Benchmarking a MOS-based Algorithm on the BBOB-2010 Noisy Function Testbed

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ABSTRACT

In this paper, a hybrid algorithm based on the Multiple Offspring Sampling framework is presented and benchmarked on the BBOB-2010 noisy testbed. MOS allows the seamless combination of multiple metaheuristics in a hybrid algorithm capable of dynamically adjusting the participation of each of the composing algorithms. The experimental results show a good performance on functions with moderate noise. However, on functions with severe noise the results deteriorate, which suggests that further research should be conducted to find more adequate control mechanisms for these types of functions.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking of algorithms, Black-box optimization, Continuous optimization, IPOP-CMA-ES, Differential Evolution, Multiple Offspring Sampling

1. INTRODUCTION

In this contribution, a hybrid algorithm constructed by means of the Multiple Offspring Sampling (MOS) framework [5] has been applied to the Black Box Optimization 2010 Noisy Function Testbed. This framework allows the combination of different evolutionary models following an HRH (High-level Relay Hybrid) approach (according to Talbi's taxonomy [8]) where the number of evaluations that each algorithm can carry out is dynamically adjusted according to their current performance. In this type of algorithms, two metaheuristics are executed in sequence, one after the other. For this paper, the IPOP-CMA-ES [1] and the DE algorithm [7] have been combined within this framework in a multistart strategy on 30 different functions. This algorithm is the same as the one presented in a complementary paper of the same proceedings [6].

2. ALGORITHM AND PARAMETERS

The algorithm and all parameters are described in the similar work on the Noiseless Testbed [6]. Due to the lack of time to do a proper parameter tuning on the noisy testbed, all the parameters values were kept the same as for the noiseless testbed.

3. RESULTS

Results from experiments according to [3] on the benchmark functions given in [2, 4] are presented in Figures 1, 2 and 3 and in Tables 1, 2 and 3.

The overall results in the noisy testbed are not as satisfactory as in the case of the noiseless one [6] in terms of achieved precision and scalability. The hybrid algorithm here presented is able to solve 30, 27, 25, 19, 16 and 10 functions out of 30 in 2, 3, 5, 10, 20 and 40 dimensions, respectively. It seems that the noise added to the functions makes the performance of the algorithm deteriorate as the number of dimensions increases. This effect is more pronounced in the case of those functions with severe noise than in those with a moderate noise.

Compared with the individual use of its composing algorithms, the DE seems not to be of much help in this type of functions. Fortunately, the regulatory mechanisms of the MOS framework are able to detect this behavior and minimize the participation of the DE technique. As a consequence of this, the overall behavior of the hybrid algorithm is quite similar to that exhibited by the CMA-ES when used individually, though it presents small variations for some groups of dimensions: in 2, 3 and 40 dimensions it seems to have a better performance, whereas IPOP-CMA-ES seems to be slightly better in the rest of the dimensions (5, 10 and 20).

