USING INTERACTIVE METHODS FOR TEACHING PROPERTIES OF PRINCIPAL AXES AND MOMENTS OF INERTIA

E. Martínez[1], M. Flórez[1], M.V. Carbonell[1]

[1] *Universidad Politécnica de Madrid. Madrid (SPAIN)*
elvira.martinez@upm.es , mercedes.florez@upm.es, victoria.carbonell@upm.es

Abstract

Engineering analysis and design often uses properties of plane sections in calculations. For example, in stress analysis of a beam under bending and torsional loads, you use the cross-sectional properties to determine the stress and displacement distributions in the beam cross section. In calculating the natural frequencies and shapes of a machine part, you also need to know the area, centroid, and various moments and products of inertia of a cross section or a composed cross section.

The centroid, moments and products of inertia of an area made up of several common shapes (rectangles usually), may thus be obtained by adding the moments of inertia of the component areas. However the parallel-axes theorems should be used to transfer each moment of inertia to the desired axis. This procedure is used to calculate the second moments of structural shapes (U-shape, L-shape, C-shape,…) because the determination of their moments of inertia is necessary for design structural members.

Many engineering students are introduced to the ideas and concepts of Mohr's Circle when studying stress states due to various loading conditions on structures or machines. Mohr's circle provides a visual representation of what is happening, and the positioning of the stress states on an element relative to a set of coordinate axes.

In this paper we propose the use of interactive methods for teaching this part of Mechanics. A set of simulations designed with the Modellus 2.5 software are used for calculating centroids, moment and product of inertia and their properties. Mohr´s circles are also drawn with interactive simulations to obtain the principal axes and moments of inertia of structural shapes. For solving problems, students can interact in the simulation, by changing shapes, size, angles, etc.

1 INTRODUCTION

In continuum mechanics, stress is a measure of the average force per unit area of a surface within a deformable body on which internal forces act. In other words, it is a measure of the intensity of the internal forces acting within a deformable body across imaginary surfaces. The stress analysis is required in engineering, e.g., civil engineering and mechanical engineering, for the study and design of structures, e.g., tunnels, dams, mechanical parts, and structural frames among others, under prescribed or expected loads.

These internal forces are produced between the particles in the body as a reaction to external forces applied on the body. External forces are either surface forces or body forces. Because the loaded deformable body is assumed as a continuum, these internal forces are distributed continuously within the volume of the material body, i.e., the stress distribution in the body is expressed as a piecewise continuous function of space coordinates. Although the forces are distributed continuously within the volume, we can consider the resultant force which may act in a point called centroid.

The second moment of area, also known as the area moment of inertia or second moment of inertia is a property of a cross section that can be used to predict the resistance of beams to bending and deflection. The deflection of a beam under load depends not only on the load, but also on the geometry of the beam's cross-section. This is why beams with higher area moments of inertia, are so often seen in building construction as opposed to other beams with the same area.

The centroid, second moment of inertia (moments and products of inertia) of an area made up of several common shapes (rectangles usually), may thus be obtained by adding the moments of inertia of the component areas. However the parallel-axes theorems should be used to transfer each moment of inertia to the desired axis. This procedure is used to calculate the second moments of structural shapes (U-shape, L-shape, C-shape,…) because the determination of their moments of inertia is necessary for design structural members.

Many engineering students have defective skills in understanding and calculating centroids, second moments of inertia (moment and product relate to axes) as well as the mean axes and mean moment

of inertia. In order to solve this problem, teachers of the Department of Physics at the Agricultural Engineering Schools of Madrid are using interactive methods in teaching this part of Mechanics.
A set of simulations designed with the Modellus 2.5 software is being used for calculating centroids, moment and product of inertia and properties of compose structural shapes.
Mohr´s circles are also drawn with interactive simulations to obtain the principal axes and moments of inertia of structural shapes.

