

ON THE PROCESS OF FLAME SPREADING OVER THE SURFACE OF PLASTIC FUELS IN AN OXIDIZING ATMOSPHERE

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A general theory of flame spreading over the surface of solid or liquid fuels in an oxidizing atmosphere is developed in the present work.

The model of the process considers heating of the fuel ahead of the flame, fuel vaporization and mixing with the gaseous oxidizer and flame propagation through this combustible mixture. The flame spreading velocity results from the balance of all these processes.

This theory is of a general type and may be applied to many kinds of flame spreading processes. However, the work has been directed to the case of flame spreading over the surface of plastic fuels in nitrogen-oxygen mixtures.

The model of the process is solved by approximate analytical methods and an experimental investigation is also carried out. Theoretical and experimental results are obtained and compared, showing the influence of initial fuel temperature, pressure and mixture composition.

1. Introduction

A complete analysis of the general problem of flame spreading along the interface between a solid or liquid fuel and a gaseous atmosphere which can react with the fuel vapors has not yet been carried out although a large number of studies, mostly experimental, have been performed on several aspects of the problem, especially in connection with fire safety regulations and fire spreading in air.¹⁻⁴ Flame spreading processes have also been studied in rocket motors, but most of these studies refer to the case of solid monopropellants, which is a problem of a different nature. On the other hand, very little information exists on flame spreading processes over the surface of solid propellants in a reactant atmosphere in connection with ignition processes in hybrid rockets.^{5,6}

A thermal theory of flame spreading along the surface dividing a solid or liquid fuel from an oxidizing atmosphere was developed by the authors of the present work.⁶

This theory was essentially based on the existence of an ignition temperature, and the flame spreading velocity was calculated on the assumption that the flame propagates to a certain location when the fuel surface temperature at that location reaches an ignition temperature.

The introduction of the concept of the ignition temperature permits a simple study of the process. However, since the ignition temper-

ature is not a real physico-chemical property of the process, its value is uncertain because it depends on the ignition procedure. Furthermore, the influence of very important variables of the process, such as the pressure and composition of the gaseous atmosphere, cannot be calculated theoretically. Another additional difficulty is that spreading velocities cannot be studied for initial temperatures of the fuel equal to, or higher than, the ignition temperature.

A more general theory of flame spreading is developed in the present work. The concept of ignition temperature has been eliminated and all the essential processes which are part of the over-all flame spreading process are taken into account: heating of the fuel by the flame, fuel vaporization, mixing of the fuel vapors with the gaseous atmosphere and flame propagation through this combustible mixture; the flame spreading velocity results from the balance of all these processes. The over-all process is extremely complex because of the complicated nature of the partial processes, and no information is available on the values of many important parameters, especially in connection with evaporation laws and chemical kinetics of combustion. Therefore, the scope of the present work will be limited to the development of a general model of the over-all process which will permit a qualitative study of the spreading velocity. This model will also permit the analysis of the influence on this velocity of each partial process as well as that of the main variables of all these

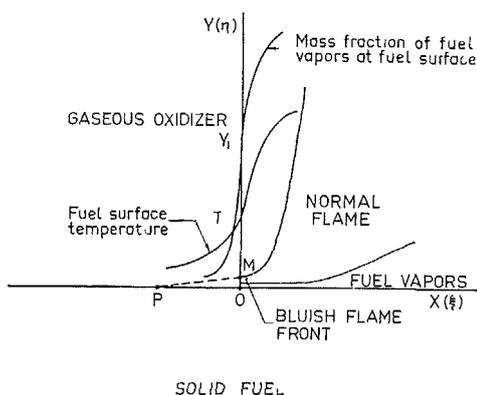


FIG 1 Flame propagation model

processes. The theoretical model of the spreading process will be of a very general type. However, the work will be directed to the case of flame spreading over the surface of plastic fuels in nitrogen-oxygen mixtures, on which an experimental program has also been carried out in order to have a qualitative comparison between theoretical and experimental results.

2. Fundamental Assumptions

2.1 The following fundamental assumptions have been made. Conditions are stationary with respect to a coordinate system moving with the flame, chemical reactions take place in the gas phase, and there is bidimensional symmetry. The flame spreading process results from heating and vaporization of the fuel, mixing of the fuel vapors with the gaseous oxidizer and flame propagation through this combustible mixture.

Starting with an assumed value u of the spreading velocity and using approximate expressions for the heat fluxes from the flame to the fuel, the temperature distribution at the fuel surface $T_s = f_1(v, u)$ can be calculated. With

these temperatures and using a vaporization law, diffusion equations give the fuel vapor mass fraction at the fuel surface, $Y_{1s} = f_2(x, u)$. Finally, the flame propagation velocity can be calculated as function of Y_{1s} , and by equating the initial and final velocities at $v = 0$, the actual spreading velocity is obtained.

The study of all these processes will be based on several assumptions which are discussed in the following paragraphs.

2.2 Heating of the Fuel

Heating of the fuel ahead of the flame may take place through radiation, conduction or convection. A point P at the fuel surface ahead of the flame will be heated directly by radiation, and by heat conduction through the gaseous atmosphere. It can also be heated through convection and radiation from the flame to the fuel and then by heat conduction within the fuel, following a path such as MOP (Fig 1). Heat conduction through the gaseous atmosphere predominates over heat conduction within the fuel, if the thermal diffusivity $\lambda/\rho c_p$ of the gas is larger than the thermal diffusivity of the fuel. This is the case, at ambient pressure, for N_2 - O_2 mixtures and plastic fuels, but the opposite occurs at high pressure or for other materials (Table I).

The radiant heat flux reaching the fuel surface ahead of the flame may be approximated by an expression of the form

$$Q_R(v) = \epsilon \sigma T_F^4 F \quad (1)$$

in which F is the geometrical factor, which may be approximated by an exponential function of the distance from the flame front. The flame emissivity ϵ depends exponentially on the flame size.

