Risk-based calibration of road sag vertical curve design guidelines on undivided highways

Alessio Rinaldi, MSc

Graduate Research Assistant.

Roma TRE University, Department of Engineering, 00146 Rome, Italy

ORCID: [0000-0002-5825-6290](https://orcid.org/0000-0002-5825-6290)

Cesar De Santos-Berbel, PhD

Assistant Professor. Departamento de Estructuras y Fisica de Edificacion

Universidad Politécnica de Madrid, 28040 Madrid, Spain

ORCID: [0000-0001-9686-8872](https://orcid.org/0000-0001-9686-8872)

Francesco Bella, PhD

Full Professor.

Roma TRE University, Department of Engineering, 00146 Rome, Italy

ORCID: [0000-0002-3724-7010](https://orcid.org/0000-0002-3724-7010)

Maria Castro, PhD (corresponding author: maria.castro@upm.es)

Associate Professor. Departamento de Ingenieria del Transporte, Territorio y Urbanismo

Universidad Politécnica de Madrid, 28040 Madrid, Spain

ORCID: [0000-0001-8941-5795](https://orcid.org/0000-0001-8941-5795)

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Abstract

Safety in highway geometric design is one of the main goals to be achieved. However, deterministic design criteria do not provide information concerning the risk associated with the design outputs proposed. To yield consistent safety levels, the sag vertical curve design model on undivided highways was calibrated using a reliability-based framework, which allows to incorporate the uncertainty associated with the model variables. The sag curve design model contemplates the features of vehicle frontlighting systems to compute the headlight sight distance (HSD), which must be equal to or greater than the stopping sight distance (SSD). A dataset of 34,238 case studies was generated. Each case study was associated with two values of the risk level, designated as probability of noncompliance (P_{nc}), one per driving direction. A Monte Carlo simulation was selected to calculate the values of P_{nc} . Through a series of interpolating surfaces of the cloud of points, contour graphs and calibrated charts were depicted. The paper provides a new methodology to verify, design and compare sag vertical curve, evaluating the risk level with the P_{nc} .

Keywords

Calibration; Geometric design; Reliability analysis; Sight distance

Introduction

Available sight distance (ASD) represents the distance that the driver can see along the path of their vehicle free from obstacles. This factor is crucial for highway safety, because when the ASD is less than the distance to perform a maneuver, such maneuver cannot be carried out safely (Gargoum and El-Basyouny 2020). Sag vertical curves are designed considering the headlight sight distance (HSD), the distance illuminated by the beam of light projected by the headlights. To ensure a safe design, the HSD must be equal to or greater than the stopping sight distance (SSD), which is defined as the distance travelled by the vehicle from the moment the driver recognizes an obstacle on the pavement that requires stopping until the vehicle comes to a complete stop. Both HSD and SSD are affected by four main factors: infrastructure, vehicle, driver capability and environmental condition. Current geometric design guides, to calculate these distances, adopt a deterministic approach ignoring the uncertainty of the factors involved. Therefore, the design guides adopt a conservative approach that does not contemplate the random nature of the variables involved.

In this study, the reliability theory was used to overcome the deficiencies associated with the geometric design of sag vertical curves based on a deterministic approach. The risk associated with sag vertical curve design is evaluated by calculating the probability of non-compliance (P_{nc}) to propose an alternative design method based on consistent design that meet a certain target reliability. To this end, the variables in the HSD and SSD equations were treated as random variables, with a probability distribution, or parameters as appropriate as opposed to the conventional deterministic methods.

The aim of this paper is to present a calibrated design model for sag vertical curves of undivided highways to yield design outputs with consistent risk levels. After the literature review, the methodology section presents basic concepts of reliability theory and provides

details about Monte Carlo sampling procedure. Next section introduces the equations used in the Limit State Function (LSF), input variable values and the generation of 34,238 sag curve cases. Results section shows the main graphs obtained and describes their use from a practical point of view. Last section summarizes the main conclusions and future research lines.

Literature review

The design of sag vertical curves in road design standards and guides is subject to a deterministic approach (AASHTO 2018; Ministerio de Fomento 2016; Ministero delle Infrastrutture e dei trasporti 2001). This approach is known to present two relevant shortcomings (Ismail and Sayed 2010). First, many model variables used in design are stochastic and not deterministic. For this reason, the deterministic approach requires to adopt conservative percentile values for uncertain design inputs, which are not based on definitive safety measures. As a result, the safety margin of the design output is unknown. Secondly, if the threshold design value imposed by the standards is not met, deviations are not admissible regardless of whether they are large or small and the risk involved in such deviations is unknown.