2 METHODOLOGY

The Modellus 2.5 software is a mathematical modelling tool freely available on the Internet. In it, initial conditions, parameters, and algebraic and differential equations are explicitly visible in mathematical notation. To build a model we write conventional mathematical equations and expressions as they would appear on paper, and programming knowledge is not necessary.

We propose the use of simulations in class as an active learning strategy [2]; for this purpose authors generated a set of simulations concerning different parts of Mechanics in a "virtual laboratory where students can access them "any where" at "any time" [3]. Other resources have been developed and included in the virtual platform Moodle, all of which students can download from the platform [4].

Teachers of "Mechanics and Mechanisms" used these simulations to explain properties of the second moments of inertia and the calculation of principal axis and moments of inertia of the structural shapes most used by engineers. Students used those to check the properties explained in class, and later, to solve problems.

Although there is other free software available to obtain the principal moment of inertia, we have selected this because students don´t need knowledge of programming and can design their own simulations [5,6].

2.1. Simulations

In this paper we present four simulations used to explain properties of moments and products of inertia and calculate the principal axes and moments of inertia.

As the structural shapes are usually composed of rectangles**, Simulation 1** has been designed in order to study properties of second moments of area of this figure. After starting it, the size and position of a rectangle can be changed by replacement of the base and height values (B1 and H1 respectively). After that, we are going to study the following properties of moment of inertia.

A property of moments of inertia states that "the moments of inertia with respect to axis through the centroid (Icx, Icy) don´t depend on the position of the shape in space. Students can check this property by moving the rectangle in space and observing that these values don´t change during movement and, check that the value is

$$I_{CX} = \frac{1}{12} A \cdot H^2 = \frac{1}{12} (7200 cm^2) \cdot (cm)^2 = 2.16 \cdot 10^6 \ cm^4 \quad (5).$$

2. Parallel-axis theorem of Steiner´s theorem. The parallel axis theorem is a relation between the moment of inertia about an axis passing through the centroid and the moment of inertia about any parallel axis.

$$I_{OX} = I_{CX} + A \cdot y_C^2 \quad (6).$$

Fig. 1 shows the rectangle in a particular position; the centroid coordinates, the moment and product of inertia with respect to axes through the centroid (CX, CY) and through origin (OX, OY) are also recorded.

Then, these values depend on the position of the rectangle in space. By moving the rectangle through space, these values are obtained. Students can also get them by using the parallel-axis theorem (Steiner´s theorem) as showed in Fig. 2. Then,

$$I_{OX} = I_{CX} + A \cdot y_C^2 = 2.16 \cdot 10^6 + (7200 cm^2) \cdot (60 cm)^2 = 7.42 \cdot 10^7 \ cm^4 \quad (7).$$

3. As the rectangle is moving, we can also watch how the centroid coordinates (xc, yc) are changing and these values are (0,0) when the centre of rectangle coincides with the origin of axes.