Heat transferred by convection to the fuel surface may be calculated by a one-dimensional

TABLE I

Approximate values of thermal diffusivities at ambient temperature

	Mixtures N_2 - O_2		Plexiglass	<i>n</i> -Heptane	Magnesium	Aluminium
	$p = 1$ atm	$p = 300$ atm				
$\lambda/\rho c_p$ cm ² °C/sec	~0.25	~0.8 × 10 ⁻³	~1.2 × 10 ⁻³	~0.9 × 10 ⁻³	~1.9	~4.5

approximation, but in the present work it will be assumed, for simplicity, that radiant heat transfer is predominant as compared with heat conduction, as usually occurs provided that the flame is not too small.

Convection and radiative heat transfer behind the flame front will not influence the process for the present case of a plastic fuel.

Once the expression of the heat flux is given, heat conduction within the fuel can be calculated by an approximate method based on the assumption that heat transfer parallel to the fuel surface is small as compared with heat transfer perpendicular to the fuel surface.

2.3 Fuel Vaporization and Mixing

Fuel vaporization as function of the fuel surface temperature will be assumed to be given by a Langmuir's expression of the type

$$\dot{m}_f/\rho_1\nu = K_1 \exp\left(-\frac{q_1/R}{T_s - T_\infty}\right). \quad (2)$$

Diffusion of vapors into the oxidizer is calculated, neglecting both convective transport perpendicular to the fuel surface and diffusion parallel to the fuel surface. Free convection effects are not considered.

2.4 Flame Propagation

It may be observed that the flame spreads over the surface of a plastic fuel as a thin bluish flame front which moves very close to the fuel surface. Combustion in this thin advancing front probably takes place at fuel-oxidizer ratios very close to the minimum value compatible with flame propagation. Behind this thin advancing flame, we find the brilliant and largest part of the flame, where combustion probably is independent of the conditions existing in the advancing flame front. Therefore, it is reasonable to assume that the spreading process is controlled by the mixture conditions at the fuel surface, and to calculate the propagation velocity for the one-dimensional case, with the mixture conditions existing at the flame front. It may be pointed out that the normal part of the flame will be responsible for the radiant heat transmitted, which will not depend on the local conditions existing at the advancing flame front.

3. Mathematical Study of the Process

3.1 The mathematical treatment of all the partial processes has been simplified considerably in order to permit analytical study of the com-

plete process. Nevertheless, numerical methods may be applied using the physical model developed in the present work.

3.2 Temperature at the Fuel Surface

The heat transport equation within the fuel is:

$$(\partial^2 T/\partial x^2) + (\partial^2 T/\partial y^2) - (\rho_f c_f u/\lambda_f)(\partial T/\partial x) = 0. \quad (3)$$

In Ref. 6, this equation was solved by an integral method. In this paper, it will be assumed that the product of the Reynolds and Prandtl numbers:

$$\text{Re Pr} = \rho_f c_f u L/\lambda_f \quad (4)$$

is large. In this expression L is a characteristic length, such as δ_f , for which the heat flux reaching the fuel surface is significant. (At low spreading velocities the approximation of this simplification is obviously questionable.)

If it is also assumed that the thickness of the thermal boundary layer in the fuel is small as compared with L , then Eq. (3), in dimensionless coordinates, reduces to:

$$\partial^2 \theta/\partial \eta^2 = (u/\nu)(\partial \theta/\partial \xi) \quad (5)$$

where u is the flame spreading velocity and ν a characteristic velocity of the process.

With the above assumptions, neglecting the heat transmitted by the fuel surface and taking as expression for the radiant heat flux reaching the fuel:

$$Q(x) = Q_R \exp[\beta(x/\delta_f)] = Q_R \exp(\beta\xi) \quad (6)$$

the boundary conditions reduce to:

$$\begin{aligned} \xi = -\infty & \quad \theta = \theta_\infty \\ \eta = -\infty & \quad \theta = \theta_\infty \end{aligned} \quad (7)$$

$$(\partial \theta/\partial \eta)_{\eta=0} = \exp(\beta\xi). \quad (8)$$

Integrating Eq. (5) for the above boundary conditions, we obtain for the temperature distribution at the fuel surface:

$$\theta_s = \theta_\infty + \exp(\beta\xi)[\beta(u/\nu)]^{-1/2}. \quad (9)$$

3.3 Mixing of the Fuel Vapor with the Oxidizer

Assuming that the diffusion coefficient and densities are constants, mixing of the fuel vapors

will be controlled by the equation:

$$V_x(\partial Y_1/\partial x) + V_y(\partial Y_1/\partial y) = D(\partial^2 Y_1/\partial x^2) + D(\partial^2 Y_1/\partial y^2). \quad (10)$$

The vaporization rate is very small, therefore, the term $V_y \partial Y_1/\partial y$ will also be very small. Assuming also that the product of the Reynolds and Schmidt numbers:

$$\text{Re Sc} = LV_x/D \quad (11)$$

is large, then diffusion parallel to the fuel surface can be neglected as compared with the convective term. Assuming finally that the thickness of the diffusion boundary layer is small as compared with L , Eq. (10) reduces to:

$$(\partial^2 Y_1/\partial \eta^2) - (u\delta_f/D)(\partial Y_1/\partial \xi) = 0. \quad (12)$$

The boundary conditions are:

$$\begin{aligned} \xi = -\infty & \quad Y_1 = Y_{1\infty} \\ \eta = \infty & \quad Y_1 = Y_{1\infty} \end{aligned} \quad (13)$$

$$\begin{aligned} (D/\nu\delta_f)(\partial Y_1/\partial \eta)_{\eta=0} & \simeq -(\dot{m}_{fs}/\rho_1\nu) \\ & = -K_1 \exp[-\tau_0/(\theta_s - \theta_\infty)] \end{aligned} \quad (14)$$

in which θ_s is given by Eq. (9).

It will be assumed that the value of $Y_{1\infty}$ in function of the fuel temperature θ_∞ , corresponds to the equilibrium conditions and that these conditions are given by an exponential function:

$$Y_{1\infty} = K_2 \exp[K_3/(\Delta - 1)] \quad (15)$$

in which Δ is the ratio of T_∞ to a reference temperature T^0 .