Reliability analysis accounts for the uncertainty in the variables involved in design models and evaluates the risk associated with a particular design feature. Although this probabilistic tool has been used mainly in structural engineering, a significant number of applications related to highway geometric design is found in literature in the last years. First, Navin (1990) suggested the use of partial safety factors and proposed the P_{nc} as a measure of reliability in design. You et al. (2012) examined the stability of different vehicles using reliability analysis. Musunuru and Porter (2014) studied the level of service on freeways under a stochastic approach to determine the required number of lanes. The risk associated to insufficient sight distance has been addressed in several

research studies. De Santos-Berbel and Castro (2015) assessed the P_{nc} associated to insufficient SSD along an existing highway, finding that the collision risk is generally higher on alignments that limit the ASD. De Santos-Berbel et al. (2017) compared the output of the reliability assessment of SSD on horizontal curves of 2D and 3D ASD models of existing highways. Ismail and Sayed (2012) utilized reliability theory to optimize the design of horizontal curves with right-of-way restrictions.

Several studies have examined the link between the P_{nc} associated with insufficient sight distance and collision frequency. Ibrahim and Sayed (2011) adjusted safety performance functions for horizontal curves, finding that predicted collisions have a statistically significant positive relationship with P_{nc} . Gargoum and El-Basyouny (2020) conducted a stochastic analysis of SSD on accident-prone highways, revealing that segments with higher non-compliance rates showed higher accident rates. Dhahir & Hassan (2019) adjusted safety performance functions considering weather conditions incorporating reliability indices of horizontal curve design according to four design criteria: vehicle stability, driver comfort, sight distance and vehicle rollover.

Concerning the calibration of design guides using reliability analysis, Hussein et al. (2014) proposed a method to design the middle ordinate offset on horizontal curves based on SSD at different P_{nc} levels. They also concluded that current design guides are conservative in this respect, especially at high speeds and on curves with small radii. Essa et al. (2016) developed system reliability calibration considering two modes of failure for the design of horizontal curves, namely SSD and vehicle stability. They proposed a more comprehensive design method for horizontal curves. The calibration outcome showed that there is a significant difference in the P_{nc} if two modes of failure are considered instead of one mode, particularly in curves with small radii. Mollashahi et al. (2017) calibrated superelevation values on horizontal curves considering the operating speed.

According to their results, superelevation values suggested by the AASTHO guide (AASHTO 2018) are generally lower than those required by vehicles in real situations, especially for horizontal curves where the design speed is less than 110 km/h.

The design of sag vertical curves is determined by headlamp features. Modern headlamp standards limit the amount of light emitted above the headlight beam axle which restricts the ability of drivers to see ahead (Gibbons et al. 2013). Whereas most design standards and guides set the upward angle of headlamp beam (α) at 1 degree, Hawkins and Gogula (2008) suggested that the value α should be reduced to a value between 0.75 and 0.9 degrees. In this respect, Andrade-Cataño et al. (2020) performed a reliability-based evaluation of the HSD on sag curve design values of the Spanish Standard. They combined several values of the upward angle α and the target height h_2 to compute the associated P_{nc} values, finding that the most conservative hypothesis in sag curve design comprised the smallest upward angle and the highest target height, which yielded significantly higher P_{nc} values. Whereas Andrade-Cataño et al. (2020) conducted a probabilistic evaluation of sag curve design outputs, this study proposes design values that no longer rely on design speeds but on the target P_{nc} and the operating speed.

Methodology

Reliability theory

By using a probabilistic approach based on reliability theory it is possible to eliminate the described deficiencies and, therefore, to have a design method that better reflects the nature of the variables involved. Instead of using prefixed values, the reliability approach uses random variables and the design equations are represented by the limit state function (LSF). The basic elements of reliability analyses are an N-dimensional vector of input variables $\mathbf{X} = x_1, x_2, \dots, x_n$ and the LSF denoted as $g(\mathbf{X})$.

141 The LSF is conventionally written in terms of the difference between supply R and
142 demand S .

$$g = R - S \quad (1)$$

143143

144 The underlying uncertainty of the input variables can be characterized by assigning each
145 input variable x_i a probability density function (PDF) $p(x)$. The reliability of a system, i.e.
146 the probability of it functioning within the acceptable performance domain, can be
147 obtained according to equation (2):

$$P_c = 1 - P_{nc} = Prob(g > 0) = 1 - \int_{g(x) \leq 0} p(x) dx \quad (2)$$

148148

149 In most cases, the integral in equation 2 cannot be solved analytically. Therefore, various
150 reliability methods can be used to achieve an approximate solution for equation 2
151 (Mahsuli and Haukaas 2013). On the one hand, analytical reliability methods are gradient-
152 based methods that require the LSF to be continuously differentiable. In addition, they
153 assume that supply and demand are independent. The most common analytical methods
154 are the first order reliability method (FORM), the second order reliability method
155 (SORM), the mean-value first order second moment (MVFOSM). On the other hand,
156 synthetic methods such as sampling methods, for instance Monte Carlo sampling, avoid
157 the main mathematical limitations of analytical methods while requiring more
158 computational time to be completed. Previous reliability-based studies concerning
159 geometric design contemplating non-independent supply and demand functions drawn on
160 Monte Carlo sampling (Andrade-Cataño et al. 2020; De Santos-Berbel and Castro 2015).

