

UNIVERSIDAD POLITÉCNICA DE MADRID
Escuela Técnica Superior de Ingeniería Aeronáutica y del Espacio



**Unsteady flow effects on droplets and spheres:
breakup and unsteady flow topology**

DOCTORAL THESIS

Submitted for the degree of Doctor by:

Pablo López Gavilán
Ingeniero Industrial

Madrid, December, 2023



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Doctoral Degree in Aerospace Engineering

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Under the supervision of:

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¿Por qué subir montañas?-Porque están ahí

George Mallory

Abstract

This thesis presents a set of experimental studies proposed to study the phenomena of droplet breakup and the unsteady flow topology and dynamics of a sphere. In particular, droplet breakup will be studied when exposed to the flow of an approaching airfoil in the environment of the stagnation line. From the point of view of the droplet, it is exposed to a continuously accelerated flow which, together with the deformation processes of the droplet, makes it an inherently non-stationary process and radically different from other flows traditionally studied. An experimental study was carried out in a rotating arm facility. Droplet diameters were of the order of 1 mm. The maximum velocity of the airfoils located at the end of the rotating arm was 91 m/s. Droplet deformation was computed using a phenomenological model developed previously by the authors. The dynamics of this deformation was coupled to an instability model based on the growth of Rayleigh-Taylor waves at the droplet surface. It was found that, within the experimental uncertainty, breakup occurs when the instability wavelength approaches the droplet hydraulic diameter assuming that it flattens and deforms as an oblate spheroid. This fact allowed for the generation of a theoretical closed-form droplet deformation and breakup model that predicts the onset of breakup with discrepancies of about $\pm 10\%$ when compared to the experimental results. Finally, as an application case, this closed-form model is used to simulate an actual situation in which the objective is to investigate whether a series of droplets that are approached by an airfoil either impact on its surface, or break prior to collision, or break without colliding, or pass through undamaged.

As a consequence of this non-stationary interaction in the phenomena of deformation and droplet breakup, the need arises to increase knowledge on the physics of the dynamics of particles subject to non-stationary flows. Therefore, an experimental study is proposed based on one of the simplest axilsymmetric shapes, the sphere, to study the flow establishment around it when it is impulsively accelerated up to an initial velocity and then decelerates freely in a fluid at rest. The diameter of the sphere was 0.04 m. Three different solid-to-fluid density ratios were considered: 0.78, 0.88, and 0.94. The submerged sphere was impulsively started upon being rammed by an actuator-mass system, being part of an innovative experimental setup defined for this purpose. Six initial velocities were considered: 0.91, 2.03, 2.54, 2.94, 3.29, and 3.78 m/s. The Reynolds number of the initial velocities based on the sphere diameter was 3.6×10^4 , 8.1×10^4 , 1.01×10^5 , 1.17×10^5 , 1.31×10^5 and 1.51×10^5 (subcritical). It was observed that both sphere dynamics and associated flow topology (identified via an optical system and a particle image velocimetry system, respectively) differed significantly from the case of an accelerating sphere. In the present case, a large vortex ring structure (with torus diameters of the order of the sphere's diameter) formed and being attached to the sphere surface. This vortex ring followed the sphere motion all the way down the falling trajectory. From the data reduction standpoint, it was found that a suitably defined dimensionless acceleration parameter allowed for collapsing the kinematics variables of the sphere trajectory, namely, position, velocity, and acceleration, into a single ordinary differential equation.

Resumen

Esta tesis presenta un conjunto de estudios experimentales propuestos para estudiar los fenómenos de ruptura de gotas y la topología y efectos del flujo no estacionario sobre una esfera. En particular, se estudiará la rotura de gotas cuando se exponen al flujo de un perfil aerodinámico que se aproxima en las proximidades de la línea de remanso. Desde el punto de vista de la gota, ésta se ve expuesta a un flujo en continua aceleración que, junto con los procesos de deformación de la gota, lo convierten en un proceso intrínsecamente no estacionario y radicalmente diferente de otros flujos tradicionalmente estudiados. Para ello, se realizó un estudio experimental en la instalación de brazo rotatorio del Instituto Nacional de Técnica Aeroespacial (INTA). Los diámetros de las gotas eran del orden de 1 mm . La velocidad máxima de las mismas en el extremo del brazo giratorio fue de 91 m/s . La deformación de las gotas se calculó utilizando un modelo fenomenológico desarrollado previamente por los autores. La dinámica de esta deformación se acopló a un modelo de inestabilidad basado en el crecimiento de inestabilidades de Rayleigh-Taylor en la superficie de las gotas. Se comprobó que, dentro de la incertidumbre experimental, la rotura se produce cuando la longitud de onda de inestabilidad se aproxima al diámetro hidráulico de la gota suponiendo que ésta se aplanar y se deforma como un esferoide oblato. Este hecho permitió generar un modelo teórico cerrado que contempla tanto la deformación como la rotura de la gota y que predice su inicio con discrepancias de aproximadamente 10% cuando se compara con los resultados experimentales. Finalmente, como caso de aplicación, este modelo se utiliza para simular una situación real en la que el objetivo es investigar si una serie de gotas que se aproximan a un perfil aerodinámico impactan contra su superficie, o se rompen antes de colisionar, o se rompen sin colisionar, o la atraviesan sin sufrir daños.

Como consecuencia de esta interacción no estacionaria en los procesos de deformación y rotura de gotas, surge la necesidad de ampliar el conocimiento sobre la física implicada en la dinámica de partículas sometidas a flujos no estacionarios. Por ello, se propone un estudio experimental basado en una de las formas axilsimétricas más simples, la esfera, para estudiar el establecimiento del flujo a su alrededor cuando se acelera impulsivamente hasta una velocidad inicial y posteriormente decelera libremente en el seno de un fluido en reposo. El diámetro de la esfera es de 0.04 m . Se consideraron tres relaciones de densidad sólido/fluido diferentes: 0,78, 0,88 y 0,94. La esfera sumergida se puso en marcha impulsivamente al ser embestida por un sistema actuador-masa que forma parte de una configuración experimental novedosa. Se consideraron seis velocidades iniciales: 0.91, 2.03, 2.54, 2.94, 3.29 y 3.78 m/s . El número de Reynolds de las velocidades iniciales en función del diámetro de la esfera fue de 3.6×10^4 , 8.1×10^4 , 1.01×10^5 , 1.17×10^5 , 1.31×10^5 y 1.51×10^5 (subcrítico). Se observó que tanto la dinámica de la esfera como la topología del flujo asociado (identificado mediante un sistema óptico y un sistema de velocimetría de imágenes de partículas TR-PIV, respectivamente) diferían significativamente del caso de una esfera acelerando. En el presente caso, se formó una gran estructura de anillo de vórtices (con diámetros de toroide del orden del diámetro de la esfera) que se adhirió a la superficie de la misma. Este anillo de vórtices acompañó a la esfera en su movimiento

durante toda la trayectoria descendente. Desde el punto de vista de la reducción de datos, se constató que un parámetro de aceleración adimensional adecuadamente definido permitía colapsar las variables cinemáticas de la trayectoria de la esfera, a saber, posición, velocidad y aceleración, en una única ecuación diferencial ordinaria.

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Abbreviations

AOI	Area of Interest
CFD	Computer Fluid Dynamics
DDB	Droplet Deformation Breakup
DRD	Droplet Ratio Deformation
DSR	Dynamic Spatial Range
FOV	Field of View
HGS	Hollow Glass Spheres
INTA	Instituto Nacional de Técnica Aeroespacial
KHI	Kelvin-Helmholtz Instability
LDV	Laser Doppler Velocimetry
LIF	Laser Induced Fluorescence
LS	Level Set
MPS	Multiple Pulse Separation
PIV	Particle Image Velocimetry
PTV	Particle Tracking Velocimetry
ROI	Region of Interest
RTI	Rayleigh-Taylor Instability
RTP	Rayleigh-Taylor Piercing
SIE	Shear Induced Entrainment
TAB	Taylor Analogy Breakup
TRPIV	Time Resolved Particle Image Velocimetry

Commonly used symbols

α	Normalized semi-major axis deformation
a	Major semi-axis
a_c	Acceleration
A_d	Surface area of droplet
Ac	Acceleration parameter
b	Minor semi-axis
Bo	Bond number
$C_{D(i)}$	Drag coefficient
C_f	Friction coefficient
C_p	Pressure coefficient
δ	Boundary layer thickness
ΔZ_L	Laser sheet thickness
Δt_{PIV}	Time distance between pair of images (PIV)
$D_{(i)}$	Diameter
D_I	Interrogation window size in px
$\overline{\overline{E}}$	Rate of strain
$E_{(i)}$	Energy
η	Normalized x displacement
$\eta_{(i)}$	Dimensionless coordinate
$F_{(i)}$	Force
F_i	In-plane loss of correlation
F_O	Out-of-plane loss of correlation
F_Δ	Loss of correlation due to velocity gradients
g	Gravity constant, $(9.81m/s^2)$
$\kappa_{(i)}$	Instability wave number

$\lambda_{(i)}$ Instability wave length
 $m_{(i)}$ Mass
 M_0 Magnification
 M_s Mach number
 μ Dynamic viscosity
 N_{ppp} Number of particles per pixel
 ν Kinematic viscosity
 Oh Ohnesorge number
 ω Vorticity
 p_{tot} Total pressure at surface
 r_0 Distance to the airfoil model
 $R_{(i)}$ Radius
 R_{arm} Radius of the rotating arm facility
 Re Reynolds number
 $\rho_{(i)}$ Fluid density
 τ Dimensionless separation time
 τ_w Wall shear stress
 θ_s Separation angle in polar coordinates
 $\overline{\tau}$ Shear stress
 $t_{(i)}$ Time
 t_e Exposure time
 t_s Separation time
 T_s Dimensionless separation time
 σ Surface tension
 Σ_d Surface between fluid phases
 $U_{(i)}$ Flow/Body velocity (depending on subscript)
 U_e Velocity out of boundary layer

U_m Velocity of the airfoil model

V_s Slip velocity

$W_{(i)}$ Work

We Weber number

x_s Separation point over the surface

ζ Dimensionless vertical droplet displacement

Commonly used subscripts (i)

d Droplet

g Gas

s Slip

c Critical

$sphere$ For a spherical body

$disk$ For a disk body

bk Breakup

sm Sampled

sp For the sphere model

Chapter 1

Introduction

1.1 Introduction

This thesis aims to complete the study previously carried out in the framework of the DFLOW project, granted by the Ministry of Economy and Competitiveness of the Spanish government in 2016, that is focused on the deformation and breakup of droplets in flows of aeronautical interest.

Specifically, this work addresses the study of the physical underlying mechanisms in the breakup of droplets exposed to the flow due to the passage of an aerodynamic airfoil, introducing a new breakup criterion, and delves into the problem of particle dynamics in non-stationary flows, through an experimental study of a sphere which, after being subjected to an impulsive acceleration, decelerates freely within a fluid at rest.

This document is divided into seven chapters. In this first chapter, this PhD dissertation and its motivation are presented, summarizing the objectives and illustrating the scientific framework in which its content is enclosed. Chapter 2 consists of a literature review related to the two main issues of this thesis. Firstly, the literature concerning droplet breakup and the physical mechanisms involved is examined. Secondly, a review of studies related to unsteady boundary layers, unsteady wakes, vortex structures and dynamics of bodies exposed to non-stationary flows, mainly spheres, is presented. Chapter 3 focuses entirely on the study of a new breakup criterion, based on the analysis of Rayleigh-Taylor instabilities, of droplets moving in the vicinity of the stagnation line of an airfoil. It includes a description of the experimental setup and cases under study, the theoretical derivation of the proposed breakup criterion and the analysis of the results. Then, in chapter 4, an experimental study of the problem of the sphere decelerating freely after an impulsive acceleration is presented. This chapter includes a detailed description of the test facility and the tools developed for data acquisition and TR-PIV analysis, an analysis of the trajectories of the different cases and the topology of the flow field and a discussion about the observed results, in particular a data reduction law that was found. Chapter 5 summarizes the results

and conclusions of this research. Finally, chapter 6 presents the publications derived from this research and in chapter 7 future work related to this work is presented.

1.2 Motivation

The deformation and breakup of droplets is a very complex process that can involve highly turbulent flows, deformable fluid interfaces and separation and coalescence of fluid volumes presenting a great scientific interest that combined with the technical interest of its applicability makes it a very active field of study within the Multiphase Flows discipline. Specifically, atomization phenomena are present in a huge number of everyday processes applied to different fields (materials engineering, environmental sciences, sprays, combustion processes, biomedical and pharmaceutical application among others). In the particular case of aerospace industry, our problem focuses on the dynamics, deformation and breakup of the dispersed phase in the vicinity of an airfoil moving under incompressible flow conditions ($M < 0.3$) which may be found when aircraft flies under severe weather conditions. Under these circumstances, the droplets may split up, leading to the formation of ice crystals on the airfoil surface. Similarly, the droplet impact dynamics on the airfoil surface, its distribution and accumulation can lead to the formation of a non-homogeneous water film on the airfoil surface that alters the pressure distribution as a result of an apparent roughness effect. Both situations can cause severe degradation of the airfoil's aerodynamic performance, dealing with massive boundary layer detachments, and drastic losses of lift.

In contrast to other flows of industrial concern, the airflow over a droplet in the vicinity of an airfoil experiences a continuous acceleration that increases as the airfoil is approached. This situation differs significantly from the results obtained in traditional experimental setups where the droplets are exposed to uniform flows, generally by the passage of a shock wave in shock-tube type experiments, or to slightly accelerated flows through the free fall of the droplet. Regardless of the above, when a droplet is exposed to an air flow, it is subjected to aerodynamic forces, which, depending on their strength, can significantly deform the droplet and cause it to break up. In relation to the latter, experimental evidence has shown that the nature of the breakage is strongly influenced by the way in which the droplet is accelerated. However, the vast majority of studies related to large aerodynamic forces are based on shock-tube setups. It is therefore of great interest to develop a physically based breakup criterion for our particular airflow conditions that could also be applicable to different ones with a certain degree of generality.

In the particular aspect of the droplet dynamics in air and the study of the forces involved, the problem has been approached from a Lagrangian representation, by means of semi-empirical linear models that evaluate the aerodynamic force at each point on the isolated particle as the sum of different components without taking into account the perturbation of the general flow due to its presence. However, in the deformation process, the droplet is exposed to severe accelerations as a consequence of its shape, which makes the problem inherently non-stationary even in the case of uniform flows. If we add to this the non-

stationary character of the flow under study, the motivation to increase the knowledge of particle dynamics in non-stationary flows is revealed. It is for this reason that the fundamental study of non-stationary dynamics becomes of great relevance to understand and increase the degree of generality of the models of droplet deformation and breakup. The aim of our work is therefore to delve into the physics of the problem of particle dynamics in non-stationary flows and, given that the vast majority of studies focus on acceleration dynamics, to focus in particular on the case of particles which are suddenly accelerated and subsequently subjected to a deceleration regime, being the latter regime where different authors have found opposite results. The uniqueness of our study is that unlike previous ones, after impulsive acceleration the body decelerates freely without being exposed to an imposed law of motion.

1.3 Thesis objectives

In conclusion, the main milestones to be achieved are as follows:

- I. Development of a breakup criterion based on the theoretical and experimental analysis of droplet breakup when exposed to continuously accelerated flows as a consequence of the passage of an airfoil.
- II. Improvement of the model of unsteady aerodynamic drag forces for droplets and particles based on the study of their fundamental problem. In particular, the case of the deceleration dynamics of a sphere freely moving in stagnant incompressible fluid will be studied.

The specific objectives for the main milestone I are:

1. To analyze the instant of breakup for droplets with initial diameters in the range of $600\ \mu m$ to $1200\ \mu m$ in the vicinity of the stagnation streamline of an approaching airfoil. We will make use of the experiments carried at the rotating arm facility of the Instituto Nacional de Técnica Aeroespacial (INTA) Experimental Aerodynamics Area, in which two self-similar symmetric airfoil models with radius of curvature of $0.103\ m$ and $0.070\ m$ approach a free falling droplet at velocities of $71\ m/s$, $81\ m/s$ and $91\ m/s$.
2. Based on literature and experimental observations, develop a breakup criterion based on the fundamental equations of the linear theory of hydrodynamic instabilities.
3. To implement this criterion in previously developed models for the deformation and trajectory of droplets in flows of aeronautical interest.
4. To validate the breakup criterion for droplets in the vicinity of the stagnation streamline of an incoming airfoil by comparing it with the experimental results. Assessing possible discrepancies.
5. Show its applicability on the basis of applied case studies.

The specific objectives for the main milestone II are:

1. To review the relevant published scientific literature to date concerning particle dynamics in non-stationary flows, focusing on those related to experimental studies.
2. To develop an experimental setup and methodology for the study of spheres decelerating freely within a fluid at rest after being subjected to impulsive motion.
3. To provide reliable experimental measurements of the sphere's trajectory in order to obtain accurate information on its instantaneous position, velocity and acceleration.
4. To study the topology of the flow field based on a Time Resolved Particle Image Velocimetry (TR-PIV) analysis identifying fluid structures in order to study their evolution in time and relate it to the sphere's dynamics.
5. To generalize the results on the basis of the problem variables, proposing a key parameter, so a data reduction law can be defined that allows the dynamics of the sphere to be modeled.
6. Validate the results by comparing them with experimental data and relate the key parameters with the flow field structures observed during the TR-PIV analysis.

1.4 Thesis plan

The thesis presents two clearly differentiated research objectives that, although related, present a quite different work schedule. Firstly, for the development of the breakup criterion for droplets exposed to continuously accelerated flows of aeronautical interest (objective I) it should be taken into account that the experimental methodology is clearly defined and validated on the basis of previous studies, therefore it is expected that most of the work will be based mainly on the following processes:

1. Development of an experimental campaign, defined by a test matrix comprising a total of 42 cases, at INTA's rotating arm facility using the test methodology applied in previous studies.
2. Analysis of the results obtained regarding droplet trajectory, deformation and breakup based on the image processing with previously developed tools. The quality of the data will be evaluated, rejecting the non valid cases and, if necessary, modifying the data analysis tool or performing new tests.
3. Study of the modes and criteria for droplet breakup available in the literature. Based the flow under study and on the analysis of the experiments carried out, a series of hypotheses and simplifications will be proposed, developing a new breakup criterion that will be combined with existing models of deformation and droplet dynamics. The complete model will be evaluated numerically in order to compare it with the experiments and based on the result make the required improvements or modifications.
4. The results will be analyzed, specifying the limits of applicability of the criterion and pointing out the uncertainties and repeatability of the analysis, and its possible sources. The applicability of the model from an engineering point of view will be

illustrated and the conclusions of the study will be drawn.

5. Dissemination of the results in the form of scientific publication. Future research lines of development, related to the generalization of the models developed for additional study conditions, will be proposed.

Figure 1.1 shows in detail the flowchart defined for developing the breakup criterion for droplets exposed to the unsteady flow generated by an incoming airfoil.

In relation to the study of the unsteady dynamics of a sphere freely decelerating within a fluid at rest, it must be taken into account that, although the study is motivated by the importance of the relative acceleration within the processes of droplet breakup, there was no previous experimental installation or methodology developed for its study. Therefore, the work plan will be rather different from the one previously described and will be organized in the following main processes:

1. Literature research on non-stationary particle dynamics, unsteady flows, unsteady fluid forces and unsteady boundary layers. It will be studied the main experimental methods, employed instrumentation and data analysis techniques, as well as the main physical variables involved.
2. Based on the previous analysis, the experimental limits will be defined and an experiment will be conceptualized. Subsequently, the experimental setup, including auxiliary elements and instrumentation, will be designed and implemented. Experimental methodology will be developed and system testing and calibration procedures will be developed.
3. A test matrix will be defined and the test campaign will be executed to analyze both the trajectory of the sphere and its first and second derivatives, validating the test results. Subsequently, the flow field around the sphere will be characterized from a TR-PIV analysis. In case the quality of any of the analyzed data is poor, the origin of its defects will be evaluated and the pertinent modifications will be performed.
4. Analysis of the results obtained in the test campaign, extracting the key variables of the problem and, taking into account the existing literature, generalizing the results by means of a data reduction law. As a result, model will be solved numerically in order to check its validity in predicting the sphere's motion, comparing it with the experimental results.
5. The results will be evaluated, drawing conclusions from the observed flow topology and its relationship with the variables and parameters of the problem. On the other hand, the applicability limits of the developed model will be analyzed. Finally, the conclusions of the study will be drawn.
6. Compilation of the results in the form of a scientific publication. Future developments and experiments will be studied making use of the developed facility and the great quantity of extracted data.

