ECCM 2010
IV European Conference on Computational Mechanics
Palais des Congrès, Paris, France, May 16-21, 2010

Modelling of the advection-diffusion equation with the generalized finite difference method

L. Gavete[1], F. Ureña[2], J.J. Benito[3] M.L. Gavete[4]

[1] *Universidad Politécnica de Madrid, Spain, lu.gavete@upm.es*
[2] *Universidad de Castilla-La Mancha Ciudad Real, Spain, francisco.urena@uclm.es*
[3] *Universidad Nacional de Educación a Distancia Madrid, Spain, jbenito@ind.uned.es*
[4] *Universidad Rey Juan Carlos, Madrid, Spain, lucia.gavete@urjc.es*

Abstract

In this paper this meshless method is applied to 2D advection-diffusion problems. The results show the stability, simplicity, applicability and accuracy of this method.

1 Explicit Generalized Finite Differences Schemes

The Generalized finite difference method (GFDM) is evolved from classical finite difference method (FDM). GFDM can be applied over general or irregular clouds of points. The basic idea is to use moving least squares (MLS) approximation to obtain explicit difference formulae which can be included in partial differential equation to establish, together with an explicit method, a recursive relationship. The authors have made many contributions to the development of this method [1], [2], [3], [4] and [5].

2 Convection-diffusion equation

We consider here an convective-diffusive equation without any sources or sinks that either create or destroy the unknown function $U(x,y,t)$

$$\frac{\partial U(x,y,t)}{\partial t} + c_x \frac{\partial U(x,y,t)}{\partial x} + c_y \frac{\partial U(x,y,t)}{\partial y} = \alpha_x \frac{\partial^2 U(x,y,t)}{\partial x^2} + \alpha_y \frac{\partial^2 U(x,y,t)}{\partial y^2} \qquad t > 0, \quad (x,y) \in \Omega \tag{1}$$

with the initial condition

$$U(x,y,0) = f(x,y) \tag{2}$$

and the boundary condition

$$aU(x,y,t) + b\frac{\partial U(x,y,t)}{\partial n} = g(t), \qquad (x,y) \in \Gamma \tag{3}$$

being $f(x,y)$ and $g(t)$ two known functions, a, b are constants and Γ is the boundary of $\Omega \subset \mathbb{R}$. The diffusion-coefficients are: $\alpha_x > 0$ and $\alpha_y > 0$; $c_x > 0$ and $c_y > 0$ are the constant velocities.

3 Conclusions

The use of the generalized finite difference method using irregular clouds of points is an interesting way of solving partial differential equations. The extension of the generalized finite difference to the explicit solution of advection-diffusion equation has been developed.
The truncation error of advection-diffusion equation in the case of irregular grids of points have been defined. The von Neumann stability criterion has been expressed in function of the coefficients of the star equation for irregular of nodes.
As is shown in the numerical results, a decrease in the value of the time step, always below the stability limits, leads to a decrease of the global error.

Acknowledgements

The authors acknowledge the support from Ministerio de Ciencia e Innovaci*ó*n of Spain, project CGL2008 – 01757/CLI.

References

[1] *Influence several factors in the generalized finite difference method.*, J. J. Benito; F. Urena; L. Gavete, Applied Mathematical Modeling (2001), vol 25, pp. 1039–1053.

[2] *An h-adaptive method in the generalized finite difference.*, J. J. Benito; F. Urena; L. Gavete; R. Alvarez, Comput. Methods Appl. Mech. Eng.(2003), vol 192, pp. 735–759.

[3] *Solving parabolic and hyperbolic equations by Generalized Finite Difference Method.*, J. J. Benito; F. Urena; L. Gavete; B. Alonso, Journal of Computational and Applied Mathematics (2007), vol 209 Issue 2, pp. 208–233.

[4] *Application of the Generalized Finite Difference Method to improve the approximated solution of pdes.*, J. J. Benito; F. Urena; L. Gavete; B. Alonso, Computer Modelling in Engineering & Sciences.(2009), vol 38, pp. 39–58.

[5] *Leading-Edge Applied Mathematical Modelling Research (chapter 7)*, J. J. Benito; F. Urena; L. Gavete, Nova Science Publishers, New York, 2008.