Monte Carlo Sampling

Although several sampling schemes are available to estimate the P_{nc} , Monte Carlo sampling is a popular approach, in which samples of the random variables x_i are generated according to their counterpart PDFs. Then, the expected value of a dependent variable Y is $\mu = E(Y)$, where $Y = g(\mathbf{X})$ and the random variables $\mathbf{X} = (x_1, x_2, \dots, x_n) \in D \subseteq \mathbb{R}^n$. The average of the values $x_1, \dots, x_n$, independently and randomly from the distribution of X , is an estimator of the random variables:

$$\hat{\mu}_n = \frac{1}{n} \sum_{i=1}^n x_i \quad (3)$$

X has a PDF $p(x)$ and g is a real-valued function defined over D . Therefore, the expected value can be noted as follows:

$$\mu = \int_{x \in X} g(x) p(x) dx \quad (4)$$

Let X be a random variable for which $\mu = E(X)$ exists, equation (5) can be written:

$$\lim_{n \rightarrow \infty} P(|\hat{\mu}_n - \mu| \leq \epsilon) = 1 \quad (5)$$

For any $\epsilon > 0$. Under the weak law of large numbers, the chance of missing by more than ϵ tends to zero.

The analytical reliability methods mentioned above are not appropriate to perform a reliability analysis when the LSF is not continuously differentiable.

Monte Carlo method is as more reliable as the number of iterations increases. The maximum number of iterations was selected as 100,000 and the P_{nc} is determined by equation (6):

$$P_{nc} = \frac{N_{pu}}{N} \quad (6)$$

Where:

- N_{pu} is the number of unsatisfactory performances
- N is the number of iterations

In order to achieve enough accuracy in sampling, the coefficient of variations of the successive P_{nc} values during the Monte Carlo iteration was monitored:

$$\delta_{P_{nc}} = \frac{\sigma_{P_{nc}}}{\mu_{P_{nc}}} \quad (7)$$

Where:

- $\delta_{P_{nc}}$ is the coefficient of variation of P_{nc}
- $\sigma_{P_{nc}}$ is the standard deviation of P_{nc}
- $\mu_{P_{nc}}$ is the mean value of P_{nc}

In this analysis, a value of 5% of the coefficient of variation was adopted to guarantee the accuracy of the sampling, which is a widely accepted value in studies that rely on sampling methods (Haukaas, 2019). For each case study, RT software (Mahsuli and Haukaas 2013) generated a vector of random variables, each one extracted from its counterpart random distribution and checked the compliance of the LSF. This process was iterated until the target coefficient of variation was obtained, or up to 100,000 iterations if otherwise.

Application to sag vertical curves

Limit state function

In design based in limit states, the system is considered to have failed or not complied with the design requirements when the demand exceeds the supply. In this calibration study, the LSF is represented by the difference between headlight sight distance (HSD) and stopping sight distance (SSD).

$$LSF = HSD - SSD \quad (8)$$

The value of HSD is calculated considering two different cases: when HSD is less or larger than the length of sag vertical curve (L) and the equations 9 and 10 was utilized respectively:

If $HSD < L$ (Figure 1, case 1)

$$HSD \cong K_V \cdot \tan \alpha \cdot \left(1 + \sqrt{1 + 2 \cdot \frac{h_h - h_2}{K_V \cdot \tan \alpha}}\right) \quad (9)$$

If $HSD > L$ (Figure 1, case 2)

$$HSD \cong \frac{K_V \theta^2 + 2 \cdot (h_h - h_2)}{2 \cdot (\theta - \tan \alpha)} \quad (10)$$

Where:

- K_V : rate of vertical curvature [m]
- α : upward divergence angle of headlamp beam [degrees]
- h_h : headlamp height [m]
- h_2 : target height [m]

- 218 • θ : absolute value of the algebraic difference between the inbound and outbound
 219 grades.