Integration of Eq. (12) is not straightforward because of boundary condition (14). It can be achieved approximately by writing the equation in integral form and by expressing the value of \dot{m}_{fs} given by (14) in the form:

$$\dot{m}_{fs}/\rho_1\nu \simeq C_1 \exp(C_2\xi) \quad (16)$$

where the values of C_1 and C_2 are numerically adjusted according to each initial value of u/ν , in order to approximate the real expression of $\dot{m}_{fs}/\rho_1\nu$.

The final solution at the origin is given by:

$$Y_{1s0} = Y_{1\infty} + C_1[BC_2(u/\nu)]^{-1/2}. \quad (17)$$

Figure 2 shows the curves of Y_{1s0} in function of

the assumed value of u/ν for different values of the initial fuel temperature represented by the parameter Δ .

3.4 Flame Propagation

3.4.1 Mathematical Model

Based on the assumptions of Sec. 2.4, flame propagation in the gaseous phase has been calculated for the one-dimensional case, using the mixture composition at the fuel surface and at the flame front ($\xi = 0$). This mixture consists of oxidizer and fuel vapors, Y_{1s0} being the mass fraction of the fuel vapors, as calculated in the preceding paragraph.

The mathematical model of this one-dimensional flame propagation process is based on the assumptions that conditions are laminar and stationary with respect to the flame front. Only two chemical species will be considered: the reactants 1, 2, formed by a mixture of fuel vapors 1 and oxidizer 2, and the combustion products 3. The oxidizer 2 can be a mixture of oxygen and nitrogen or pure oxygen. Constant and equal values of the specific heats and thermal conductivities are assumed for both species.

Finally, an over-all reaction rate of the form:

$$w_3 = \Omega\rho^n(1 - Y_3)^n \exp[-(E/RT)] \quad (18)$$

will be considered.

Under these assumptions, the equations of energy, diffusion and continuity in dimensionless coordinates are given by:

$$d\tau/dz = \tau - \epsilon_3 \quad (19)$$

$$\text{Le}(dY_3/dz) = Y_3 - \epsilon_3 \quad (20)$$

$$d\epsilon_3/dz = \Lambda(1 - Y_3)^n \frac{\exp[-\tau E/(\tau + \tau_r)]}{(\tau + \tau_r)^n} \quad (21)$$

in which the parameter Λ , given by:

$$\Lambda = \dot{m}_{12}^{-1}\Omega\rho_0^n T_\infty^n (\bar{\lambda}/\bar{c}_p) (\bar{c}_p/q_r)^n \quad (22)$$

is the "eigenvalue" of the system.

The boundary conditions are as follows:

$$\begin{aligned} z = -\infty & \quad Y_3 = 0 (Y_{12} = 1 = Y_{1s0} + Y_{2s0}) \\ (\xi = -\infty) & \quad \epsilon_3 = 0 \\ & \quad \tau = 0 \text{ (or } \theta = \theta_\infty) \end{aligned} \quad (23)$$

$$\begin{aligned} z = +\infty & \quad Y_3 = 1 \\ (\xi \pm +\infty) & \quad \epsilon_3 = 1 \\ & \quad \tau = 1 \end{aligned} \quad (24)$$

