Risk-based calibration of road sag vertical curve design guidelines on undivided highways

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Manuscript accepted in Journal of Transportation Engineering, Part A: Systems
https://doi.org/10.1061/JTEPBS.0000572
Abstract

Safety in highway geometric design is one of the main goals to be achieved. However, deterministic design criteria do not provide information concerning the risk associated with the design outputs proposed. To yield consistent safety levels, the sag vertical curve design model on undivided highways was calibrated using a reliability-based framework, which allows to incorporate the uncertainty associated with the model variables. The sag curve design model contemplates the features of vehicle frontlighting systems to compute the headlight sight distance (HSD), which must be equal to or greater than the stopping sight distance (SSD). A dataset of 34,238 case studies was generated. Each case study was associated with two values of the risk level, designated as probability of noncompliance ($P_{nc}$), one per driving direction. A Monte Carlo simulation was selected to calculate the values of $P_{nc}$. Through a series of interpolating surfaces of the cloud of points, contour graphs and calibrated charts were depicted. The paper provides a new methodology to verify, design and compare sag vertical curve, evaluating the risk level with the $P_{nc}$.

Keywords

Calibration; Geometric design; Reliability analysis; Sight distance
Introduction

Available sight distance (ASD) represents the distance that the driver can see along the path of their vehicle free from obstacles. This factor is crucial for highway safety, because when the ASD is less than the distance to perform a maneuver, such maneuver cannot be carried out safely (Gargoum and El-Basyouny 2020). Sag vertical curves are designed considering the headlight sight distance (HSD), the distance illuminated by the beam of light projected by the headlights. To ensure a safe design, the HSD must be equal to or greater than the stopping sight distance (SSD), which is defined as the distance travelled by the vehicle from the moment the driver recognizes an obstacle on the pavement that requires stopping until the vehicle comes to a complete stop. Both HSD and SSD are affected by four main factors: infrastructure, vehicle, driver capability and environmental condition. Current geometric design guides, to calculate these distances, adopt a deterministic approach ignoring the uncertainty of the factors involved. Therefore, the design guides adopt a conservative approach that does not contemplate the random nature of the variables involved.

In this study, the reliability theory was used to overcome the deficiencies associated with the geometric design of sag vertical curves based on a deterministic approach. The risk associated with sag vertical curve design is evaluated by calculating the probability of non-compliance ($P_{nc}$) to propose an alternative design method based on consistent design that meet a certain target reliability. To this end, the variables in the HSD and SSD equations were treated as random variables, with a probability distribution, or parameters as appropriate as opposed to the conventional deterministic methods.

The aim of this paper is to present a calibrated design model for sag vertical curves of undivided highways to yield design outputs with consistent risk levels. After the literature review, the methodology section presents basic concepts of reliability theory and provides
details about Monte Carlo sampling procedure. Next section introduces the equations used in the Limit State Function (LSF), input variable values and the generation of 34,238 sag curve cases. Results section shows the main graphs obtained and describes their use from a practical point of view. Last section summarizes the main conclusions and future research lines.

**Literature review**

The design of sag vertical curves in road design standards and guides is subject to a deterministic approach (AASHTO 2018; Ministerio de Fomento 2016; Ministero delle Infrastrutture e dei trasporti 2001). This approach is known to present two relevant shortcomings (Ismail and Sayed 2010). First, many model variables used in design are stochastic and not deterministic. For this reason, the deterministic approach requires to adopt conservative percentile values for uncertain design inputs, which are not based on definitive safety measures. As a result, the safety margin of the design output is unknown. Secondly, if the threshold design value imposed by the standards is not met, deviations are not admissible regardless of whether they are large or small and the risk involved in such deviations is unknown.

Reliability analysis accounts for the uncertainty in the variables involved in design models and evaluates the risk associated with a particular design feature. Although this probabilistic tool has been used mainly in structural engineering, a significant number of applications related to highway geometric design is found in literature in the last years. First, Navin (1990) suggested the use of partial safety factors and proposed the $P_{nc}$ as a measure of reliability in design. You et al. (2012) examined the stability of different vehicles using reliability analysis. Musunuru and Porter (2014) studied the level of service on freeways under a stochastic approach to determine the required number of lanes. The risk associated to insufficient sight distance has been addressed in several
research studies. De Santos-Berbel and Castro (2015) assessed the P_{nc} associated to insufficient SSD along an existing highway, finding that the collision risk is generally higher on alignments that limit the ASD. De Santos-Berbel et al. (2017) compared the output of the reliability assessment of SSD on horizontal curves of 2D and 3D ASD models of existing highways. Ismail and Sayed (2012) utilized reliability theory to optimize the design of horizontal curves with right-of-way restrictions.

