.5 N/mm². Now he compares this value with the stress due to the dead load of the bridge, which he estimates in 2 tons/sqin or 31 N/mm². The change of temperature will more than double this stress. The concern is that such a concentration of stresses may crack the supporting stones in the abutments and Young praises Rennie's solution of "causing the blocks to bear somewhat more strongly on the abutments at the middle than at the sides, so as to allow some little latitude of elevation and depression, in the nature of the joint". I have not been able to see the actual details, but it appears that Young is referring to making the supporting joint with a little concavity, a disposition much used in the second half of the 19th [19].

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