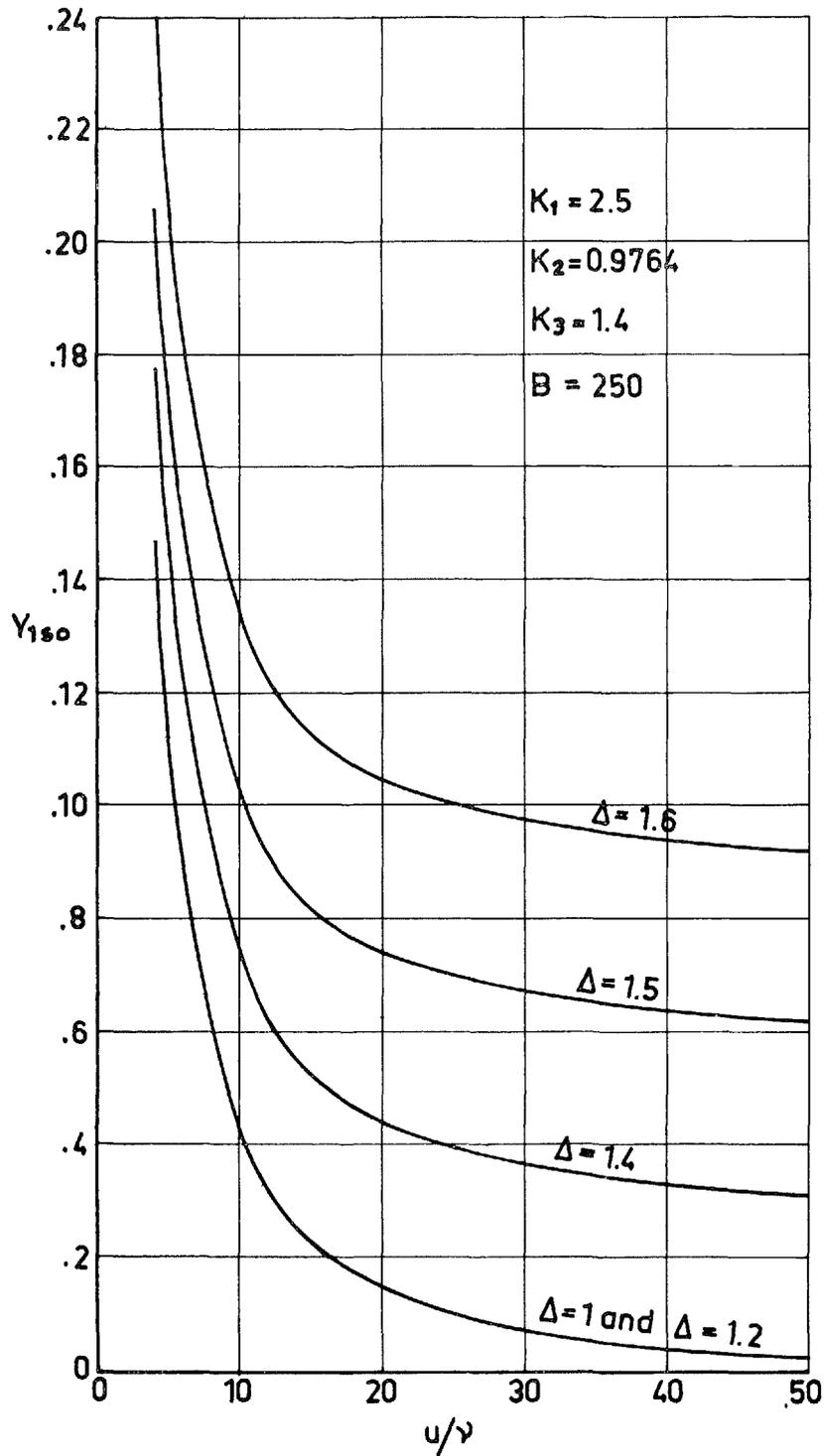


FIG. 2. Fuel vapor mass fraction $Y_{1,0}$ at the fuel surface and at the origin as a function of the assumed value of the flame spreading velocity, for several values of the fuel temperature at infinity.

These conditions imply that derivatives dY_3/dz , $d\epsilon_3/dz$ and $d\tau/dz$ must also be equal to zero for $z = \pm \infty$

All these derivatives, except $d\epsilon_3/dz$, are automatically equal to zero for $z = -\infty$. This contradiction at the cold boundary is a well known formal difficulty in theories of laminar flame propagation. In our case, where the existence of a reaction zone of finite thickness is assumed, that condition is automatically satisfied.

3.4.2 Solution of the System

Systems (19), (20), and (21) are solved by an approximate analytical method based on the assumption that the reaction zone has a finite thickness $\zeta = z_{II} - z_I$, and by approaching the flux fraction ϵ_3 within that zone by means of parabolic expressions.

A similar method was used in Refs. 7 and 8 to study flames with spherical symmetry, and gives an excellent approximation of the process.

For simplicity, it is assumed that the Lewis-Semenov number is equal to one. Then, the flux fraction Y_3 will be equal to the dimensionless temperature τ .

Therefore, in the region ahead of the flame ($-\infty \leq z \leq z_I$) ϵ_3 is considered equal to zero, and the flux fraction and temperature are given by

$$Y_3 = \tau = \exp z \quad (25)$$

Behind the flame ($z_{II} \leq z \leq +\infty$) ϵ_3 is equal to unity and τ and Y_3 are also equal to one.

Within the flame, by taking

$$\epsilon_3 = -(\epsilon_3'^*/2z_I)(z_I - z)^2 \quad (z_I \leq z \leq 0) \quad (26)$$

and

$$\epsilon_3 = 1 - (\epsilon_3'^*/2z_{II})(z_{II} - z)^2 \quad (0 \leq z \leq z_{II}) \quad (27)$$

where $\epsilon_3'^*$ is the maximum value of $d\epsilon_3/dz$, the following expressions are obtained for the temperatures and mass fractions

$$\begin{aligned} \tau = Y_3 = \exp z + (\epsilon_3'^*/z_I) \exp(z - z_I) \\ - (\epsilon_3'^*/2z_I)[1 + (z_I - 1 - z)^2] \\ (z_I \leq z \leq 0) \end{aligned} \quad (28)$$

and

$$\begin{aligned} \tau = Y_3 = (\epsilon_3'^*/z_{II}) \exp(z - z_{II}) \\ - (\epsilon_3'^*/2z_{II})[1 + (z_{II} - 1 - z)^2] + 1 \\ (0 \leq z \leq z_{II}) \end{aligned} \quad (29)$$

By equalizing both values of ϵ_3 at $z = 0$, we obtain

$$[(z_{II} - z_I)/2]\epsilon_3'^* = 1 \quad (30)$$

The equality of the values of ϵ_3 also implies the equality of the derivatives of temperature and mass fractions at $z = 0$.

Parameters $\epsilon_3'^*$, z_I , and z_{II} are obtained by stating that at $z = 0$, the value $\epsilon_3'^*$ is given by expression (21), that is

$$\epsilon_3'^* = \Lambda(1 - Y_3^*)^n \frac{\exp[-\tau_{II}/(\tau^* + \tau_r)]}{(\tau^* + \tau_r)^n} \quad (31)$$

and that at $z = 0$, $\epsilon_3'^*$ is a maximum

$$\begin{aligned} -\frac{n Y_3^* - \epsilon_3^*}{\text{Le}(1 - Y_3^*)} + \frac{\tau_{II}(\tau^* - \epsilon_3^*)}{(\tau^* + \tau_r)^2} \\ - \frac{n(\tau^* - \epsilon_3^*)}{\tau^* + \tau_r} = 0 \end{aligned} \quad (32)$$

Finally, by equating the temperatures at $z = 0$, the additional condition needed to obtain the eigenvalue Λ of the system is obtained. This eigenvalue gives the mass flow m_{12} per unit area, and from it the spreading velocity is calculated.

From (26) and (27), we obtain

$$\begin{aligned} \frac{\exp(-z_{II})}{z_{II}} - \frac{\exp(-z_I)}{z_I} - \frac{1}{2} \\ \times \left[\frac{1 + (z_{II} - 1)^2}{z_{II}} - \frac{1 + (z_I - 1)^2}{z_I} \right] = 0 \end{aligned} \quad (33)$$

Systems (31), (32), and (33) together with condition (30) give the values of z_I , $z_{II}\epsilon'^*$, and Λ .

The flame propagation model as described here gives the dimensionless flame spreading velocity $u/(\Omega\bar{\lambda}/\bar{c}_p)^{1/2}$ in function of the heat of reaction q_r and the initial fuel or gas temperature Δ , as shown in Fig. 3(a). However, to correlate these results with the values of u/v given in Fig. 2, it is necessary to know the law of variation of the heat of reaction with the mixture composition, represented by the mass fraction $Y_{1,0}$ of fuel vapors in oxygen or in a given mixture of oxygen and nitrogen.