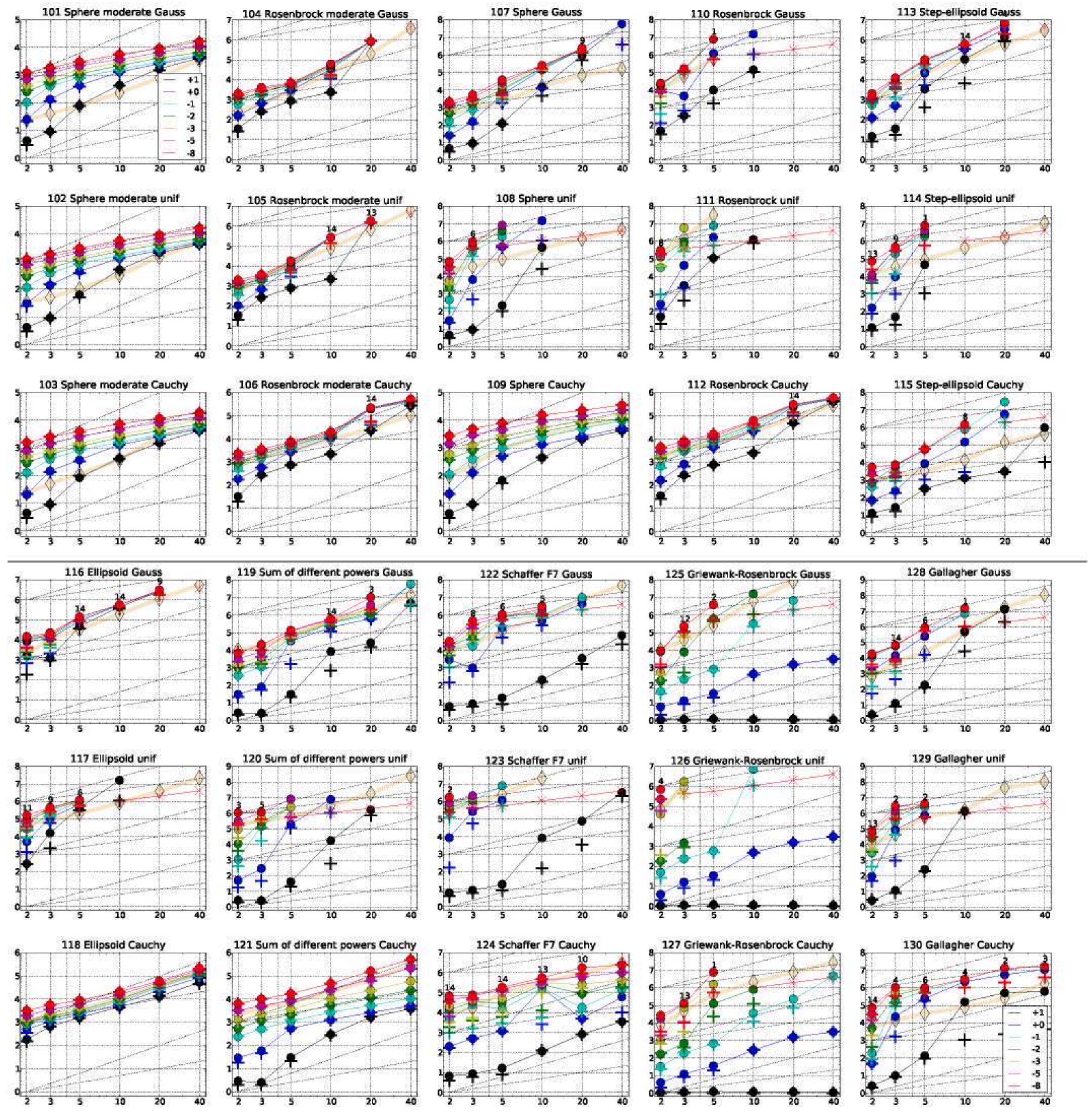

Figure 1: Expected Running Time (ERT, ●) to reach $f_{\rm opt} + \Delta f$ and median number of f-evaluations from successful trials (+), for $\Delta f = 10^{\{+1,0,-1,-2,-3,-5,-8\}}$ (the exponent is given in the legend of f_{101} and f_{130}) versus dimension in log-log presentation. For each function and dimension, $\mathrm{ERT}(\Delta f)$ equals to $\#\mathrm{FEs}(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\rm opt} + \Delta f$ was surpassed. The $\#\mathrm{FEs}(\Delta f)$ are the total number (sum) of f-evaluations while $f_{\rm opt} + \Delta f$ was not surpassed in the trial, from all (successful and unsuccessful) trials, and $f_{\rm opt}$ is the optimal function value. Crosses (×) indicate the total number of f-evaluations, $\#\mathrm{FEs}(-\infty)$, divided by the number of trials. Numbers above ERT-symbols indicate the number of successful trials. Y-axis annotations are decimal logarithms. The thick light line with diamonds shows the single best results from BBOB-2009 for $\Delta f = 10^{-8}$. Additional grid lines show linear and quadratic scaling.

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{101} in 5-D, N=15, mFE=3301					f_{101} in 20-D, N=15, mFE=9903				
10	15	8.0e1	2.0e0	1.5e2	8.0e1	15	1.7e3	1.3e3	2.2e3	1.7e3
1	15	4.3e2	2.4e2	6.9e2	4.3e2	15	2.5e3	2.4e3	2.6e3	2.5e3
1e−1	15	8.3e2	7.1e2	9.8e2	8.3e2	15	3.0e3	2.8e3	3.2e3	3.0e3
1e−3	15	1.5e3	1.4e3	1.6e3	1.5e3	15	5.4e3	5.1e3	5.6e3	5.4e3
1e−5	15	2.1e3	1.8e3	2.3e3	2.1e3	15	6.6e3	6.0e3	7.2e3	6.6e3
1e−8	15	3.1e3	2.9e3	3.2e3	3.1e3	15	9.2e3	8.8e3	9.9e3	9.2e3

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{102} in 5-D, N=15, mFE=3339					f_{102} in 20-D, N=15, mFE=9897				
10	15	6.3e1	2.0e0	1.1e2	6.3e1	15	2.1e3	2.0e3	2.2e3	2.1e3
1	15	4.4e2	2.0e2	7.0e2	4.4e2	15	2.5e3	2.4e3	2.7e3	2.5e3
1e−1	15	8.3e2	7.3e2	9.3e2	8.3e2	15	3.0e3	2.8e3	3.2e3	3.0e3
1e−3	15	1.5e3	1.4e3	1.6e3	1.5e3	15	5.3e3	5.2e3	5.5e3	5.3e3
1e−5	15	2.1e3	1.8e3	2.3e3	2.1e3	15	6.5e3	6.0e3	7.2e3	6.5e3
1e−8	15	3.0e3	3.0e3	3.2e3	3.0e3	15	9.2e3	8.8e3	9.6e3	9.2e3

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{103} in 5-D, N=15, mFE=4544					f_{103} in 20-D, N=15, mFE=13768				
10	15	8.4e1	2.0e0	1.4e2	8.4e1	15	1.5e3	1.2e3	2.1e3	1.5e3
1	15	3.6e2	2.0e2	6.4e2	3.6e2	15	2.5e3	2.4e3	2.6e3	2.5e3
1e−1	15	8.2e2	7.3e2	9.1e2	8.2e2	15	2.9e3	2.8e3	3.1e3	2.9e3
1e−3	15	1.5e3	1.4e3	1.6e3	1.5e3	15	5.4e3	5.2e3	5.7e3	5.4e3
1e−5	15	2.5e3	2.1e3	2.8e3	2.5e3	15	8.0e3	7.8e3	8.3e3	8.0e3
1e−8	15	4.0e3	3.5e3	4.4e3	4.0e3	15	1.2e4	1.1e4	1.2e4	1.2e4