Several studies have examined the link between the P_{nc} associated with insufficient sight distance and collision frequency. Ibrahim and Sayed (2011) adjusted safety performance functions for horizontal curves, finding that predicted collisions have a statistically significant positive relationship with P_{nc}. Gargoum and El-Basyouny (2020) conducted a stochastic analysis of SSD on accident-prone highways, revealing that segments with higher non-compliance rates showed higher accident rates. Dhahir & Hassan (2019) adjusted safety performance functions considering weather conditions incorporating reliability indices of horizontal curve design according to four design criteria: vehicle stability, driver comfort, sight distance and vehicle rollover.

Concerning the calibration of design guides using reliability analysis, Hussein et al. (2014) proposed a method to design the middle ordinate offset on horizontal curves based on SSD at different P_{nc} levels. They also concluded that current design guides are conservative in this respect, especially at high speeds and on curves with small radii. Essa et al. (2016) developed system reliability calibration considering two modes of failure for the design of horizontal curves, namely SSD and vehicle stability. They proposed a more comprehensive design method for horizontal curves. The calibration outcome showed that there is a significant difference in the P_{nc} if two modes of failure are considered instead of one mode, particularly in curves with small radii. Mollashahi et al. (2017) calibrated superelevation values on horizontal curves considering the operating speed.
According to their results, superelevation values suggested by the AASTHO guide (AASHTO 2018) are generally lower than those required by vehicles in real situations, especially for horizontal curves where the design speed is less than 110 km/h.

The design of sag vertical curves is determined by headlamp features. Modern headlamp standards limit the amount of light emitted above the headlight beam axle which restricts the ability of drivers to see ahead (Gibbons et al. 2013). Whereas most design standards and guides set the upward angle of headlamp beam ($\alpha$) at 1 degree, Hawkins and Gogula (2008) suggested that the value $\alpha$ should be reduced to a value between 0.75 and 0.9 degrees. In this respect, Andrade-Catanño et al. (2020) performed a reliability-based evaluation of the HSD on sag curve design values of the Spanish Standard. They combined several values of the upward angle $\alpha$ and the target height $h_2$ to compute the associated $P_{nc}$ values, finding that the most conservative hypothesis in sag curve design comprised the smallest upward angle and the highest target height, which yielded significantly higher $P_{nc}$ values. Whereas Andrade-Catanño et al. (2020) conducted a probabilistic evaluation of sag curve design outputs, this study proposes design values that no longer rely on design speeds but on the target $P_{nc}$ and the operating speed.

**Methodology**

**Reliability theory**

By using a probabilistic approach based on reliability theory it is possible to eliminate the described deficiencies and, therefore, to have a design method that better reflects the nature of the variables involved. Instead of using prefixed values, the reliability approach uses random variables and the design equations are represented by the limit state function (LSF). The basic elements of reliability analyses are an N-dimensional vector of input variables $X = x_1, x_2, \ldots, x_n$ and the LSF denoted as $g(X)$. 
The LSF is conventionally written in terms of the difference between supply $R$ and demand $S$.

$$g = R - S$$  \hspace{1cm} (1)

The underlying uncertainty of the input variables can be characterized by assigning each input variable $x_i$ a probability density function (PDF) $p(x)$. The reliability of a system, i.e. the probability of it functioning within the acceptable performance domain, can be obtained according to equation (2):

$$P_c = 1 - P_{nc} = Prob(g > 0) = 1 - \int \ldots \int_{g(x) \leq 0} p(x) dx$$  \hspace{1cm} (2)

In most cases, the integral in equation 2 cannot be solved analytically. Therefore, various reliability methods can be used to achieve an approximate solution for equation 2 (Mahsuli and Haukaas 2013). On the one hand, analytical reliability methods are gradient-based methods that require the LSF to be continuously differentiable. In addition, they assume that supply and demand are independent. The most common analytical methods are the first order reliability method (FORM), the second order reliability method (SORM), the mean-value first order second moment (MVFOSM). On the other hand, synthetic methods such as sampling methods, for instance Monte Carlo sampling, avoid the main mathematical limitations of analytical methods while requiring more computational time to be completed. Previous reliability-based studies concerning geometric design contemplating non-independent supply and demand functions drawn on Monte Carlo sampling (Andrade-Cataño et al. 2020; De Santos-Berbel and Castro 2015).
Monte Carlo Sampling