There are no data available for the combustion of the fuel vapors of plexiglass in oxygen. Therefore, a reasonable assumption will be made which consists in taking $q_r/\bar{c}_p = 2000^\circ\text{C}$ for the case of combustion in oxygen, for $Y_{1,0} = 0.1$, which is probably close to the stoichiometric condition, and then using a linear law of variation of q_r/\bar{c}_p from that value down to zero for $Y_{1,0} = 0$.

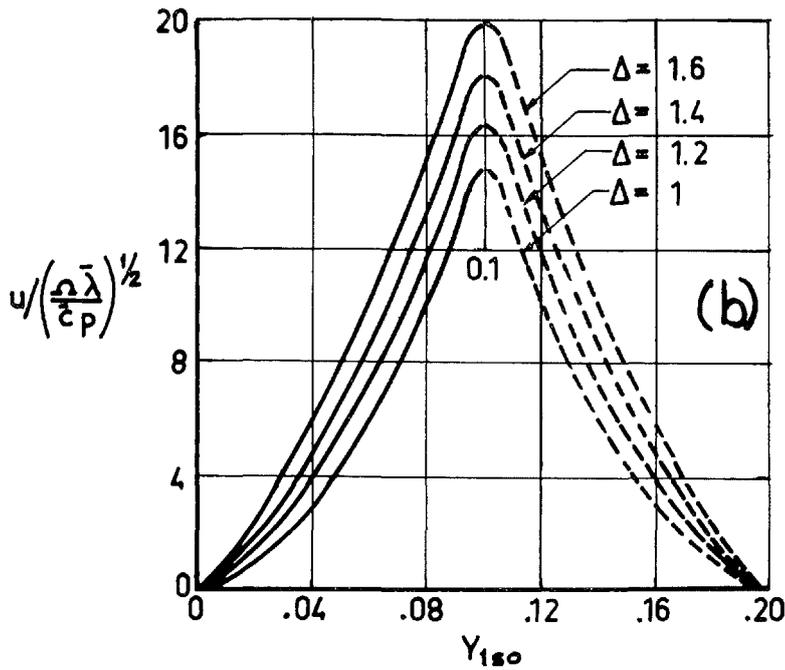
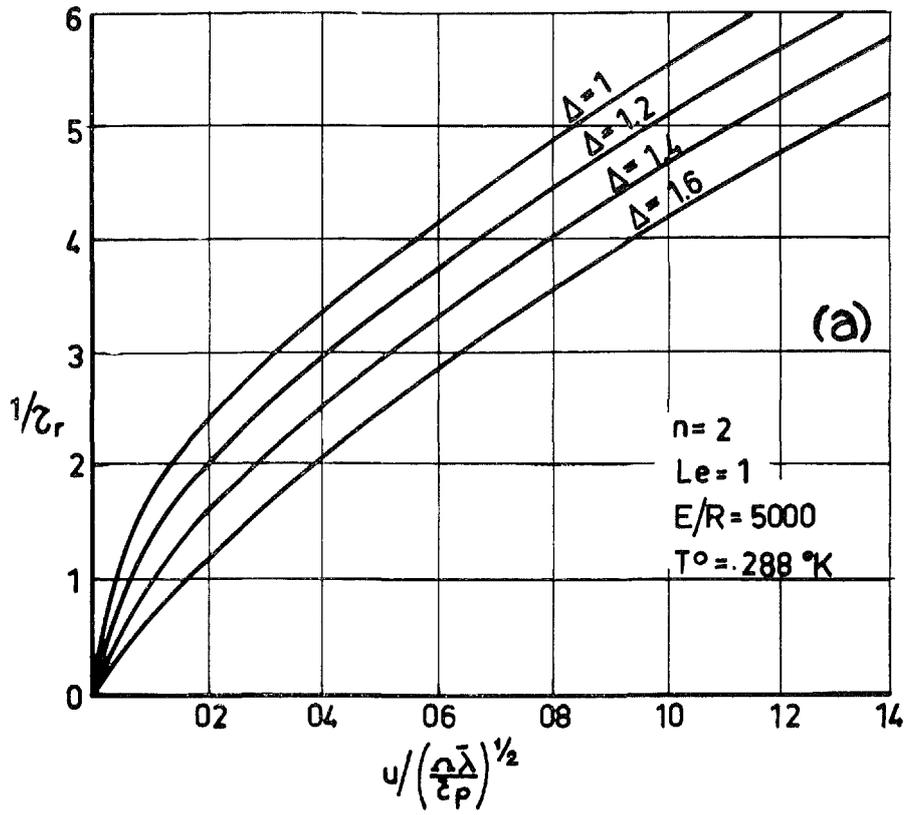


FIG. 3. (a) Flame spreading velocity as a function of the heat of reaction for several initial temperatures of the oxidizer at infinity, equal to the initial fuel temperature. (b) Flame spreading velocity calculated as a function of the fuel vapor mass fraction $Y_{1,0}$ for several values of the fuel temperature at infinity.