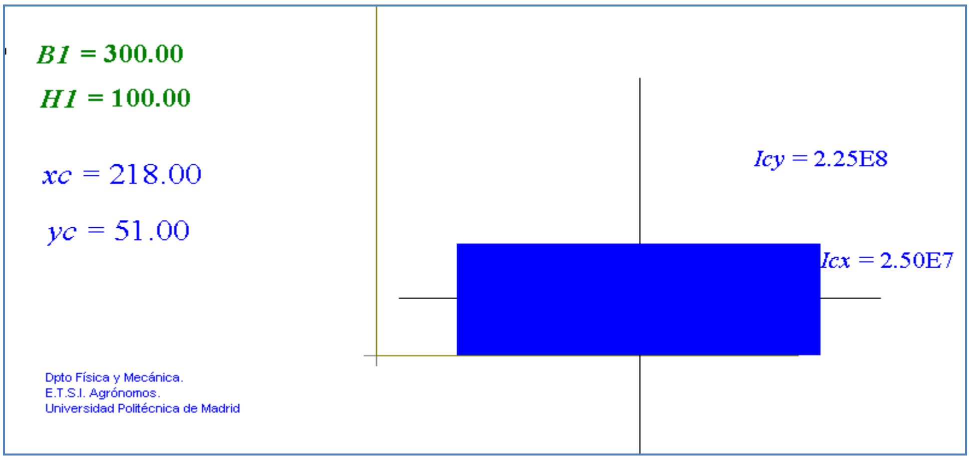

Figure 1. the moments of inertia of axes located at the centroid (Icx, Icy) don´t depend on the location of the shape in the space.

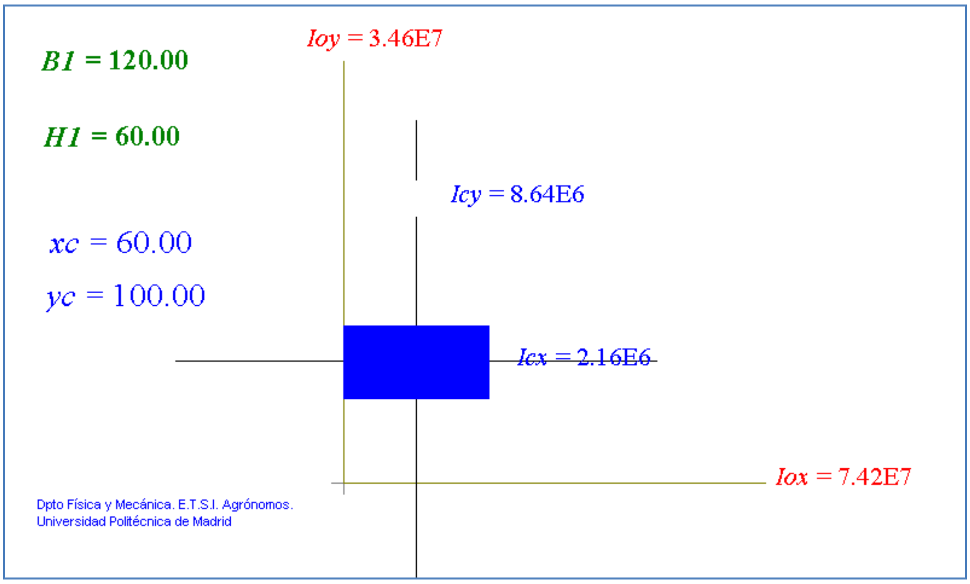

Figure 2. Steiner´s Theorem

Simulation 2. The objective is determine the moment and product of inertia respect different axis.

The moment of inertia with respect to the axis R can be calculated by applying the formula

$$I_R = I_{Ox}\cos^2\alpha + I_{OY}\cos^2\beta - 2P_{Xy}\cos\alpha\cos\beta \quad (8)$$

and the product of inertia with respect to axes R1 and R2, according to

$$P_{R_1R_2} = P_{XY} sen(\alpha_1+\alpha_2) - I_{Ox}\cos\alpha_1\cos\alpha_2 - I_{OY}\, sen\alpha_1\, sen\alpha_2 \quad (9)$$

The simulation also can be used to obtain the moments and product of inertia with respect to axes R1 and R2, substituting the new values for the angles as shown in fig 3. In this figure we have introduced values α1=20º and α2=70º and the moment and product of inertia with respect to axes R1 and R2 have been calculated at the top of the figure

I1=5.46·107 cm4   I2=7.82·107 cm4   and P12=5.68·107 cm4.

When the axes R1 and R2 coincide with the OX and OY axes, the I1 and I2 values are the as the IOX and IOY, while the product P12 is equal the P value.

Fig. 3 shows the moment of inertia respect and axe and the product of inertia respect axis R1 and R2 respectively.