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{104} in 5-D, N=15, mFE=11171					f_{104} in 20-D, N=15, mFE=1.33e6				
10	15	8.8e2	7.2e2	1.3e3	8.8e2	15	8.2e5	4.2e5	1.2e6	8.2e5
1	15	3.1e3	2.1e3	3.8e3	3.1e3	15	8.4e5	4.4e5	1.2e6	8.4e5
1e−1	15	4.6e3	3.8e3	5.6e3	4.6e3	15	8.4e5	4.4e5	1.2e6	8.4e5
1e−3	15	5.6e3	4.5e3	6.7e3	5.6e3	15	8.5e5	4.4e5	1.2e6	8.5e5
1e−5	15	6.2e3	5.2e3	7.3e3	6.2e3	15	8.5e5	4.5e5	1.2e6	8.5e5
1e−8	15	6.9e3	6.0e3	8.1e3	6.9e3	15	8.5e5	4.5e5	1.3e6	8.5e5

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{105} in 5-D, N=15, mFE=127943					f_{105} in 20-D, N=15, mFE=2.02e6				
10	15	8.0e2	6.6e2	9.3e2	8.0e2	13	1.8e6	1.2e6	3.4e6	1.5e6
1	15	8.5e3	2.2e3	3.0e4	8.5e3	13	1.8e6	1.2e6	3.3e6	1.5e6
1e−1	15	1.1e4	3.5e3	5.3e4	1.1e4	13	1.8e6	1.2e6	3.4e6	1.5e6
1e−3	15	1.6e4	4.4e3	5.9e4	1.6e4	13	1.8e6	1.2e6	3.4e6	1.5e6
1e−5	15	1.7e4	5.2e3	5.9e4	1.7e4	13	1.8e6	1.2e6	3.4e6	1.5e6
1e−8	15	1.8e4	5.8e3	6.0e4	1.8e4	13	1.8e6	1.3e6	3.4e6	1.5e6

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{106} in 5-D, N=15, mFE=12092					f_{106} in 20-D, N=15, mFE=2.01e6				
10	15	7.6e2	6.5e2	9.0e2	7.6e2	15	2.3e4	2.0e4	2.8e4	2.3e4
1	15	2.8e3	2.1e3	3.5e3	2.8e3	14	2.0e5	3.6e4	1.9e5	5.2e4
1e−1	15	4.2e3	3.5e3	4.6e3	4.2e3	14	2.1e5	4.2e4	2.6e5	6.3e4
1e−3	15	5.4e3	4.5e3	5.8e3	5.4e3	14	2.1e5	4.7e4	2.6e5	6.8e4
1e−5	15	6.4e3	5.3e3	6.7e3	6.4e3	14	2.1e5	4.8e4	2.7e5	7.2e4
1e−8	15	7.6e3	6.7e3	8.7e3	7.6e3	14	2.2e5	5.3e4	2.7e5	7.6e4

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{107} in 5-D, N=15, mFE=180258					f_{107} in 20-D, N=15, mFE=2.06e6				
10	15	1.2e2	2.0e0	2.7e2	1.2e2	13	9.4e5	1.7e4	2.1e6	6.2e5
1	15	5.4e3	1.2e3	6.1e3	5.4e3	9	2.2e6	4.6e5	5.0e6	8.1e5
1e−1	15	9.2e3	2.1e3	1.6e4	9.2e3	9	2.3e6	4.9e5	5.0e6	9.1e5
1e−3	15	1.7e4	3.0e3	4.8e4	1.7e4	9	2.4e6	5.3e5	5.3e6	1.0e6
1e−5	15	2.7e4	4.0e3	8.8e4	2.7e4	9	2.4e6	5.4e5	5.4e6	1.0e6
1e−8	15	3.7e4	5.5e3	9.0e4	3.7e4	9	2.4e6	6.1e5	5.4e6	1.1e6

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{108} in 5-D, N=15, mFE=556071					f_{108} in 20-D, N=15, mFE=2.08e6				
10	15	2.2e2	2.0e0	8.3e2	2.2e2	0	*43e+0*	*26e+0*	*74e+0*	5.0e5
1	10	4.9e5	1.2e4	1.1e6	2.1e5	.	.	.	.	.
1e−1	4	1.8e6	8.8e4	3.9e6	2.9e5	.	.	.	.	.
1e−3	1	8.3e6	1.1e6	1.8e7	5.1e5	.	.	.	.	.
1e−5	1	8.3e6	1.1e6	1.9e7	5.4e5	.	.	.	.	.
1e−8	0	*89e-2*	*36e-4*	*22e-1*	1.8e5	.	.	.	.	.

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{109} in 5-D, N=15, mFE=9797					f_{109} in 20-D, N=15, mFE=28724				
10	15	6.7e1	2.0e0	1.3e2	6.7e1	15	2.1e3	2.0e3	2.2e3	2.1e3
1	15	5.0e2	2.5e2	7.1e2	5.0e2	15	2.6e3	2.5e3	2.7e3	2.6e3
1e−1	15	1.1e3	8.9e2	1.4e3	1.1e3	15	4.9e3	4.7e3	5.2e3	4.9e3
1e−3	15	2.7e3	2.2e3	3.0e3	2.7e3	15	9.6e3	8.3e3	1.1e4	9.6e3
1e−5	15	4.6e3	3.6e3	5.3e3	4.6e3	15	1.5e4	1.4e4	1.7e4	1.5e4
1e−8	15	7.9e3	6.7e3	8.9e3	7.9e3	15	2.3e4	2.0e4	2.6e4	2.3e4

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{110} in 5-D, N=15, mFE=581226					f_{110} in 20-D, N=15, mFE=2.06e6				
10	15	9.8e3	8.0e2	4.8e4	9.8e3	0	*19e+0*	*17e+0*	*19e+0*	1.8e6
1	5	1.2e6	8.9e3	3.0e6	1.2e5	.	.	.	.	.
1e−1	1	7.9e6	6.9e5	1.8e7	1.4e5	.	.	.	.	.
1e−3	1	7.9e6	7.0e5	1.8e7	1.4e5	.	.	.	.	.
1e−5	1	7.9e6	7.0e5	1.9e7	1.5e5	.	.	.	.	.
1e−8	1	7.9e6	7.0e5	1.8e7	1.5e5	.	.	.	.	.