1) $HSD < L$

2) $HSD > L$

Figure 1. Geometric scheme of the two cases of sag vertical curve 1) $HSD < L$; 2) $HSD > L$

The value of SSD is calculated in accordance with Spanish geometric design standard (Ministerio de Fomento 2016):

$$SSD = \frac{V \cdot t_{pr}}{3.6} + \frac{V^2}{254(f_l + G)} \quad (11)$$

Where:

- V : initial speed of the braking maneuver [km/h]
- t_{pr} : perception-reaction time [s]
- f_i : tire-pavement friction
- G : longitudinal grade of the highway along the stopping maneuver

Input variables

In the calibration process, the variables included in the LSF were considered as calibration parameters, constant values, or random variables as discussed below.

- Rate of vertical curvature (K_v): this calibration parameter defines the geometry of the sag curve.
- Upward angle of headlamp beam (α): this value was set constant, equal to 0.75 degrees, according to the results discussed in the literature review (Hawkins and Gogula 2008).
- Headlamp height (h_h): The height of the headlight axle above the pavement surface is considered as a random variable following a normal distribution to cover the range of heights of the vehicles in the market. A previous study based on the 11 best-selling vehicles in Spain in 2015 indicated that the mean height was 0.731 m, and the standard deviation was 0.0052 m (De Santos-Berbel et al. 2016).
- Target height (h_2): in accordance to the Spanish geometric design standard the value of this parameter was set constant at 0.50 m (Ministerio de Fomento 2016). Moreover, as discussed in the literature review, the selection this value of the target height represents the most critical situation (Andrade-Cataño et al. 2020). The target height value aims to represent a portion of a potential obstacle on the roadway that permits the driver to recognize it. Due to the difficulty of setting a

statistical distribution of target height values, especially in relation to collision occurrence, a constant value was selected instead.

- Absolute value of the algebraic difference between the inbound and outbound grades (θ): This is a calibration parameter defined taking into account the restrictions of Spanish geometric design (Ministerio de Fomento 2016). A sample of grade values is generated at steps of 0.5 %, enclosed within the maximum grade values allowed in the standard, including the exceptional increases. In addition, the smaller grade in absolute value was taken as 0.2 %, the minimum grade allowed in the design standard to permit drainage to the road pavement. Table 1 shows the different range of grades in relation to design speed.

Table 1. Correlation between design speed and range of grades

Design Speed (km/h)	Range of grades (%)
100-140	-6 to 5
80-90	-7 to 7
60-70	-8 to 8
40-50	-10 to 10

- Initial speed of the braking maneuver (V): The speed in this calibration is assumed as a random variable of normal distribution and is derived from operating speed distributions. On sag vertical curves, the operating speed is assumed to be determined by the horizontal alignment. Given a mean speed value (V_{50}), the counterpart standard deviation (σ_V) is required to characterize the uncertainty associated to this variable. In this sense, the relation set in the operating speed model of Pérez-Zuriaga (2012) was adopted, which is given by equation (12).

$$\sigma_V = \sqrt{14.8194 + \exp\left(\frac{V_{50}^2 + 15760.216}{4841.26}\right)} \quad (12)$$

Table 2 shows the values of σ_V and V_{50} in relation with the design speed (V_D).

Table 2. Values resulting from model application.

V_D (km/h)	V_{50} (km/h)	σ_V (km/h)
40	32.872	6.878
50	42.487	7.249
60	51.958	7.759
70	61.253	8.439
80	70.333	9.328
90	79.150	10.469
100	87.655	11.911
110	95.797	13.703
120	103.528	15.893
130	110.805	18.520
140	117.601	21.611

- Perception-reaction time (t_{pr}): This variable was assumed as a random variable of lognormal distribution. The values from the study of Lerner (Lerner 1995), which are widely accepted in numerous studies, were also considered in the current application. The distribution is features by a mean value of 1.5 seconds and the standard deviation is 0.4 seconds.
- Tire-pavement friction (f_i): It was assumed to be a random of beta distribution since its value must be enclosed between 0 and 1. Considering that the Spanish geometric design standard (Ministerio de Fomento 2016) provides the 95th percentile value of this variable, the probability distribution function can be fully

characterized by means of a measure of deviation. Given that the random value of this variable is known to be conditioned by speed, each value of the speed V_{50} is associated to a beta distribution with its respective mean ($f_{l,50}$) and standard deviation (σ_{fl}). Therefore, the results of the research conducted by Bühlmann et al. (1991) were used to derive the relationship between the speed and the standard deviation of the tire-pavement friction, which lead to the following equation:

$$\sigma_{fl} = 0.0343 + 2.2 \cdot 10^{-3} \cdot V - 3.2 \cdot 10^{-5} \cdot V^2 + 1.35 \cdot 10^{-7} \cdot V^3 \quad (13)$$

Accordingly, the resulting beta PDFs at different speeds led to the following relationship:

$$f_{l,50} = 0.6673 - 2.8 \cdot 10^{-3} \cdot V + 1.1 \cdot 10^{-5} \cdot V^2 \quad (14)$$

The values obtained from equation (14) are displayed in figure 2 and are compared with other percentile values of the longitudinal friction in relation to the design speed.

