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An application of HODMD to predict flutter using NeoCASS

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Abstract

We present a method based on higher order dynamic mode decomposition (HODMD) to predict aeroelastic modes in signals representing fluid-structure interactions. For such aim, some data are generated using an open source solver called NeoCASS (Next generation Conceptual Aero-Structural Sizing Suite), which reproduces real flight test conditions. Symmetric and anti-symmetric aeroelastic modes have been excited in two different numerical experiments. The HODMD-based method is able to detect the frequencies, damping rates and modal shapes of the leading aeroelastic modes with high accuracy. The good performance of this method provides a new and efficient tool, which can be extended to predict flutter in flight test in real time.

1. Introduction

Performing flutter flight test experiments is a part of the process required for designing commercial aircrafts. The main goal of this practice focuses on the detection of aeroelastic instabilities, which are developed during the flight under specific velocities, heights and manoeuvres. The origin of flutter, an aerodynamic instability, lies in the complex interaction between the structure and the fluid, and could evolve in undesirable effects. The wings, the stabilizers and some other minor parts of the airplane such as the aileron or the elevator are some of the most sensible areas where this instability develops. If flutter is not controlled, it can lead to the aircraft destruction.

Prediction of flutter in flight test experiments is an expensive and dangerous practice that requires using efficient tools to reduce the time of flight under the performance of this experiment, minimizing the waste of fuel. The detection of the frequencies and damping rates of the structural modes provides information about the fluid-structure interaction and the modes producing flutter. If the origin of flutter is understood, the it is possible to control it.

Flutter flight test involves real-time evaluation of the structure response. When particular control surface are excited (manoeuvre), the response is measured with sensors, and the analysis of the data is made on the ground, typically studying the evolution of the frequencies, damping rates, and modal shapes of the structural modes susceptible from become unstable. This information is used to decide the possibility of repeating the test with different flight conditions. Using a highly efficient tool for the analysis of the data, providing accurate results in reduced time is one of the key points to ensure the success of this practice. During the last decade researchers have paid especial attention in the development of new accurate techniques to predict flutter. The conventional methods used in these problems assume data extrapolation in the sense that their approaches are based on extrapolating damping trends from available aeroelastic damping data.[6] A review of some of these methods is presented in[9, 14] .

This article introduces a new application of HODMD to predict flutter. For such aim, we use the open source code NeoCASS (Next generation Conceptual Aero-Structural Sizing Suite)[1,2] to generate an artificial signal response to different inputs. NeoCASS is a code based on the software Nastran ®[11] , also suitable to develop this analysis. In this paper we use NeoRESP, which is a module of NeoCASS dedicated to simulate aeroelastic dynamic models, combining the beam model with a double lattice method (DLM) model, which provides the oscillatory aerodynamic forces. The DLM calculates the forces directly in the frequency domain. Thus, the simplest way to solve the problem in the time domain is to transform the frequency responses by Fourier analysis. The perturbation travels along the flight path and stays constant in time; the perturbation field is not sensitive to the aircraft passing and flows interfering phenomena.

For the analysis presented in this paper, two different manoeuvres have been carried out, and the output given by some sensors, located over the surface of the airplane, are analysed using an HODMD-based method. The accuracy and good performance of this method provides a new tool potentially used to predict flutter in real time flight test.

The article is organized as follows. Sections 2 and 3 briefly introduces a general description of HODMD and the solver NeoCASS (setup and main equations), while the main results are presented in 4. Finally, Section 5 describes the main conclusions.

2. Mathematical modelling: Higher order dynamic mode decomposition

The state variables describing a temporal signal, $\boldsymbol{v}_k$, can be decomposed as an expansion of M Fourier-like modes with damping rate δ_m and frequency ω_m, as

$$\boldsymbol{v}_k \simeq \sum_{m=1}^{M} a_m \mathbf{u_m} e^{(\delta_m + i\omega_m)(k-1)\Delta t}, \quad k = 1, \ldots, K, \tag{1}$$

where $\mathbf{u_m}$ are the spatial coefficients or modes (unitary vector) and a_m are their corresponding amplitudes. The data, composed by K state vectors, are equidistant in time with Δt. The number of terms of the expansion M, can be referred to as the *spectral complexity* and the dimension of the subspace generated by the M modes is the *spatial complexity* N. The expansion on the top can be obtained using dynamic mode decomposition (DMD)[12] if $N = M$.