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{111} in 5-D, N=15, mFE=556613					f_{111} in 20-D, N=15, mFE=2.08e6				
10	15	1.1e5	1.3e3	2.5e5	1.1e5	0	*28e+1*	*57e+0*	*12e+2*	1.1e6
1	4	1.7e6	1.3e5	4.1e6	1.8e5	.	.	.	.	.
1e−1	1	7.9e6	6.8e5	1.9e7	1.3e5	.	.	.	.	.
1e−3	0	*20e-1*	*27e-2*	*32e-1*	2.8e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{112} in 5-D, N=15, mFE=21025					f_{112} in 20-D, N=15, mFE=2.00e6				
10	15	7.8e2	6.4e2	9.5e2	7.8e2	15	5.0e4	4.1e4	6.2e4	5.0e4
1	15	4.4e3	2.9e3	5.7e3	4.4e3	14	2.4e5	8.1e4	1.3e5	9.9e4
1e−1	15	7.6e3	5.9e3	9.7e3	7.6e3	14	2.6e5	9.2e4	2.1e5	1.2e5
1e−3	15	1.1e4	8.3e3	1.3e4	1.1e4	14	2.9e5	1.0e5	3.4e5	1.5e5
1e−5	15	1.3e4	9.8e3	1.5e4	1.3e4	14	3.0e5	1.0e5	3.5e5	1.6e5
1e−8	15	1.6e4	1.3e4	1.8e4	1.6e4	14	3.1e5	1.1e5	3.6e5	1.7e5

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{113} in 5-D, N=15, mFE=411864					f_{113} in 20-D, N=15, mFE=2.07e6				
10	15	3.4e3	1.1e2	1.9e3	3.4e3	12	1.4e6	4.2e5	2.8e6	8.4e5
1	15	2.2e4	1.4e3	5.8e4	2.2e4	8	3.4e6	1.2e6	6.1e6	1.6e6
1e−1	15	6.0e4	3.1e3	1.6e5	6.0e4	5	5.7e6	1.4e6	1.2e7	1.6e6
1e−3	15	9.8e4	4.2e3	2.0e5	9.8e4	5	5.7e6	1.4e6	1.2e7	1.6e6
1e−5	15	9.8e4	4.2e3	2.0e5	9.8e4	5	5.7e6	1.4e6	1.2e7	1.6e6
1e−8	15	1.0e5	4.3e3	2.4e5	1.0e5	4	7.3e6	1.8e6	1.6e7	1.7e6

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{114} in 5-D, N=15, mFE=600813					f_{114} in 20-D, N=15, mFE=2.08e6				
10	15	4.6e4	4.2e2	2.1e5	4.6e4	0	*15e+1*	*29e+0*	*33e+1*	7.9e5
1	4	1.9e6	2.7e5	4.4e6	3.8e5	.	.	.	.	.
1e−1	4	1.9e6	2.8e5	4.2e6	3.9e5	.	.	.	.	.
1e−3	2	4.0e6	5.4e5	9.3e6	3.7e5	.	.	.	.	.
1e−5	2	4.0e6	5.4e5	8.9e6	3.7e5	.	.	.	.	.
1e−8	1	8.0e6	7.6e5	1.8e7	2.0e5	.	.	.	.	.

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{115} in 5-D, N=15, mFE=174750					f_{115} in 20-D, N=15, mFE=2.01e6				
10	15	3.4e2	1.8e2	6.8e2	3.4e2	15	3.2e3	2.7e3	4.8e3	3.2e3
1	15	8.9e3	7.2e2	5.5e3	8.9e3	4	5.8e6	1.1e4	1.4e7	3.7e5
1e−1	15	5.7e4	1.3e3	1.2e5	5.7e4	1	2.9e7	3.0e6	6.8e7	1.0e6
1e−3	15	5.8e4	2.2e3	1.3e5	5.8e4	0	*17e-1*	*24e-2*	*29e-1*	1.4e6
1e−5	15	5.8e4	2.2e3	1.3e5	5.8e4	.	.	.	.	.
1e−8	15	5.9e4	3.6e3	1.3e5	5.9e4	.	.	.	.	.