Although several sampling schemes are available to estimate the $P_{nc}$, Monte Carlo sampling is a popular approach, in which samples of the random variables $x_i$ are generated according to their counterpart PDFs. Then, the expected value of a dependent variable $Y$ is $\mu = E(Y)$, where $Y = g(X)$ and the random variables $X = (x_1, x_2, \ldots, x_n) \in D \subseteq \mathbb{R}^n$. The average of the values $x_1, \ldots, x_n$, independently and randomly from the distribution of $X$, is an estimator of the random variables:

$$\hat{\mu}_n = \frac{1}{n} \sum_{i=1}^{n} x_i$$  \hspace{1cm} (3)

Let $X$ be a random variable for which $\mu = E(X)$ exists, equation (5) can be written:

$$\lim_{n \to \infty} P(|\hat{\mu}_n - \mu| \leq \varepsilon) = 1$$  \hspace{1cm} (5)

For any $\varepsilon > 0$. Under the weak law of large numbers, the chance of missing by more than $\varepsilon$ tends to zero.

The analytical reliability methods mentioned above are not appropriate to perform a reliability analysis when the LSF is not continuously differentiable.
Monte Carlo method is as more reliable as the number of iterations increases. The maximum number of iterations was selected as 100,000 and the $P_{nc}$ is determined by equation (6):

$$P_{nc} = \frac{N_{pu}}{N}$$  (6)

Where:

- $N_{pu}$ is the number of unsatisfactory performances
- $N$ is the number of iterations

In order to achieve enough accuracy in sampling, the coefficient of variations of the successive $P_{nc}$ values during the Monte Carlo iteration was monitored:

$$\delta_{P_{nc}} = \frac{\sigma_{P_{nc}}}{\mu_{P_{nc}}}$$  (7)

Where:

- $\delta_{P_{nc}}$ is the coefficient of variation of $P_{nc}$
- $\sigma_{P_{nc}}$ is the standard deviation of $P_{nc}$
- $\mu_{P_{nc}}$ is the mean value of $P_{nc}$

In this analysis, a value of 5% of the coefficient of variation was adopted to guarantee the accuracy of the sampling, which is a widely accepted value in studies that rely on sampling methods (Haukaas, 2019). For each case study, RT software (Mahsuli and Haukaas 2013) generated a vector of random variables, each one extracted from its counterpart random distribution and checked the compliance of the LSF. This process was iterated until the target coefficient of variation was obtained, or up to 100,000 iterations if otherwise.
Application to sag vertical curves

Limit state function

In design based in limit states, the system is considered to have failed or not complied with the design requirements when the demand exceeds the supply. In this calibration study, the LSF is represented by the difference between headlight sight distance (HSD) and stopping sight distance (SSD).

\[
LSF = HSD - SSD \tag{8}
\]

The value of HSD is calculated considering two different cases: when HSD is less or larger than the length of sag vertical curve (L) and the equations 9 and 10 was utilized respectively:

If HSD < L (Figure 1, case 1)

\[
HSD \approx K_v \cdot \tan \alpha \cdot (1 + \sqrt{1 + 2 \cdot \frac{h_h - h_2}{K_v \cdot \tan \alpha}}) \tag{9}
\]

If HSD > L (Figure 1, case 2)

\[
HSD \approx \frac{K_v \theta^2 + 2 \cdot (h_h - h_2)}{2 \cdot (\theta - \tan \alpha)} \tag{10}
\]

Where:

- \(K_v\): rate of vertical curvature [m]
- \(\alpha\): upward divergence angle of headlamp beam [degrees]
- \(h_h\): headlamp height [m]
- \(h_2\): target height [m]
- $\theta$: absolute value of the algebraic difference between the inbound and outbound grades.