Figure 2. Percentiles of longitudinal friction distribution as a function of the speed

- Longitudinal highway grade along the stopping maneuver (G): This input was computed as the average grade along the stopping maneuver. Depending on whether the vertical curve is longer or shorter than the SSD, two different equations apply. On either case, the counterpart mathematical expression is derived from the equation of a second-degree parabola of vertical axis:

$$z = z_0 + G_1(s - s_0) + \frac{(s - s_0)^2}{2 \cdot K_V} \quad (15)$$

Where:

- z_0 : elevation at the beginning of the sag curve [m].

- G_1 : longitudinal grade at the beginning of the sag curve.

306 - s : stationing on the sag curve [m]

307 - s_0 : stationing at the beginning of the sag curve [m]

308 - K_V : radius of the osculating circle of the sag curve parabola at its vertex [m]

309 Therefore, G is evaluated as per equations (16) and (17).

310 If $L < SSD$:

$$G = G_1 + \theta - \frac{K_V \cdot \theta^2}{2 \cdot SSD} \quad (16)$$

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312 If $L > SSD$:

$$G = G_1 + \frac{SSD}{2 \cdot K_V} \quad (17)$$

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314 As evidenced by equations (11), (16) and (17), a recursive dependence exists between the
 315 value of G and the SSD. Thus, a method of successive substitutions to approximate G and
 316 SSD was implemented in the calibration procedure. Substituting equation (16) or (17) in
 317 equation (11) as applicable, the scheme of this iteration process yields a non-linear
 318 equation as follows:

$$SSD^{(k+1)} = f(SSD^{(k)}) \quad k = 1, 2, 3 \dots \quad (18)$$

319319

320 Where k is the number of iterations. The value G_1 is used as initial guess and the
 321 calculation is iterated until the desired convergence is achieved.

$$|SSD^{(k+1)} - SSD^{(k)}| < 0.01 \text{ m} \quad (19)$$

322322

323 Figure 3 illustrates a directed graph characterizing the variables and parameters above
 324 described as well as their relations.

Figure 3. Variables involved in the calculation process of P_{nc} .

Generation of sag curve cases

The procedure devised for this calibration study proposes the generation of a sample of sag curves and the calculation of the counterpart P_{nc} considering first one travelling direction. The sample of sag curves comprised all possible combinations of parameters within certain ranges (Table 1 and 2) complying a set of conditions relating to operating and appearance criteria of the Spanish geometric design standard (Ministerio de Fomento 2016):

- 1) The values of K_V (m) selected were the following: 700; 1,030; 1,380; 1,760; 2,160; 2,590; 3,050; 3,540; 4,070; 4,630; 5,230; 5,870; 6,560; 7,290; 8,080; 8,920; 9,820; 10,780; and 11,810.
- 2) The inbound and outbound grades were enclosed within the maximum grade values allowed by the standard, including the exceptional increases, in relation to the corresponding design speeds (Table 2).
- 3) The combinations of inbound and outbound grades were to result in lengths equal to or greater than the counterpart design speed ($L \geq V_D$). The length of a sag curve is given by:

$$L = K_V \cdot \theta \quad (20)$$

In addition, a preliminary computation of the P_{nc} was performed to limit the number of cases to those that are associated to admissible levels of risk, ruling out datasets where all combinations of parameters yielded P_{nc} values higher than 0.2. Therefore, the domain in Figure 4 was finally considered. Under these considerations, 66,384 sag vertical curves were generated and the counterpart P_{nc} values were retrieved. To evaluate the overall P_{nc} on a sag curve considering both travelling directions, pairs of case studies that correspond to identical geometric designs viewed from opposing travelling directions were merged into one case, assuming that speed distributions are identical on both travelling directions and collision risk due to insufficient HSD are independent on either travelling direction. To this end, pairs of sag curves with the same K_V value and same speed distribution where the inbound grade value of one sag was the opposite to that of the outbound grade of the other sag and vice versa were combined, and the average of both P_{nc} values was associated to the sag curve considering both travelling directions. Symmetrical sags were incorporated the same into the results dataset. Following these criteria, the results dataset comprised a total of 34,238 sag vertical curves.

Results and discussion

The goal of calibration is to propose design outputs that meet both the design constraints and a prespecified target P_{nc} value.

The 34,238 case studies were split according to the domain in figure 4 and 146 combinations of K_V and V_{50} were considered. To analyze the data, the same procedure was used for each combination:

- 3D scatter plot was created to represent the cloud of points that resulted from depicting G_2 on the x axis, the corresponding G_1 on the y axis, and the P_{nc} on the vertical axis (Figure 5).