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{116} in 5-D, N=15, mFE=525920					f_{116} in 20-D, N=15, mFE=2.08e6				
10	15	4.8e4	2.6e3	1.2e5	4.8e4	10	2.6e6	1.3e6	5.3e6	1.6e6
1	15	8.8e4	3.5e3	2.3e5	8.8e4	10	2.7e6	1.6e6	5.7e6	1.7e6
1e−1	15	1.1e5	4.3e3	2.7e5	1.1e5	10	2.8e6	1.6e6	5.8e6	1.7e6
1e−3	14	1.4e5	4.9e3	3.7e5	1.0e5	10	2.8e6	1.7e6	5.8e6	1.8e6
1e−5	14	1.4e5	5.9e3	3.7e5	1.0e5	10	2.8e6	1.7e6	5.8e6	1.8e6
1e−8	14	1.4e5	6.7e3	3.7e5	1.1e5	9	3.2e6	1.8e6	6.0e6	1.8e6

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{117} in 5-D, N=15, mFE=580716					f_{117} in 20-D, N=15, mFE=2.04e6				
10	12	4.1e5	8.9e4	8.4e5	2.7e5	0	*22e+2*	*11e+2*	*10e+3*	5.6e5
1	10	6.5e5	2.1e5	1.3e6	3.7e5	.	.	.	.	.
1e−1	7	9.6e5	2.3e5	2.1e6	3.3e5	.	.	.	.	.
1e−3	7	9.8e5	2.4e5	2.0e6	3.5e5	.	.	.	.	.
1e−5	6	1.2e6	2.5e5	2.6e6	3.3e5	.	.	.	.	.
1e−8	6	1.2e6	2.8e5	2.6e6	3.6e5	.	.	.	.	.

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{118} in 5-D, N=15, mFE=10625					f_{118} in 20-D, N=15, mFE=173902				
10	15	1.6e3	1.3e3	2.1e3	1.6e3	15	1.3e4	1.1e4	1.4e4	1.3e4
1	15	2.2e3	2.0e3	2.7e3	2.2e3	15	2.5e4	1.7e4	2.6e4	2.5e4
1e−1	15	2.6e3	2.2e3	2.9e3	2.6e3	15	3.4e4	2.3e4	3.2e4	3.4e4
1e−3	15	4.2e3	3.7e3	5.0e3	4.2e3	15	4.6e4	3.3e4	4.2e4	4.6e4
1e−5	15	6.1e3	5.0e3	7.4e3	6.1e3	15	5.3e4	3.9e4	5.1e4	5.3e4
1e−8	15	8.9e3	7.3e3	1.0e4	8.9e3	15	6.0e4	4.6e4	6.0e4	6.0e4

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{119} in 5-D, N=15, mFE=430531					f_{119} in 20-D, N=15, mFE=2.10e6				
10	15	3.0e1	1.0e0	6.9e1	3.0e1	15	2.6e4	4.8e3	6.3e4	2.6e4
1	15	3.3e4	1.0e3	8.8e4	3.3e4	15	6.5e5	2.1e4	1.3e6	6.5e5
1e−1	15	3.8e4	1.5e3	8.8e4	3.8e4	15	1.0e6	1.6e5	1.5e6	1.0e6
1e−3	15	6.9e4	4.8e3	1.7e5	6.9e4	11	2.4e6	1.3e6	4.0e6	1.6e6
1e−5	15	1.2e5	7.3e3	4.1e5	1.2e5	8	3.7e6	1.8e6	6.2e6	1.9e6
1e−8	15	1.3e5	4.3e4	4.2e5	1.3e5	3	1.0e7	2.0e6	2.1e7	2.0e6

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{120} in 5-D, N=15, mFE=558152					f_{120} in 20-D, N=15, mFE=2.08e6				
10	15	4.1e1	1.0e0	9.0e1	4.1e1	9	1.6e6	8.4e3	4.2e6	2.6e5
1	13	1.8e5	8.0e2	5.9e5	9.8e4	0	*92e-1*	*43e-1*	*14e+0*	7.9e5
1e−1	3	2.4e6	2.1e5	5.7e6	2.1e5	.	.	.	.	.
1e−3	3	2.5e6	2.3e5	5.3e6	2.4e5	.	.	.	.	.
1e−5	1	8.0e6	8.1e5	1.9e7	2.5e5	.	.	.	.	.
1e−8	0	*48e-2*	*11e-6*	*12e-1*	2.0e5	.	.	.	.	.

Table 1: Shown are, for functions f_{101}-f_{120} and for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{opt} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value (RT_{succ}). If $f_{opt} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.