**Figure 1.** Geometric scheme of the two cases of sag vertical curve 1) HSD < L; 2) HSD > L

The value of SSD is calculated in accordance with Spanish geometric design standard (Ministerio de Fomento 2016):

$$SSD = \frac{V \cdot t_{pr}}{3.6} + \frac{V^2}{254(f_i + G)}$$ (11)
Where:

- $V$: initial speed of the braking maneuver [km/h]
- $t_{pr}$: perception-reaction time [s]
- $f_l$: tire-pavement friction
- $G$: longitudinal grade of the highway along the stopping maneuver

**Input variables**

In the calibration process, the variables included in the LSF were considered as calibration parameters, constant values, or random variables as discussed below:

- Rate of vertical curvature ($K_v$): this calibration parameter defines the geometry of the sag curve.
- Upward angle of headlamp beam ($\alpha$): this value was set constant, equal to 0.75 degrees, according to the results discussed in the literature review (Hawkins and Gogula 2008).
- Headlamp height ($h_h$): The height of the headlight axle above the pavement surface is considered as a random variable following a normal distribution to cover the range of heights of the vehicles in the market. A previous study based on the 11 best-selling vehicles in Spain in 2015 indicated that the mean height was 0.731 m, and the standard deviation was 0.0052 m (De Santos-Berbel et al. 2016).
- Target height ($h_2$): in accordance to the Spanish geometric design standard the value of this parameter was set constant at 0.50 m (Ministerio de Fomento 2016). Moreover, as discussed in the literature review, the selection this value of the target height represents the most critical situation (Andrade-Cataño et al. 2020). The target height value aims to represent a portion of a potential obstacle on the roadway that permits the driver to recognize it. Due to the difficulty of setting a
statistical distribution of target height values, especially in relation to collision occurrence, a constant value was selected instead.

- Absolute value of the algebraic difference between the inbound and outbound grades ($\theta$): This is a calibration parameter defined taking into account the restrictions of Spanish geometric design (Ministerio de Fomento 2016). A sample of grade values is generated at steps of 0.5 %, enclosed within the maximum grade values allowed in the standard, including the exceptional increases. In addition, the smaller grade in absolute value was taken as 0.2 %, the minimum grade allowed in the design standard to permit drainage to the road pavement. Table 1 shows the different range of grades in relation to design speed.

<table>
<thead>
<tr>
<th>Design Speed (km/h)</th>
<th>Range of grades (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100-140</td>
<td>-6 to 5</td>
</tr>
<tr>
<td>80-90</td>
<td>-7 to 7</td>
</tr>
<tr>
<td>60-70</td>
<td>-8 to 8</td>
</tr>
<tr>
<td>40-50</td>
<td>-10 to 10</td>
</tr>
</tbody>
</table>

- Initial speed of the braking maneuver ($V$): The speed in this calibration is assumed as a random variable of normal distribution and is derived from operating speed distributions. On sag vertical curves, the operating speed is assumed to be determined by the horizontal alignment. Given a mean speed value ($V_{50}$), the counterpart standard deviation ($\sigma_V$) is required to characterize the uncertainty associated to this variable. In this sense, the relation set in the operating speed model of Pérez-Zuriaga (2012) was adopted, which is given by equation (12).
Table 2. Values resulting from model application.

<table>
<thead>
<tr>
<th>(V_D) (km/h)</th>
<th>(V_{50}) (km/h)</th>
<th>(\sigma_V) (km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>32.872</td>
<td>6.878</td>
</tr>
<tr>
<td>50</td>
<td>42.487</td>
<td>7.249</td>
</tr>
<tr>
<td>60</td>
<td>51.958</td>
<td>7.759</td>
</tr>
<tr>
<td>70</td>
<td>61.253</td>
<td>8.439</td>
</tr>
<tr>
<td>80</td>
<td>70.333</td>
<td>9.328</td>
</tr>
<tr>
<td>90</td>
<td>79.150</td>
<td>10.469</td>
</tr>
<tr>
<td>100</td>
<td>87.655</td>
<td>11.911</td>
</tr>
<tr>
<td>110</td>
<td>95.797</td>
<td>13.703</td>
</tr>
<tr>
<td>120</td>
<td>103.528</td>
<td>15.893</td>
</tr>
<tr>
<td>130</td>
<td>110.805</td>
<td>18.520</td>
</tr>
<tr>
<td>140</td>
<td>117.601</td>
<td>21.611</td>
</tr>
</tbody>
</table>

- Perception-reaction time (\(t_{pr}\)): This variable was assumed as a random variable of lognormal distribution. The values from the study of Lerner (Lerner 1995), which are widely accepted in numerous studies, were also considered in the current application. The distribution is features by a mean value of 1.5 seconds and the standard deviation is 0.4 seconds.