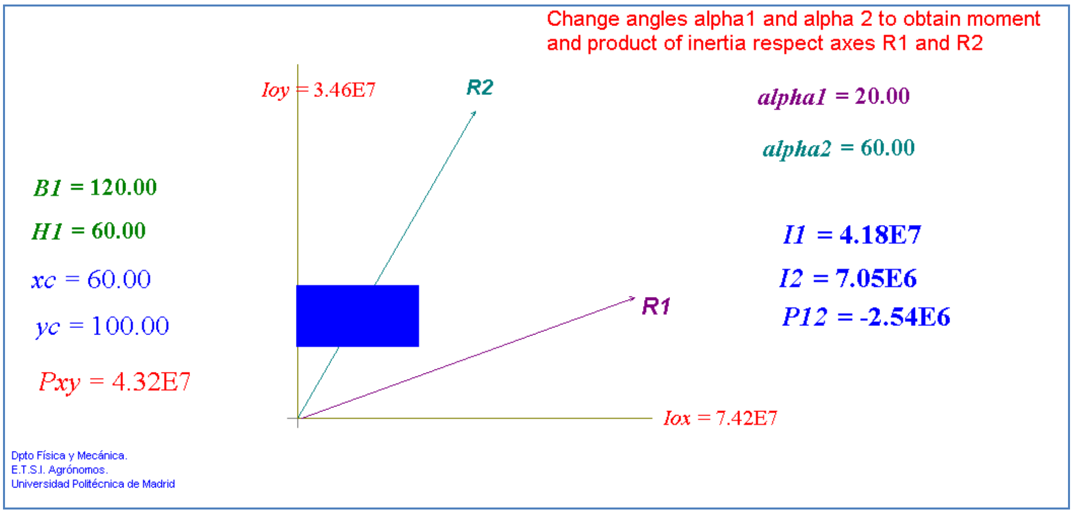

Figure 3. Moment and product of inertia respect axes R_1 and R_2

In accordance with the properties of moments of inertia of plane shapes, the moment of inertia with respect to a point O (IO) is the sum of moments of inertia with respect to two perpendicular axes passing through this point; then

$$I_O = I_{OX} + I_{OY} = I_{R_1} + I_{R_2} \quad (10)$$

Choosing angles α2=90+α1, students can check this property as shown in fig. 4.