- An interpolating surface of the cloud of points corresponding to P_{nc} was computed (Figure 5).

- The contour graph of P_{nc} derived from the interpolating surface was depicted (Figure 6).

The domain of the graphs in Figures 5 and 6 is illustrated by the blue points that represent the cases analyzed.

Figure 5. Example of cloud of points and interpolated surface using regression. Data corresponding to $K_v = 2160$ m and $V_{50} = 61.253$ km/h.

For each pair of K_V and V_{50} considered, a contour graph was generated. These graphs are the final result of the study and can be used in various ways:

1) Verify in an existing sag vertical curve the P_{nc} being known the variables involved: G_1 , G_2 , K_V , V_{50} . The V_{50} shall be derived from the horizontal alignment, being known the speed design (V_D) as shown in table 3.

For example, in the case of a sag vertical curve in an existing two-way road which have $V_D = 70$ km/h, $K_V = 2160$ m, $G_1 = -5.5\%$, $G_2 = 3\%$. The V_{50} associated at $V_D = 70$ km/h is $V_{50} = 61.253$ km/h. Finding the intersection point between $G_1 = -5.5\%$ and $G_2 = 3\%$ in the figure 6 it is possible to know the P_{nc} , which is equal to 18 %.

2) Design a new sag vertical curve using the P_{nc} as an indicator of good design choices. Known the speed distribution that derive from the horizontal alignment, a limit value of P_{nc} can be set and G_1 , G_2 , K_V selected so that their associated P_{nc} is less than the limit value of P_{nc} .

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390 **Figure 6.** Example of contour graph of P_{nc} for $K_v = 2160\text{m}$ and $V_{50} = 61.253\text{ km/h}$.

391 For example, in the case of designing a sag vertical curve in a two-way highway which
 392 have $V_D = 70\text{ km/h}$. If a $K_v = 2160\text{ m}$, $G_1 = -5.5\%$ and $G_2 = 3\%$ are selected, the P_{nc}
 393 associated is 18%. It is a value too high to design in safety, so other design parameters
 394 can be selected and then verified.

395 3) Compare two or more design choices using the P_{nc} as a comparative parameter.

396 The known P_{nc} values are only those represented by points in figure 4. In case it is
 397 necessary to know the P_{nc} associated with different K_v and V_{50} values the weighted
 398 average can be used.

399399

400 **Figure 4.** Domain of $K_V - V_D$.

401 In general, it is important to emphasize that the choice of G_1 and G_2 must respect the
 402 conditions 1 and 2 described in Sample generation. It can happen that the choice respects
 403 the conditions but the value of G_1 is not present in the x axis. This is because the case of
 404 two-way and only one direction of slope was considered. It is therefore sufficient to
 405 consider the opposite case of G_1 and G_2 . For example, referring to the case in the Figure
 406 6, the P_{nc} of $G_1 = 4\%$ and $G_2 = 8\%$ wants to be calculate. The value of 4% is not present
 407 on the x axis, so the opposite case is considered ($G_1 = -0.08$ and $G_2 = -0.04$).

408 Other interesting results were obtained using a prespecified (target) P_{nc} in the calibration
 409 process. The goal of the calibration process then is to find values of design parameters,
 410 G_1 , G_2 , K_V , V_{50} , such that the P_{nc} of the limit state function equals the prespecified value.

411 The choice of a target P_{nc} is a paramount decision that or the designer or the roadway
 412 agency are required to take. For this purpose, a real highway should be selected and an

accident analysis should be carried out. Once road characteristics and accident data are known, a safety performance function (SPF) can be developed and the maximum value of P_{nc} can be set, as the acceptable risk level.

Not being possible to choose a priori a specific P_{nc} target, calibrated charts with different target were depicted (figure 7). This calibrated chart may offer designers a tool to know the safety consequences of their decisions in terms of safety level.

From this chart it is evident that the variable that has the most impact on the calculation of P_{nc} is the speed. The almost horizontal lines show that fixed the average speed and the target of P_{nc} , the range of K_v varies very little with varying G_1 . For example, the line associated with a target $P_{nc} = 1\%$ and $V_{50} = 70$ km/h, has a range of K_v from 4119 m to 4165 m. If the same case with the target of $P_{nc} = 0.0001\%$ is considered, the range of K_v varies from 7610 m to 8781 m. This means that as the target of P_{nc} decreases, the influence of the G_1 increases.

