FINITE GEOMETRY EFFECTS IN THE OSCILLATORY INSTABILITY

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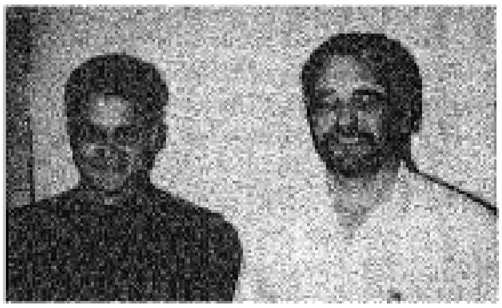

Abstract

In this work we study the effect of the boundary conditions in the oscillatory instability in large domains with perfectly reflecting sidewalls (that is the case when the original physical system has isolated boundary conditions). As is well known [1], at the onset of this instability a pair of counter propagating and nonlinearly interacting wavetrains appears. The equations [2] governing the evolution (in a slow time scale) of the envelopes of these wavetrains are:

$$
\begin{aligned}
A_t &= (1+ia_1)A_{xx} + \frac{1}{\sqrt{|\epsilon|}}A_x + A - (1+ia_2)A|A|^2 - (a_3+ia_4)A|B|^2 \\
B_t &= (1+ia_1)B_{xx} - \frac{1}{\sqrt{|\epsilon|}}B_x + B - (1+ia_2)B|B|^2 - (a_3+ia_4)B|A|^2
\end{aligned}
\qquad (1)
$$

together with [3,4] :

$$
\begin{aligned}
B &= Ae^{i\theta}, \quad B_x = -A_x e^{i\theta} \quad \text{at } x = 0, \\
A &= Be^{i\theta}, \quad A_x = -B_x e^{i\theta} \quad \text{at } x = L\sqrt{|\epsilon|},
\end{aligned}
\qquad (2)
$$

where $\epsilon \ll 1$ is the bifurcation parameter and $L \gg 1$ is the length of the spatial domain, that is large compared with the basic wavelength. We assume $1 + a_3 > 0$ to ensure supercriticality.

Two distinguished limits are considered:

1. Averaged coupling limit: $\epsilon L^2 \sim 1$.

The slow scales are $t \sim \frac{1}{\sqrt{|\epsilon|}}$, $t \sim \frac{1}{|\epsilon|}$ and $x \sim \frac{1}{\sqrt{|\epsilon|}}$. Applying the multiple-scale method to (1)+(2), the resulting equation written in characteristic variables and appropiately scaled is [5]:

$$W_t = (1+ia_1)e^{i\theta x}(e^{-i\theta x}W)_{xx} + W(\beta - (1+ia_2)|W|^2 - (a_3+ia_4)\tfrac{1}{2}\int_0^2 |W(u,t)|^2 du)$$
$$W(x+2,t) = W(x,t) \tag{3}$$

where $\beta = \epsilon L^2$ and the effect of the interaction between the counterpropagating wave trains is represented by the integral averaging term.

The analysis of the stability of the solutions with constant modulus leads, for appropiante values of the parameters (i.e. $1 + a_1 a_2 < 0$), to modulational instability (as the standard Ginzburg-Landau equation does[6]) and phase turbulence. Also, nonuniform steady solutions of (3) provide an explanation of the blinking states[3] that are frecuently observed in experiments.

2. Hyperbolic limit: $\epsilon L \sim 1$.

Now the slow scales are $t \sim \frac{1}{\sqrt{|\epsilon|}}$, $t \sim \frac{1}{|\epsilon|}$, $x \sim \frac{1}{\sqrt{|\epsilon|}}$ and $t \sim \frac{1}{|\epsilon|}$. If we consider that only the slowest scales appear we can neglect the diffusion terms in (1) and the rescaled equations for $(a,b) = (|A|^2, |B|^2)$ are ($\beta = \epsilon L$):

$$\begin{aligned} &a_t + a_x = 2a(\beta - a - a_3 b) \\ &b_t - b_x = 2b(\beta - b - a_3 a) \\ &\text{with} \quad a = b \quad \text{at} \quad x = 0, 1. \end{aligned} \tag{4}$$

This system of first order hyperbolic equations has been integrated numerically using a predictor corrector method along the characteristic lines.

Increasing β from 0 we found uniform steady solutions, nonuniform steady solutions (confined states), unaymmetrical periodic solutions (beating states), symmetrical periodic solutions (alternating states) and successive period doubings leading to a chaotic behavior.

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