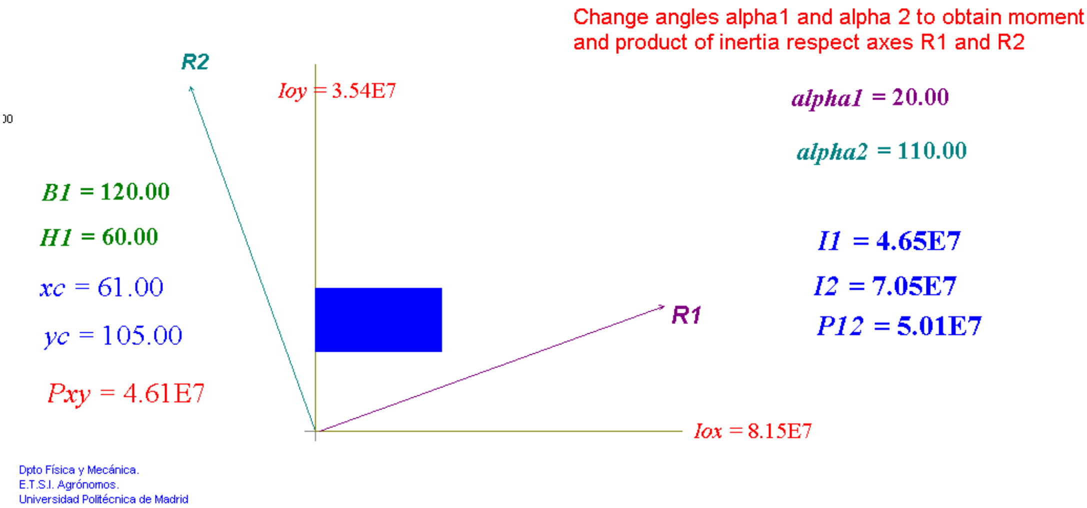

Figure 4. The moment of inertia respect two perpendicular axes

Simulation 3. Drawing Mohr´s circle. Simulation 3 allows students to obtain the principal moment and axes of inertia by using Mohr´s circle. For that, we´ll call I1 and I2 the moment of inertia with respect to axis OX and OY respectively. By introducing these values and the P value in the simulation, Mohr´s circle is drawn and the student obtains the maximum and minimum value of the moments of inertia as well as the angles necessary to rotate the OX-OY axes to obtain the principal moment. In order to draw Mohr´s circle using the appropriate scale, we´ll use a factor. Then, if IOX=4.6·10^6 cm^4 we must write I1=460 and factor=10000.

Fig. 5 shows the Mohr´s circle. After introducing the moment and product of inertia, the circle is drawn, the maximum and minimum values are calculated and the principal axis of inertia is also drawn.

If In this we can observe that the maximum value of the moment of inertia is Imax=255.69 cm^4, obtained for the R1-axe, rotating 25.45º the x-axis clockwise. The minus value of moment of inertia, Imin =8.31 cm^4 was obtained for the R2 axe, rotating 25.45º the y-axis clockwise.

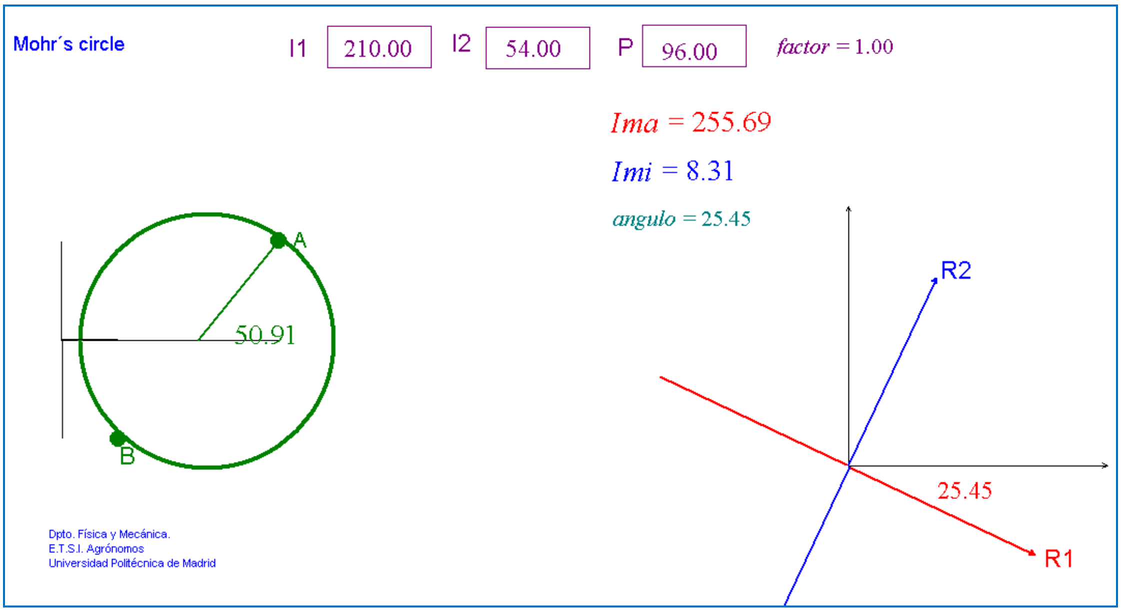

Figure 5. Mohr´s circle.

Simulation 4 allows the determination of principal moment and axes of inertia by using the Lagrange´s multipliers Method as an alternative method.

By introducing these values and the P value in the simulation, the equations are changed and the maximum and minimum values are calculated and the principal axis of inertia are drawn as showed in fig 6.