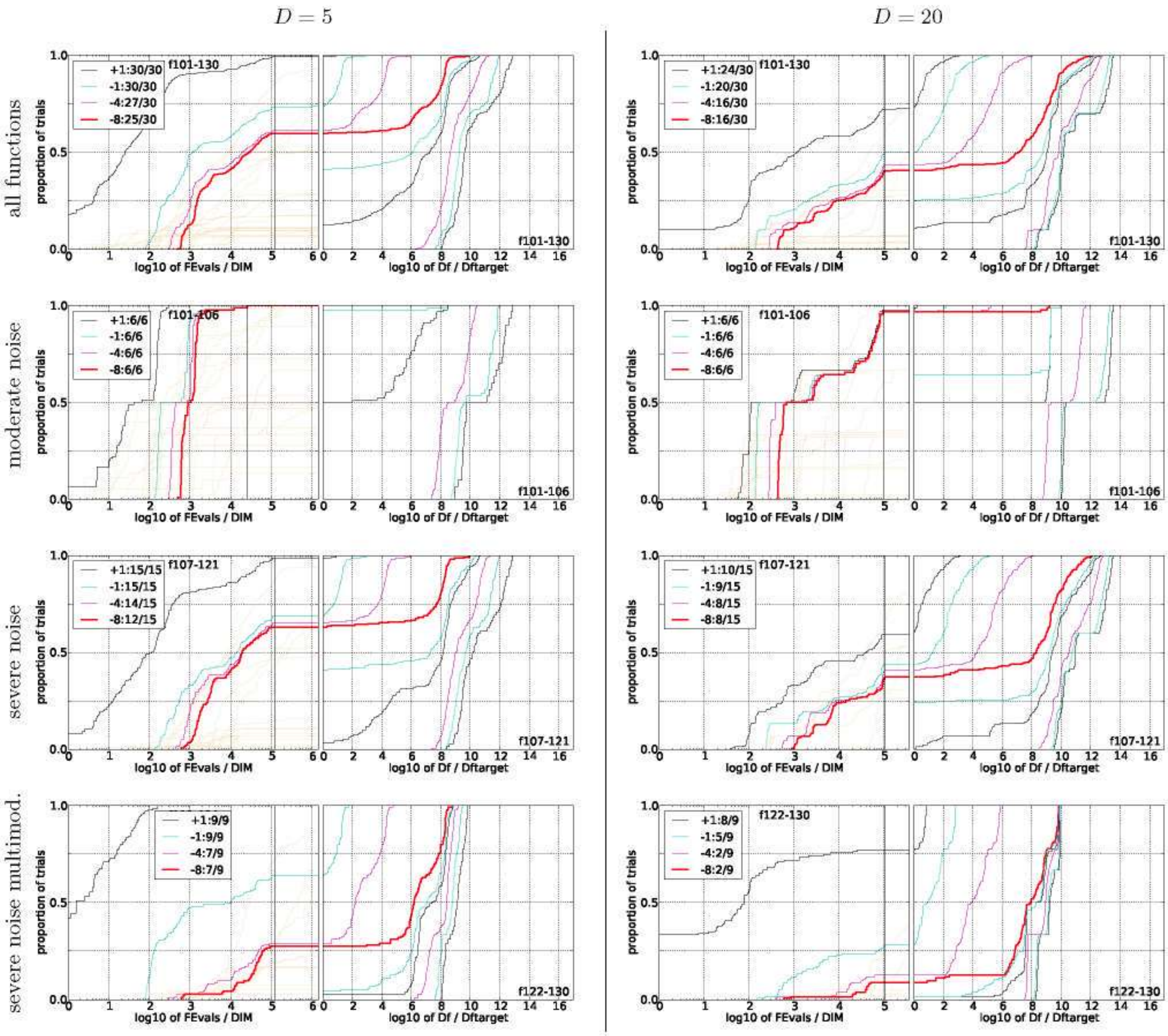

Figure 2: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus Δf (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D, to fall below $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10\,D, 100\,D \ldots$ function evaluations (from right to left cycling black-cyan-magenta). The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value. Light brown lines in the background show ECDFs for target value 10^{-8} of all algorithms benchmarked during BBOB-2009.

Δf	f_{121} in 5-D, N=15, mFE=22717					f_{121} in 20-D, N=15, mFE=282998				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	3.0e1	1.0e0	1.0e2	3.0e1	15	1.6e3	8.5e2	1.9e3	1.6e3
1	15	5.5e2	3.3e2	7.2e2	5.5e2	15	2.6e3	2.4e3	2.8e3	2.6e3
1e−1	15	1.1e3	7.7e2	1.3e3	1.1e3	15	5.3e3	5.0e3	5.7e3	5.3e3
1e−3	15	3.6e3	3.0e3	4.3e3	3.6e3	15	2.4e4	2.1e4	2.7e4	2.4e4
1e−5	15	8.9e3	6.8e3	1.0e4	8.9e3	15	7.6e4	5.2e4	1.7e5	7.6e4
1e−8	15	1.7e4	1.4e4	2.0e4	1.7e4	15	1.6e5	1.2e5	2.8e5	1.6e5

Δf	f_{123} in 5-D, N=15, mFE=555518					f_{123} in 20-D, N=15, mFE=2.08e6				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.9e1	2.0e0	2.9e1	1.9e1	15	7.5e4	1.5e3	4.0e5	7.5e4
1	5	1.1e6	6.4e3	2.8e6	4.2e4	0	*68e−1*	*58e−1*	*90e−1*	5.0e5
1e−1	1	7.8e6	6.1e5	1.9e7	5.9e4	.	.	.	.	.
1e−3	0	*18e−1*	*20e−2*	*26e−1*	5.0e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{125} in 5-D, N=15, mFE=552901					f_{125} in 20-D, N=15, mFE=2.05e6				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.2e0	1.0e0	2.0e0	1.2e0	15	1.1e0	1.0e0	2.0e0	1.1e0
1	15	3.4e1	1.2e1	7.7e1	3.4e1	15	1.5e3	1.5e3	1.5e3	1.5e3
1e−1	15	8.1e2	4.2e2	1.4e3	8.1e2	4	6.8e6	1.3e6	1.5e7	1.2e6
1e−3	2	3.9e6	2.8e5	8.8e6	2.7e5	0	*18e−2*	*65e−3*	*29e−2*	1.8e6
1e−5	2	3.9e6	2.9e5	8.5e6	2.8e5	.	.	.	.	.
1e−8	2	3.9e6	2.9e5	9.1e6	2.8e5	.	.	.	.	.

Δf	f_{127} in 5-D, N=15, mFE=536028					f_{127} in 20-D, N=15, mFE=2.01e6				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.2e0	1.0e0	2.0e0	1.2e0	15	1.1e0	1.0e0	2.0e0	1.1e0
1	15	3.5e1	1.2e1	8.1e1	3.5e1	15	1.5e3	1.5e3	1.5e3	1.5e3
1e−1	15	6.6e2	4.3e2	1.3e3	6.6e2	14	2.2e5	2.3e4	2.3e5	8.0e4
1e−3	4	1.6e6	1.8e4	3.7e6	1.6e5	0	*44e−3*	*17e−3*	*56e−3*	7.1e5
1e−5	1	7.6e6	8.4e5	1.7e7	3.3e5	.	.	.	.	.
1e−8	1	7.6e6	8.5e5	1.8e7	3.3e5	.	.	.	.	.