The flowchart defined for the experimental study on the free decelerating motion of an

impulsively accelerated sphere is shown in figure 1.2.

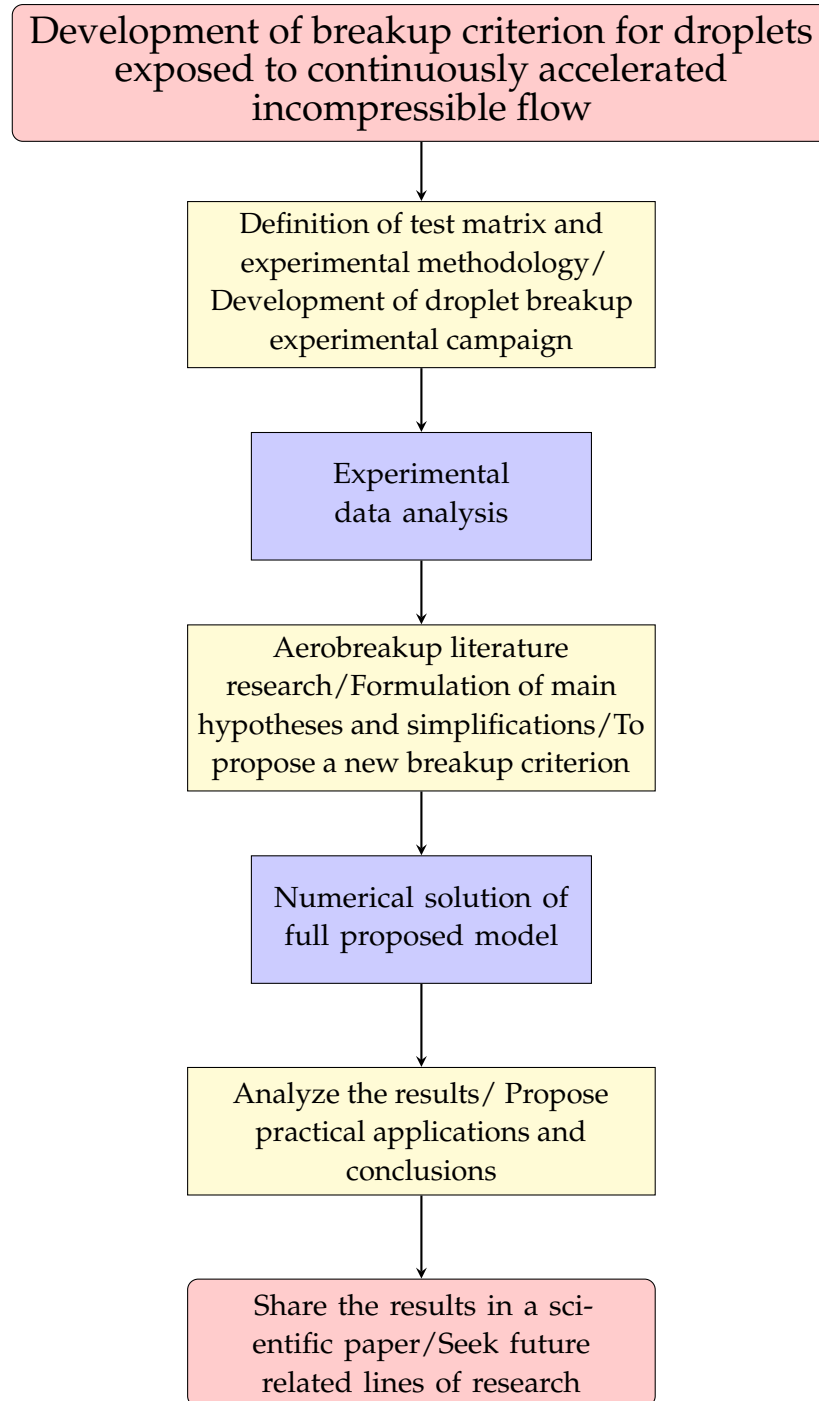


Figure 1.1: Flowchart of the Milestone I. Development of breakup criterion for droplets exposed to the unsteady flow generated by an incoming airfoil.

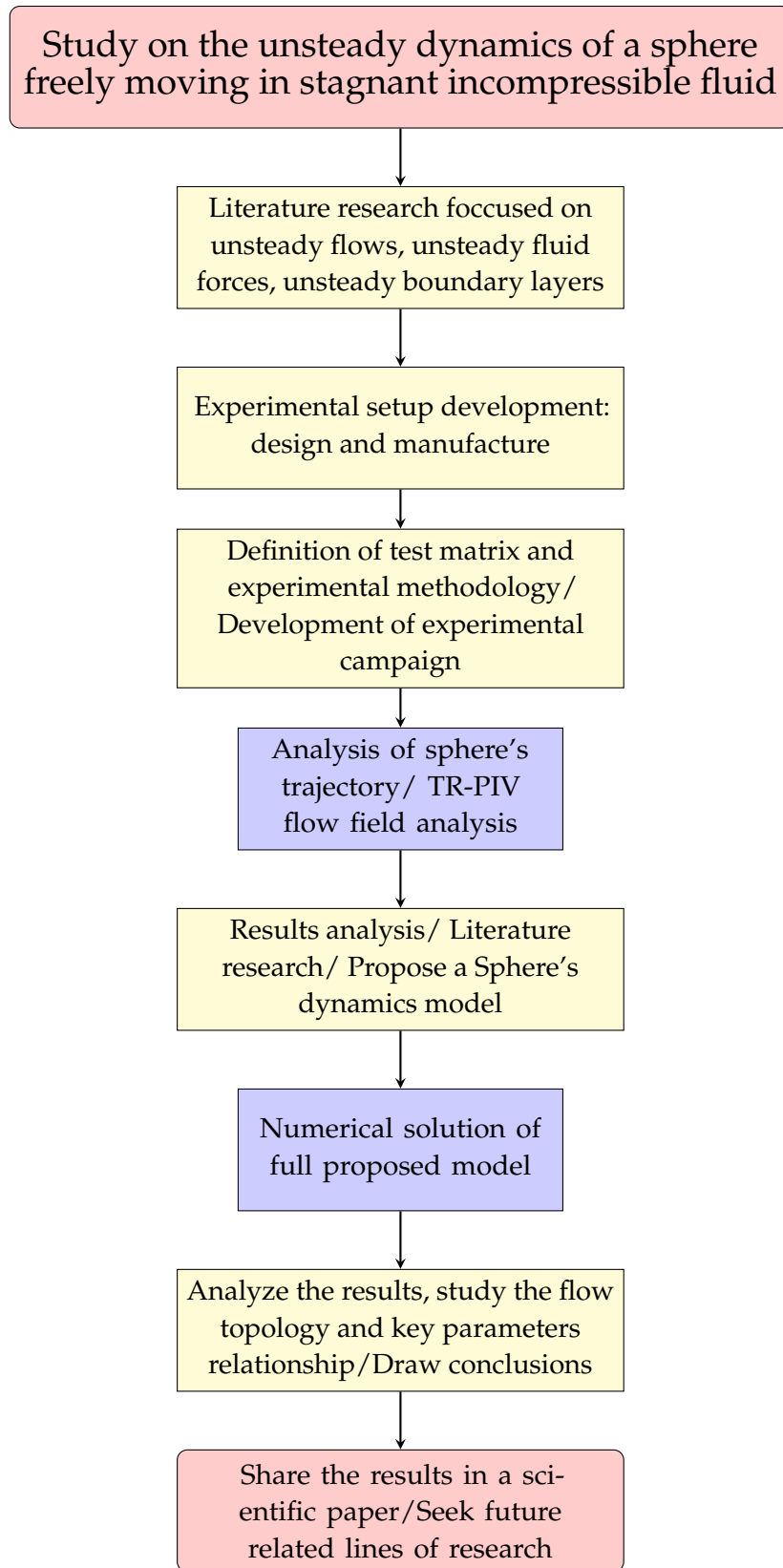


Figure 1.2: Flowchart of the Milestone II. Experimental study on the free decelerating motion of a impulsively accelerated sphere.

Chapter 2

Background

2.1 A review on droplet aerobreakup dynamics

This section will be divided as follows; first, the basic concepts related to aerobreakup and the facilities commonly used in experimental studies available in the literature will be described (2.1.1). In the section 2.1.2 the problem of droplet breakup, its morphology and the physical mechanisms involved will be analyzed in detail, and the experimentally observed deformation processes will be discussed. In the following section 2.1.3, fundamental droplet deformation models for the problem of droplet breakup are discussed. This will be followed by a brief review of the problem of non-stationary droplet and particle dynamics (2.1.4). Finally, the problem of droplet breakup in flows of aeronautical interest and its technical implications will be put into context (2.3).

2.1.1 Basic aspects of droplet deformation and breakup

The study of the breakup of droplets exposed to a gas flow, commonly referred to as aerobreakup, when dealing with high velocity flows, or secondary atomization, when referring to the fragmentation of primary droplets generated as a consequence of the breakup of a liquid jet, has been a prolific area of study for more than sixty years. To a large extent, this productivity is due to the technical implications it has in a large number of industrial applications and the large number of physical processes in which it is involved. At industrial level we can find it in combustion processes of reciprocating engines or gas turbines (Chrigui et al. [1], Bazdidi-Tehrani and A.Teymoori [2], Bazdidi-Tehrani et al. [3], Sula et al. [4] and Qu et al. [5]), in atomization processes applied to the pharmaceutical field (Martin and Finlay [6] and Scharfman et al. [7]), in the dissemination of chemical agents in the atmosphere (Theofanous [8]), in the interaction of aircrafts, airfoil and ballistics with hydrometeors (Zhao et al. [9], Bartolome and Teuwen [10], Gohardani [11] and Daniel et al. [12]) and many more fields of application. In fact, in nature, it is directly present in the formation of raindrops, where, when exposed to air currents and the flow resulting from free fall, they fragment, accounting for the richness of these processes and the great diversity of sizes present in raindrops, Villermaux and Bossa [13]. A very complete source

for the engineering application of sprays and atomization, mainly focused on combustion processes, material science and pharmaceutical drugs delivery, can be found at Ashgriz [14].

From the point of view of fundamental fluid mechanics, this problem is of special concern given the complexity of the physical phenomena involved. Thus, the main problem lies in knowing the aerodynamic forces that are developed over the droplet, from their distribution and subsequent normal and tangential balance, the dynamics, deformation and possible breakup of the droplet are derived. Thus, in addition to the inherent difficulty of studying the dynamics of a blunt body, in this case it involves interfaces that can deform significantly, which makes the problem an eminently non-stationary one where different length and time scales can arise [14].

Let's define some concepts that will appear throughout this dissertation. Firstly, when the fluid domain is made up of two or more fluids, a transition region appears in the form of a discontinuity surface known as an interface (Σ_d) that can change its shape under the application of fluid forces or body forces. When we refer to droplets, any deviation of the droplet from its volume with a spherical shape of radius R_d is called deformation. The existence of this deformable interface therefore introduces more degrees of freedom that require defining new boundary conditions with respect to the solid-fluid case. One of these unknowns that needs to be solved is the shape of the interface itself, we will define a scalar function of the interface $F(\vec{x}_{\Sigma_d}, t) \equiv 0$, being $\vec{x}_{\Sigma_d}$ all the points belonging to the interface Σ_d . From its definition it is possible to express the normal vector to this surface as $\vec{n}_{\Sigma_d} = F/|\nabla F|$. With these definitions, the kinematic boundary condition at the interface takes the form:

$$\left(\frac{1}{|\nabla F|} \frac{\partial F}{\partial t} \right)_{x=\vec{x}_{\Sigma_d}} + \vec{u} \cdot \vec{n}_{\Sigma_d} = 0 \quad (2.1)$$

On the other hand, when there is a boundary between two fluids, a stress condition must be defined based on the force balance at the interface. Considering its simplest definition, the equilibrium of forces is divided into a set of bulk pressure and stresses, that act on the faces of the surface and are proportional to the area, and a tensile force resulting from the surface tension between the phases (σ_{12}), that acts on the edges of the surface and are therefore forces per unit length. Thus, the surface tension force is a direct measure of the free energy associated with the surface per unit area, so that an increase in area always requires the application of work. This set of forces is usually decomposed into surface-normal and tangential components. The latter is often referred to as the shear-stress balance which, for constant surface tension at the interface and a Newtonian fluid, takes the form:

$$\left[\left(\bar{\bar{E}}_2 - \lambda \bar{\bar{E}}_1 \right) \cdot \vec{t}_{(i)} \right] = 0 \quad (2.2)$$

Where the sub-indices 1,2 denote the different fluid phases, $\bar{\bar{E}}$ represents the rate-of-strain tensor, $\vec{t}_{(i)}$ the vectors tangential to the surface (and orthogonal to each other) and λ the ratio of dynamic viscosity μ_1/μ_2 . If we consider a bubble or a droplet, the difference in

viscosity makes $\lambda \approx 0$ and therefore in the phase of higher viscosity we have $\bar{\bar{E}}_2 \cdot \vec{t}_{(i)} \approx 0$, which is commonly referred to as the "zero-shear-stress" condition. On the other hand, the normal stress balance would be:

$$p_{tot_1} - p_{tot_2} + \{ [(\bar{\tau}_2 - \bar{\tau}_1) \cdot \vec{n}_{\Sigma_d}] \cdot \vec{n}_{\Sigma_d} \} - \sigma_{12} (\nabla \cdot \vec{n}_{\Sigma_d}) = 0 \quad (2.3)$$

Where p_{tot} is the total pressure at the surface for each of the phases, and $\bar{\tau} = 2\mu\bar{E}$ for Newtonian fluids. This equilibrium condition for static fluids gives rise to the Young-Laplace equation which establishes the pressure jump at the interface as a consequence of the curvature which, for a spherical surface, would be $\Delta p_{\Sigma_d} = 2\sigma/R_d$. From this result one can define several of the relevant nondimensional numbers that govern the problems of droplet breakage and deformation:

- **Weber number** (We): arises from the balance between the disruptive aerodynamic forces, which tend to deform or break the interface, and the surface tension forces which tend to restore it. From their ratio:

$$We = \frac{2\rho_g U_\infty^2 R_d}{\sigma} \quad (2.4)$$

- **Ohnesorge number** (Oh): considering that viscosity acts as a dissipative force, reducing the effect of aerodynamic forces and thus reducing deformation and the likelihood of breakage, it defines the ratio between the viscous forces and those derived from surface tension.

$$Oh = \frac{\mu_d}{\sqrt{2\rho_d R_d \sigma}} \quad (2.5)$$

The sub-indexes d and g refer to the droplet and the surrounding gas respectively. Where ρ is the density and U_∞ is the velocity of the gas stream. It is important to mention that these numbers, when referring to droplets subtly exposed to air stream, are usually referenced to the initial conditions. On the other hand, in the case where the droplet is in free fall, they usually refer to terminal conditions. In addition to these numbers, given the non-stationary character and the importance of aerodynamic forces, the **Reynolds number** (Re) and the **Acceleration parameter** (Ac) are also relevant.

$$Re = \frac{2\rho_g V_s R_d}{\mu_g} \quad (2.6)$$

$$Ac = \frac{2R_d}{V_s^2} \frac{dV_s}{dt} \quad (2.7)$$

The first compares the relevance of the convective terms with respect to the viscous ones, while the second is the ratio between the local acceleration term and the convective acceleration term in the Navier-Stokes equations and is a reference to the non-stationary character of the flow. Sometimes they refer to instantaneous values for which the relative magnitudes

between the droplet and the gas referenced from the "s" sub-index are maintained.

As will be seen in the section 2.1.2, breakup mechanisms seem to be related to hydrodynamic instability mechanisms arising at the interface. Hydrodynamic instability is the response of a fluid system to a perturbation in velocity, pressure, temperature or concentration of solutes in chemical mixtures. As the Nobel laureate Chandrashekhhar pointed out in his treatise on hydrodynamic and hydromagnetic stability [15]: *"The equations of Hydrodynamics, in spite of their complexity, allow some simple patterns of flow...as stationary solutions. These patterns of flow can, however, be realized only for certain ranges of the parameters characterizing them. Outside these ranges, they cannot be realized. The reason for this lies in their inherent instability, in their inability to sustain themselves against small perturbations..."*. Thus, when this disturbance grows in time, it is said to be unstable. The opposite occurs when the perturbation decays in time, returning to its original state and therefore being stable. The analysis of the range of dimensional parameters for which the problem is stable or unstable allows us to separate the critical value, which is considered the stability limit. Traditionally, the study of these transitions is carried out under the hypothesis of small perturbations, which allows a linear approximation of the Navier-Stokes equations to be used, hence the name linear hydrodynamic stability. The limitation of the theory is mainly due to two points; the first is that as a consequence of assuming infinitesimally small perturbations, it is not valid for studying the time evolution of unstable responses; the second limitation is that the number of basic problems to be studied is usually limited to those that have an analytical or static solution. There are different instability problems depending on the base flow, specifically in relation to droplet breakup, the following instability mechanisms have been found:

- **Rayleigh-Taylor Instability (RTI):** This type of instabilities traditionally has its origin in a continuous dense fluid with density variations in the direction of the volumetric forces, generally gravity. Let us suppose that the fluid varies its density with height, if there is a displacement of a portion of fluid from a height z_1 to another z_2 being $z_1 > z_2$, the fluid will be surrounded by less dense phase and will precipitate under the gravitational action. In the opposite case, we would find a portion of lower density surrounded by heavier one, which will tend to rise due to buoyancy, and therefore become unstable in the face of any perturbation. Another manifestation of this type of instability mechanism occurs when there are two immiscible homogeneous phases separated by a flat interface, perpendicular to the gravitational axis in such a way that one of the phases is superimposed with respect to the other. In this case, if the denser phase is located on top of the lighter one, we would be in an equivalent situation as the case of heterogeneous phase were it not for the existence of surface tension. This last acts as a restoring force that tends to reduce any displacement of the interface from its planar shape. A variation of the latter case was developed in detail by Taylor [16] for the case of one phase accelerated over the other, which states that this surface can be stable or unstable according to whether the acceleration is directed from the heavier to the lighter fluid or vice versa. It is this latter manifestation of Rayleigh-Taylor instability that is considered to be involved in droplet breakup under certain flow field conditions, Re , We and Oh numbers. Apart from droplet breakup,

this type of instability mechanism can also be found in astrophysics in supernova modeling (Kuran et al. [17]), in meteorology in models of mammatus formation (Tremblin et al. [18]), in the formation of mushroom bomb clouds and even in the distribution of lithospheric mantle plates (Neil and Houseman [19]).

- **Kelvin-Helmholtz Instability (KHI):** In this case, the instability arises either from a heterogeneous fluid with properties that vary in the vertical direction, i.e., arranged in horizontal layers with parallel relative motion, or between two vertically superimposed homogeneous fluids separated by a horizontal plane with parallel relative motion between them. This is an instability originating in the energy stored in the velocity difference between layers or phases, so the higher the prevalence of shear dU/dz the greater the tendency of the system to instability. In the case of fluids with continuously varying properties, it is counteracted by the inertia of the phases/layers acting as a restoring force by means of a pressure gradient that prevents mixing. For separated phases, on the other hand, we find that the stabilizing behavior has its origin in the surface tension, which suppresses instabilities under the following stability criterion:

$$(U_1 - U_2)^2 < \frac{2(\rho_1 + \rho_2)}{\rho_1 \rho_2} \sqrt{\sigma g (\rho_1 - \rho_2)} \quad (2.8)$$

Where U_1 is the velocity of the lower fluid, U_2 is the velocity of the upper fluid and ρ_1 and ρ_2 are the densities of the lower and upper phases respectively. It should be noted that the derivation of this result is performed for a non-accelerating reference frame. It is considered that this instability mechanism arises in drop breakup phenomena when the air has a high relative velocity with respect to the drop and is created at its interface, and therefore, as in RTI, it is a function of the flow field and the key parameters of the problem. This instability can be also found in several geophysical processes, such as formation of ocean waves (Miles [20]), liquid jet primary breakup (Varga et al. [21]) or cloud formation (Lee and Kim [22]).

As will be discussed below, both mechanisms can arise simultaneously and compete during the course of the droplet breakup process, as the phenomenon of air acceleration over the droplet and a difference in velocities between the two phases ($U_1 - U_2$) can appear in a combined form. On the other hand, given the characteristics of the regime studied in this thesis document, the RTI is of great importance for the development of the breakup criterion, so it will be derived in detail in the section 3.5.1.