Although the good performance of DMD has been shown in a wide range of applications (e.g. Ref.[4,5]), in the analysis of noisy signals composed by a large number of frequencies and damping rates, measured in a small quantity of points, the spectral complexity is larger than the spatial complexity, $N < M$, and DMD fails for the analysis of the data. Higher order dynamic mode decomposition (HODMD),[7] an extension of DMD for the analysis of complex signals and noisy experimental data,[8] can be used instead to address such problem. The method is highly effective in terms of computational cost. Additionally, it has been shown to provide more accurate results with a shorter length signal than DMD (see Figure 5.1 in Ref.[7]).

In flutter flight test, a limited number of sensors mounted in strategic locations over the surface of the airplane, provide the evolution of a signal measuring the effect and evolution of aerodyamical instabilities. Analysing this signal it is possible to calculate the frequencies and damping rates of the modes leading to flutter. Using the expansion (1) it is also possible to calculate the shape of the modes, providing a deeper insight of the physical problem and the effects of such aeroelastic instabilities. Nevertheless, the signal analysed is noisy and and complex, composed by a large number of frequencies, and the number of sensors collecting information is generally small (less than 100). This fact makes HODMD as a suitable tool to analyze and predict flutter. The efficiency of this tool provides accurate results in a reduced computational cost (see more details in Refs.[6,10]). In this work HODMD is applied to a set of dynamic responses obtained from the solver NeoCASS.

HODMD algorithm is described as follows. For convenience, a set of K time equidistant (with Δt) snapshots (vector states obtained for instance in flight test) are collected in the following matrix

$$\boldsymbol{V}_1^K = [\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k, \boldsymbol{v}_{k+1}, \ldots, \boldsymbol{v}_{K-1}, \boldsymbol{v}_K], \tag{2}$$

where each vector $\boldsymbol{v}_k$ is composed by the signal at time instant t_k collected in each one of the sensors mounted over the surface of the airplane. The dimension of this matrix is $J \times K$, where J is the number of sensors. Then, the HODMD algorithm proceeds in two main steps.

- *Step 1: Dimension reduction.* In a first step, the spatial dimension J (number of sensors) of the original data set of snapshots is reduced to a set of linearly dependent vectors of dimension N, reducing the noise of the signal. In this way, singular value decomposition[13] (SVD) is applied to the snapshots matrix as

  $$\boldsymbol{V}_1^K \simeq \boldsymbol{W} \boldsymbol{\Sigma} \boldsymbol{T}^\top, \tag{3}$$

  where $\boldsymbol{W}^T \boldsymbol{W} = \boldsymbol{T}^T \boldsymbol{T} =$ the $N{\times}N$ unit matrix and the diagonal of matrix $\boldsymbol{\Sigma}$ contains the singular values $\sigma_1, \cdots, \sigma_K$. The number of retained SVD modes, N, is calculated through the standard SVD-error estimated for a certain tolerance ε (set by the user) as

  $$\sigma_{N+1}/\sigma_1 \leq \varepsilon. \tag{4}$$

  Then the *reduced snapshots matrix* of dimension $N \times K$ is written as

  $$\hat{\boldsymbol{V}}_1^K = \boldsymbol{\Sigma} \boldsymbol{T}^\top, \quad \text{with} \;\; \boldsymbol{V}_1^K = \boldsymbol{W} \hat{\boldsymbol{V}}_1^K \tag{5}$$

- *Step 2: DMD-d algorithm.* The following higher order Koopman assumption (written in matrix form)

  $$\hat{\boldsymbol{V}}_{d+1}^K \simeq \hat{\boldsymbol{R}}_1 \hat{\boldsymbol{V}}_1^{K-d} + \hat{\boldsymbol{R}}_2 \hat{\boldsymbol{V}}_2^{K-d+1} + \ldots + \hat{\boldsymbol{R}}_d \hat{\boldsymbol{V}}_d^{K-1}. \tag{6}$$

is applied to the reduced snapshot matrix. The main system dynamics (frequencies, damping rates and DMD modes) are contained in all these linear operators that can be encompassed in the *modified Koopman matrix* (that contains the dynamics of the system). Solving the eigenvalue problem of such matrix it is possible to calculate expansion (1), where the eigenvalues represent the frequencies and growth rates, and the eigenvectors approximate the modes.

