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Defect Detection Combining Multi-frequency Ultrasonic Guided Waves and Topological Derivative

A. Martínez[1], J.A. Güemes[2], J.M. Perales[3] and J.M. Vega[1]

[1] Dpto. Matemática Aplicada a la Ingeniería Aeroespacial, E.T.S.I. Aeronáutica y del Espacio, Universidad Politécnica de Madrid, Madrid, Spain.
[2] Dpto. Materiales y Producción Aeroespacial, E.T.S.I. Aeronáutica y del Espacio, Universidad Politécnica de Madrid, Madrid, Spain.
[3] Dpto. Aeronaves y Vehículos Espaciales, E.T.S.I. Aeronáutica y del Espacio, Universidad Politécnica de Madrid, Madrid, Spain.

Summary

Ultrasonic guided waves are an attractive alternative to conventional methods because the elastic waves emitted at one location travel over a long distance. By analyzing these waves, the presence of flaws may be detected. Mathematically, this is an inverse problem. A large variety of mathematical methods to solve inverse problems consist in minimizing an instrumental objective function, which gives the difference between the measured and calculated signals. Among these, the topological derivative describes the sensitivity of the objective function to infinitesimal inclusions on the material.

INTRODUCTION

- Formulate, by means of elasticity equations, the **direct problem** in the frequency domain.
- **FEM method** for the integration of the elasticity equations.
- Defining the functional $\mathbf{J} = \frac{1}{2}\sum_{i=1}^{Nreceptors} ||\vec{u} - \overrightarrow{u_{exp}}||^2$.
- The functional **J** measures the difference between the received signal in the **piezo-electric sensors** and the one theoretically computated.
- Estimation of the defects minimizing the functional **J**.
- **Adjoint formulation** for the computation of the sensitivity of **J**.

METHODS

Structural flaws produce variations in the elastic properties of a material such as density and Young's module. The methods to be developed intend to estimating those variations from the structural response to elastic excitations. Thus, we are trying to solve an **inverse problem**.

This inverse problem is ill-posed, it could not have a solution or it could have infinite approximate solutions. Then, we use a less demanding formulation which is looking for the defects within the structure that minimize functional **J**.

The **topological sensitivity** of a functional is a scalar spatial distribution such that its pronounced negative values, occur at those regions where defects are more likely to be located.

More precisely, the topological sensitivity of an objective functional **J**(R) is defined at each point **x** ϵ R as follows:

$$\mathcal{D}(\mathbf{x}) := \lim_{\varepsilon \to 0} \frac{\mathcal{J}(\mathcal{R} \setminus B_\varepsilon(\mathbf{x})) - \mathcal{J}(\mathcal{R})}{h(\varepsilon)}, \qquad \mathbf{x} \in \mathcal{R},$$

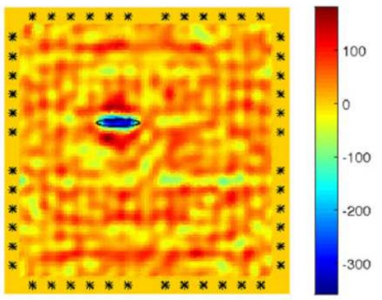
where B_ϵ is a ball with radius ε centered at **x**, and h(ε) > 0 is a monotone decreasing function satisfying $\lim_{\varepsilon \to 0} h(\varepsilon) = 0$ and selected such that the previous limit exists and is non-zero. That limit can equivanlently be written as:

$$\mathcal{J}(\mathcal{R} \setminus B_\varepsilon(\mathbf{x})) = \mathcal{J}(\mathcal{R}) + h(\varepsilon)\mathcal{D}(\mathbf{x}) + o(h(\varepsilon)), \quad \text{as } \varepsilon \to 0,$$

which motivates the idea of agglutinating points where the topological derivative is large and negative, since in such points:

$$\mathcal{J}(\mathcal{R} \setminus B_\varepsilon(\mathbf{x})) < \mathcal{J}(\mathcal{R})$$

As a result, topological sensitivvity could also be seen as the sensitivity of J with respect to density and Young's module perturbations inside the structure.



Topological sensitivity in an acoustic problem

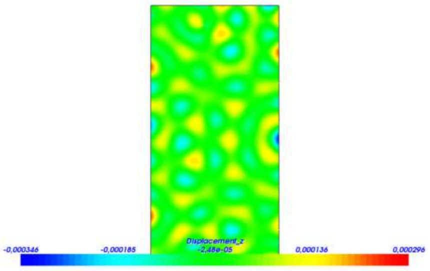
RESULTS

In order to compute the topological sentitivity we need to solve the next three problems:

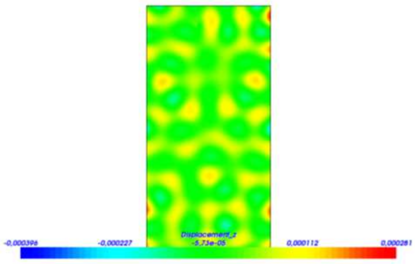
- **Direct** problem, in the domain with defects.
- **Unperturbed direct** problem, in the domain without defects.
- **Adjoint** problem, in the domain without defects. This problem includes a forcing term proportional to $\vec{u} - \overrightarrow{u_{exp}}$, where $\vec{u}$ is the solution of the unperturbed problem and $\overrightarrow{u_{exp}}$ is the solution of the direct one.

In this work, pre-processing, processing and post-processing steps are being developed by means of open source software such as **Gmsh**, **Elmer and Paraview**.

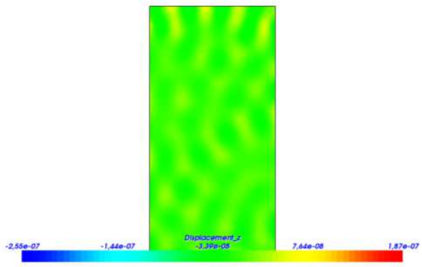
This is an example of these three problems in an aluminium plate excited with a tangential load at a frequency of 7300hz :



Direct problem. Displacement along z-axis



Unperturbed direct problem. Displacement along z-axis



Adjoint problem. Displacement along z-axis

CONCLUSIONS

- Our aim is to develop a **NDT method based on topological sensitivity** to compete with conventional ultrasonic testers.
- For the sake of simplicity, in this preliminary study, the excitation of the plate is being done by point loads. Subsequently, the excitation will be the one which best simulates the piezoelectric emitters in order to generate proper **Lamb waves**.
- After the numerical simulations, we will perform an **experimental implementation** of the problem.