Δf	f_{129} in 5-D, N=15, mFE=553898					f_{129} in 20-D, N=15, mFE=2.08e6				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	2.5e2	2.3e1	6.0e2	2.5e2	0	*59e+0*	*42e+0*	*67e+0*	1.3e5
1	7	7.3e5	3.3e3	1.9e6	9.8e4	.	.	.	.	.
1e−1	3	2.5e6	2.2e5	5.7e6	3.1e5	.	.	.	.	.
1e−3	2	3.8e6	2.3e5	9.1e6	2.2e5	.	.	.	.	.
1e−5	2	3.8e6	2.3e5	9.1e6	2.2e5	.	.	.	.	.
1e−8	2	3.8e6	2.4e5	8.5e6	2.3e5	.	.	.	.	.

Δf	f_{122} in 5-D, N=15, mFE=554519					f_{122} in 20-D, N=15, mFE=2.08e6				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.9e1	2.0e0	5.1e1	1.9e1	15	3.4e3	3.5e2	8.8e3	3.4e3
1	12	1.9e5	8.4e2	6.0e5	5.2e4	6	4.4e6	1.0e6	9.6e6	1.3e6
1e−1	8	6.0e5	5.2e4	1.7e6	1.1e5	3	1.0e7	1.9e6	2.2e7	1.8e6
1e−3	7	7.9e5	5.6e4	1.9e6	1.6e5	0	*14e−1*	*24e−3*	*37e−1*	2.0e6
1e−5	7	8.3e5	1.1e5	1.9e6	2.0e5	.	.	.	.	.
1e−8	6	1.1e6	2.5e5	2.5e6	2.8e5	.	.	.	.	.

Δf	f_{124} in 5-D, N=15, mFE=606117					f_{124} in 20-D, N=15, mFE=2.07e6				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.6e1	2.0e0	3.6e1	1.6e1	15	8.3e2	3.8e2	1.3e3	8.3e2
1	15	1.2e3	7.3e2	1.8e3	1.2e3	15	4.9e3	2.9e3	6.0e3	4.9e3
1e−1	15	1.5e4	2.1e3	6.2e4	1.5e4	15	1.9e4	8.4e3	1.8e4	1.9e4
1e−3	15	5.0e4	7.2e3	1.3e5	5.0e4	15	2.7e5	1.2e5	5.8e5	2.7e5
1e−5	14	1.5e5	5.0e4	2.4e5	1.1e5	15	6.2e5	3.5e5	9.1e5	6.2e5
1e−8	14	1.9e5	6.3e4	2.5e5	1.4e5	10	1.8e6	4.0e5	4.6e6	7.3e5

Δf	f_{126} in 5-D, N=15, mFE=554060					f_{126} in 20-D, N=15, mFE=2.08e6				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.2e0	1.0e0	2.0e0	1.2e0	15	1.1e0	1.0e0	2.0e0	1.1e0
1	15	3.4e1	1.2e1	7.2e1	3.4e1	15	1.5e3	1.5e3	1.5e3	1.5e3
1e−1	15	5.7e2	4.1e2	7.3e2	5.7e2	0	*39e−3*	*32e−3*	*48e−3*	5.6e4
1e−3	0	*23e−3*	*13e−3*	*40e−3*	2.2e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{128} in 5-D, N=15, mFE=553761					f_{128} in 20-D, N=15, mFE=2.08e6				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.9e2	2.3e1	3.9e2	1.9e2	2	1.3e7	9.1e4	3.1e7	6.6e4
1	11	2.4e5	3.0e2	7.1e5	4.4e4	0	*50e+0*	*99e−1*	*66e+0*	4.0e5
1e−1	6	8.4e5	2.0e3	2.2e6	1.7e4	.	.	.	.	.
1e−3	6	8.5e5	2.3e3	2.2e6	2.4e4	.	.	.	.	.
1e−5	6	8.6e5	2.8e3	2.2e6	2.9e4	.	.	.	.	.
1e−8	6	8.6e5	3.3e3	2.2e6	3.3e4	.	.	.	.	.

Δf	f_{130} in 5-D, N=15, mFE=558853					f_{130} in 20-D, N=15, mFE=2.01e6				
	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
10	15	1.3e2	2.3e1	2.5e2	1.3e2	12	5.0e5	2.0e3	2.0e6	2.6e3
1	13	2.4e5	5.6e2	5.6e5	1.5e5	4	5.5e6	2.5e3	1.4e7	3.8e3
1e−1	8	6.7e5	7.0e2	1.6e6	1.9e5	2	1.3e7	4.5e3	3.0e7	3.8e3
1e−3	6	9.9e5	1.4e3	2.4e6	1.7e5	2	1.3e7	7.5e3	3.2e7	6.6e3
1e−5	6	9.9e5	2.7e3	2.3e6	1.7e5	2	1.3e7	1.1e4	3.2e7	9.6e3
1e−8	6	9.9e5	3.7e3	2.4e6	1.8e5	2	1.3e7	1.7e4	3.2e7	1.5e4

Table 2: Shown are, for functions f_{121}-f_{130} and for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\rm opt} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value (RT_{succ}). If $f_{\rm opt} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.