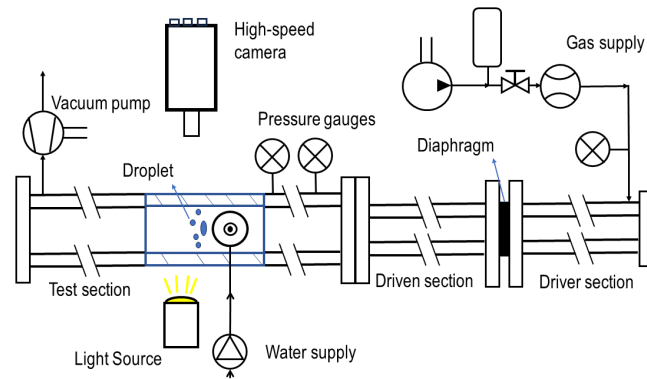
In relation to the studies present in the literature, thanks to the advances made in the numerical modeling of multiphase fluids, coupled with the exponential increase in the computing power of electronic equipment, the number of publications related to CFD problems has increased significantly over the last two decades (Kékesi et al. [23], Yang et al. [24], Lakhfif et al. [25] and Stefanitsis et al. [26] and Strotos et al. [27]). Contrary to what one might think, this has increased the interest in the experimental study of physical models of the phenomena related to droplet deformation and breakup. On the one hand, the experimental study allows the validation of specific methodologies applied to Computer

Fluid Dynamics studies and the development of specific codes oriented to the resolution of this problems, and on the other hand, it allows engineering applications to be tackled from reduced order models and phenomenological basis at an affordable computational cost. Given the complex nature of the physical phenomenon of aerobreakup there are a large number of experimental studies available in the literature. In order to illustrate the most common types of experiments that appear in most of the works that are presented in this chapter, it is of great interest to describe them first. In particular, three main types of experimental set-ups can be distinguished for the study of particle/droplet dynamics and droplet deformation and break-up:

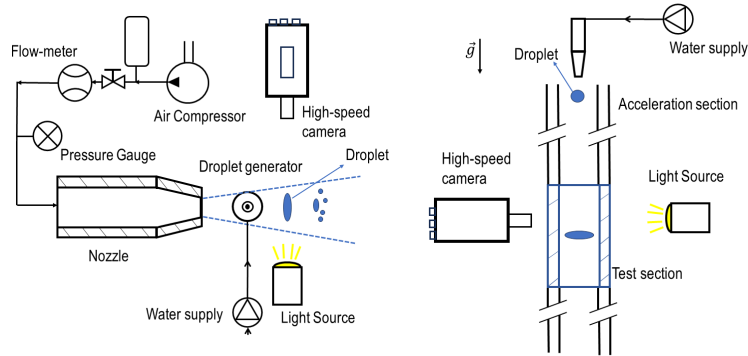
- **Shock-Tube type setup:** These are small-scale versions of the facilities used to study blast wave impacts and detonations. These set-ups are based on the generation of a uniform stream as a consequence of the passage of a shock wave inside a duct of usually rectangular cross-section. There are different ways of generating this shock wave, the most common being the division of the tube from a diaphragm into two sections, one of low pressure (usually atmospheric) and the other of high pressure, where gas is pumped. When the desired pressure is reached, the test sample is dropped into a region of the low-pressure section designed for the visualization of the phenomenon. As a result of the sample being dropped or passing through a presence sensor, a device is triggered which suddenly bursts the diaphragm and, as a result of the pressure difference, generates a shock wave which travels towards the test area. The evolution of the shock wave through the duct is usually recorded by a series of pressure transducers and its passage over the sample is recorded by high-speed cameras that are able to record the phenomenon. The imaging system is often based on shadowgraphy techniques to generate contrast between the sample and the gaseous medium (Simpkins and Bales [28] and Joseph et al. [29] and [30]). Other authors combine it with Laser-Induced Fluorescence (LIF) illumination techniques to study the interface evolution in droplet breakup experiments (Theofanous and Li [31]). A schematic diagram of the elements that make up this setup is shown in figure 2.1a. A particularly interesting variant of this type of setup is the ALPHA installation, described in detail in Theofanous et al. [32], where rarefied gas conditions, equivalent to heights of 70 km above sea level, are generated using a set of vacuum pumps to reduce the pressure in the downstream section to 10^{-4} atm absolute. Because of the importance of this type of installation, the state-of-the-art review by Balan and Raj [33] is very relevant.
- **Air jet facility:** These are systems where the droplet/particle is exposed to a air stream at the outlet of a nozzle, usually of circular cross-section, generated by an air blower. It achieves significantly lower velocities than in the case of shock tube type systems, and is therefore usually limited for studies at moderate We . The insertion of the test sample into the jet is usually done by free fall in the region close to the jet exit to minimize the shear layer. Due to the highly turbulent nature of the flow, authors such as Zhao et al. [34] and Zhao et al. [35] suggest that special attention should be paid to the values of the root mean square velocity fluctuations of the gas (u') to ensure that shear is generated as a consequence of the mean velocity (U_g) and not by fluxes of velocity $u'/U_g < 0.33$. Thus, as a rule, the evolution of the sample in the

gas is recorded by means of high-speed cameras in combination with backlighting techniques by light screens. A schematic of such a setup is shown in figure 2.1b.

- **Drop Towers:** This is one of the simplest set-ups and usually consists of installations developed in the direction of the gravitational axis, where a droplet generated at the upper end falls over long distances. In order to avoid any interaction with the surroundings, the drop usually falls inside a tube, generally made of a transparent material, in order to be able to visualize the phenomena of breakup and deformation. The use of this type of installation is intended for quasi-stationary deformation and breakup regimes. Therefore, it corresponds to conditions completely different from those found at the shock-tube type installations which present an almost impulsive character in the passage of the shock wave through the drop. A schematic of such a setup is shown in figure 2.1c.



(a) Sketch of shock-tube facility



(b) Sketch air jet facility

(c) Sketch of drop tower facility

Figure 2.1: (a) Sketch of shock-tube facility. Some of the elements shown, such as the vacuum pump or the gas supply system, depend on the configuration employed. (b) Sketch of an air jet facility. The system includes gas supply systems in combination with a nozzle, the diameter of which determines the flow to which the free-falling droplet is exposed. (c) Sketch of a drop tower facility. This is one of the simplest configurations used mainly to study the deformation and break-up of the drop in the quasi-steady limit.

2.1.2 Droplet deformation and breakup in high speed flows

To date, there is an almost general agreement in recognizing that the droplet breakup phenomenon is mainly related to the generation of hydrodynamic instabilities on its interface, either as a consequence of the interaction of the droplet with the flow field or as a consequence of deviations from its equilibrium shape in combination with the application of aerodynamic forces. Thus, in the deformation process and as a consequence of aerodynamic forces, different instability mechanisms have been found to coexist and compete with each other, although there is controversy related to their prevalence and effect on the breakup mode. This conclusion, however, has been mainly supported by a series of experimental works that have been carried out for more than 70 years. This section aims to compile some of this work.

One of the first studies found in the literature that is noteworthy for its historical significance is the work of Lane [36] in which atomization of liquid droplets in stationary air flows inside vertical wind tunnels was studied. A series of photographs were made using the resources available at the time and as a result, it was found that the drop flattened and expanded in a transverse direction to the flow. Then, a belt was formed where it concentrated a large amount of the original volume to subsequently form a bag on the inside, bursting into small fragments. Finally, the outer belt breaks into larger droplets, completing the parent drop breakage process. The authors tested water droplets with initial radius (R_d) ranging from 0.25 to 2.5 mm and reach a conclusion considered fundamental in the study of droplet breakup, namely that since the deformation is resisted by surface tension forces and the deformation has its origin in the dynamic pressure, the breakup must be related by expressions of the form $2R_d(U_{air} - v_{bk})^2$, where v_{bk} is the velocity of the droplet at the instant of breakup and U_{air} the free-stream mean velocity. Moreover, they found a critical magnitude for the breakage defined as $2R_d(U_{air} - v_{bk})^2 = 612$. Although not noticed by the author, this represents one of the first evidences of the dependence of the droplet breakup on the Weber number and of the existence of a critical value for the breakup onset. The study was then further extended to a shock wave generator, similar to a shock-tube test facility, except that the droplet was tested once the shock was discharged at ambient conditions acting like an air blast. The air velocities varied between those realized at rest and $M_s = 2.1$ at normal conditions. As a result, the authors identified a new breakup mode, different from the previous one, which they attributed to the instability of the drop. By analyzing the photographs and relying on the work of Taylor [16], the authors attribute one of the possible explanations for the breakup mechanism to the appearance of instability waves on its surface, which they believe to be a consequence of acceleration (RTI). After applying the analogy suggested by Taylor of considering the droplet as a mass-spring system, they compared the velocities at which the breakage took place, concluding that, when the non-stationary force was applied impulsively and reached the same transverse extension of the droplet as in the stationary case, the ratio between the transient and stationary velocity should be equal to 0.71. This relationship was partially fulfilled for the experimental results of droplets with diameters between 0.05 and 2 mm, attributing the departures from the expected value for the larger droplets to the time duration of the air blast.

This dependence of the droplet breakup on the Weber number was described by Hinze [37], where it is defined as $We = 2\rho_g V_s^2 R_d / \sigma$, being σ de surface tension between phases, V_s the air velocity relative to the drop and ρ_g de gas phase density. Also a critical Weber number (We_c) was defined, below which no breakup is observed as a result of air flow, having two different values depending on the transient character of the flow: $We_c = 13$ for shock-tube flows and $We_c = 22$ for free-falling drops. Later, Hanson et al. [38] employed a shock-tube facility in combination with a particularly interesting pressure wave droplet levitation system to test five different types of liquid exposed to shock waves. Their findings partly contradict the work of Lane [36], finding that the bag breakup mode also appears in non-stationary conditions, but no longer presents itself when it exceeds its critical Weber value significantly. When this is the case, the authors found a new form of breakup that forms a stamen in the center of the deformed droplet. Thus the study is particularly interesting for two reasons; firstly, it provides the basis for the dependence of the breakup on the Ohnesorge number by finding variations of the critical breakup Weber number as a function of the dynamic viscosity of the droplet, and secondly, it shows that the breakup is a consequence of the exposure of the droplet to a stream with relative velocity for a significant period of time.

Improving the experimental designs of shock-tube experimental facilities Ranger and Nicholls [39] tested water droplets with radius from $375 \mu m$ to $2200 \mu m$ for shock-wave Mach numbers between 1.5 and 3.5 corresponding to measured dynamic pressures of approximately $69 kPa$ and $2.2 MPa$, discharging at ambient pressure. The authors made use of a shadowgraph image system with short time exposure to take photographs of the deformation and breakup process, describing in detail the phases found as a function of the Mach number. In the case of droplets tested at $M_s = 2.0$ the droplet first underwent a period of deformation where it flattened, which was termed the dynamic stage, and then as a consequence of the shear flow it produced atomization in the form of a fog at the droplet equator. A direct conclusion they drew from the analysis of the breakup instant is that the dynamic pressure plays a key role, revealing its dependence on the Weber number. Finally, the authors made use of Taylor [40] series of studies, focusing on those related to the stripping phenomena over a droplet deformed as a lenticular object by an airstream, in order to approximate the problem to a two superimposed boundary layers. Thus, the authors integrate in time the total mass that is dragged as a consequence of shearing at the droplet equator or what is the same at $\pi/2 rad$ of the frontal region. Since the integral cannot be solved analytically, the authors used experimental data related to the final instant of breakup, and obtained a mass close to the original one of the droplet but departing around 10%. In addition, the model retains the experimentally observed relationships of breakup time with diameter, stream velocity and the ratio of densities between phases, namely; $t_{bk} \propto D_d$, $t_{bk} \propto V_s^{-1/2}$ y $t_{bk} \propto \sqrt{\rho_d / \rho_g}$ respectively. Another interesting point of the paper is the definition of the characteristic time $t^* = 2R_d(\rho_g / \rho_d)^{1/2} / V_s$ which will be widely used to define different instants of the breaking process in later studies.

Bringing together all these results and a vast amount of related research, the review paper of Pilch and Erdman [41] thoroughly documents the different breakup mechanisms for low-viscosity droplets exposed to a high velocity air stream. The authors summarize the most relevant key dimensionless groups and parameters that characterize droplet breakup, namely the Weber number (We), the Ohnesorge number (Oh) and the characteristic breakup time as defined by [39], $T = t_{bk}/t^*$, and according to these they classify the observations of droplet breakup found in the literature. Thus, the authors classify the different types of documented rupture mechanisms according to the Weber number and shown in the famous figure depicted in this article, figure 2.2. These mechanisms are listed below:

- Vibrational breakup ($We \leq 12$): this mechanism arises when the droplet develops oscillations with frequencies of the order of the natural frequency, generating large fragments.
- Bag breakup ($12 < We \leq 50$): after an initial deformation stage where the droplet adopts a spheroidal shape, flattening in the direction of the flow, the droplet forms an outer toroidal ring and inside it expands like a soap bubble, forming a bag. At one point the bag breaks into small fragments and then the outer rim breaks into larger fragments.
- Bag and stamen ($50 < We \leq 100$): this is a breakup mechanism similar to the bag breakup, but with the difference that after the toroidal ring is formed, a column (stamen) is also generated at the centre. The two structures remain joined by a pocket that grows until it disintegrates into smaller droplets. Subsequently, the ring and the column break up into larger droplets.
- Sheet stripping ($100 < We \leq 350$): in this case, when the droplet is deformed as a result of aerodynamic forces, the stream is able to continuously drag a sheet from the periphery, stretching it and finally breaking part of it into droplets. This continuous process of mass removal from the droplet causes the droplet shape and breaking mechanisms to be preserved throughout the process.
- Wave crest stripping ($350 \leq We$): Under these conditions it is proposed that small wavelengths penetrate the surface and, as a consequence of the ripples generated on the interface, a part of it is continuously torn away. Under certain circumstances, atomization occurs as a chain reaction, decomposing the droplets formed in previous breaks in what is known as catastrophic breakup. The authors link this catastrophic phenomenon to the penetration of longer wavelengths linked to higher amplitudes.

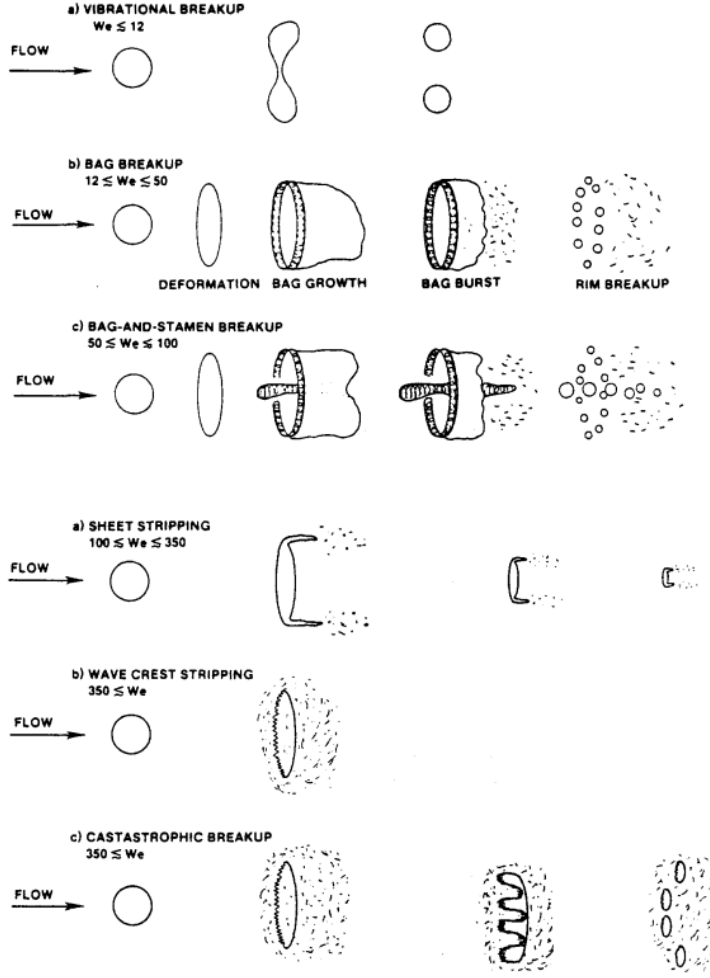


Figure 2.2: Breakup mechanisms. Reprinted from [41] M. Pilch and C. A. Erdman. ‘Use of Breakup Time Data and Velocity History Data to Predict the Maximum Size of Stable Fragments for Acceleration Induced Breakup of a Liquid Drop’. In: *International Journal of Multiphase Flow* 13.6 (1987), pp. 741–757. DOI: [10.1016/0301-9322\(87\)90063-2](https://doi.org/10.1016/0301-9322(87)90063-2), with permission from Elsevier

Regarding the critical Weber number, the authors point to the large amount of experimental studies that have shown that for $Oh < 0.1$ its value is approximately $We_c = 12$. Above this value, the dependence of the Ohnesorge number on the critical Weber number increases significantly, with an empirical correlation widely accepted in the literature of the form:

$$We_c = 12 \left(1 + 1.077 Oh^{1.6} \right) \quad (2.9)$$

This correlation, known as Brodkey’s correlation, establishes the limit of the Ohnesorge number at which breakup would be impossible, which was $Oh = 2$. Another particularly interesting point of the article is the determination of the breakup times or, in other words, the instant at which the process begins and when it terminates. Given the celerity of the phenomena, as well as the different breakup mechanisms, it is presented as a very arbitrary

definition. Regarding primary breakup, the most widely used definition in the literature is the time required until the disappearance of the residual drop. In the same sense, the total breakup time is known as the time that defines the instant in which the droplets and their fragments do not lead to further breakup. According to the authors, these definitions present the problem of not being able to be quantified efficiently under the application of shadowgraph images where atomization masks the processes. They also indicate the existence of other definitions such as the one used by Simpkins and Bales [28] which is related to the appearance of instabilities on the surface of the droplet as well as other definitions based on the deformation of the droplet. On the other hand, something that almost all authors agree with, is that the higher the Weber number, the shorter the time required until the onset of breakup. Moreover, for Ohnesorge numbers greater than 0.1, there is an increase in the time required until the onset of breakup, without altering the mechanisms found for this Weber number. Finally, the paper also gives an expression for the maximum possible fragment size after rupture called the maximum stable diameter, which is based on the critical Weber number, the dynamic pressure, the surface tension and the fragment velocity at the instant of total rupture. This expression gives reasonable results when compared to experimental data from previous studies for $We \geq 350$.

The dependence of the critical Weber number on the Ohnesorge number was explored in depth in Hsiang and Faeth [42] where the deformation and breakup phenomena for droplets of different liquids was studied. For this purpose, a 3.1 m long, 38 mm wide $\times$ 64 mm high, ambient-discharge Shock-Tube type installation was used, being able to generate shock waves with Mach numbers of 1.01 and 1.24. The droplets were produced through a monodisperse droplet generator by Rayleigh breakup with initial radius between 250 μm and 750 μm . To reduce interaction between adjacent drops, the authors employed electrostatic charges to deflect part of the drop set. In addition, thanks to the different liquids used, the authors performed tests for a wide range of Ohnesorge numbers, between 0.00006 – 4, and density ratios ρ_d/ρ_g in the range of 580 – 12000, with ρ_g being evaluated after the passage of the shock wave. Under these conditions Reynolds numbers of the droplet (Re_d) ranged from 300 – 16000 and Weber numbers from 0.5 to 1000. To measure the deformation, they use a pulsed shadowgraphy system, capable of taking 20 images at up to 8 kHz and a holocamera system to obtain measurements of the drop after breakup with spatial resolutions of 25 μm and an accuracy of 5%. This advanced setup allows the authors to characterize transitions between different levels of limit strain (for the non-breaking cases) and different breaking mechanisms as a function of Weber and Ohnesorge numbers. Regarding the critical Weber number, the authors find that its value increases with the Ohnesorge number, which is in agreement with previous observations [41]. However, the authors dissent on the existence of a breakup limit at $Oh > 2$, finding that for $We < 1000$ it is inhibited at values of $Oh > 4$. Consequently, they reject the existence of an Ohnesorge limit value for breakup. In relation to the mechanism that inhibits it, they conclude that this has its origin in the viscous damping that makes the drop require more time to be significantly deformed and to allow the formation of instabilities. When this delay is high enough, it allows the droplet to adapt its velocity to that of the flow and therefore to avoid breakup. This dependence on the Ohnesorge number therefore also affects other key parameters

such as the breakup time (t_{bk}), as reported in [41]. In the same way, the distortion of the drop is affected by the Ohnesorge number, which tends to retard it. Thus for shock-tube tests and $Oh < 0.1$, the maximum distortion is reached at the normalized time according to [39], $t/t^* = 1.6$. Regarding the classification of breakup mechanisms, the authors referred to multi-mode breakup instead of Bag and Stamen to group all those breakups with significant deformation of the tips and centres. Having extended the experimental ranges with respect to previous studies, the authors provide an expression of the best empirical fit for the critical Weber and for the normalized strain time in the ranges $We < 1000$ and $Oh < 3.5$.