- Tire-pavement friction (\(f_i\)): It was assumed to be a random of beta distribution since its value must be enclosed between 0 and 1. Considering that the Spanish geometric design standard (Ministerio de Fomento 2016) provides the 95th percentile value of this variable, the probability distribution function can be fully
characterized by means of a measure of deviation. Given that the random value of
this variable is known to be conditioned by speed, each value of the speed \( V_{50} \) is
associated to a beta distribution with its respective mean \((f_{l,50})\) and standard
deviation \((\sigma_{fl})\). Therefore, the results of the research conducted by Bühlmann et
al. (1991) were used to derive the relationship between the speed and the standard
deviation of the tire-pavement friction, which lead to the following equation:

\[
\sigma_{fl} = 0.0343 + 2.2 \cdot 10^{-3} \cdot V - 3.2 \cdot 10^{-5} \cdot V^2 + 1.35 \cdot 10^{-7} \cdot V^3
\]  

(13)

Accordingly, the resulting beta PDFs at different speeds led to the following
relationship:

\[
f_{l,50} = 0.6673 - 2.8 \cdot 10^{-3} \cdot V + 1.1 \cdot 10^{-5} \cdot V^2
\]  

(14)

The values obtained from equation (14) are displayed in figure 2 and are compared
with other percentile values of the longitudinal friction in relation to the design
speed.
Figure 2. Percentiles of longitudinal friction distribution as a function of the speed

- Longitudinal highway grade along the stopping maneuver ($G$): This input was computed as the average grade along the stopping maneuver. Depending on whether the vertical curve is longer or shorter than the SSD, two different equations apply. On either case, the counterpart mathematical expression is derived from the equation of a second-degree parabola of vertical axis:

$$z = z_0 + G_1(s - s_0) + \frac{(s - s_0)^2}{2 \cdot K_V}$$

Where:

- $z_0$: elevation at the beginning of the sag curve [m].
- $G_1$: longitudinal grade at the beginning of the sag curve.
- $s$: stationing on the sag curve [m]

- $s_0$: stationing at the beginning of the sag curve [m]

- $K_V$: radius of the osculating circle of the sag curve parabola at its vertex [m]

Therefore, $G$ is evaluated as per equations (16) and (17).

If $L < SSD$:

$$G = G_1 + \theta - \frac{K_V \cdot \theta^2}{2 \cdot SSD}$$  \hspace{1cm} (16)

If $L > SSD$:

$$G = G_1 + \frac{SSD}{2 \cdot K_V}$$  \hspace{1cm} (17)

As evidenced by equations (11), (16) and (17), a recursive dependence exists between the value of $G$ and the SSD. Thus, a method of successive substitutions to approximate $G$ and SSD was implemented in the calibration procedure. Substituting equation (16) or (17) in equation (11) as applicable, the scheme of this iteration process yields a non-linear equation as follows:

$$SSD^{(k+1)} = f(SSD^{(k)}) \hspace{1cm} k = 1,2,3 \ldots$$  \hspace{1cm} (18)

Where $k$ is the number of iterations. The value $G_1$ is used as initial guess and the calculation is iterated until the desired convergence is achieved.

$$|SSD^{(k+1)} - SSD^{(k)}| < 0.01 \text{ m}$$  \hspace{1cm} (19)

Figure 3 illustrates a directed graph characterizing the variables and parameters above described as well as their relations.
Figure 3. Variables involved in the calculation process of $P_{nc}$.

**Generation of sag curve cases**

The procedure devised for this calibration study proposes the generation of a sample of sag curves and the calculation of the counterpart $P_{nc}$ considering first one travelling direction. The sample of sag curves comprised all possible combinations of parameters within certain ranges (Table 1 and 2) complying a set of conditions relating to operating and appearance criteria of the Spanish geometric design standard (Ministerio de Fomento 2016):

1) The values of $K_V$ (m) selected were the following: 700; 1,030; 1,380; 1,760; 2,160; 2,590; 3,050; 3,540; 4,070; 4,630; 5,230; 5,870; 6,560; 7,290; 8,080; 8,920; 9,820; 10,780; and 11,810.

2) The inbound and outbound grades were enclosed within the maximum grade values allowed by the standard, including the exceptional increases, in relation to the corresponding design speeds (Table 2).