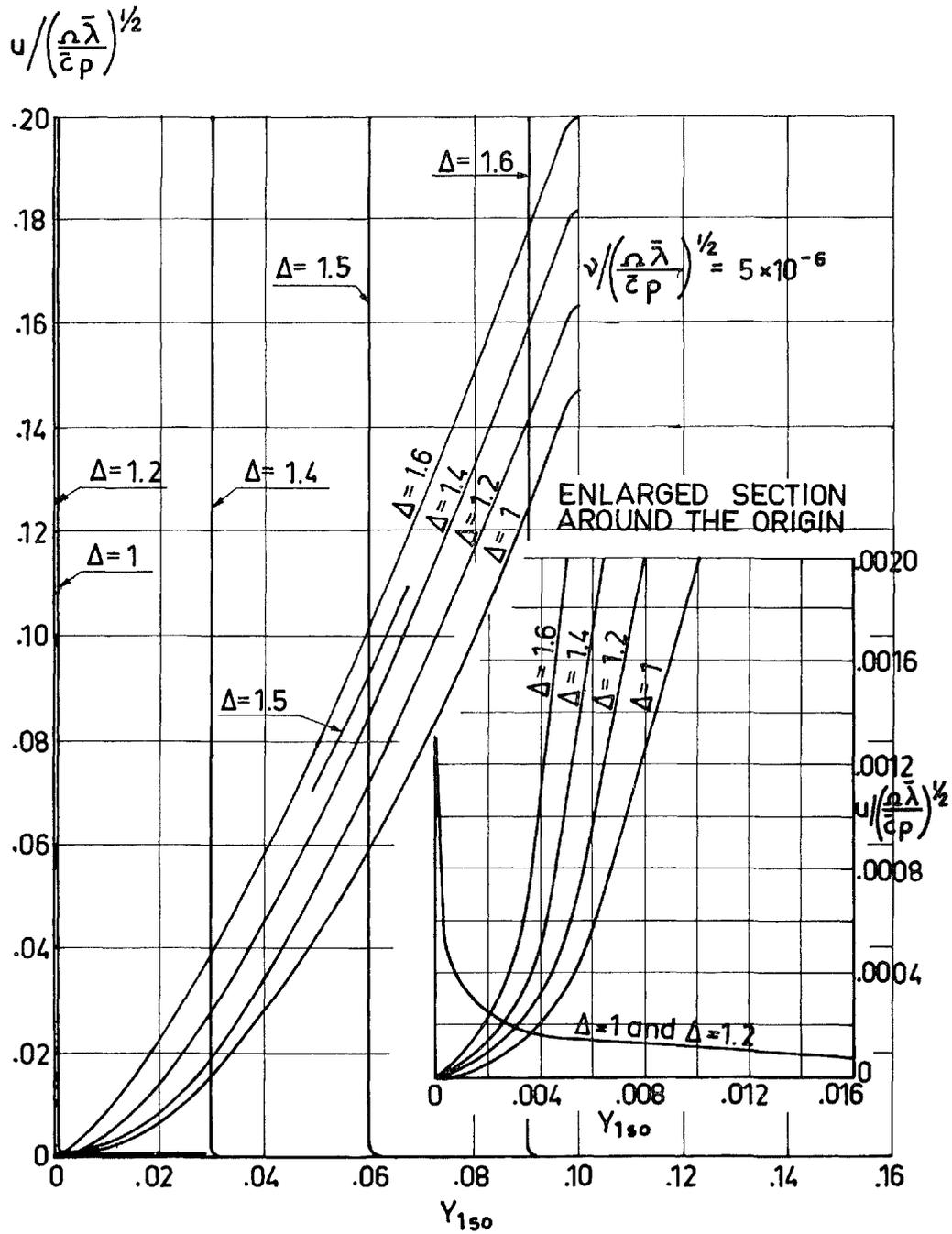


Fig. 4. Comparison of the assumed (Fig. 2) and calculated (Fig. 3b) spreading velocities.

and for $Y_{1s0} = 0.2$. We have not acknowledged with this law of variation the lower and upper propagation limits for fuel vapors-oxygen mixtures.

With these linear laws of variation of $q_r/\bar{c}_p =$

$f(Y_{1s0})$, the curves of Fig. 3(b) have been calculated, starting from the data of Fig. 3(a). These curves give the dimensionless spreading velocity $u / \left(\frac{\Omega \bar{\lambda}}{\bar{c}_p} \right)^{1/2}$ in function of fuel vapor mass fraction Y_{1s0} .