Extending these results, in Hsiang and Faeth [43] the authors carry out an experimental study for Ohnesorge numbers from 0.0006 to 600 and density ratios from 1.15 to 12000. For this purpose, they use three different experimental set-ups, the previously described shock-tube set-up, a 10 m high drop tower with a circular cross-section of 300 mm diameter and a liquid-liquid drop tower of 1.2 m height and a square cross-section of 150 mm $\times$ 150 mm. The study confirms the results found in [42] and yields a very interesting result for situations below the critical Weber number, where the authors focus on two final deformation lines; one for the $We = 5 - 10$ range and the other for the $We = 10 - 20$ range. The result is that the final deformation is mainly dependent on the Weber number and independent of the Ohnesorge number, the latter exclusively influencing the strain rate. The authors summarize the relationship of the transition Weber numbers to the Ohnesorge number from the graph reproduced in figure 2.3. Another interesting finding found in the free-fall tests is related to the shape of deformation. Two main shapes can be distinguished; a bowl-shaped one, where the leeward region is flattened, and a dome-shaped one, where the windward region is flattened. Both situations are limited by a dividing line with the shape $We = C'Re^{1/2}$ being $C' = 0.5$ for steady disturbances and $C' = 0.7$ for shock-wave disturbances. Thus for situations above the dividing line bowl-shaped deformation is found while below dome-shaped shapes are adopted. Both studies show the great importance of the drop history, and therefore the non-stationary character of the problem and its dependence on the base flow field in characterizing the breakup.

Also related to the role of viscosity in the critical Weber number, it is very interesting the paper by Cohen [44] which, unlike previous works, relies on energy dissipation approaches to obtain a semi-empirical relation. Thus, the energy required to break a droplet (E_{bk}) would be the sum of the energy required to break it in the inviscid case, $E_{bk}(\mu_d \rightarrow 0)$, with the viscous dissipation energy (E_μ). Relating the breakdown energy in the inviscid limit to the kinetic energy imparted by the gas and to the Weber number of rupture We_{bk} , the authors propose a correlation for the ratio of the breaking Weber number We_{bk} to $We_c(\mu_d \rightarrow 0)$ of the form $(1 + K^*Oh)$ where K^* can be approximated as a constant or as a function of Oh . Comparing with the work of Hinze [37], Pilch and Erdman [41] and Hsiang and Faeth [42], it is found that the parameter K^* lies in the range 1.0 to 1.8 thus indicating an underlying dependence on Oh which the authors indicate should be studied further.

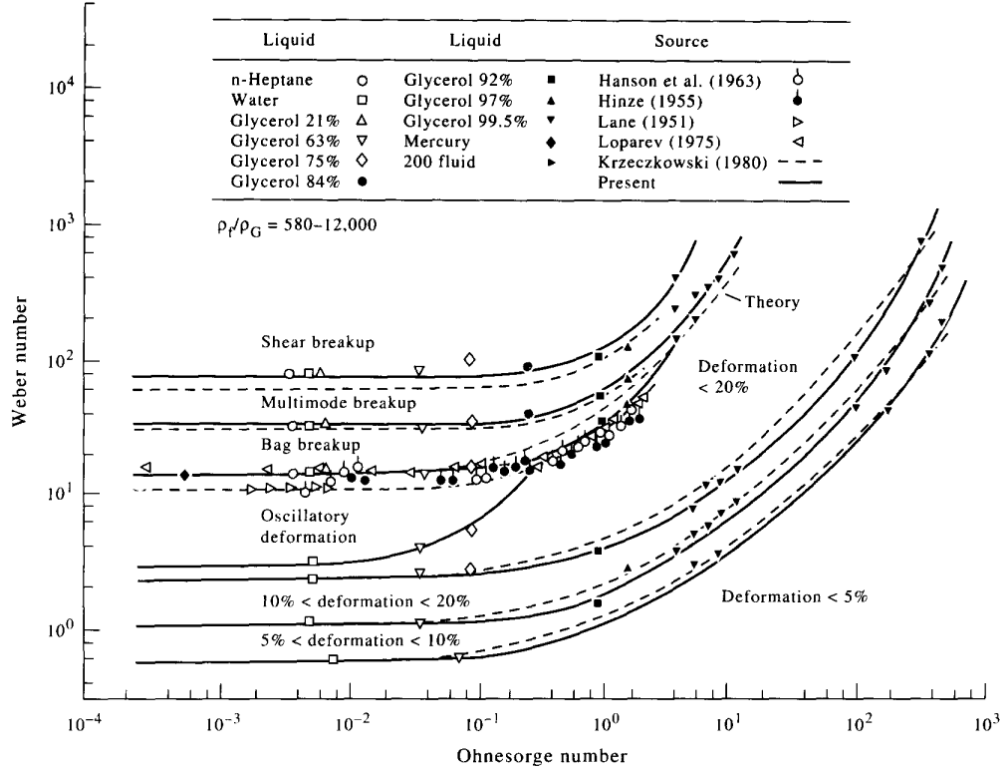


Figure 2.3: Drop deformation and breakup regime map. Reprinted from [43] L. P. Hsiang and G. M. Faeth. ‘Drop Deformation and Breakup due to Shock-Wave and Steady Disturbances’. In: *International Journal of Multiphase Flow* 21.4 (1995), pp. 545–560. DOI: [10.1016/0301-9322\(94\)00095-2](https://doi.org/10.1016/0301-9322(94)00095-2), with permission from Elsevier

Focusing on the shear breakup regime, Chou et al. [45] employ the [42] facility to characterize its temporal properties and evolution. Thus, the authors employ droplets of water, alcohol and glycerol mixtures with initial diameters up to 1 mm, Ohnesorge numbers between 0.003 and 0.040 and Reynolds numbers in the range of 3070 – 11240. Under these conditions the Weber number lies between 125 – 375. Considering the instant $t/t^* = 0$ as the last instant in which the drop retains its spherical shape, the authors define the breakage stages as follows; at $t/t^* = 1$ the drop is flattened, at the instant $t/t^* = 1.5$ ligaments of the parent drop begin to be stripped, finally for $t/t^* = 5$ the breakup is completed. The characterization of the breakup mechanism is based on [39], considering a boundary layer originating at the frontal stagnation point and of thickness $\delta(t)$ function of time. This thickness would be related to the size of the droplets that are produced in the fracture. The results for the case of $Oh < 0.1$ show two different regimes, a transient one where $\delta(t) \sim (v_d t)^{1/2}$ and a quasi-steady one where $\delta(t) \sim 0.092 R_d$ being the diameter of the droplet generated by shear tearing of the order of $\delta(t)$. The authors suggest that the shear breakup should be considered a rate process rather than a subite one.

As can be seen, most of the previously mentioned works focused on the different modes and linked it to We as a function of the initial conditions. In this way, the breaking mo-

ment was determined on the basis of characteristic times based mainly on experimental correlations based on We and Oh , figure 2.3. On the other hand, the different breakup modes were classified according to their morphology, based on experimental observations via Shadowgraph, LIF or holography techniques. A very interesting alternative approach, which has allowed the development of a more or less generic framework for droplet breakup, is based on considering that the formation of these breakup modes is related to underlying hydrodynamic instabilities. Thus, the previously observed modes (vibrational, bag, multimode, sheet stripping and catastrophic) are in fact the result of a competition between RTI and KHI in combination with shear stresses at their interface.

One of the most relevant articles in relation to the application of the analysis of instabilities to characterize the instant of breakup is that of Joseph et al. [29]. In this article, the authors propose a criterion for the breakup mechanism based on Rayleigh-Taylor instabilities. In order to achieve this criterion, and in contrast to previous studies, the authors rely on high sampling frequencies, $5 \mu s$, to capture the break-up process, allowing a large number of points to reproduce the droplet's trajectory. The tests are performed in a shock-tube facility discharging to ambient pressure with a total length of $10.36 m$, a test section of $0.076 m$ and a double diaphragm configuration, that allows varying the length of the driver section between $1.83 m$ and $4.62 m$. Two discharge pressures and two different gases were tested; a first one at $50 kPa$ and using Helium which gives a shock wave of $M_s = 3$, and a second one at $58 kPa$ with Nitrogen with a Mach number of 2. In total, droplets of more than 13 different liquids with diameters between 2.1 and $3.2 mm$ were used, which were generated by means of a flexible tube connected to a needle actuated by a solenoid and controlled by a micrometer. Thus, the experimental range would be $0.002 \leq Oh \leq 82.3$, Weber numbers in the interval between 9.6×10^3 and 1.69×10^5 and Reynolds numbers $4 \times 10^4 \leq Re \leq 1.34 \times 10^5$. For the calculation of accelerations the authors fit the trajectory to practically constant accelerations of the order $10^4 - 10^5$ times the acceleration of gravity, finding an inversely proportional relationship with R_d . The authors link this to the loss of the transfer of the momentum from the gas to the droplet in order to strain the droplet's interface. Thanks to the large set of images, the breakup stages are classified into three consecutive first stages, consistent with other studies and apparently universal, which are described below:

- Flattening: during this initial stage the effects of viscosity do not become significant and the flow is virtually potential, so that due to its transverse plane-symmetric pressure distribution the droplet is squashed. This stage is apparently independent of the We and Oh numbers.
- Fluid entrainment towards the equator: as a consequence of the development of viscous stresses at the interface, a flow is generated from the droplet front towards the ends, which is increased by the relative roughness developed at the interface as a consequence of the instabilities.
- Liquid accumulation at the equator: The liquid continues to accumulate at the equator and begins to generate a ring.

Regarding the subsequent stages, the authors determine that they are dependent on the initial conditions and on the Oh number, that allow undergoing large deformations. They also point out that these stages are not sequential and that alternation from one to the other is possible during the breaking process. At the same time, by looking at the images, the authors are able to distinguish some ripples on the surface of the droplet, which they linked to the formation of Rayleigh-Taylor instabilities as a consequence of the acceleration of the gas on the liquid phase, 2.1.1. On the other hand, in relation to the shear flow developed in the second stage, the authors apply a controversial assumption, which will later be discussed by [31], which assumes that although Kelvin-Helmholtz instabilities are likely to be expected, they will not be dominant when compared to RTI. Thus applying this assumption, the authors use [15] derivation for RTI to characterize the breakup which is supposed to be a unique consequence of RTI mechanisms. In doing so, they assume a flat drop surface and simplify the problem under the viscous potential flow analysis (Joseph and Liao [46]) which assumes that the disturbance velocity is derived from a potential. Assuming $\rho_d \gg \rho_g$, the growth rate can be approximated as:

$$n = -\frac{k^2 \mu_d}{\rho_d} \pm \sqrt{ka_c - \frac{k^3 \sigma}{\rho_d} + \frac{k^4 \mu_d^2}{\rho_d^2}} \quad (2.10)$$

Where k is the wavenumber, related to the wavelength $\lambda = 2\pi/k$, and a_c is the acceleration of the droplet. The instability edge will therefore be when $n = 0$ is equal to:

$$k_c = \frac{2\pi}{\lambda_c} = \sqrt{\frac{\rho_d a_c}{\sigma}} \quad (2.11)$$

Being k_c the critical wavenumber, λ_c the critical wavelength and independent as expected to the viscosity. The authors also solve the waves of highest growth rate $dn/dk = 0$ to be able to identify them in the images and considering that the waves of $\lambda < \lambda_c$ are stable they establish the condition that for the breakup to occur, $\lambda_c \leq 2R_d$. Regarding the breakup time, they relate it to the wave of greater growth rate, when the amplitude reaches 10 times its initial value, relating it to the blow-off instant. From the theory developed, the authors relate viscosity to the results of [42] where the effect of the Ohnesorge number on the critical Weber is a manifestation of the retarding effect of viscosity that allows the drop to accommodate the flow velocity and therefore avoid breakup. It is therefore a very complete article that analyses in detail the stages of deformation and breakup, presenting a detailed development of the break-up model that has been widely used in subsequent works by other authors under different starting hypotheses.

Later, in the same approach as [45], the subsequent work by Dai and Faeth [47] focuses on the temporal properties of the multimode breakup regime, however in this case the authors distinguish the competition between the different instability mechanisms. Thus, employing again the same setup, they test 0.25 to 0.4 mm droplets with $Oh < 0.1$ and Weber numbers of $15 \leq We \leq 150$. Under the test conditions, Reynolds numbers range from 1500 to 3300. The authors find a deformation stage up to $t/t^* = 2$ and a bag period growing up

to t/t^* in the interval 3.0 – 3.5. It is further determined that in this, the times to the end of the breakup dilate as a function of We until the plume finishes breaking ($t/t^* \sim 6$ in the shear breakup regime limit). However, this growth is not monotonically increasing but exhibits a maximum at $t/t^* \sim 8$ for $We = 40$ in what the authors relate to the slow growth of the RTI and a transition of subsequent KHI breakup mechanisms, which may call into question the assumption made by [29] regarding the prevalence of RTI over KHI.

As a first step towards generalizing the results, the article by Theofanous et al. [32] explores droplet breakup based on surface instabilities under conditions of high velocity rarefied air. The experimental study is carried out in the ALPHA facility, which consists of a vertically enclosed supersonic wind tunnel connecting two tanks at different pressures. The tanks are initially separated by a diaphragm which is detonated at the start of the test. Thanks to a set of vacuum pumps, rarefied conditions are generated with static pressures between 15 – 2500 Pa, generating back pressure ratios between the tanks higher than 50 which, combined with the convergent-divergent nozzle, allow reaching $M_s > 3$ in rarefied conditions. Specifically, the tests were carried out at $M_s = 3$ and flow velocities of 200 m/s. In relation to the droplets used, the authors used glycerine and Tributyl-Phosphate droplets in the range of $3.7 < D_d < 4.5$ mm, both newtonian and with vapour pressures below the test pressures. By modifying the static pressure the authors achieve Weber numbers in the range below 26 up to 2600 and Reynolds numbers between 200 and 3.16×10^4 . The authors identify a new breakup regime, known as pulling, below the Weber number of 26 where the instability manifests itself in the form of a stretching of the droplet in the opposite direction to the flow. Increasing Weber numbers between 26 and 44 gives way to bag or multibag modes. For Weber numbers above 44 and up to 10^3 a mode, that the authors link to a pure manifestation of Rayleigh-Taylor instabilities, called Piercing appears at Weber numbers one or two orders of magnitude smaller than predicted in the theory and observed in previous experiments. Thus, the difference is that for Weber larger than 10^3 the authors observe that the mode that appears is Stripping. These results contradict previous results in which the piercing mode appears for Weber numbers larger than the stripping mode. In a very interesting further discussion, the authors identify the wavelength for the fastest-growing wave described by [15] as $\lambda_d = 2\pi\sqrt{3\sigma/a_c\rho_d}$, where a is the acceleration of the droplet based on the stationary sphere drag coefficient multiplied by two empirical fitting parameters that take into account the deformation and shape of the droplet. Thus, the number of waves in a droplet deformed as a disc of diameter d_{max} would be $n_\lambda = 2d_{max}/\lambda_d$. The authors link the number of waves to the different manifestations of the bag and multimode mechanisms. Thus for $n_\lambda = 1$ we find the bag breaking mode and as n_λ increases we progress to multibag modes with number of lobes as a function of it. By extrapolating the results for higher Weber numbers, the authors discuss the observations of [28] and those of [29], finding that the observed manifestations for the so called catastrophic regime cannot have its origin in RTI, since the wavelengths of such tests would be in the order of $\lambda_d = 27 \mu m$ and would not be observable with the imaging systems. Furthermore, they note that the penetration velocities of the RTI are $U \sim \sqrt{\lambda_d a_c}$ and that it grows with $We^{1/4}$. The shear regime on the other hand grows with $We^{1/2}$ and therefore the authors conclude that for the balance between both rupture mechanisms, the shear on the interface

is favorable for $We \rightarrow \infty$. The cascade of breaking mechanisms would therefore be viscous drag for pulling regime, Rayleigh-Taylor instabilities for bag and multiwave (multibag and piercing) and shear effects at the interface (in possible combination with Kelvin-Helmholtz and turbulence) for stripping.

The discussion is extended in the later work of Theofanous and Li [31] where they again use the ALPHA facility with the combination of shadowgraphy and laser-induced fluorescence (LIF) visualization techniques to answer the break-up mechanisms of the different modes and to resolve the inconsistencies observed in their experiments when compared to others. In this case, tests are performed for Weber numbers up to 3×10^4 for a single droplet diameter and $Oh = 0.013$. The authors compare the experimental results for the case of $We = 5.4 \times 10^3$ to illustrate the Shear Induced Entrainment (SIE) regime, mainly consequence of KHI or shear flow effects, and compare it with the Rayleigh Taylor Piercing (RTP) case for 75 Weber number. The LIF visualization reveals that the ripples previously referred to in earlier works, are artifacts generated by optical phenomena at the interface, revealing a mirror surface in the frontal regions of the droplet. Moreover, the authors indicate that the blow-up appearance in shadowgraphies is mainly due to radial flow. By using LIF technique, the authors also observe that the breakage is generated long before the formation of mist at the periphery, due to the loss of droplet coherence. On the other hand, according to the authors, the LIF visualization for the case $We = 75$ leaves no doubt and shows an RTP regime with the penetration of multiple waves. The analysis of the results gives rise to a very interesting discussion, firstly they determine that the multiwave piercing phenomena is present thanks to the rarefied conditions that allow low values of the Weber number with higher aerodynamic accelerations. Secondly, they compare their results with the findings of [29], where it was found that the instants shown by the latter correspond to instants in which the breakup has already occurred and indicating that the roughness appearing as waves therefore correspond to the aforementioned optical artifacts. Because of the latter, they conclude that this catastrophic regime does not exist and is a consequence of the experimental techniques applied. On the other hand, authors are also critical of the hypothesis that the tangential velocity is zero and therefore the KHI do not intervene significantly so the breakup is mainly due to RTI. Regarding the latter, they indicate that according to their visualizations it is shown that the breakup occurs by the SIE mechanism, being dominated by the shear that dampens the RTI generated and therefore contradicting this assumption. As already stated in [32], authors also indicate that when considering the gas temperatures of the driver gas for the conditions of [29] the wavelengths of the fastest growing wave would be an order of magnitude below the reflected ones and therefore indistinguishable given the spatial resolution of the images.

Other mechanisms have been proposed to explain breakup apart from that based on combinations of hydrodynamic instabilities with aerodynamic forces, such as that proposed by Khosla et al. [48] where a set of 3D Volume of Fluid (VOF) simulations were used to explain the breakup modes found for $We < 150$, with a constant physical droplet diameter of $600 \mu m$ and varying the velocity to adapt the Weber number to the desired deformation and breakup conditions. Their conclusion is that the breakage is mainly due to internal flows

towards the equator that accumulate fluid in a toroid shape, so that when there is several deformation and the surface is very thin, the surface tension force is insufficient and it is subjected to elongation until rupture at the thinnest point. Part of these breakup mechanisms are studied in a very comprehensive review article related to secondary atomization processes, Gueldenbecher et al. [49]. The article shows the discrepancies in the literature and the lack of common agreement on some observations in particular, but highlights the value of breakup criteria based on hydrodynamic instabilities combined with deformation models.