From the same chart (figure 7) it can be observed that when the target of P_{nc} increases, for the same value of G_1 , it is possible to adopt a smaller value of K_v . This is an expected trend which confirms that the more restrictive the value of the target of P_{nc} is, the greater the value of K_v associated will be. The effect of target P_{nc} is little for low-speed values and it increases as the speed value increases. For example, for $V_{50} = 40$ km/h, the calibrated K_v value varies between approx. 1170 and 1580 m; for $V_{50} = 110$ km/h, it ranges from 7630 to 11200 m. In addition, the calibrated K_v value is almost constant because both driving directions are weighted for the output K_v value.

Figure 7. Calibrated chart with different target P_{nc} , fixed $G_2=3\%$

Conclusions

In this research study, it is proposed a new method to design, verify and compare sag vertical curves using reliability analysis. The LSF was represented by the difference between HSD and SSD. Given that LSF is not a function continuously differentiable, Monte Carlo method was selected to compute the P_{nc} associated to 34,238 cases of sag vertical curves, considering the roadways traveled in both directions. This study proposes design values that no longer rely on design speeds but on the target P_{nc} and the operating speed. The variables involved were characterized and considered as model parameters or random accordingly. To analyze the data, the 34,238 cases were split according to the possible combinations K_V - V_{50} . For every possible combination, a contour graph was depicted where it is possible to calculate the P_{nc} for each pair of G_1 - G_2 that observes specific conditions. The contour graphs derived from an interpolating surface of the cloud of points that represent the sample of cases. Using the contour graphs, it is possible to calculate the P_{nc} setting the values of K_V , V_{50} , G_1 and G_2 , where the value of V_{50} is derived

from the horizontal alignment. Using the approach described, the risk associated with the choice of the parameters of design sag vertical curve can be evaluated.

Other interesting application was obtained using a prespecified (target) P_{nc} in the calibration process. Using the calibrated charts (such as Figure 7) it is possible to know the design values associated with a prespecified target. In addition, it was found that the variable that has the most significant impact on the P_{nc} calculation is the speed.

As future line of research, the acceptable risk levels can be investigated and a comparison with the Spanish standard can be conducted. To know the acceptable risk level, so the maximum value of P_{nc} , a safety performance function (SPF) should be developed.

To incorporate adequately the uncertainty of the factors involved in the road design, the use of reliability theory is recommended in the development of geometric design standards and guides, and particularly for sag vertical curves.

Data Availability Statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request.

References

AASHTO. 2018. *A Policy on Geometric Design of Highways and Streets*. Washington, DC: AASHTO.

Andrade-Cataño, F., De Santos-Berbel, C., and Castro, M. 2020. "Reliability-based safety evaluation of headlight sight distance applied to road sag curve standards." *IEEE Access*, 8 (Feb), 43606–43617.

<https://doi.org/10.1109/ACCESS.2020.2977258>.

Bühlmann, F., Lindenmann, H. P., and Spacek, P. 1991. *Überprüfen der Grundlagen*

- zur VSS-Norm SN 640 090 Projektierungsgrundlagen, Sichtweiten. ETH Zurich Research Collection, Zurich, Switzerland.
- De Santos-Berbel, C., and Castro, M. 2015. "Stopping-sight-distance simulation using a new probabilistic approach." *5th International Symposium on Highway Geometric Design*, 1–13. Washington, DC: Transportation Research Board.
- De Santos-Berbel, C., Castro, M., and Iglesias, L. 2016. "Influence of headlamp lighting parameters on nighttime sight distance." In *Proc. 3rd Int. Conf. on Traffic and Transport Engineering (ICTTE)*, 88–94. Belgrade, Serbia: City Net Scientific Research Center Ltd.
- De Santos-Berbel, C., Essa, M., Sayed, T., and Castro, M. 2017. "Reliability-based analysis of sight distance modelling for traffic safety." *J. Adv. Transp.* 2017: 5612849. <https://doi.org/10.1155/2017/5612849>.
- Dhahir, B., and Hassan, Y. 2019. "Using horizontal curve speed reduction extracted from the naturalistic driving study to predict curve collision frequency." *Accid. Anal. Prev.*, 123 (Feb 2018), 190–199. <https://doi.org/10.1016/j.aap.2018.11.020>.
- Essa, M., Sayed, T., and Hussein, M. 2016. "Multi-mode reliability-based design of horizontal curves." *Accid. Anal. Prev.* 93 (Aug): 124–134. <https://doi.org/10.1016/j.aap.2016.04.023>.
- Gargoum, S. A., and El-Basyouny, K. 2020. "Analyzing the ability of crash-prone highways to handle stochastically modelled driver demand for stopping sight distance." *Accid. Anal. Prev.*, 136 (Mar 2019), 105395. <https://doi.org/10.1016/j.aap.2019.105395>.
- Gibbons, R. B., Medina, A., Williams, B., Du, J., and Rakha, H. 2013. *Sag Vertical*