3) The combinations of inbound and outbound grades were to result in lengths equal to or greater than the counterpart design speed ($L \geq V_D$). The length of a sag curve is given by:
\[ L = K_V \cdot \theta \] (20)

In addition, a preliminary computation of the \( P_{nc} \) was performed to limit the number of cases to those that are associated to admissible levels of risk, ruling out datasets where all combinations of parameters yielded \( P_{nc} \) values higher than 0.2. Therefore, the domain in Figure 4 was finally considered. Under these considerations, 66,384 sag vertical curves were generated and the counterpart \( P_{nc} \) values were retrieved. To evaluate the overall \( P_{nc} \) on a sag curve considering both travelling directions, pairs of case studies that correspond to identical geometric designs viewed from opposing travelling directions were merged into one case, assuming that speed distributions are identical on both travelling directions and collision risk due to insufficient HSD are independent on either travelling direction. To this end, pairs of sag curves with the same \( K_V \) value and same speed distribution where the inbound grade value of one sag was the opposite to that of the outbound grade of the other sag and vice versa were combined, and the average of both \( P_{nc} \) values was associated to the sag curve considering both travelling directions. Symmetrical sags were incorporated the same into the results dataset. Following these criteria, the results dataset comprised a total of 34,238 sag vertical curves.

**Results and discussion**

The goal of calibration is to propose design outputs that meet both the design constraints and a prespecified target \( P_{nc} \) value.

The 34,238 case studies were split according to the domain in figure 4 and 146 combinations of \( K_V \) and \( V_{50} \) were considered. To analyze the data, the same procedure was used for each combination:
- 3D scatter plot was created to represent the cloud of points that resulted from depicting $G_2$ on the x axis, the corresponding $G_1$ on the y axis, and the $P_{nc}$ on the vertical axis (Figure 5).

- An interpolating surface of the cloud of points corresponding to $P_{nc}$ was computed (Figure 5).

- The contour graph of $P_{nc}$ derived from the interpolating surface was depicted (Figure 6).

The domain of the graphs in Figures 5 and 6 is illustrated by the blue points that represent the cases analyzed.

![3D scatter plot and contour graph](image)

**Figure 5.** Example of cloud of points and interpolated surface using regression. Data corresponding to $K_V = 2160$ m and $V_{50} = 61.253$ km/h.
For each pair of \( K_V \) and \( V_{50} \) considered, a contour graph was generated. These graphs are the final result of the study and can be used in various ways:

1) Verify in an existing sag vertical curve the \( P_{nc} \) being known the variables involved: \( G_1 \), \( G_2 \), \( K_V \), \( V_{50} \). The \( V_{50} \) shall be derived from the horizontal alignment, being known the speed design (\( V_D \)) as shown in table 3.

For example, in the case of a sag vertical curve in an existing two-way road which have \( V_D = 70 \) km/h, \( K_V = 2160 \) m, \( G_1 = -5.5 \% \), \( G_2 = 3 \% \). The \( V_{50} \) associated at \( V_D = 70 \) km/h is \( V_{50} = 61.253 \) km/h. Finding the intersection point between \( G_1 = -5.5 \% \) and \( G_2 = 3 \% \) in the figure 6 it is possible to know the \( P_{nc} \), which is equal to 18 \%.

2) Design a new sag vertical curve using the \( P_{nc} \) as an indicator of good design choices. Known the speed distribution that derive from the horizontal alignment, a limit value of \( P_{nc} \) can be set and \( G_1 \), \( G_2 \), \( K_V \) selected so that their associated \( P_{nc} \) is less than the limit value of \( P_{nc} \).
Figure 6. Example of contour graph of $P_{nc}$ for $K_V = 2160$ m and $V_{50} = 61.253$ km/h.

For example, in the case of designing a sag vertical curve in a two-way highway which have $V_D = 70$ km/h. If a $K_V = 2160$ m, $G_1 = -5.5\%$ and $G_2 = 3\%$ are selected, the $P_{nc}$ associated is 18%. It is a value too high to design in safety, so other design parameters can be selected and then verified.

3) Compare two or more design choices using the $P_{nc}$ as a comparative parameter.

The known $P_{nc}$ values are only those represented by points in figure 4. In case it is necessary to know the $P_{nc}$ associated with different $K_V$ and $V_{50}$ values the weighted average can be used.
Figure 4. Domain of $K_v - V_d$.