Based on the previous review by Theofanous [50], which attempted to unify criteria regarding the regimes and breakup mechanisms for newtonian and viscoelastic fluids, in Theofanous et al. [51] the authors extend the research work contained in [31], carrying out a detailed study of the breakup limits for the SIE and RTP regimes. The study, which comprises an experimental and a numerical study, makes use of the linear analysis theory of RTI and KHI instabilities in combination with the gas-induced shear flow for different Ohnesorge numbers in order to determine the limits of the RTP and SIE regimes. Direct Numerical Simulations (DNS) are used to complement the information obtained from experiments and to understand the richness of the observed phenomena. To obtain experimental data, the authors build on the ALPHA facility to develop a second facility called ASOS with the same test section geometry but capable of operating at 2 MPa and $M_s = 3.5$ and with a fully transparent test section made of polycarbonate. The test conditions are varied by changing the dynamic pressure and the liquid of the drop, which are chosen to have an extremely low vapour pressure. Under these conditions, different values of M_s , Re , We and Oh are obtained, as well as different viscous, $t_{v0}^+ = (\nu_d/a_{c0}^2)^{1/3}$, and inertial, $t_I^+ = 2R_d(\rho_d/\rho_g)^{1/2}/U_\infty$, scales. The conclusions are that even in the limit of viscous fluids, the main rupture modes for Newtonian fluids are RTP and SIE, dominated by RTI and KHI respectively. The authors describe a first criticality in the $Oh - We$ space, linked to large wavelength phenomena, from which RTP appears, figure 2.4a. For the same Ohnesorge, the larger the Weber number in the RTP region, the higher modes are to be expected. It is also confirmed that for higher Oh longer times, or distances traveled by the droplet, are necessary to break up as the inertial time scales are of the order of the viscous ones. The authors also indicate that the RTP is connected to the degree of droplet deformation, which becomes more evident for higher Ohnesorge numbers. On the other hand, they describe a limit for the second criticality, figure 2.4b, which the authors relate to liquid ejections at the equator. Thus, analyzing the second criticality based on the maximum shear stress found (at around 60 deg from the droplet's frontal stagnation point) they find that for non-viscous conditions, $Oh \ll 1$, the boundary scales with the capillary velocity, $u_c^+ = \sqrt{\sigma/2\rho_d R_d}$. Since this second criticality is above the first, there is a competition between KHI and RTI, which given the growth rate and considering sufficiently long times always wins the Kelvin-Helmoltz instability. What is interesting is that, if the conditions for SIE rupture cease to exist, we would fall into the RTP regime, which, quoting the authors, is equivalent to *"RTP can be seen as the result of R-T instability left to act alone, and in concert with drop deformation"*. This suggests that, according to the experimental results and the evaluation of DNS simulations carried out by the authors, the breakup mode is strongly

influenced by the deformation history, so understanding the non-stationary forces on the droplets becomes highly relevant. This series of works is completed by analyzing the case of the physics of aerobreakup for viscoelastic liquids in the article Theofanous et al. [52].

Under this set of studies, the breakup mechanisms are somewhat unified in terms of the

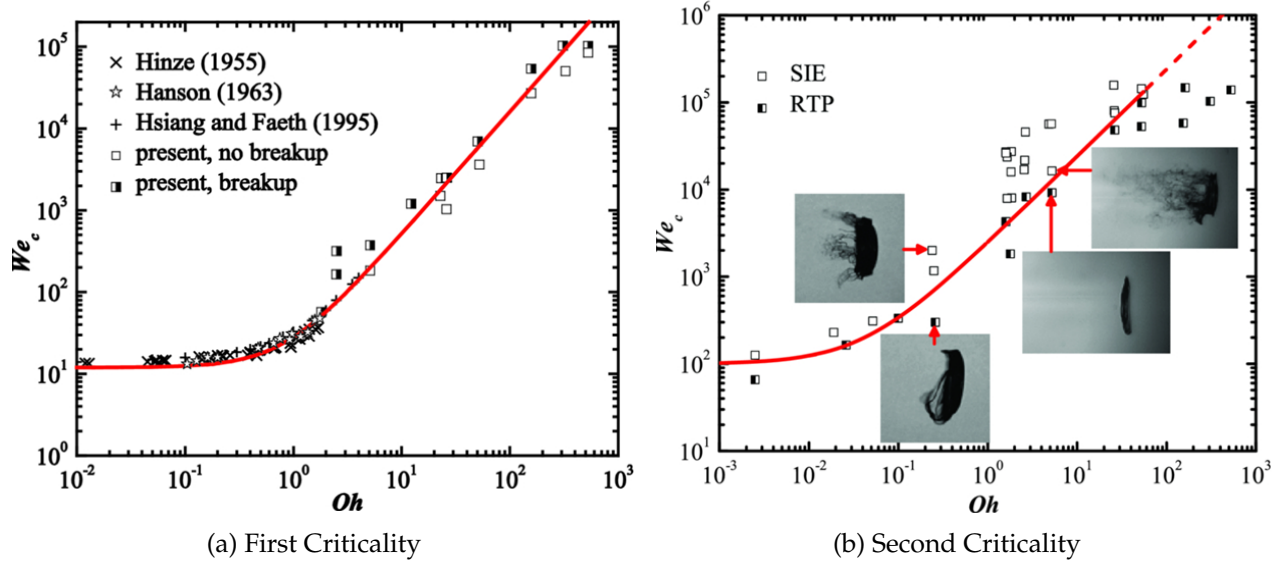


Figure 2.4: (a) The first criticality Rayleigh-Taylor piercing in classical Oh - We coordinates. Open symbols denote no breakage and half-full RTP. (b) The second criticality (shear-induced entrainment). Open symbols refer to SIE, while half-full RTP. Reprinted from [51] T. G. Theofanous et al. 'The physics of aerobreakup. II. Viscous liquids'. In: *Physics of Fluids* 24.2 (Feb. 2012), p. 022104. ISSN: 1070-6631. DOI: [10.1063/1.3680867](https://doi.org/10.1063/1.3680867), with permission from AIP

hydrodynamic instabilities that develop on their surfaces and characterized for more or less isothermal uniform flows acting on newtonian liquid droplets, in what can be designated as a canonical problem García-Magariño et al. [53]. Therefore, the manifestation of breakup modes of different morphologies for problems that depart from the canonical problem is not so much due to the manifestation of different physical mechanisms, but to their manifestation under different conditions.

One of these works that relates breakup mechanisms based on instability analysis but for the secondary breakup of a liquid jet, is that of Varga et al. [21]. In this article the authors analyze the breakup of a water jet that is coaxially exposed to a high velocity air flow. The transfer of kinetic energy from the air stream to the liquid jet causes atomization as a consequence of instabilities at its interface whose wavelength does not seem to be related to the most unstable wavelength of a Kelvin-Helmholtz mechanism. The authors find that this secondary breakup on the ridges generated in the jet as a consequence of the KHI is attributable to Rayleigh Taylor instabilities and that the analysis of [29] is valid to characterize their breakup.

Using other types of experimental setups that change the flow field history around the droplet, other authors have been able to distinguish new breakup regimes in regions corresponding to the multimode one. In Cao et al. [54] the authors study a new breakup mode using a uniform air jet flow installation for water and ethanol droplets injected 50 *mm* from the outlet with Weber numbers between 10 and 100. Under these conditions the droplets evolve in a so-called dual bag breakup corresponding to a first bag breakup followed by a second bag breakup in the ring. The different breakup modes illustrate the richness of processes and their complexity of the droplet breakup phenomenon in particular for the multimode region.

In relation to the bag breakup Zhao et al. [34] employ an experimental setup similar to [54] to generate an air jet stream of diameter 63 *mm* where water and ethanol droplets with diameters between 1.2 and 6.6 *mm* are tested at a distance of 20 *mm* from the outlet within a stream whose average velocity, U_m is tested in the ranges 6.5 – 55 *m/s*. In addition, the fluctuating component (u') is measured with a hot-wire anemometer to ensure that the ratio $u'/U_m < 0.33$, which is the critical condition for the breakup to occur due to turbulence. The authors use the classic linear stability analysis for Rayleigh Taylor instabilities, estimating the critical wavelengths with the highest growth rate for the inviscid case, based on the measured acceleration. Thus, the authors rely on the observations of [32] regarding the number of waves inside the deformed drop, n_λ . Based on this definition, the breakage would occur for the bag mode in the interval from $n_\lambda = 1/\sqrt{3}$ to 1, bag and stamen between $n_\lambda = 1$ and 2 and dual bag between $n_\lambda = 2$ and 3. This is therefore an alternative breakage criterion to Weber's number based on RTI.

Another particularly interesting work, which studies the transition of breakup modes of droplets exposed to an air jet while including PIV visualizations of the near-field of the droplet at different instants, is that of Flock et al. [55]. The authors test ethyl alcohol droplets with initial diameters on the order of 2.33 *mm* exposed to a 25.4 *mm* diameter air jet, whose Reynolds numbers for bag breakage and sheet-thinning are 1500 and 2500 and with Weber numbers of 12 and 32 respectively. The authors use Shadowgraph techniques to calculate the trajectory and acceleration of the droplet and PIV to characterize the flow field. For the PIV analysis they rely on 2000 instantaneous realizations of the droplet trajectory, which gives information on the average flow field. The analysis of the results shows that the wake formation does not seem to be related to the morphology and, on the other hand, the morphology seems to have very little correlation with the Re of the airstream, thus reinforcing the hypothesis of transition between morphologies as a consequence of the growth of instabilities.

In Jain et al. [56] the authors combine an experimental and numerical study to analyze in detail the transition within the bag-multimode-shear regions, distinguishing an interesting region with multiple lobes prior to the shear breakup region. Droplets generated with diameters in the order of 200 μm are exposed to the flow generated by a 2 *mm* nozzle, with Weber numbers between 8 and 300. Experimental results are evaluated with simulations

for the different morphologies observed and compared with previous results. In addition, the rim retraction velocity is evaluated by comparing it with the Taylor-Culick retraction velocity, finding that it is very close to the experimental observations and simulations. The authors conclude that the bag, multibag and multilobe modes obtained in tests and simulations are explainable from the point of view of RTI development.

Of numerical nature in Yang et al. [57] a simulation of droplets for Oh from 0.001 to 2 is performed for continuous flows with Weber numbers from 13 to 110 and density ratios of 800, in order to study the transition between bag and bag-stamen. These constant velocity flows depart from the canonical situation where the droplet accelerates as a consequence of drag forces. The authors propose a RTI model that includes viscous dissipation effects taking the form:

$$\lambda_d = 2\pi \left(\sqrt{\frac{3\sigma}{a_c \mu_d}} + C_2 \sqrt[3]{\frac{\mu_d^2}{a_c \rho_d^2}} \right) \quad (2.12)$$

When comparing the application of this model with previous simulations and studies, the authors determine that being λ_d the wavelength of the highest growth rate, for $0.5 \leq 2R_d/\lambda_d \leq 1$, the bag breakup mode is found and for $1.24 \leq 2R_d/\lambda_d$ bag and stamen. The results are therefore similar to those contained in [32].

It is interesting to mention the works of Sher and Sher [58] and Sichani and Emami [59] that propose breakup criteria based on different principles. For the former, an analytical criterion is proposed based on an energy balance between the energy of the deformed surface resulting from the potential flow around the droplet and the sum of the surface energy of the product droplets and a dissipative term linked to the viscosity. In the second one, a deformation and breakup model is proposed for the bag regime based on the principle of virtual works considering the potential flow around the droplet, $Re \sim 1000$ and high density ratio. The droplet dynamics are modeled based on two degrees of freedom and three variants of the base model are proposed which are able to follow the bag formation regime and the rupture. As a result, they are able to predict the critical Weber number as a function of Oh for the aforementioned regime with a maximum error of 20%

In an attempt to bring together some of the results described for different flow conditions in relation to the critical Weber numbers that separate different regimes based on their morphology, Strotos et al. [60] proposes an effective Weber number that includes multidimensional dependence on Re , Oh and Ca , the latter being the Capillarity number from a single expression. This expression includes fitting coefficients that are determined based on simulations and experimental data in the literature. Although the model lacks a physical foundation, being an extension of existing correlation models, it is able to obtain results below the 20% error when compared to the literature.

Finally, it is worth mentioning the article by Xu et al. [61] where, using an air jet flow installation, the authors are able to characterize a new break-up mode for droplets exposed to a base shear flow, which they call butterfly. For this purpose, ethanol droplets with diameters of 2.55 mm are exposed to an air flow resulting from a jet generated by a 30 mm diameter nozzle, with Reynolds numbers based on the nozzle diameter from 18000 to 23000. The authors define the breakup regimes based on a ratio of Weber numbers based on the jet velocity, We_g considering the frontal area of the droplet and Weber number based on the drop velocity and the initial radius of the droplet, We_d . It is found that the deflection angle is a function of $\sqrt{We_g/We_d}$. The authors elaborate a regime map based on We_d vs We_g delimiting the regions of vibrational breakup, Bag breakup, Bag-stamen breakup and Butterfly breakup, finding that the rim deformation and the shear effect have a significant weight.

Directly linked to flows of aeronautical interest, García-Magariño et al. [62] establish a breakup criterion focused on droplets approaching regions close to the stagnation line of an airfoil, where the flow is continuously accelerating. For this purpose a rotating arm installation (which will be defined in detail in section 3.2) is used, at the end of which self-similar airfoils with 200, 470 and 690 mm chord are placed, moving at tangential velocities between 50 and 90 m/s and hitting droplets produced by a monodisperse generator with diameters in the range of $300 - 3600 \text{ }\mu\text{m}$. A high-speed camera in combination with the shadowgraph technique is used to measure the deformation and trajectory. The velocity of the flow field is based on previous PIV measurements. This study is based on García-Magariño et al. [63] and García-Magariño et al. [64] where it was observed that for similar flows the droplets experienced much larger deformations than expected in Shock-Tube type installations. Thus, for these test conditions, the authors observe that in this range of tests, the droplets break in bag and bag and stamen modes. Considering the instant of breakup as that in which the minor droplet semi-axis starts to increase, causing the droplet to no longer deform as an oblate spheroid and therefore presenting a hat-like shape, the authors are able to obtain the following empirical correlation for the instant of breakup:

$$We_{BK} = 17.5 + 17.9\tau_{bk} \quad (2.13)$$

Being the parameter We defined for the instantaneous conditions of relative velocity and being τ the ratio between the characteristic time of deformation, $t_{def} = \sqrt{m_d/\sigma}$, and of acceleration $t_{ff} = V_s/\dot{V}_s$. The conclusion is very interesting since for this type of continuously accelerated flows, in contrast to the steady airflow problem, larger droplets are able to absorb more energy before breaking and therefore break at higher velocities.

Using the same setup, but with velocities ranging from $30 - 60 \text{ m/s}$ and using ethyl-alcohol droplets with diameters of 500, 1000 and $1500 \text{ }\mu\text{m}$, in García-Magariño et al. [53] the authors find that for intermediate droplet sizes, the droplets deform into a cone shape downstream, presenting a completely new mechanism. The conclusion is that, given the ability of these droplets to deform significantly, the higher accelerations to which they are exposed must vary the flow topology of the liquid phase. Thus the history of how the droplet deforms

and how it is accelerated significantly influence both the evolution of the deformation and breakup phases as well as the morphology of the droplets. A summary of some of the breakage mechanisms that appear in the literature based on their morphology is shown in the figure figure 2.5.

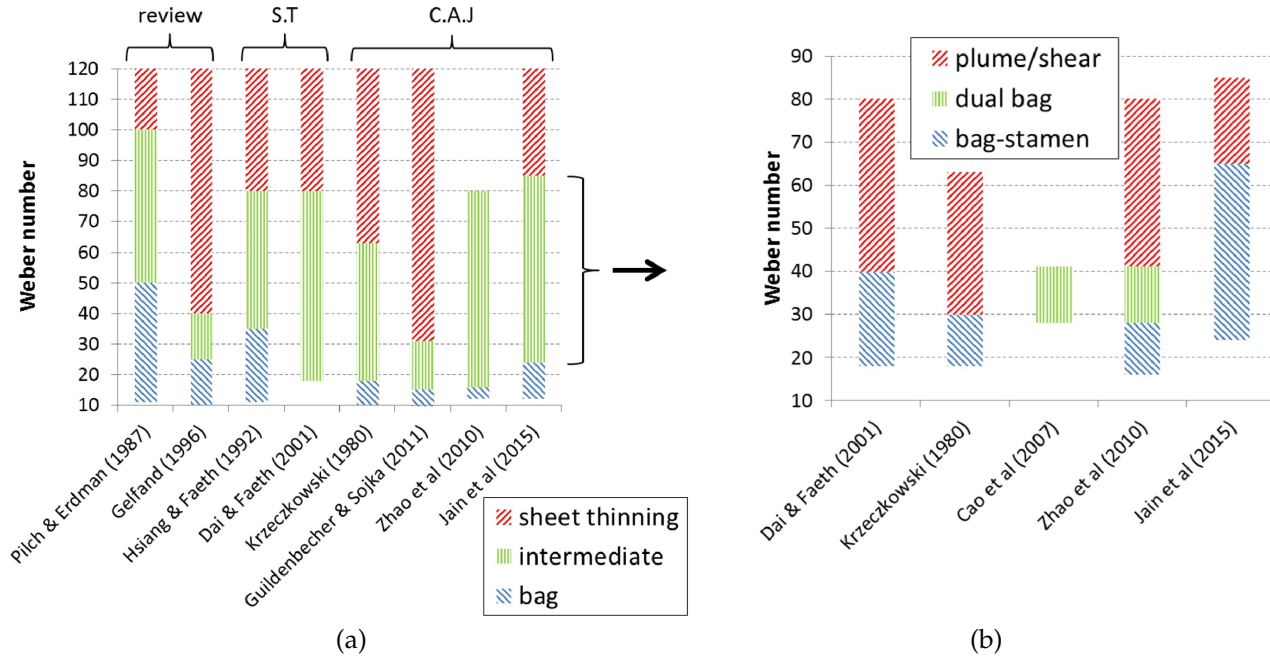


Figure 2.5: Ranges of We numbers for $Oh < 0.1$ corresponding to the breakup regimes as a function of their morphology grouped by baseflow. Please note that the authors originally grouped Bag breakup mode and sheet thinning breakup mode into the so called "intermediate breakup", that is classified in detail in figure (b). Data from (a) is grouped depending on the source: review study, shock-tube (S.T) studies and continuous air jet (C.A.J) experiments. Reprinted from [27] G. Strotos, I. Malgarinos, N. Nikolopoulos and M. Gavaises. 'Predicting droplet deformation and breakup for moderate Weber numbers'. In: *International Journal of Multiphase Flow* 85 (2016), pp. 96–109. ISSN: 0301-9322. DOI: <https://doi.org/10.1016/j.ijmultiphaseflow.2016.06.001> with permission from Elsevier.

2.1.3 Droplet deformation models

From most of the literature reviewed, however, it is obvious that regardless of the resulting breakup morphology or empirical correlation, there is a deformation regime prior to the onset of breakup, with almost universal characteristics, which can be defined as a function of the normalized time t/t^* [42]. This relation can be expressed through empirical correlations that usually take the general form $a/R_d = A + B(t/t^*)^{c_1} + C(t/t^*)^{c_2} + D(t/t^*)^{c_3}$, where a is the semi-major axis of the droplet. The most common coefficients are shown in the table 2.1. As can be seen from their values, one of the problems with these forms is that there is lack of generality as they depend on the type of experiment giving valid

results within the regime and mode of rupture for which they were calibrated. In order to avoid these shortcomings, some deformation and breakup models have been developed based on semi-analytical expressions derived by applying the Navier-Stokes equations in conjunction with a series of simplifications. One of the best known is that of Gonor and Zolotova [65] which used a power series to obtain the axial and radial velocities and integrate them in a time interval to obtain the deformation. The main disadvantage of the latter is the loss of accuracy as the integration time interval increases, generating divergences in the deformation calculation for numbers of $We > 20$, well before the instant of rupture. Apart from the above approaches, most of the models developed and incorporated in commercial software for atomization and spray modeling are based on Taylor's analogy (shape and acceleration of a drop in a high speed stream). In his work Taylor suggested that the deformation of the drop can be modeled as a mass-spring system. Among the main advantages of this approach are its simplicity of implementation and relatively low computational cost. Traditionally there are three models based on this analogy that are considered fundamental; the TAB (Taylor Analogy Breakup) model [66], the Clark model [67] and the DDB (Droplet Deformation Breakup) model [68]. In the following we will explain the basis of each of them.