More details can be found in Ref.[7]

3. Next generation Conceptual Aero-Structural Sizing Suite - NeoCASS

NeoCASS is a numeric analysis tool particularly used in the design of aircrafts. Based on a physical insight, the software provides information to built the structure of the aircraft under development and to investigate its aeroelastic behaviour using structural and aerodynamic numerical methods.

The dynamic response analysis is made using the NeoRESP solver. NeoRESP is based on a second-order frequency domain formulation, which solves the dynamic response of the aircraft to some external controlled inputs. The main advantage of this solver is that aerodynamic matrices are directly used in their original form. Fourier transform converts inputs into the frequency domain. Frequency responses are evaluated over the frequency range of interest, and inverse transformations are finally applied to recover time-histories of states and outputs.

For NeoRESP three parameters are required in order to define the flight reference condition[21] :

- VREF: flight speed V_∞ in m/s.
- RHOREF: air density ρ_∞ in Kg/m^3.
- MACH: flight Mach number M_∞.

These parameters are required to define all the terms for the aeroelasltic response:

$$\left(-\omega^2 M + i\omega C + K - q_\infty H_{mm}(k, M)\right) q(\omega) = q_\infty H_{mg} v_g(\omega) \tag{7}$$

where ω is the frequency, M the mass matrix, C damping matrix, K the stiffness matrix, q_∞ the dynamic pressure and the aerodynamics matrices are H_{mm} and H_{mg} (where H_{mg} only appears if the external force is a gust). Three stages are necessary to obtain the dynamic response of the aircraft, described below.

- *Stage 1: Defining the mesh.* A model is introduced in NeoRESP. This information is provided in an .xml file. For this paper we use a Boeing 747 model (B747-100). At this point, inner subroutines provide the aerodynamic model, the structural model, and the aeroelastic model. The B747-100 node distribution is presented in Fig.1.

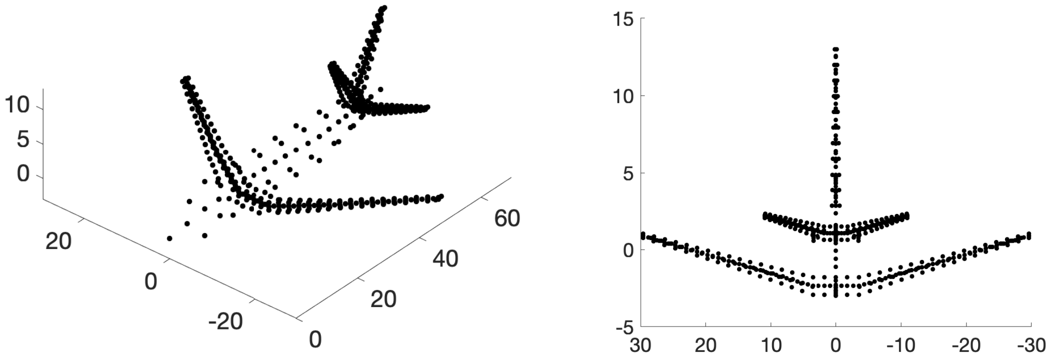


Figure 1: Computational domain and mesh in a Boeing 747 introduced in NeoRESP. Node representation. Three-dimensional (left) and frontal (right) views.

- *Stage 2: Input data file.* The main parameters are defined in this file: control surface input, flight conditions, options for output (node displacements, velocities, accelerations and inertial forces), modal analysis parameters (number of tracking modes and range of detectable frequencies), doublet lattice method parameters, like max speed used in flutter tracking algorithm, reference length to reduced frequency, density, symmetry for the planes, the order of kernel for doublet lattice and range of velocities. In this work, we use VREF= 170 m/s, RHOREF= 1.225 kg/m^3, MACH=0.500. For the eigenvalue analysis, we define the first two modes and the upper-frequency limit as 300 Hz,this range is selected due to the crucial modes appear in these limits.[3] Two different manoeuvres are carried out, symmetric input in the elevator, and anti-symmetric input in the aileron, as presented in Tab.1.

Table 1: Maneuvers with the number of retained modes

Surface	Type of excitation	Number of modes
Elevator	Symmetric	2
Aileron	Anty-symmetric	2

The input function is sinusoidal, as shown in Fig.2 and represents the surface deflection with time. We use the same functions for the two manoeuvres. HODMD is applied to detect the structural modes susceptible for flutter as result of the two manoeuvres.