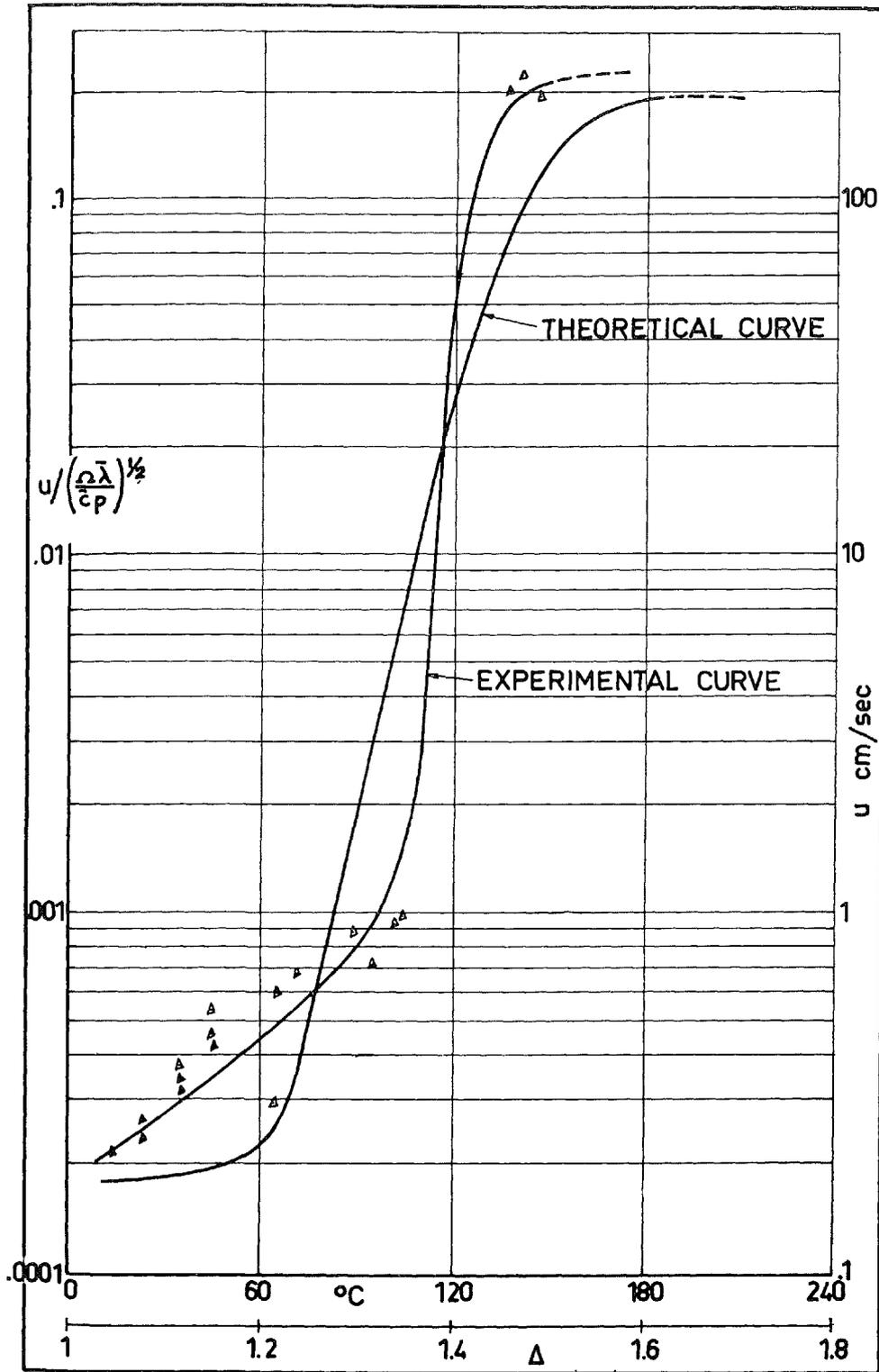


Fig. 5. Flame spreading velocities, calculated and measured over Plexiglass rods in oxygen at variable initial fuel temperature ($p = 1 \text{ atm}$). The qualitative agreement is satisfactory.

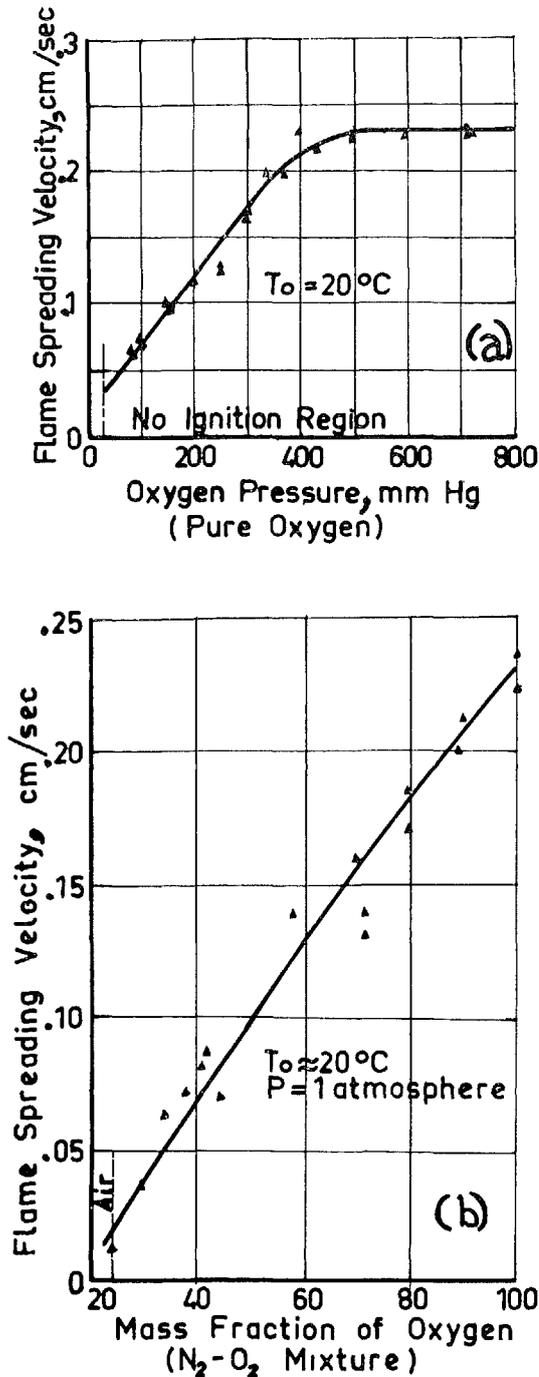


FIG. 6. (a) Flame spreading velocities over Plexiglass rods in oxygen at variable pressure ($T \sim 20^\circ\text{C}$) (b) Flame spreading velocities over Plexiglass rods in $\text{N}_2\text{-O}_2$ mixtures ($p = 1$ atm, $T \sim 20^\circ\text{C}$).

4. Theoretical Results

Theoretical results have been obtained giving the flame spreading velocity in function of the initial fuel temperature, by equating the values of u/ν given in Fig. 2 with those of $u/(\Omega\bar{\lambda}/\bar{c}_p)^{1/2}$ of Fig. 3(b), as shown in Fig. 4.

To do this, a value must be given to parameter $\nu/(\Omega\bar{\lambda}/\bar{c}_p)^{1/2}$ and in the absence of data for the frequency factor Ω , a value has been assigned to this parameter with the condition that the theoretical value of u in the region of its maximum value will be of the same order of magnitude as the highest measured values of this velocity.

Figure 5 shows the resulting flame spreading velocity as a function of the initial fuel temperature. In the region of low temperatures, the flame spreads very slowly, there is no significant combustible mixture ahead of the flame and heating of the fuel controls the spreading process. At a certain temperature, because of the exponential form of the vaporization functions, the fuel vapor mass fraction at the fuel surface increases rapidly, and the flame spreading velocity increases very sharply reaching values two or three orders of magnitude higher over a very short temperature interval. Actually, this temperature must be close to the "ignition temperature of the fuel," as usually defined. Above this temperature, a combustible mixture exists over the entire fuel surface and the flame propagates through this mixture, and heating of the fuel does not influence the process appreciably.

If the combustion properties of the fuel vapors and the oxidizers were known, it would be possible to calculate the influence of the mixture composition and gas pressure, at least approximately, neglecting possible variations of the flame size.