The first one was described by O'Rourke and Amsden [66] which represented the droplet as an oblate spheroid and evaluated the oscillator mass displacement based on the displacement of the droplet's equator (x_{eq}), defined by its semi-major axis (a) perpendicular to the flow, with respect to its initial position (figure 2.6a). The equation took the harmonic oscillator form expected by Taylor:

$$\ddot{y} + \frac{C_d N}{K Re} \dot{y} + \frac{C_k}{K We} y = \frac{C_F}{C_b K} \quad (2.14)$$

Being $y = x_{eq}/C_b R_d$ and $t_{TAB}^* = t V_s / R_d$ the dimensionless variables for the equator radius displacement and the oscillation time, with respect to which derivatives are defined. The rest of the elements of the equation are the Reynolds number, $Re = \rho_g V_s R_d / \mu_g$, the Weber number, defined as $We = \rho_g V_s^2 R_d / \sigma$, the ratio of densities $K = \rho_d / \rho_g$, the ratio of dynamic viscosity $N = \mu_d / \mu_g$. To obtain the rest of the constants, the authors define a breakup criterion based on the instant in which the amplitude of the oscillation reaches the value of the initial radius of the drop, $y = 1$. In addition, the authors assume that there is only one fundamental mode of oscillation. Thus, solving the equation 2.14 by considering constant V_s and employing the previously defined breakup criterion, it is straightforward that $C_k C_b / C_F = 2 We_c$ which assumes $We_c = 6$ for $Oh < 0.1$. Analyzing the period and frequency of the fundamental mode we obtain $C_k = 8$ and $C_d = 5$. Supposing that it deforms symmetrically with respect to the minor axis of the spheroid, it is straightforward that $C_b = 1/2$ and $C_F = 1/3$. Substituting the parameters and solving the equation for $y = 1$ the authors obtain that the breakup time can be defined as $t_{bk} = \sqrt{3} \sqrt{K} R_d / U_{slip}$. This result agrees with experimental observations regarding the linear dependence of the breakup time on $\sqrt{K} R_d / U_{slip}$ but differs significantly in the proportionality constant. The authors attributed this to the aforementioned mismatch of criteria for the designation of the instant of rupture. The model was also originally used to calculate the droplet velocity

Table 2.1: Empirical correlations for droplet deformation

Empirical correlation for general equation: $a/R_d = A + B(t/t^*)^{c_1} + C(t/t^*)^{c_2} + D(t/t^*)^{c_3}$										
Research	A	B	C	D	c_1	c_2	c_3	Conditions of applicability 1		
								Non-dimensional numbers	Breakup Mode	Time range
Simpkins and Bales [28]	1	21/80	-7/625	30/1741	2	3	4	7.3<We<20 ; Oh<0.1	Bag	$t/t^* \leq \sqrt{10\rho_g/\rho_d}$
Hsiang and Faeth [42] Dai and Faeth [47]	1	$0.19We^{1/2}$	0	0	1	0	0	10<We<100 ; Oh<0.1	Bag Multimode Stripping	$t/t^* \leq 1.6$
Chou et al. [45]	1	0.5	0	0	1	0	0	13<We<20; Oh<0.05	Bag	$t/t^* \leq 2$
Cao et al. [54]	1	0	0	0	0	0	0	28<We<41 ;	Multimode	$t/t^* \leq 0.3$
	0.59	1.34	0	0	1	0	0	Oh<0.003		$0.3 < t/t^* \leq 1$
Zhao et al. [69]	1	0.54	0	0	1.67	0	0	16<We<26 ; Oh<0.4	Multimode	$t/t^* \leq 1.5$
Xu et al. [61]	1	0.732	0	0	1	0	0	1<We _d <100;	Bag	$t/t^* \leq 1.2$
								12<We _g <20	Bag-stamen Bag-butterfly	

after break-up, allowing the estimation of the spray opening in combustion processes. To estimate the droplet size after break-up, the authors apply pre- and post-break-up energy conservation criteria, defining the former as the sum of the interface energy and the oscillation energy. Regarding the post-breakup energy, it is considered that the drop does not oscillate but has gained velocity perpendicular to the current and is therefore the sum of the interface energy and the transverse kinetic energy. Finally, the authors group all this set of equations in a numerical method called KIVA, originally intended for the calculation of diesel injector cones.

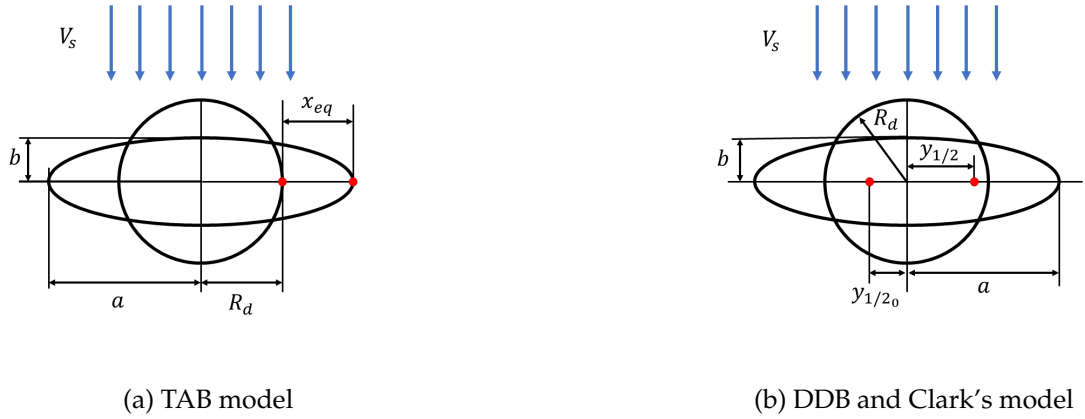


Figure 2.6: Sketch of droplet deformation models. (a) Drop deformation scenario for TAB model, [66]. (b) Drop deformation scenario for Clark's and DDB model, [67] and [68].

One of the most controverted points of the TAB model is that, although its results when compared with experiments are satisfactory, the forces do not apply on the centre of mass of the harmonic oscillator. In this respect, Clark [67] uses the simplification of assuming the problem to be flat, representing the droplet as a cylinder with an elliptical cross-section. In addition, he references the deformation of the droplet based on the displacement of the centre of mass, $y_{1/2}$ contained in the volume of the half-droplet (divided by the plane of symmetry with versor perpendicular to the semi-major axis of the ellipsoid) and where the forces are applied (figure 2.6b). Thus, the droplet is deformed as a consequence of a pressure force generated by the external fluid and liquid is assumed as a pure extensional flow. The system will be governed by this pressure force, a dissipative force linked to the viscosity, an elastic force linked to the surface tension and an inertial force linked to the acceleration of the mass of the semi-droplet. Thus the authors start from the energy dissipated per unit length for an incompressible flow (dE_μ) as a consequence of the viscous forces (F_μ) which under the assumption of purely extensional flow can be modelled as $dE_\mu = F_\mu dy_{1/2}$. Then, the dimensionlessness $y = y_{1/2}/R_d$ is applied, which they linearise as $y = y_0 + \Delta y$, being $y_0 = 4/3\pi$ the position of the centre of mass of the undeformed half-drop. After applying a series of simplifications valid for small deformations the viscous forces take the form:

$$F_\mu = -\frac{9}{8}\pi^3\mu_d R_d \dot{y} \quad (2.15)$$

On the other hand, to obtain the forces related to surface tension, the authors start from the differential work necessary to increase the area of the interface, $dW = \sigma dA$, and relate it to the force assuming the mechanical model $F_\sigma = dW/2dy$. Furthermore, by applying an approximation for the area of the ellipsoid of constant cross-section and linearising around the equilibrium point, they allow the surface tension forces to be expressed as:

$$F_\sigma = -\frac{9}{8}\pi^3\sigma y \quad (2.16)$$

For the half-droplet subjected to the constant-pressure excitation force $F_p = \rho_g R_d U_d^2$ and using both equations (2.15 and 2.16), if we adimensionalise the time as t_{TAB}^* it can be represented as an oscillator of the form:

$$(K+1)\ddot{y} + \frac{9\pi^2(N+1)}{4Re}\dot{y} + \frac{9\pi^2}{4We}\left(y - \frac{4}{3\pi}\right) = \frac{2}{\pi} \quad (2.17)$$

Where K , N , Re and We are those defined for the equation 2.14. This equation 2.17 is very similar to that of [66] but includes an additional dependence on the Weber number in the zeroth order term. One of the main differences with the TAB model is that all the terms are defined on the basis of theoretical development and simplifications, so it is not necessary to adjust them on the basis of experimental observations. Regarding the breakup criterion, the authors simply use a value of the ultimate deformation, which is similar to the one used in the TAB model, but suggesting that under this criterion the breakup may not be a unique function of the Weber number.

In order to avoid the linearisation applied by Clark [67] and to ensure the application of the resultant of forces on the centre of mass, in Ibrahim et al. [68] the DDB model is proposed based on the balance of energy variation on the half-droplet and considering that it deforms as a consequence of an extensional flow in the form of an oblate spheroid with major and minor semi-axes a and b respectively, as described in figure 2.6b. Thus, the authors propose that if heat generation is neglected, the internal energy variation dE/dt must be equal to the power developed by the forces $-dW/dt$ on the half-droplet with centre of mass $y_{1/2}$. The variation of internal energy on the half-mass therefore has a kinetic energy component and a potential energy component linked to the interface, taking the form:

$$\frac{dE}{dt} = \frac{2}{3}\pi R_d^3 \rho_d \frac{dy_{1/2}}{dt} \left(\frac{d^2 y_{1/2}}{dt^2} \right) + \frac{1}{2}\sigma \frac{A_d}{dt} \quad (2.18)$$

Where A_s is the surface of the oblate spheroid which is approximated as $A_d = 2\pi(a^2 + b^2)$. On the other hand, the power developed by the forces on the droplet is decomposed into a part consequence of the pressure forces and another part as a result of the viscous forces:

$$\frac{dW}{dt} = -\frac{1}{2}P A_p \frac{dy_{1/2}}{dt} + \frac{8}{3}\pi R_d^3 \mu_d \left(\frac{1}{y_{1/2}} \frac{dy_{1/2}}{dt} \right)^2 \quad (2.19)$$

Where $P = \rho_g U_{slip}^2 / 2$ is the dynamic pressure of the gas and A_p is the projected area of the spheroid in the direction of the gas stream incident on the droplet. Applying the constant volume condition on the 2.18 and 2.19, and using the previously described dimensionlessness of the DDB model, the equation of the droplet dynamics takes the following harmonic oscillator form:

$$K\ddot{y} + \frac{4N}{Re} \frac{1}{y^2} \dot{y} + \frac{27\pi^2}{16We} y \left[1 - 2 \left(\frac{3\pi}{4} y \right)^{-6} \right] = \frac{3}{8} \quad (2.20)$$

As a consequence of its derivation, the model incorporates non-linearities that have been simplified in Clark's model and overcomes the inconsistency of applying forces on the equator (TAB model). Finally, the authors establish a breakup criterion considering that in the previous instants, both the kinetic energy of the mass and the viscous dissipation are negligible. The result is therefore $y_{bk} = 2We_{bk}/9\pi^2$, where y_{bk} is the position of the centre of mass of the semidroplet at the instant of breakage and We_{bk} is the Weber number suggested by Krzeczowski [70] for the breakage of drops of different sizes.

The three droplet models mentioned above are reviewed and analyzed in Lee et al. [71] where a number of errors present in the original TAB and DDB model formulations are listed and corrected. The authors analyze the corrected models by integrating them numerically and comparing the result with the experimental data of [70]. The results are that, after their correction, the TAB model appears to perform better when compared to Clark's and DDB models for the given experimental data set. In general, the TAB model and its variants are well suited for implementation in CFD codes due to the versatility of the empirical fit constants. On the other hand, the DDB model, thanks to its rationale, seems to be better adapted to large deformations.

Thus, modifications have been made to the aforementioned models to adapt them to specific situations of droplet breakup. This would be the case of the MTAB (Modified TAB) model presented by Senda et al. [72], which optimized the constants of the TAB model to adapt it to experimental observations of diesel sprays. In the same year, [73] presented the ETAB (Enhanced TAB) model, which assumes spheroid-shaped deformed droplets and proposes an oscillator equation based on the energy balance before and after breakup. Its performance is compared in Lakhfif et al. [25] in CFD simulations for diesel spray applications, finding superior performance in predicting vapour penetration with respect to other models studied. Also based on the TAB model, Schmehl [74] presented the NLTAB3 model, which couples the equations for the deformation and dynamics of the droplet through the relative velocity obtained from the balance of aerodynamic forces on the droplet. For this purpose, it assumes an oblate or prolate spheroid deformation and represents the surface deformation ratio based on polynomial correlations as a function of the equator radius. In relation to the kinetic energy balance, it is defined for the masses of the half-droplet. On the other hand, the pressure forces are defined as a function of the deformation from the equator displacement but assuming a cylindrical shaped droplet. The model is able to include non-linear effects and has yielded good results when compared

with the experimental work of Hsiang and Faeth [42], Hsiang and Faeth [43] and Temkin and Kim [75]. Another model that modifies the TAB model to include the effects of deformation on aerodynamic forces is that of Park et al. [76]. The authors couple the deformation model with a breakup criterion based on the balance of pressures on the liquid phase between the equator and the poles. According to the authors, the model generates better results for the shear stripping and bag breakup regimes. In Bartz et al. [77] is proposed a methodology that uses the NLTAB3 model to meet a breakup criterion based on the critical deformation observed in experiments. To solve the breakup process, the authors adapt the model for other aerodynamic loading conditions than those found in shock-tube tests. Finally, they compare the full model for cross-flow diesel injection conditions. Another model that introduces another breakup criterion is the WAVE-MTAB model, Kitaguchi et al. [78], that incorporated experimental results regarding the changes of the break-up modes as a function of the Weber number. Focused on bag breakup, Opfer et al. [79] present a model that takes into account the pressure forces on a droplet that deforms as a cylinder to describe the ring motion as a function of Weber number and density ratio. The authors calibrated the model in their wind tunnel droplet breakup experiments. Based on the DDB model, Rimbert et al. [80] study the possibility of incorporating the RTI criterion into the deformation model [68], finding that the pressure terms must be reconsidered as in the original one it wasn't considered to be independent of the deformation. Thus, the authors propose two approaches, one based on the potential flow and the other on an empirical drag coefficient.

It is worth mentioning the work of Stefanitsis et al. [26] which defines a deformation model based on the TAB model, which the authors call ITAB (improved TAB), optimizing its coefficients for different breakup mechanisms (Bag, Multi-mode and Sheet-thinning) according to the results of VOF-CFD simulations for different Weber numbers. In addition, the article is of great interest because it includes a detailed synthesis of the most commonly used correlations and models for droplet deformation, introducing a unified deformation model, which allows synthesizing all the expressions contained in the literature and previous developments as:

$$\left(f_{kin} \ddot{y} + \frac{1}{2} \frac{df_{kin}}{dy} \dot{y}^2 \right) + f_{vis} \frac{Oh}{\sqrt{We}} \frac{\dot{y}}{y^2} + \frac{f_{st}}{We} \frac{dS^*}{dy} = f_p u_{slip}^{*2} \quad (2.21)$$

Being y the semi-major axis variable normalized by R_d and the normalized time t^* according to the definition of [39] where $u_s^* = V_s/U_\infty$ is the normalized relative velocity, S^* is the droplet surface normalized by $4\pi R_d^2$ and f_{kin} , f_{vis} , f_{st} and f_p are fit functions depending on the model.

Finally, a model that involves a reinterpretation of the fundamentals of the TAB model is that of Marek [81] known as DMTAB (Double Mass Taylor Analogy Breakup). This model proposes a description of the drop with two degrees of freedom; one related to deformation and the other to translation. Thus, the droplet is presented as a system of two equal masses, separated by the vertical plane with two degrees of freedom, which, joined by a spring-damper assembly, are excited by an aerodynamic force of unequal way. The author's

hypothesis is that the total aerodynamic forces affect only the semi-mass related to the front mass of the droplet. Two coordinates describe the problem, x_1 for the global centre of mass of the droplet and x_2 for the radius on the axis of displacement. A relative velocity linked to the combined effect between the deformation and the displacement of the droplet appears. The authors solve the system analytically under a linearization of the aerodynamic force $F_d \propto V_r$ and numerically for $F_d \propto V_r^2$. From these solutions he obtains critical surfaces in the space $(We - Oh - \epsilon)$, being $\epsilon = \rho_g / \rho_d$, which characterize stability and rupture assuming O'Rourke's criterion $|x_2| = 1$. From the analysis of these surfaces $|x_2| = 1$ the authors are able to approximate curves as a function of $We_{bk} = A(1 + B(\epsilon)Oh^{C(\epsilon)})$ proposed in [44]. It should be noted that the system results are analyzed as a response to a force step (impulsive) input. The analysis of the DMTAB model in comparison to TAB has shown it to be more effective in analyzing the vibrational and pocket break-up regimes for conditions of low density ratios of $\epsilon \ll 1$ and high Ohnesorge numbers under which much of the flow energy is transferred to kinetic energy of the droplet, energy transfer which is not represented by a single degree of freedom.

As it has been seen, most of the models are focused on industrial applications and therefore optimized, directly related with aeronautical practical applications a key article in the development of the breakup criterion described in this thesis is Sor and García-Magariño [82], where a deformation model based on [67] and [68] is presented, but with a particular focus on droplets approaching in the regions close to the stagnation line of an airfoil. Same setup and approach than that showed on Garcia-Magariño et al. [62] and García-Magariño et al. [63] is used, but with water droplet of diameters in the range of $400 - 1400 \mu m$. From the results, the authors evaluate the performance of the TAB, Clark and DDB models and find that they diverge significantly when trying to represent the observed data. Their conclusion is therefore that they are not appropriate for this particular situation, so a new Droplet Ratio Deformation (DRD) model is proposed. From the energy dissipation in an extensional flow, the work of the restoring surface tension force and modelling the pressure forces from a coefficient of pressures (C_p), the DRD model takes the form of a harmonic oscillator:

$$K(1 + B_v t^*)^2 \ddot{y} + \left[K(2B_v + B_a t^*) + \frac{4N}{Re} \frac{1}{y^2} (1 + B_v t^*) \dot{y} \right] + \frac{8}{4We} \frac{dA'_d}{da'} = \frac{3}{4} C_p \quad (2.22)$$

The normalized variables y and t^* and constants K, N are the same as those defined for the Clark and DDB models. Regarding the variables $a' = a/R_d$ and $A'_d = A_d(a)/\pi R_d^2$ are the normalized variables of the semi-major axis and area of the oblate spheroid respectively. Regarding the rest of the variables, $B_v = Bo/KWe$ being Bo an equivalent Bond number defined as $Bo = (\rho_d R_d^2 dV_s/dt)/\sigma$ and $B_a = (R_d^2 d^2 V_s/dt^2)/V_s^3$. In order to model the pressure forces appropriately for this case, the authors estimate the pressure proportionality coefficient from the experimental data of 159 cases and obtain the value that minimizes its error. The value obtained is $C_p = 0.93$. It is therefore a calibrated model for this particular situation and hence its value for predicting the deformation of the droplet prior to droplet break-up around the stagnation line of an airfoil moving under subsonic and incompressible conditions.

2.1.4 Droplet and particles dynamics

As it has been deduced from the previous sections 2.1.2 and 2.1.3 the forces involved in the dynamics of the droplet and their history derived from the non-stationary character of the problem play a key role in the deformation process and subsequent breakup. Thus, in relation to the problem of drop dynamics, the traditional approach is based on the application of reduced order models from a Lagrangian representation. In this way, it is assumed that the flow field remains unchanged in the presence of the droplet, and the aerodynamic forces are deduced from Newton's second law, by separating the forces resulting from the undisturbed flow from those originating from the disturbance of the flow, as a sum of their different contributions for a given position. Thus, as a rule, most of the proposed models are based on the expression for the force on an isolated solid particle, proposed independently by Basset [83], Boussinesq [84] and Oseen [85], and therefore known as Basset-Boussinesq-Oseen (BBO) equation, which takes the original form:

$$m_p \frac{d\vec{u}_p}{dt} = -6\pi R_p \mu \vec{u}_p - \frac{2}{3} \rho \pi R_p^3 \frac{d\vec{u}_p}{dt} - 6R_p^2 \sqrt{\pi \nu} \int_{-\infty}^t \frac{1}{\sqrt{t-\xi}} \frac{d\vec{u}_p}{dt} \Big|_{t=\xi} d\xi \quad (2.23)$$

The three terms on the right are usually designated as quasi-stationary term, which in this case is constituted by the Stokes law for the creeping flow limit, added mass term and the last one on the right the viscous-unsteady or Basset force. Regarding the rest of the variables and constants; m_p is the mass of the particle, $\vec{u}_p$ is the relative velocity vector between the particle and the undisturbed flow field at its location at the given instant t , R_p the radius of the particle and ρ , μ and ν the density, dynamic and kinematic viscosity for the carrier fluid respectively. Later Faxén [86] studied the effect of non-homogeneities on the Stokes term for the stationary drag on the sphere, which would take the form $6\pi R_p \mu \vec{u}^s$. The study of non-uniform flows was dealt with in the doctoral thesis of Tchen [87], where a series of modifications to the BBO equation were proposed for its application to spheres moving as a function of time in non-uniform flows. Since then, adaptations and extensions of this expression have been made, being the most widely used form for $Re < 1$ the one that groups the results of Maxey and Riley [88] and Gatignol [89] that correct the equation for the limit of non-stationary and non-uniform Stokes flows.