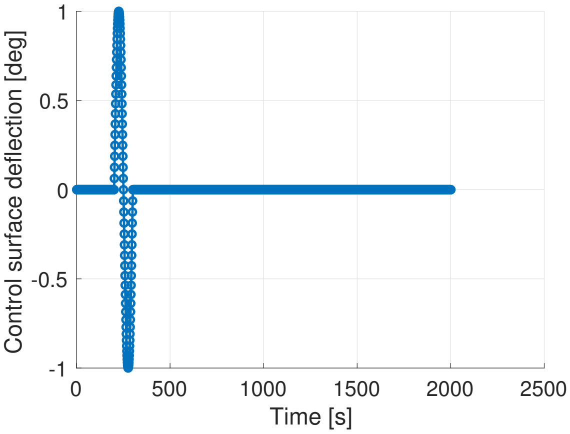


Figure 2: Control surface input for the elevator and the aileron.

- *Stage 3: Running NeoRESP.* The solver loads all the inputs necessary to solve eq.(7). At this stage, all the calculations related to eigenvalue analysis, mode shapes, aerodynamics matrices, and flutter analysis are carried out. These calculations are mandatory to create the data necessary for solving he dynamic equations. During the calculations, flutter detection analysis it is carried out in NeoRESP, providing the frequencies and damping rates that will be compared with the HODMD results.

  HODMD is applied in the signal response describing the acceleration of every node, as presented in Fig.3, where we can differentiate the symmetric and anti-symmetric response comparing two nodes on the opposite side of the wing (located symmetrically and equi-distant in two sides of the airplane) and evaluating their evolution with time. In the symmetric manoeuvre (Fig.3 top) the signal evolves identical in time, while in the anti-symmetric manoeuvre (Fig.3 bottom), it is possible to distinguish the opposite movement of the signal every time instant. The structural modes will also show the same symmetric/anti-symmetric tendency, as it will be presented in the following section. The information contained in these two signals is equivalent to the information extracted from a sensor in real experiments (for instance accelerometer in real flight test). In the HODMD analysis, we use the first three degrees of freedom defining these signals (acceleration in x, acceleration in y and acceleration in z) to determinate the frequencies, damping rates and shape of the modes.

The input of the symmetric and anti-symmetric manoeuvres is defined by the two modal frequencies presented in Tab. 2.

4. Results

HODMD has been applied to a set of data coming from NeoCASS simulations to calculate frequencies, damping rates, and their associated mode shape of the modes susceptible to produce flutter. The results are compared with the inner