- 496 *Curve Design Criteria for Headlight Sight Distance*. NCHRP Project No. 15-41.
497 Blacksburg, VA: Virginia Tech Transportation Institute.
- 498 Haukaas 2019. "Sampling". Accessed June 10, 2020. [http://civil-](http://civil-terje.sites.olt.ubc.ca/files/2019/06/Sampling.pdf)
499 [terje.sites.olt.ubc.ca/files/2019/06/Sampling.pdf](http://civil-terje.sites.olt.ubc.ca/files/2019/06/Sampling.pdf)
- 500 Hawkins, H. G., and Gogula, M. 2008. "Assessment of sag curve design criteria
501 considering modern headlamp performance." *Transp. Res. Rec.* 2060 (1), 3–9.
502 <https://doi.org/10.3141/2060-01>.
- 503 Hussein, M., Sayed, T., Ismail, K., and Van Espen, A. 2014. "Calibrating Road Design
504 Guides Using Risk-Based Reliability Analysis." *J. Transp. Eng.* 140 (9):
505 04014041. [https://doi.org/10.1061/\(ASCE\)TE.1943-5436.0000694](https://doi.org/10.1061/(ASCE)TE.1943-5436.0000694).
- 506 Ibrahim, S. E. B., and Sayed, T. 2011. "Developing safety performance functions
507 incorporating reliability-based risk measures." *Accid. Anal. Prev.* 43 (6): 2153–
508 2159. <https://doi.org/10.1016/j.aap.2011.06.006>
- 509 Ismail, K., and Sayed, T. 2010. "Risk-based highway design: Case studies from British
510 Columbia, Canada." *Transp. Res. Rec.* 2195 (1): 3–13.
511 <https://doi.org/10.3141/2195-01>.
- 512 Ismail, K., and Sayed, T. 2012. "Risk-optimal highway design: Methodology and case
513 studies." *Saf. Sci.* 50 (7): 1513–1521. <https://doi.org/10.1016/j.ssci.2012.02.001>
- 514 Lerner, N. 1995. "Age and driver perception-reaction time for sight distance design
515 requirements." In *Proc., 1995 Compendium of Technical Papers*. Institute of
516 Transportation Engineers 65th Annual Meeting, 624–628. Washington, DC:
517 Institute of Transportation Engineers.
- 518 Mahsuli, M., and Haukaas, T. 2013. "Computer program for multimodel reliability and

- 519 optimization analysis.” *J. Comput. Civ. Eng.* 27 (1): 87–98.
520 [https://doi.org/10.1061/\(ASCE\)CP.1943-5487.0000204](https://doi.org/10.1061/(ASCE)CP.1943-5487.0000204).
- 521 Ministerio de Fomento. 2016. *Norma 3.1 -Instrucción de Carreteras Trazado*. [In
522 Spanish.] Madrid, Spain: Ministerio de Fomento.
- 523 Ministero delle Infrastrutture e dei trasporti. 2001. *Norme Funzionali E Geometriche*
524 *Per la Costruzione Delle Strade*. [In Italian.] Rome: Ministero delle Infrastrutture e
525 dei trasporti.
- 526 Mollashahi, H. F., Khajavi, K., and Ghaeini, A. K. 2017. “Safety Evaluation and
527 Adjustment of Superelevation Design Guides for Horizontal Curves Based on
528 Reliability Analysis.” *J. Transp. Eng. Part A. Syst.* 143 (6): 04017013.
529 <https://doi.org/10.1061/JTEPBS.0000037>
- 530 Musunuru, A., and Porter, R. J. 2014. “Reliability-Based Geometric Design Approach
531 for Selecting Basic Number of Freeway Lanes.” *Transp. Res. Rec.* 2436 (1): 70–
532 80. <https://doi.org/10.3141/2436-08>.
- 533 Navin, F. P. D. 1990. “Safety Factors for Road Design: Can They Be Estimated?”
534 *Transportation Research Record*, 1280 (1), 181–189.
- 535 Pérez-Zuriaga, A. M. 2012. “Caracterización y modelización de la velocidad de
536 operación en carreteras convencionales a partir de la observación naturalística de la
537 evolución de vehículos ligeros.” [In Spanish.] Ph.D. dissertation, Departamento de
538 Ingeniería e Infraestructura de los Transportes. Universidad Politécnica de
539 Valencia.
- 540 You, K., Sun, ; Lu, and Gu, W. 2012. “Reliability-Based Risk Analysis of Roadway
541 Horizontal Curves. *J. Transp. Eng.* 138 (8): 1071–1081.

542 [https://doi.org/10.1061/\(ASCE\)TE.1943-5436.0000402](https://doi.org/10.1061/(ASCE)TE.1943-5436.0000402).

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