Focusing on the problem of particles exposed to non-stationary and non-uniform flows at higher Reynolds numbers, there are two approaches in the literature (Peng and Aggarwal [90]), one based on the modification of the BBO equation and the other based on the use of correlations where the effects of non-uniformity, non-stationarity and stationary forces are divided as the sum of different terms. Regarding the former, the well-known works of Mei and Adrian [91] and Kim et al. [92] are worth mentioning, which extend the validity of the BBO equations for finite Reynolds numbers less than 200 by modifying the history kernel. Subsequently, Mei [93] performs a numerical study for a sphere immersed in a fluid undergoing a step jump in velocity for low and moderate Reynolds numbers (0.1 to 100), comparing the results with analytical expressions based on BBO and with experimental results. Their conclusion is that, for particles suddenly accelerated to $Re \gg 1$, if the density of the carrier phase is much smaller than that of the particle, it is possible to neglect both

the added mass term and the Basset term. Finally, regarding the generalization of the BBO equations to take into account the effects of flow compressibility, the series of works by Parmar et al. [94], Parmar et al. [95], applicable to particles, and Parmar et al. [96], applied to droplets and bubbles, are worth mentioning.

Considering the works based on separating stationary and non-stationary effects, it is relatively common to use the acceleration parameter (Ac) when evaluating the relative importance of the non-stationary effects of the flow in the particle/droplet dynamics. One of the first works to consider this separation of terms is that of Karanfilian and Kotas [97] who studied the forces on a sphere immersed in a tank of water and forced into a linear harmonic motion. The sphere, approximately 40 mm in diameter, is attached to a rod with a force transducer installed which gives it a harmonic motion with Reynolds numbers between 10 and 6000 and acceleration numbers between 0.1 and 10.5. The authors pool the data and fit them to an empirical relationship formed by the product of the stationary coefficient of the sphere with a term of the form $(1 + Ac)^{1.2}$. The conclusion is therefore that for the described non-stationary motion, since $Ac > 0$, the drag coefficient of the sphere when subjected to acceleration is always larger than the stationary one. In the same way, it quantifies the increase of 1% on the C_D for Ac of the order of 0.01, being therefore sufficient a quasi-stationary model of the aerodynamic coefficient for its evaluation. To provide an interpretation of the correlation, the authors present a visualization of the flow field by exposing the image of alumina particles on photographic paper, being able to distinguish a vortex shedding phenomena to which they attribute the increase in resistance.

In this line, Temkin and Kim [75] study the dynamics of droplets with diameters between 87 and 575 μm in a shock-tube setup and Reynolds numbers in the range of $3.2 < Re < 77$ and attempt to fit it on the basis of a semi-empirical drag coefficient model. The droplet trajectory was measured by repeatedly exposing the droplet using stroboscopic lights at 2.5 kHz. Then, trajectories are fitted to a third-degree polynomial from which their velocity and acceleration are derived. In order to extract the horizontal velocity after the shock wave, u_s , a pressure measurement is used in combination with the equations for normal shock waves in a perfect gas. The results are that the drag coefficient was always larger than the stationary one. Thus, considering that it is a weak shock wave, the authors neglected the compressibility effects and tried to fit it on the basis of a drag coefficient model separated into a stationary and a non-stationary term which depends on the relative acceleration parameter. Thus the expression takes the form:

$$C_D = C_{D_{sphere}}(Re_s) - k_1 \left(\frac{\rho_d}{\rho_g} - 1 \right) \frac{2R_d}{V_s^2} \frac{dV_s}{dt} \quad (2.24)$$

Where k_1 is a fit constant, which the authors find to be of order unity, Re_s is the Reynolds number based on the relative velocity between the drop and the gas and $C_{D_{sphere}}$ is the drag coefficient for a sphere in stationary conditions. The authors indicate that this is a valid model for situations of spherical drops moving after the passage of a shock wave and indicate that it is only validated for $dV_s/dt < 0$, which is the case of their experiments.

In later work, Temkin and Mehta [98] repeat the analysis for Reynolds numbers between 9 and 115, diameters in the range $115 - 187 \mu\text{m}$ and Weber numbers of 0.137, but including situations of relative droplet acceleration in the opposite direction to the flow ($dV_s/dt > 0$). For this purpose, they modify the shock-tube setup of [75] by allowing the interaction of N waves, so that the initial acceleration of the droplet is allowed to subsequently decelerate in the stream. The results are particularly interesting as the authors find a different behavior depending on the direction of the relative acceleration with respect to the stream, finding that when the particle accelerates, accommodating its velocity to that of the flow, the drag coefficient is higher when compared to the steady state, while in the opposite case, the drag coefficient decreases with respect to the steady drag coefficient. The authors attribute this to the establishment of the recirculation region, assuming that for the first case it is larger and for the second case it is smaller. In particular, they specify that for the case of ($dV_s/dt > 0$) the correlation takes the form:

$$C_D = C_{D_{\text{sphere}}}(Re_s) - K_2 \left\{ \left(\frac{\rho_d}{\rho_g} - 1 \right) \frac{2R_d}{V_s^2} \frac{dV_s}{dt} \right\}^{-1} - K_3 \quad (2.25)$$

The fitting coefficients K_2 and K_3 are, for $5.9 < Ac < 25$, 3.83 and 0.20 respectively. The behavior is therefore completely different from that of the equation 2.24 and the correlation found in [97]. The authors are cautious in assessing the generalization of their correlations and indicate that in particular for this case further studies are required under these conditions, where the acceleration is counter to the motion and involves an increase in the relative velocity.

In a somewhat different approach, Sommerfeld [99] uses a vertical shock-tube system to generate a plane shock wave on a gas-particle mixture in order to characterize the propagation of shock waves in this medium and to describe the particle dynamics. Particles of average diameter $27 \mu\text{m}$ with densities of 2500 kg/m^3 are coaxially premixed in a chamber and transported inside the shock-tube test section continuously before the generation of the shock wave, which reaches Mach numbers up to 1.70. Using a laser-Doppler-Velocimeter (LDV) system, the authors find that in the initial stages the particles have a drag coefficient below stationary, although it should be noted that the results refer to dust clouds, so their correlation may not be appropriate for isolated particles. In a later study, Igra and Takayama [100] study the non-stationary dynamics of solid spheres, with sizes between 0.5 mm and 4.8 mm , when exposed to a shock wave with Mach numbers between 1.27 and 1.80 in a shock-tube type installation. As a consequence of the large size of these samples, the authors are able to gather the dynamics of particles exposed to flows with Reynolds numbers between 6×10^3 and 10^5 , which is a much larger range than previous studies. The trajectories, recorded by double exposure and laser holographic illumination, are fitted to a third-degree polynomial and their acceleration is obtained by direct derivative and used to characterize the aerodynamic forces and drag coefficient. With these results, the authors pool the data, obtaining the following correlation for the drag coefficient as a function of

Reynolds number valid for $Ac \leq 0.1$ and Reynolds numbers between 200 and 10^5 :

$$\log_{10} C_D = 7.823 - 5.814 \log_{10} Re + 1.413 (\log_{10} Re)^2 - 0.115 (\log_{10} Re)^3 \quad (2.26)$$

The study is interesting not only for increasing the range of Reynolds numbers studied, but also for exploring the region of small acceleration numbers. Thus, they find that the drag coefficient for the non-stationary conditions tested is always higher, with the difference with respect to the stationary case being greater the lower the Reynolds number.

In another experimental study, Suzuki et al. [101] employ a horizontal shock-tube setup to study the drag coefficient of spheres with diameters in the range of $0.3 - 5.57 \text{ mm}$. The Mach numbers of the test are $M_s = 1.10, 1.25, 1.30$ and 1.40 . To improve the method of particle insertion, a pneumatic actuator pushes the sphere which is launched synchronously with the shock wave towards the centre of the test section. The trajectories are obtained by photographic exposure through three flash lamps. As a result, a significant reduction in data scattering is achieved and a conclusion similar to that of [100] is reached, which is that the drag coefficient is an average of 20% higher than stationary for Reynolds numbers tested from 10^3 to 10^5 . The authors however indicate that the particles are strongly affected by the experimental setup, pointing to shock wave reflection and wall roughness as possible sources of discrepancy between studies.

In the same experimental line as the previous mentioned, Jourdan et al. [30] uses a shock-tube type facility equipped with more modern techniques to study the non-stationary effects in spheres of different materials and dimensions between $0.29 - 6.5 \text{ mm}$ and in subsonic and supersonic regimes with Reynolds numbers between 500 and 10^4 . The authors indicate that the vast majority of the studies found subsonic flows after the passage of the shock wave. To analyse this effect, the authors use gas mixtures to achieve Mach numbers in the vicinity of 4 that guarantee supersonic conditions after the passage of the shock wave. One of the most relevant conclusions of the study is that, given the characteristics of the particles, the acceleration number is in the range $2.5 \times 10^{-5} \leq Ac \leq 3.5 \times 10^{-2}$, so the contribution of this parameter on the particle dynamics should not be significant. The results however show the opposite, with values of the non-stationary drag coefficient that are on average 50% larger than the stationary one. The authors suggest that the Ac parameter is not an adequate parameter to describe the problem and that the differences with $C_{D_{\text{phere}}}$ are probably due to the mechanism of flow establishment around the particle, as previously suggested by [98], for the initial instants.

Grouping this result in Wagner et al. [102] they review results obtained in the experiments [98], [100] and [101], for which the acceleration parameter is of the order of 0.01, indicating that under these conditions the relevance of non-stationarity must come from another source. The authors rely on [95] to justify that only in the initial instants non-stationarity is relevant, with quasi-stationary dynamics being imposed later, and that previous analyses could be erroneous as they refer to data well after the passage of the shock wave, without considering the effects of the compressibility of the flow. To evaluate their hypotheses, the

authors perform tests in a horizontal shock-tube installation for Mach numbers between 1.68 and 2.04 and stainless steel particles with a diameter of 1 *mm* that are introduced by gravity. In order to evaluate possible sources of non-stationarity, the authors incorporate a Particle Track Velocimetry (PTV) system for drag coefficient determination, high-frequency pressure sensors and a Schlieren high-speed imaging system to evaluate reflected shock waves. The authors compare the cases where particle interactions with other shock waves are not significant, with the expression [95] and find that it agrees with the observed particle dynamics. Considering the results they conclude that the non-stationary effects on the particle previously observed in other studies, and the scattering of data, must come from sources related to the compressibility of the gas and to non-stationary effects linked to the establishment of boundary layer on the walls of the facility.

Considering this article, in Martinez et al. [103] the authors design a new horizontal shock-tube facility whose objective is to characterize the non-stationary dynamics of particles in the initial stages of the shock-wave passage over them. Thus, they develop a diaphragmless design, based on a double-acting pneumatic cylinder, which avoids the intrusion of diaphragm debris into the test section and is capable of generating shock waves of Mach numbers between 1.1 and 2. The facility is capable of working with particle diameters between 2 and 90 μm and Reynolds numbers between 6 and 3500. The authors use 0.5 μm glycol particles for the PIV visualizations to calculate the acceleration of the flow field in 38 *mm* regions. Analysis of the visualization shows an absence of wave reflections and the absence of boundary layers in the test region, which would contradict the hypotheses of [102]. In addition, the authors employ very high-speed cameras to record the particle trajectory, fitting their position to third-order polynomials. Their conclusions are that the drag coefficients are always larger than quasi-stationary and similar to other results found in the literature.

Using the same setup, Bordoloi et al. [104] take measurements of the particle trajectory after the passage of the shock wave in the relaxation zone interval (i.e. when the particle accommodates its velocity to that of the flow). Particles with Reynolds numbers ranging from 0.31 to 56.4 and incident wave Mach numbers from 1.2 to 1.4 are tested. Furthermore, to avoid compressibility effects on the particle, the Mach number defined on the particle is always less than 0.5 for the measured times. For the calculation of the acceleration a piecewise is used for fitting that is integrated three times in order to adjust the trajectory in order to obtain the drag coefficient. The authors define a modified Knudsen number (Kn^*) which represents the time dependence of the dynamics on the particle relative velocity and is defined as the ratio of the Mach number of the incident shock wave divided by the instantaneous Reynolds number of the particle $M_s/Re(t)$. The results are that for ($Kn^* = 1$) the contribution of the non-stationary drag coefficient to the total increases and then decreases until for ($Kn^* = 2$) are of the order of quasi-stationary one. The authors found that for the regions studied the previously developed linearised models fail as a consequence of the non-linear relationships linked to the development of the flow around the particle.

In general, all the previously cited works were applied to situations in which both the particle and the droplet assumed almost spherical geometries. However, as was seen in the sections 2.1.2 and 2.1.3, under certain circumstances of Weber number and flow field to which the droplet is exposed, the droplet can deform monotonically before breakup, departing from the situation studied for spherical geometries. Thus, in Simpkins and Bales [28], focusing on their analysis in relation to the impulsive acceleration of the droplet, the displacement of its stagnation point is analyzed by the authors and compared it with the drag coefficients obtained in previous studies of droplet breakup in similar installations, [39]. Their conclusion is that for Reynolds numbers greater than 10^3 the drag coefficient is significantly larger than that of a sphere and closer to the results for a disc-shaped particle in similar flows.

In Pilch and Erdman [41], the authors also indicate that the deformation of the droplet plays a significant role in the evolution of the drag coefficient with time, pointing out the great difficulty for the experimentalist in determining the centre of mass of the droplet, since the breakup can induce errors when applying shadowgraph techniques showing larger areas than the real one. The authors indicated that to date there was no valid correlation for all situations. On the other hand, obtaining the velocity and acceleration based on finite differences or fits, also has the inherent difficulties of the processes of volume loss, deformation and breakage that modify the mass of the drop when applying Newton's second law. In Hsiang and Faeth [43] the authors also discuss the drag coefficient, finding that it depends mainly on the deformation of the droplet, with values between the sphere and the disc. The main impact is that the combination of shape variation and the extension of its frontal area causes increases in drag force of up to 13 times that of the sphere, both in steady disturbances and shock-wave disturbances, which clearly indicates the weight of the deformation on the dynamics of the droplet.

Another very interesting study is that of Ortiz et al. [105], where the authors seek to obtain a correlation based on previous studies for liquid drops of Ohnesorge numbers from 0.002 to 44 in shock-tube type installations with shock Mach numbers between 2.0 and 4.7. Based on the dimensionless parameters of the problem the authors propose a correlation for the drag coefficient in the form $C_D = A + K_1 Oh^p We^q$, this correlation contains therefore on the one hand the liquid properties and on the other hand the deformation ratio based on the Weber number. Thus, the values of the correlation coefficients for Weber numbers in the interval from 1000 to 1.62 are $A = 1.6$, $K_1 = 0.4$, $p = 0.08$ and $q = 0.01$. This, as the authors indicate, presents a weak dependence for the lower range of the Weber number and may tend to underestimate the drag coefficient when compared to fit data from previous studies. Subsequently, Loth [106] analyze the literature data on bubble and droplet drag and deformation under quasi-steady conditions. The authors note that while one would expect a dependence on the instantaneous Weber number, the high viscosity ratio at the interface would result in conditions similar to non-slip boundary conditions. Finally, they suggest a correlation based on the dimensionless group $WeRe_p^{0.2}$ which is valid for quasi-stationary conditions in the limit of $400 < Re_p < 7000$.

Directly linked to the study of the dynamics of super-cooled droplets in flight-equivalent conditions, Luxford et al. [107] analyze previous results for droplets in free-falling air reaching terminal conditions. The authors explore a geometric interpolation between the drag coefficient of the disc and the sphere of the form $C_d = C_{d_{sphere}}^k C_{d_{Disk}}^{(1-k)}$. A drag coefficient for a droplet is represented by observing that its values lie between those of the disc and the sphere, so using different values of the parameter k allows the curve to be calibrated to fit the results of the droplet. The authors determine that the interpolation factor depends on the deformation of the droplet and analyze their result for two different deformation models, finding differences of less than 1%.

In addition to experimental studies, there are a number of numerical studies of the drag of droplets exposed to an air flow. Such numerical studies are complicated, as they arise from the interaction between the external flow and the deformable interface, thus requiring specific discretization schemes combined with unstructured and adaptive meshing. However, their number has increased due to their capability to provide clues about the physical mechanisms involved. For example, in Sirignano [108] it is considered that the internal circulation of the fluid inside the droplet has a significant weight on the aerodynamic drag, according to the results of Prakash and Sirignano [109]. In Han and Tryggvason [110], the case of a 1.15 and 10 density ratio droplet free-falling medium is examined. The most relevant conclusion is that as the density ratio increases, the droplets with Bond numbers ($Bo = \rho_d g D_d^2 / \sigma$) of $Bo > 36$ pass from breaking in the form of a frontal pocket to breaking in a shear breakup mode, showing very similar mechanisms to those with lower density ratios. Later the same authors extend the results to impulsively accelerated drops in Han and Tryggvason [111], where the authors find similar results but indicate that it is the sustained application of aerodynamic forces and the flow separation and wake formation which may determine the breakup modes.

In a subsequent numerical study Wadhwa et al. [112] examine the transient deformation and aerodynamic drag of decelerating droplets within an axisymmetric flow. A first study is performed for $We = 1, 10$ and 100 , keeping $Oh = 0.01$, which are typical numbers in aerobreakup problems, and a second study is performed at $We = 10$ and varying Oh between 0.001 and 0.1 . For the first one, the authors find that the drag increases as the droplet deforms into an oblate spheroid and that for the case of low or intermediate Weber we can find oscillations in the drag coefficient if it varies between oblate and prolate spheroids. On the other hand, higher Weber numbers imply higher deformation rates and therefore higher transfer of the momentum from the stream to the droplet. The Oh number plays an inverse role, increasing the viscous dissipation and reducing the deformation rate. This result contradicts the dependence found in [105] on the Ohnesorge number.

A study entirely devoted to the droplet falling at terminal velocity under stationary and axisymmetric conditions is that of Feng [113]. For this analysis, the author uses a Galerkin finite element method in combination with boundary-fitted quadrilateral scaling. Density ratios up to 1000 , Weber numbers of $5, 10$ and 100 and viscosity ratios between 50 and 1000

are tested. It follows that the simulation can yield prolate spheroid shapes under certain ρ_d/μ_d^2 conditions, which, as reported by Helenbrook and Edwards [114], are axisymmetrically unstable and should result in oblate spheroids as reported by experimental tests. The author links these situations to the hypothesis of stationary and axisymmetric flow that eliminates the possibility of forming Vortex shedding phenomena. Finally, for the cases studied, it is found that the drag coefficient adjusts quite well to that of a sphere, but considering the solid sphere with a radius equivalent to that of the frontal area, being invariant, therefore, to the density of the droplet.

Finally, it is worth to highlight Shao et al. [115] where the authors carry out a numerical study by performing a non-stationary simulation of a deformable droplet using the Level set (LS) method, which captures the interface with a mass conservation method proposed by the authors. The effects of the We number and the density ratio are also investigated, being the latter the most relevant according to the authors. Thus, the non-stationary coefficient is always larger than the stationary one as a consequence of the deformation of the droplet, which is in agreement with the experiments of [75]. Based on the results, the authors give a correlation as a function of the Ac parameter, which includes the Weber number from the form:

$$Ac_{Shao} = \left(\frac{\rho_d}{\rho_g} - 1 \right) \frac{\sigma We}{\rho_g V_s^4} \frac{dV_s}{dt} \quad (2.27)$$

This new acceleration parameter allows them to define a correlation equivalent to that proposed by Temkin and Mehta [98] for $Oh \leq 0.1$ and $We \leq 0.15$. Specifically; $C_D = C_{Ds} - 1.3Ac_{Shao} - 0.4$ for $0 < Ac_{Shao} \leq 2$ and $C_D = C_{D_{sphere}} - 0.7Ac_{Shao} - 1$ for $2 < Ac_{Shao} \leq 6$. Although the authors do not study the case of a $dV_s/dt > 0$, they agree with [98] that if the relative velocity decreases, $dV_s/dt < 0$ the coefficient will be less than stationary.