Table 3: ERT loss ratio (see Figure 3) compared to the respective best result from BBOB-2009 for budgets given in the first column. The last row RL_{US}/D gives the number of function evaluations in unsuccessful runs divided by dimension. Shown are the smallest, 10%-ile, 25%-ile, 50%-ile, 75%-ile and 90%-ile value (smaller values are better).

	f_{101}–f_{130} in **5-D**, maxFE/D=121223					
#FEs/D	best	10%	25%	**med**	75%	90%
2	1.1	1.3	1.9	2.8	5.6	8.1
10	1.2	2.1	3.2	5.0	6.6	29
100	1.1	3.6	5.6	7.8	12	2.7e2
1e3	2.3	2.9	7.3	25	40	2.7e3
1e4	0.75	2.4	2.9	29	93	2.4e2
1e5	1.6	2.2	2.9	17	72	2.5e2
RL_{US}/D	1e5	1e5	1e5	1e5	1e5	1e5

	f_{101}–f_{130} in **20-D**, maxFE/D=104808					
#FEs/D	best	10%	25%	**med**	75%	90%
2	0.73	1.0	2.4	34	40	40
10	1.0	3.0	18	1.4e2	2.0e2	2.0e2
100	1.0	4.6	7.2	18	34	2.0e3
1e3	0.45	2.6	5.6	24	1.1e2	2.0e4
1e4	1.8	3.2	9.2	66	2.8e2	2.0e5
1e5	1.8	2.6	4.4	28	1.6e2	1.0e6
1e6	1.8	2.6	4.3	27	2.7e2	1.0e7
RL_{US}/D	1e5	1e5	1e5	1e5	1e5	1e5

4. CONCLUSIONS

In this paper a hybrid algorithm combining Differential Evolution and IPOP-CMA-ES has been benchmarked on the BBOB-2010 noisy testbed. The experimental results show a good performance on functions with a moderate noise. On the other hand, functions with severe noise seem to be harder to solve with this algorithm. A more thorough study on the control mechanisms, specially those related to the detection of the stagnation and the restart of the search process, should be conducted on these functions. The selection of the parameter values was done based on the similar work of the noiseless testbed. Therefore, the proposed algorithm should also benefit of a proper parameter tuning process. Finally, the combination of additional techniques could be of great help in order to improve current results.

5. ACKNOWLEDGMENTS

This work was supported by the Madrid Regional Education Ministry and the European Social Fund and financed by the Spanish Ministry of Science TIN2007-67148. The authors thankfully acknowledge the computer resources, technical expertise and assistance provided by the Centro de Supercomputación y Visualización de Madrid (CeSViMa) and the Spanish Supercomputing Network.

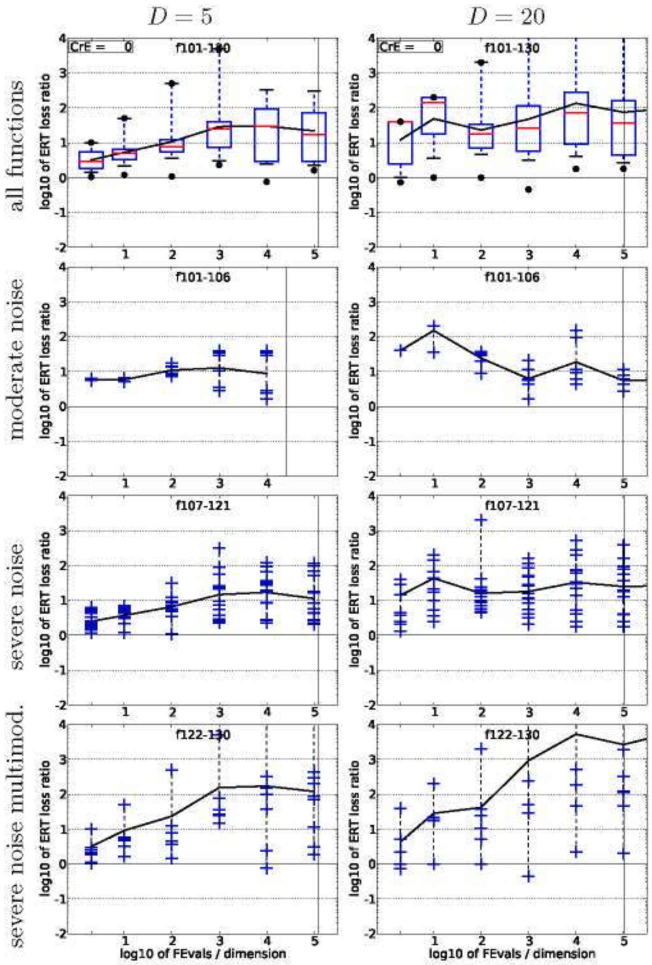

Figure 3: ERT loss ratio versus given budget FEvals. The target value f_t for ERT (see Figure 1) is the smallest (best) recorded function value such that $\mathrm{ERT}(f_t) \leq \mathrm{FEvals}$ for the presented algorithm. Shown is FEvals divided by the respective best $\mathrm{ERT}(f_t)$ from BBOB-2009 for functions f_{101}–f_{130} in 5-D and 20-D. Each ERT is multiplied by $\exp(\mathrm{CrE})$ correcting for the parameter crafting effort. Line: geometric mean. Box-Whisker error bar: 25-75%-ile with median (box), 10-90%-ile (caps), and minimum and maximum ERT loss ratio (points). The vertical line gives the maximal number of function evaluations in this function subset.

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