The flame temperature depends on mixture composition through the heat of reaction, and in this way the variation of the heat fluxes might be studied.

The influence of pressure is mainly exerted through the eigenvalue Λ , which is proportional to ρ^n . It also influences the heat transfer processes by conduction and convection, and if the pressure were reduced sufficiently, it would also influence chemical kinetics of the process and the flame temperature.

5. Experimental Results

An experimental program was carried out in which flame spreading velocities were measured over the surface of plastic fuels in a chamber filled with oxygen-nitrogen mixtures at pressures

from 1 to 1/25 atm and at compositions ranging from air to pure oxygen.

The experiments were performed with plexiglass rods, held in a vertical position by a central wire which was electrically heated in order to control the initial temperature of the fuel. At temperatures above 90–100°, horizontal strips were also utilized because the rods became too soft to be held by the wire.

Flame spreading velocities as a function of the initial fuel temperature are shown in Fig. 5, where the theoretical curve has also been included. It may be seen that they are in satisfactory qualitative agreement and that even the order of magnitude of the ratio of the values of the spreading velocities from the upper region to the lower region is very similar for both curves.

Figure 6 shows spreading velocities as functions of pressure and mixture composition. If these results are compared it may be seen that in pure oxygen, flame spreading velocities at pressures of the order of the corresponding partial pressure of the oxygen in air are much higher than the spreading velocities in air.

List of Principal Symbols

- B = $D/\nu\delta_f$ parameter
- C_1, C_2, K_1, K_2, K_3 parameters and coefficients
- c_f specific heat of solid fuel
- \bar{c}_p specific heat of mixture of gases
- D diffusion coefficient
- E activation energy
- Le = $\bar{p}D\bar{c}_p/\lambda$, Lewis–Semenov’s number
- \dot{m} mass flow per unit area of reactants or reaction products
- \dot{m}_{fs} mass flow of fuel evaporated per unit area and fuel surface
- n order of over-all reaction rate
- q_1 heat of vaporization
- q_r heat of reaction
- Pr Prandtl number
- Q_R radiant heat
- R gas constant
- Re Reynolds number
- Sc Schmidt number
- T temperature
- T^0 reference temperature
- u spreading velocity
- x, y coordinates
- Y mass fraction
- w specific reaction rate
- z = $\dot{m}_{12}\bar{c}_p/\lambda x$ dimensionless coordinate
- δ_f = $\lambda_f/\rho_f c_f \nu$ characteristic length
- Δ = $T_\infty/T^0 = 1 + \theta_\infty$ temperature parameter
- ϵ flux fraction of chemical species
- η = y/δ_f dimensionless coordinate

- ξ = x/δ_f dimensionless coordinate
- λ_f thermal conductivity of fuel
- $\bar{\lambda}$ mean value of thermal conductivity of mixture of gases
- Λ eigenvalue of flame spreading system of equations
- Ω frequency factor
- ρ_f fuel density
- \bar{p} mean density of mixture of gases
- θ = $(T - T^0)/T^0$ dimensionless fuel temperature
- τ = $\bar{c}_p(T - T_\infty)/q_r = \tau_r\theta$ dimensionless gas temperature
- τ_E = $E\bar{c}_p/q_r R$ parameter
- τ_r = $\bar{c}_p T_\infty/q_r$ parameter

Subscripts

- f solid fuel
- s at fuel surface
- 0 at origin
- 1 fuel vapors
- 2 oxidizer
- 3 reaction products
- ∞ at infinity

Superscripts

- ' derivative with respect to z
- * at center of reaction zone

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COMMENTS

A. F. Roberts, Safety in Mines Research Establishment. In the regime where rate of spread of flame is controlled by preheating of the fuel, care must be exercised in applying your analysis to the spread of a flame across a liquid surface.

The analysis considers the conduction of heat along the fuel bed ahead of the flame. However in experiments with flames on liquid surfaces, I have observed well-defined convection currents in the liquid which serve to preheat the liquid ahead of the flame. The rate of heat transfer due to these currents has been estimated as 1000 times greater than that due to conduction.¹

In the regime where an ignitable concentration of vapor exists above the fuel surface initially, my impression is that your analysis leads to a maximum value for the rate of spread close to the laminar burning velocity of the stoichiometric fuel/air mixture. However, in experiments with propanol ($\bar{S}_u \simeq 0.45$ m/sec), I measured rates of spread up to 2 m/sec, which is approximately the velocity at which the hot products of combustion leave the flamefront, i.e., $(S_u \cdot T_b)/T_u$, although this may be a coincidence.

REFERENCE

1. ROBERTS, A. F.: Ph.D. thesis, University of London, 1959.

R. F. McAlevy III, Stevens Institute of Technology. The independent measurement of flame spreading velocity (V) and distance along the surface affected by the spreading flame (δ),

that has been made in my laboratory, might be useful in testing the validity of the particular means of nondimensionalizing the conservation equations employed by the authors. For example, it appears that the product $V\delta$ must equal the thermal diffusivity for the approach to be valid, and this could be checked quite readily.

W. A. Sirignano, Princeton University. I cannot agree with the statement that your flame-spreading model applies to liquids as well as solids. For a variety of reasons, motion in the liquid can occur, modifying thermal-transfer processes and the spreading rate which destroys the model. For example, convection occurs since the heat-transfer and vaporization rates at the surface are not uniform. Also, if the liquid expands near the flame, gravity causes a flow along the surface. Another possibility is the effect of surface tension which is variable along the surface and can, therefore, produce motion along the surface.

C. Sanchez Tarifa, INTA, Madrid. We agree, indeed, that the processes of flame spreading over the surface of liquid or solid fuels might be different. However, the essential phenomena— heating of the fuel by the flame, fuel vaporization and mixing with the oxidizer, and flame propagation through this combustible mixture—are the same in both cases.

The effects mentioned by Prof. Sirignano probably are of secondary importance as compared with other effects disregarded in the model, such as the existence of tridimensional conditions and free-convection effects over the fuel surface.