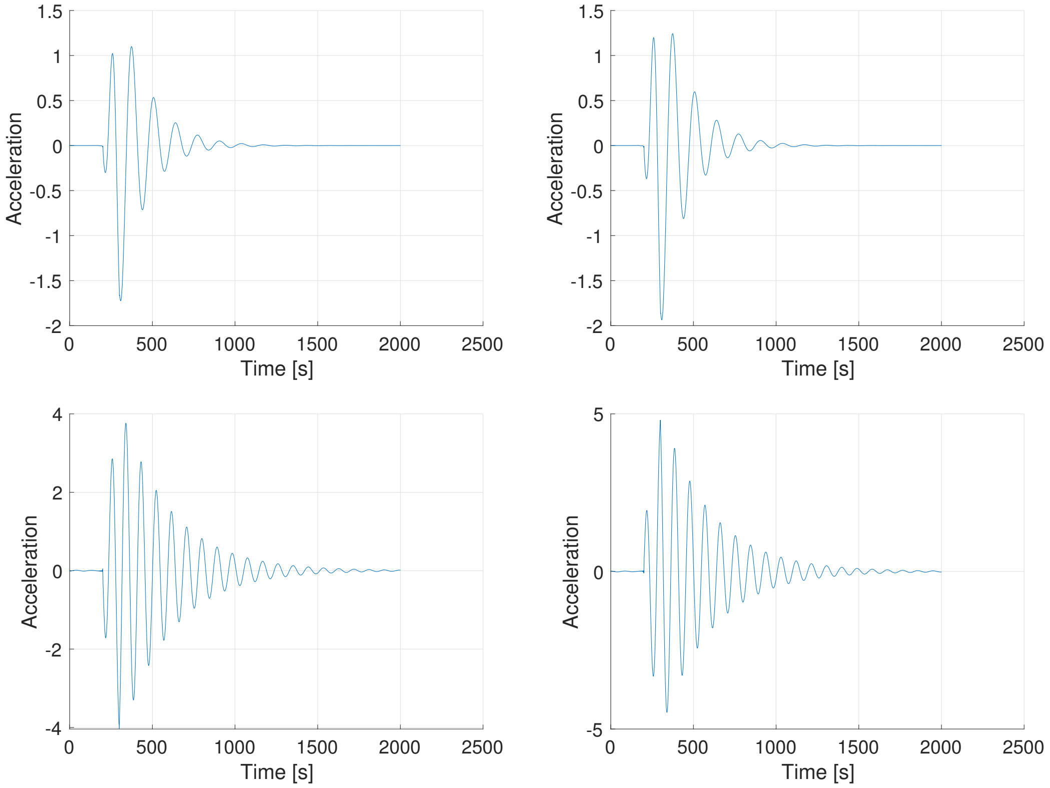


Figure 3: Acceleration response for two symmetric nodes on the wings. Top: elevator excitation (symmetric). Bottom: aileron excitation (anti-symmetric).

Table 2: Modal frequencies

Modal frequency (Hz)
1,3837 (symmetric)
2,1058 (anti-symmetric)

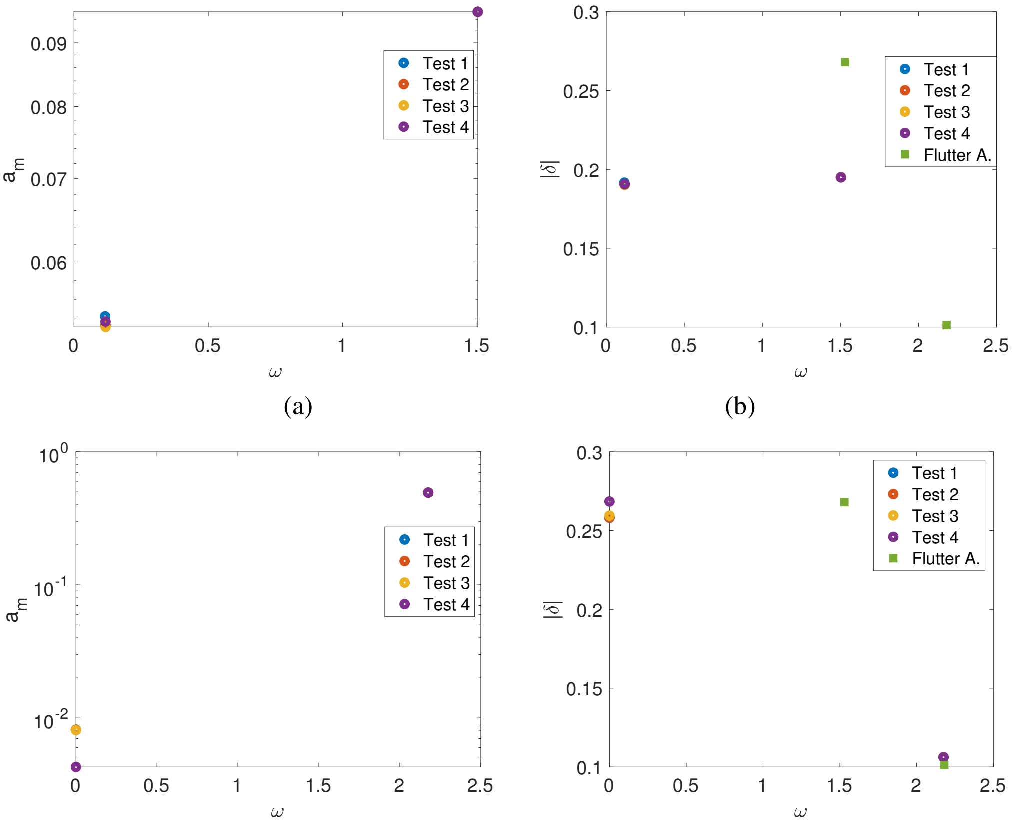
analysis carried out in NeoCASS. Four different flight test have been carried out, in a mesh composed by 512 nodes, considering 3 degrees of freedom in each node. The data are organized for each node as $n_i = \begin{pmatrix} a_{xi} \\ a_{yi} \\ a_{zi} \end{pmatrix}$ from $i = 1$ to $i = 512$. In order to calibrate the code for this data analysis and to test the robustness of this methodology we set the HODMD parameters as $5 \leq d \leq 50$ and $\varepsilon = \varepsilon_1 = 1e-3$, obtaining similar results in all cases. We present the results according to the control surface excited, elevator and aileron.