In general, it is important to emphasize that the choice of $G_1$ and $G_2$ must respect the conditions 1 and 2 described in Sample generation. It can happen that the choice respects the conditions but the value of $G_1$ is not present in the x axis. This is because the case of two-way and only one direction of slope was considered. It is therefore sufficient to consider the opposite case of $G_1$ and $G_2$. For example, referring to the case in the Figure 6, the $P_{nc}$ of $G_1 = 4\%$ and $G_2 = 8\%$ wants to be calculate. The value of 4\% is not present on the x axis, so the opposite case is considered ($G_1 = -0.08$ and $G_2 = -0.04$).

Other interesting results were obtained using a prespecified (target) $P_{nc}$ in the calibration process. The goal of the calibration process then is to find values of design parameters, $G_1$, $G_2$, $K_v$, $V_{so}$, such that the $P_{nc}$ of the limit state function equals the prespecified value. The choice of a target $P_{nc}$ is a paramount decision that or the designer or the roadway agency are required to take. For this purpose, a real highway should be selected and an
accident analysis should be carried out. Once road characteristics and accident data are known, a safety performance function (SPF) can be developed and the maximum value of $P_{nc}$ can be set, as the acceptable risk level.

Not being possible to choose a priori a specific $P_{nc}$ target, calibrated charts with different target were depicted (figure 7). This calibrated chart may offer designers a tool to know the safety consequences of their decisions in terms of safety level.

From this chart it is evident that the variable that has the most impact on the calculation of $P_{nc}$ is the speed. The almost horizontal lines show that fixed the average speed and the target of $P_{nc}$, the range of $K_V$ varies very little with varying $G_1$. For example, the line associated with a target $P_{nc} = 1\%$ and $V_{50} = 70$ km/h, has a range of $K_V$ from 4119 m to 4165 m. If the same case with the target of $P_{nc} = 0.0001\%$ is considered, the range of $K_V$ varies from 7610 m to 8781 m. This means that as the target of $P_{nc}$ decreases, the influence of the $G_1$ increases.

From the same chart (figure 7) it can be observed that when the target of $P_{nc}$ increases, for the same value of $G_1$, it is possible to adopt a smaller value of $K_V$. This is an expected trend which confirms that the more restrictive the value of the target of $P_{nc}$ is, the greater the value of $K_V$ associated will be. The effect of target $P_{nc}$ is little for low-speed values and it increases as the speed value increases. For example, for $V_{50}=40$ km/h, the calibrated $K_V$ value varies between approx. 1170 and 1580 m; for $V_{50}=110$ km/h, it ranges from 7630 to 11200 m. In addition, the calibrated $K_V$ value is almost constant because both driving directions are weighted for the output $K_V$ value.
Conclusions

In this research study, it is proposed a new method to design, verify and compare sag vertical curves using reliability analysis. The LSF was represented by the difference between HSD and SSD. Given that LSF is not a function continuously differentiable, Monte Carlo method was selected to compute the $P_{nc}$ associated to 34,238 cases of sag vertical curves, considering the roadways traveled in both directions. This study proposes design values that no longer rely on design speeds but on the target $P_{nc}$ and the operating speed. The variables involved were characterized and considered as model parameters or random accordingly. To analyze the data, the 34,238 cases were split according to the possible combinations $K_V-V_{50}$. For every possible combination, a contour graph was depicted where it is possible to calculate the $P_{nc}$ for each pair of $G_1-G_2$ that observes specific conditions. The contour graphs derived from an interpolating surface of the cloud of points that represent the sample of cases. Using the contour graphs, it is possible to calculate the $P_{nc}$ setting the values of $K_V$, $V_{50}$, $G_1$ and $G_2$, where the value of $V_{50}$ is derived.
from the horizontal alignment. Using the approach described, the risk associated with the
choice of the parameters of design sag vertical curve can be evaluated.

Other interesting application was obtained using a prespecified (target) $P_{nc}$ in the
calibration process. Using the calibrated charts (such as Figure 7) it is possible to know
the design values associated with a prespecified target. In addition, it was found that the
variable that has the most significant impact on the $P_{nc}$ calculation is the speed.

As future line of research, the acceptable risk levels can be investigated and a comparison
with the Spanish standard can be conducted. To know the acceptable risk level, so the
maximum value of $P_{nc}$, a safety performance function (SPF) should be developed.

To incorporate adequately the uncertainty of the factors involved in the road design, the
use of reliability theory is recommended in the development of geometric design
standards and guides, and particularly for sag vertical curves.

**Data Availability Statement**

Some or all data, models, or code that support the findings of this study are available from
the corresponding author upon reasonable request.

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