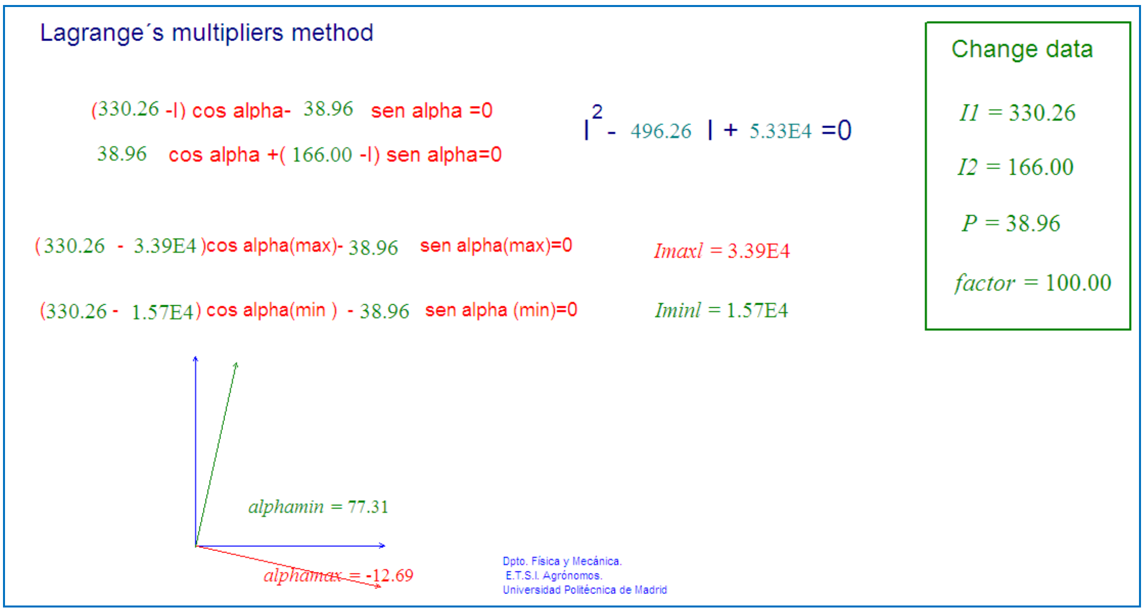

Figure 6. Langrange´s multipliers equations

3 CONCLUSIONS

1. Use of simulations helps to deepen and broaden the understanding of mechanical concepts and laws; the software has allowed us, the teachers to make our own didactic design, paying attention to the key points we consider most remarkable.

2. Mechanics teachers have used these to explain properties of moment of inertia. They used these in class and interacted by changing size and position to check the properties. Students preferred the interactive simulations to the classical explanations.

3. Teachers used these to solve problems in face-to-face classes and students used these to solve problems at home by using mathematical methods. Students who used simulations spent less time than students who used mathematical methods.

4. Students can use these for other subjects, to design their own simulations for this subject and others.

5. We hope other professors will make use of these resources, useful in the development of critical analysis in students.

4 REFERENCES

[1].Moodle. http://www.moodle.org

[2]E. Martinez, M.V. Carbonell, M. Flórez, J.M. Amaya. Simulations as a new Physics Teaching tool. Computer Applications in Engineering Education 1, 2009 (1-4)

[3] C. Romero, E. Martinez ¡Aprende Física con Simulaciones" y otras ramas de la Física. Congreso de Estudiantes. Universidad Politécnica de Madrid. Libro de Actas. Mayo 2010. (Accepted)

[4]V.T. Duarte Modellus. http://phoenix.sce.fct.unl.pt/modellus/

[5]Aulatematica. http://www.aulamatematica.com/Classpad/VCP/vcp.htm. Last visit: 07/03/2008

[6]Círculo de Mohr. http://www.unalmed.edu.co/~jmfernan/hp.html