4.1 Control surface input: Elevator

The results for the input introduced in the elevator (symmetric manoeuvre) are presented in Fig. 4 (a) and (b). We observe that HODMD captures the lowest frequencies $f_i < 1\ Hz$, which are the modes representing the rigid body, and the symmetric mode with frequency $1.5029\ Hz$, as introduced in Tab.2. In Fig. 4 (b) we can see the comparison between the HODMD results and the real results obtained with NeoRESP for the flutter analysis (green square). The relative error made in the calculation of the frequency is very small ($\sim 1\%$), while the relative error made in the calculation of the damping rates is larger ($\sim 35\%$). The shape of the modes calculated using HODMD and provided by NeoRESP is presented in Fig. 5 (a)-(b). The Modal assurance criterion (MAC) (see Ref.[10] for more detail) between these two solutions is very high (~ 0.93), showing the high accuracy of this calculations.

4.2 Control surface input: Aileron

The results for the input introduced in the aileron (anti-symmetric manoeuvre) are presented in Fig. 4 (c) and (d). As in the previous case HODMD captures the lowest frequencies $f_i < 1$ Hz (rigid body modes) and the anti-symmetric mode 2.1752 Hz introduced in Tab.2. Fig.4 (d) compares the results obtained using NeoRESP with the results calculated using HODMD. Again the relative error made in the calculations of the frequencies is very low, ~ 1%, and the error made in the calculations of the damping rates is larger, ~ 5%, although much smaller than in the previous analysis. This fact suggests that the method could improve the results obtained in these two test cases (i) re-calibrating HODMD and (ii) re-organizing the information before carrying out the analysis of the data as in Ref.[10] As presented in Fig. 5 (c)-(d), the shape of the modes calculated with both NeoRESP and HODMD are in good agreement. The MAC is also high, ~ 0.95.



(a) (b)

(c) (d)

Figure 4: Results comparing deferents HODMD parameters of d from elevator signals, amplitude vs frequency (left) and damping vs frequency (right)

Table 3 summarizes the results for this paper. It shows the capability of HODMD to detect symmetric or anti-symmetric modes with high accuracy comparing the results with the structural modes calculated using NeoRESP. Based on the shape of the signal analyzed (previously shown in Fig. 3), in the symmetric manoeuvre, HODMD only detects the symmetric modes, while in the anti-symmetric manoeuvre, the method only detect the anti-symmetric modes. The analysis carried out in this article is similar to the one carried out during a flight test campaign to predict flutter, comparing the modes obtained to the one calculated in a ground vibration test (GTV).

Table 3: Frequencies and damping rates obtained with HODMD.

Modal frequency	$Frequency_{aileron}$	$Damping_{aileron}$	MAC_{abs}	$Frequency_{elevator}$	$Damping_{elevator}$	MAC_{abs}
1,3837	-	-	-	1,5029	0,1950	0.9532
2,1058	2,1752	0,1062	0,9363	-	-	-

5. Conclusions

This paper applies HODMD to predict aeroelastic modes in a set of data calculated using NeoCASS. This work shows the capability of HODMD to detect frequencies, damping rates, and modal shapes in the analysis of signals composed by the acceleration of the aero-elastic structure. These signals are provided by NeoRESP dynamic response, a module from NeoCASS. Two manoeuvres have been carried out to excite symmetric and anti-symmetric modes in the elevator and aileron, respectively. Results show the good capability of the method to discriminate symmetric/anti-symmetric signals and to calculate the modal shapes. These results are promising and establish HODMD as suitable tool that can be used in future application to predict flutter in real time flight test.

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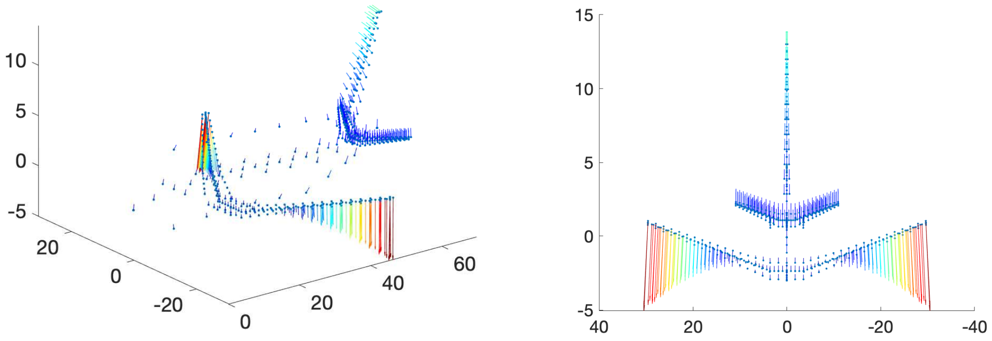
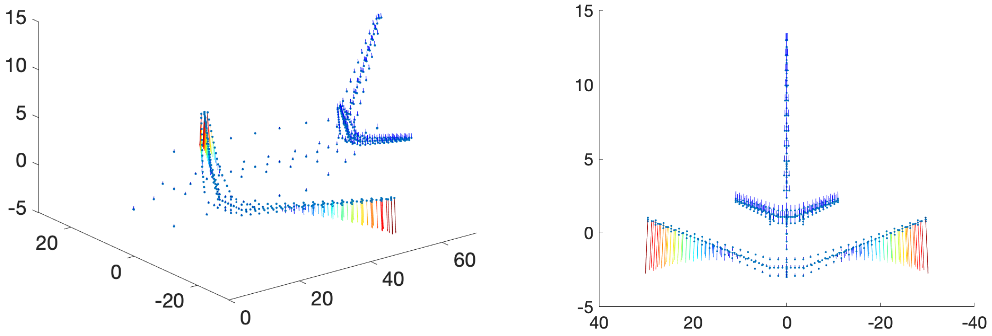
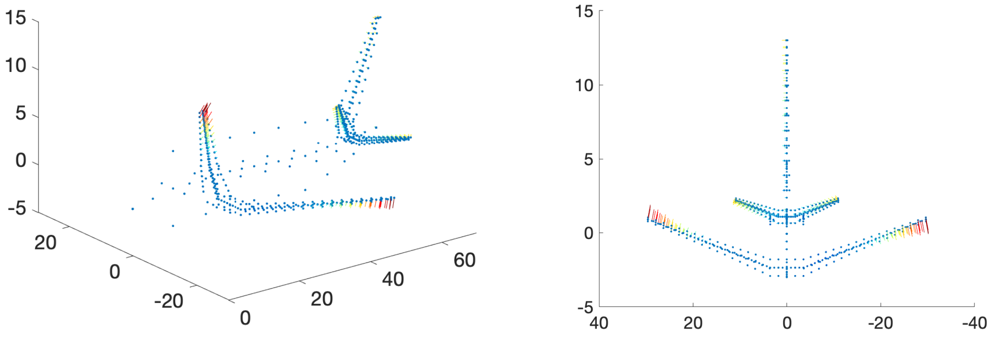


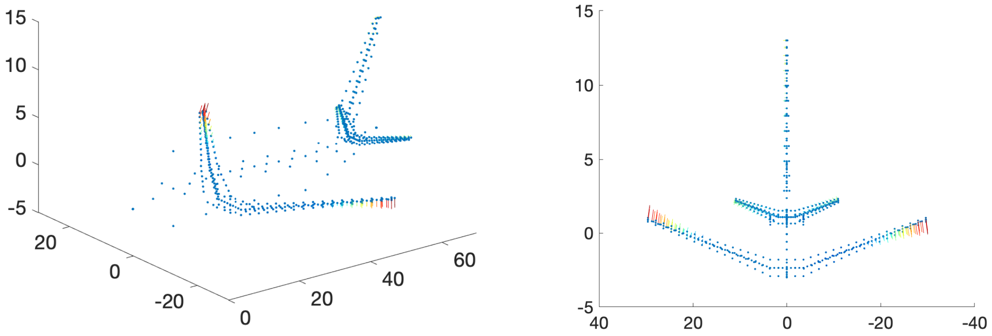
(a) Symmetric mode



(b) Symmetric mode approximated using HODMD



(c) Anti-symmetric mode



(d) Anti-symmetric mode approximated using HODMD

Figure 5: Vector field visualization of the structural modes calculated using NeoRESP and HODMD. From left ro right: three-dimensional mode representation and frontal view of the aircraft.