Specifically for flows of aeronautical interest, the model developed in Sor et al. [116] is very relevant, in which the authors extend the experimental study of [82] and seek to complete the deformation model with a model of the droplet dynamics. Thus, based on the trajectories and the deformation of the droplet, the authors describe the drag coefficient as the sum of a stationary and a non-stationary term that depends on the acceleration parameter. To account for the effects of droplet deformation, the stationary coefficient is modeled from a geometric interpolation, similar to that proposed in [107], between the two extremes of possible geometry, the sphere and the disk. The authors include the deformation of the droplet from the exponent, that is a function of the aspect ratio between the semi-major and semi-minor axis (a and b respectively) of an ellipse that fits the cross section of the droplet. The equation of the drop dynamics would take the form:

$$F_{drag_x} \approx \frac{1}{2} \rho_{air} V_{s_x}^2 \pi a^2 \left(C_{D_{sphere}}^{b/a} C_{D_{disk}}^{1-b/a} + k \frac{b}{V_{s_x}^2} \frac{d^2 V_{s_x}}{dt^2} \right) \quad (2.28)$$

Where V_{s_x} is the relative horizontal velocity between the drop and the air, $C_{D_{sphere}}$ the drag coefficient of the sphere as a function of Re and $C_{D_{disk}} = 1.17$ the drag coefficient of the disk. On the other hand, the non-stationary coefficient includes an empirical fit constant $k = 9$, which is obtained by minimizing the error on the observed droplet path. The results

of applying this model for the trajectory calculation show average discrepancies when compared to the experimental results of 3.6%. Subsequently, in Garcia-Magariño et al. [117] an attempt is made to analyze the generality of the equation 2.28. For this purpose, the same experimental setup is used but with airfoils of different geometry so the validity of the model for different velocity fields and therefore different acceleration of the flow field is tested, finding mean errors of the order of 6% and 5%.

2.2 Unsteady flow dynamics past an sphere

In Theofanous et al. [52] it was clearly stated the importance of modeling deformation of impulsive phenomena: *"We can see the quantitative role of acceleration (also tied to deformation) as the driver for breakup, and how liquid inertia and viscosity enter as retarding parameters"*. Following on from the previous section 2.1.4, it is interesting to understand the flow field and its evolution in the environment of bodies that are subjected to accelerations or decelerations. From its evolution it will be possible to understand the interaction of the fluid with the solid boundaries and to model its effects on particle and droplet dynamics.

As a starting point, much of the research has focused on the non-stationary dynamics of spherical particles, and the fact that flow past a sphere is one of the canonical problems in Fluid Mechanics. Journal articles on the subject and on its multiple variants are being published since the middle of the 20th century, not to mention earlier works that were disseminated by other means. It is somewhat difficult to attempt a classification but, broadly speaking, studies could be organized according to the Reynolds number (laminar, transitional, or turbulent), the Mach number (compressible or incompressible), the Strouhal number (steady or unsteady), the state of the sphere (moving or unmoving), and whether the sphere motion (if any) is prescribed or free. A research line that still presents many open questions involves the case when the sphere accelerates or decelerates after an impulsively started motion. One reason for this fact could be associated with the difficulties of experimental testing in such conditions. It is worth noting that while pioneering experimental work was carried out nearly 60 years ago, Odar and Hamilton [118], very accurate new experiments in the field have been published recently, see, for example, Fernando et al. [119, 120], Jetly et al. [121], Cross [122] and Galler et al. [123]. Some of these experiments are detailed in section 2.2.2.

Indeed, the non-stationary flow around the body, and in particular the evolution of its wake, is of great interest in engineering applications such as droplet and particle dynamics (see section 2.1.4), transient flow control (Lorite-Díez et al. [124]), the free motion of marine systems and structures (Terziev et al. [125]), buoys, marine moorings (Pascoal et al. [126]), and to the entry of blunt bodies into a fluid (Hasanloo et al. [127]). In general, as a consequence of the non-stationary dynamics of the body, the drag force is dominated initially by the added mass and then reaches a transitional period in which it becomes dominated by vortex growth, the origin of which is the establishment of the boundary layer and the shape of the body itself, and its subsequent detachment. These force distributions

will therefore be closely related to the near wake and vortex structures and therefore also closely related to the production of vorticity leading away from the inviscid solution. In the following section it will be reviewed how these aspects are directly related to the topology of the flow field around the body undergoing acceleration/deceleration.

2.2.1 Unsteady flow topology past a bluff body

The general boundary layer theory was originally hypothesized in 1904 by Prandtl [128], who roughly assumed that the flow around a solid is divided into two regions: an inviscid bulk region governed by the Euler equations, and a very thin region around the solid surface where viscous effects are not negligible. Although for practical purposes all boundary layers can be considered non-stationary, Prandtl overcame this situation by assuming that the changes across the boundary layer were much faster than the changes along the boundary layer. This early contribution has led to the development of an entire subject, with the most widely recognised academic text being that of Schlichting and Gersten [129].

When dealing with the establishment in time of this thin viscous region around the wall, it is usual to turn at first to the solution posed by Stokes [130] and Lord Rayleigh [131] where the solution of the flow for an incompressible fluid around a flat infinite wall that is set into motion impulsively at a constant velocity U_0 is dealt with. This problem is known as the first Stokes problem. Under these conditions, the pressure distribution around its surface is constant, and the problem can be solved from the dimensionless similarity variable $\eta_y = y/2\sqrt{\nu t}$, where y is the normal component to the wall, ν the kinematic viscosity and t the elapsed time since the start-up process. The solution takes the form $u/U_0 = 1 - \text{erf}(\eta)$, being erf the error function. This solution is relevant because it is the local general solution of the unsteady boundary layer of a body that is impulsively set in motion within a fluid at rest. Considering the above, it is possible to apply the perturbation theory to find the approximate solution of the development of the non-stationary boundary layer as a consequence of a start-up process, so that after the acceleration the body will move at constant velocity U_0 in the limit of $Re \rightarrow \infty$. These solutions take the form of the expansion power series in $t_R^* = \nu t/l_c^2$, where l_c is the characteristic length of the body. As a result of its application, we find that the solution u/U_0 behaves locally like the first Stokes problem, as we had anticipated. From this approach, the solution to the problem has been found for different geometries and number of expansion terms, more details can be found in section 4.6.2 on the impulsively accelerated sphere and in Schlichting and Gersten [129] for different geometries and conditions. In relation to the sphere, one of the first works employing such a methodology to obtain its solution in the limit of $Re \rightarrow \infty$ is that of Boltze [132], where a four-term expansion in combination with the potential solution of the sphere is employed to address the first instant at which the wall shear stresses (τ_w) become zero at the back stagnation point. This instant is expressed as the dimensionless separation time and defined as $T_s = t_s U_0/R$, which according to Boltze was 0.39.

One of the problems of applying the aforementioned series expansion is that it is only valid for Reynolds numbers $Re \rightarrow \infty$ and for the initial instants $t \rightarrow 0$. Overcoming this

limitation, years later, Wang [133] analysed the flow around a circular cylinder started impulsively from rest using the method of inner and outer expansions. The analysis focused on times short enough to ensure that the cylinder had moved a small percentage of its radius only. The inner expansion was treated by means of boundary layer-like formulation, while a stream function was used for the outer expansion. The author also uses the boundary layer equations expressed in cylindrical polar coordinates for the inner expansion, which according to the author allows to keep a curvature term that does not appear in the Blasius equations. The article points out that the advantage of applying this methodology instead of the usual boundary layer equations is that it allows the impact of the outer region to be transferred to the inner region and vice versa. Possibly, the main finding was that for $10^5 > Re > 10^3$ (Re based on the cylinder radius, R), the dimensionless separation time, defined as $T_s = t_s U_0 / R$, was 0.35, increasing as we increased the Reynolds number. As in Boltze [132], separation was defined by Wang [133] in Sec. 4 of its article as the time when zero shear appears on the body surface. Useful information is presented in Fig. 7, where the dimensionless separation time is drawn as a function of the Reynolds number.

Proceeding in a similar way, Dennis and Walker [134] solve the flow around an impulsively started sphere by extending the expansion series in the limit of $Re \rightarrow \infty$ to the eighth order term and thus improving the accuracy of [132] work. The authors employ the criterion of $\tau_w = 0$ at the rear stagnation point to characterize the first instant of separation. Their analytical solution for $Re \rightarrow \infty$ had the form of a perturbation expansion valid for the velocity components U, V and vorticity (ω), again, for short times only, giving a dimensionless separation time of $T_s = 0.396$. On the other hand, the authors solve numerically by means of Newton's method the separation instant for finite Reynolds number finding that in the range of $10^5 > Re > 10^3$, the dimensionless separation time, defined as before, varied between 0.406 and 0.499. An interesting point of the article is that in addition to the separation instant the authors obtain numerically the drag coefficient and divide it into its pressure contribution C_p and viscous stress contribution C_f , expressing its evolution in time. In addition, the evolution with time of the separation point is monitored, which is compared with the Blasius series applied on steady axylsymmetric bodies (Schlichting and Gersten [129]), finding that for the limit $Re \rightarrow \infty$ it exceeds its steady value of 109.6° at $T_s = 0.875$. Another finding of the authors is that the convergence region, defined from the dimensionless time variable $T = tU_0 / R$, decreases with Reynolds number, making that for numbers of $Re = 40$ only the time window $T = 0.05$ can be considered. Based on this work, in Dennis and Walker [135] the authors solve the same problem by applying a similar methodology but implementing a transformation of variables that saves the singularity at the initial instants. They also apply Legendre series for the streamline and vorticity functions with arguments in polar coordinates ($z = \cos(\theta)$), where θ is the polar angle. Thus, they solve the problem in the interval of $20 < Re < 1000$ and $Re \rightarrow \infty$, employing iterative Crank-Nicolson schemes. The work therefore partly replicates the methodology of their previous work [134] with one main difference, which is that the order of the terms increases as time evolves, so that the accuracy of the solution increases as we move away from the initial instant, allowing convergence in the solutions up to $T \sim 50$ for $Re = 20$. For higher Reynolds numbers the number of terms grows dramatically due to the complexity

of the flow, limiting its applicability. The authors compare it with experimental results such as Taneda [136] and other asymptotic expansion based works, validating their findings. Referenced in Collins and Dennis [137] the authors addressed the impulsively started motion of a cylinder with a similar methodology.

One of the aspects that most significantly influences the distribution of pressures around a body is the separation of the boundary layer. As has been seen in the previously mentioned works, the criterion usually used is that of $\tau_w(x_s) = 0$, where x_s is the separation point over the surface. In a very interesting introduction Telionis and Tsahalis [138] state the following: *"...above evidence indicate that in unsteady flow, the region of reversed flow may be thin and may remain within the viscous layer while the phenomenon of separation may appear further down-stream where the wall shear vanishes"*. Thus, under the assumption of a thin recirculation region, it would be possible to continue to apply the thin boundary layer approach when solving the problem. Under these conditions, the authors found that the definition of the separation point must be redefined in non-stationary flows, finding more appropriate its definition based on the appearance of a Goldstein singularity, which they assume to be *"...an actual response of the boundary layer to separation of outer flow."*. Solving by means of asymptotic expansions for the initial instants and subsequently solving the boundary layer differential equation numerically, the authors calculate the problem for the case of a cylinder with outer potential flow $U_e(s)$ in order to compare the evolution with time $\tau_w = 0$ with other works. In their case, it was found that the point of zero wall shear stress at $\theta_s = \pi$ appears for the first time for $T_s \simeq 0.35$. From that point onwards the numerical solution shows an evolution of the reverse flow region, which the authors do not consider separation, and which asymptotically leads to the stationary solution $\theta = 104.5^\circ$. On the other hand, at $T = 0.65$, the authors show the development of a singularity at $\theta = 140^\circ$ with a significant velocity magnitude and normal to the surface that evolves with time towards the point of zero shear stress. The authors conclude that this singularity is the natural response of the boundary layer to the separation.

In a later review paper, Telionis [139] discusses the laminar and turbulent boundary layer problems for sinusoidally oscillating incompressible flow conditions. One of the sections of this review paper collects the findings to date related to unsteady separation, again emphasising the shortcomings of the $\tau_w = 0$ criterion for characterizing the separation. Again, the Goldstein singularity is proposed as an indication of the separation (understood in the numerical solutions as the point at which there is a sharp growth of the normal velocity component to the surface or of the local derivatives $\partial/\partial s$, where s is the tangential component to the surface). According to the authors, the M.R.S. criterion (Moore [140], Rott [141] and Sears [142]) constitutes, according to the literature present at that time, one of the criteria that has shown the best results for determining the singularity point. One of the problems highlighted is the difficulty of experimentally characterizing the boundary layer given the thickness of the viscous region, which is why a large part of the experimental efforts to study the evolution of the separation are directed towards studying non-stationarity in impulsively accelerated flat plates. Thus they indicate that many of the questions related to the physics of unsteady separation remain open, although it was

already established that the criterion of $\tau_w = 0$ was not directly related to the separation, but it did favour the appearance of flow reversal and served as a precursor for the emergence of a localized point of separation. Focusing on the instants after the separation, most of the experimental studies establish that as a consequence of the shear layer, there is a transfer of energy towards vortical structures that generate a separation region. Finally, the conclusions are interesting in that they indicate the value of the experimental facilities for evaluating the analytical models proposed and in particular with regard to the formation of wakes with large recirculation bubbles, given their difficulty in being analytically predicted, and the value of using simple unsteady flows that depart from the classical oscillatory problem, which in the author's opinion are not applicable to engineering purposes.

A further very interesting review linked to the separation of boundary layers at high Reynolds number is that of Smith [143], where he revisits the distinction between large-scale separations and small-scale separations. On the other hand, they analyze the various descriptions of Large-Scale Eddies and its closure. These large-scale vortical structures are, according to the author, the most relevant issue for the study of boundary layers at high Reynolds numbers. In relation to unsteady separation, the gap between theory and experiments is pointed out, which in the author opinion requires a deeper understanding of linear and nonlinear instabilities and transition, in particular for $T \rightarrow \infty$, where the classical theory of unsteady boundary layers fails.

As can be seen, the classical boundary layer theory is limited by the separation between the irrotational regions and those with vorticity, $\vec{\omega} \neq 0$. One of the most relevant theoretical works that explores the production of vorticity at solid boundaries is the well-known paper of Morton [144]. In this paper the Helmholtz vorticity equation for incompressible and homogeneous fluids is analyzed:

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{u} \cdot \nabla) \vec{\omega} = (\vec{\omega} \cdot \nabla) \vec{u} + \nu \nabla^2 \vec{\omega} \quad (2.29)$$

where the second term corresponds to vorticity amplification or concentration effects, and the last one to the vorticity spread due to viscosity. However, Morton points out that equation (2.29) does not contain any vorticity generation terms, so all sources must relate to the boundaries of the problem or interior regions of inhomogeneous fluids. A number of questions are raised and answered in the article: *i) what are the boundary conditions on vorticity and why are they so generally overlooked, ii) does pressure play any role in vorticity dynamics, iii) is vorticity generated by wall shear stress, iv) what if any is the role of torque in fluid dynamics, v) what is the physical mechanism or mechanisms for the generation of vorticity at boundaries and vi) what are the mechanisms for loss of vorticity, and in particular can vorticity be lost by diffusion at boundaries?* . Regarding the first question, the author mentions that traditionally the generation of vorticity at solid boundaries has been posed as a distributed sources or sinks (Lighthill [145]) or as a consequence of solid boundary motion (Batchelor [146]), assuming that vorticity is lost by diffusion at boundaries. To clarify the remaining issues, they study a set of cases with exact solutions, see: the first Stokes problem, for which they conclude that vorticity is generated at the initial instant at its surface, and then

diffuses with a flux density:

$$-\nu \frac{\partial \eta_\omega}{\partial z} = -U_0 z (2\pi\nu)^{-1/2} t^{-3/2} e^{-z^2/2\nu t} \quad (2.30)$$

Where η_ω is the component normal to the 2D space, z is the component normal to the wall, and U_0 is the final velocity at which the plate moves. The circulation per unit of length around the curve ψ will be constant and of value $-U_0$. Another relevant case studied is that of a uniformly accelerated flat plate. Under these conditions there is a similarity solution whereby vorticity is generated at the same rate as diffusion with a vorticity rate per unit time of $-A$, being the circulation per unit length of the along ψ equal to $-At$, being A the acceleration. In addition to these cases, which are particularly interesting for our problem, it is also studied in detail the problem of a plate oscillating in its plane, plane couette-flow, plane Poiseuille flow and the Blasius boundary layer. In relation to the latter, the discussion about the production of vorticity, which must be generated locally at the singularity of the leading edge and being subsequently advected downstream to a $U_0^2/2$ ratio and diffused into a thickening boundary layer, is very relevant. It is therefore concluded that vorticity is generated instantaneously as a consequence of acceleration (either as a consequence of a pressure gradient or as a consequence of surface acceleration) and the viscosity term $\nu \nabla^2 u$ has no effect on its generation. Thus, it follows at the same time that τ_w does not produce vorticity, that vorticity does not vanish at the boundaries and that the only way for vorticity to decay is by diffusion or by annihilation of vorticity of opposite sign.

A very interesting numerical study is that of Wang and Dalton [147] where the flow around a decelerating cylinder is studied. The numerical solution is initialized from the potential solution around the cylinder on a cylindrical and symmetric mesh. A solver based on the High-Order Difference approximation with Identity Expansion (HODIE) method is used, solving the streamfunction from the evolution of the vorticity near the wall. To evaluate the method, the solution of the impulsively started cylinder for finite Reynolds numbers is used and compared with other experimental and numerical results, finding necessary to stimulate the asymmetry of the wake when compared with experimental studies. Cases of continuously decelerating spheres and steeply decelerating spheres are both simulated. When compared with the results of impulsive spheres obtained from their simulation, they determine that the drag coefficient is higher for the decelerated case than for the accelerated case. Moreover, for the accelerated case, the force changes its direction of application twice because of the wake vortex structure and its interaction with the cylinder. Finding that force tends to zero when the vortical structures diffuses away. For the case of continuous deceleration this change is not so abrupt and for deceleration parameters value of $aD^3/\nu^3 = 139$, where a is the flow acceleration, there is a growth of the wake structure that exceeds the sphere. The evolution of this wake length with $t_a = at/U$ is compared with the work of Taneda [136], finding similar results. The authors emphasize the importance of deceleration studies since as they mention: *"...an aspect of unsteady flow about a circular cylinder which apparently has escaped attention in numerical studies is a decelerated flow"*.

In relation to the wake generated by bodies in impulsive motions, the work of Lawrence and Mei [148] is particularly relevant, in which the long-time behaviour of a blunt body of arbitrary shape is studied when moving from a velocity U_1 to another U_2 . The results are referred to the characteristics of the wake in the far field, so that the authors are able to dodge the geometry of the body under the assumptions that there is no lift and therefore that the hydrodynamic force is parallel to the velocity of the incoming stream. The authors determine that for a hydrodynamic force decomposed into a transient and a stationary part, the transient part (Φ_H) is a function with a proportionality relation $\Phi_H \propto \tau^{-2}$ where $\tau = tU_2/R$, and being R the radius for the spheres case. This relation is produced by the existence of a single source generated by the impulse and advected downstream of the bluff-body by the velocity flux U_2 . The authors point out that this result is practically universal with the exception of the cases of flux reversal and impulsive stop, where the non-stationary force has an inversely proportional relationship to τ . The results present the limitation of being for moderate Reynolds. On the other hand, the authors obtain a numerical solution for the case of the sphere subjected to an impulsive change in velocity or viscosity, in agreement with the asymptotic results. The results are presented in a separated paper (Mei and Lawrence [149]) and are of particular interest since they show the flow topology and results for impulsively reversed or stopped flow. One of the main conclusions of the latter paper is that it is a complex phenomena very problem dependent and that reaching a general expression becomes challenging. Wakaba and Balachandar [150] studied numerically the sudden acceleration/deceleration of a sphere starting from a constant velocity. They found that sudden acceleration from preexisting flow, Re jumping from 50 to 62, generated a potential flow that superposes the previous flow pattern. In the case of a weak deceleration, the strength of the superposed potential flow may not be sufficient to modify the preexisting flow pattern.

In general, both analytical as well as numerical or experimental studies show the presence of vortex ring structures behind the accelerated sphere. A very interesting work on the formation of vortex rings is that of Gharib et al. [151], where the authors study their formation as a consequence of the displacement of a piston inside a tube that ejects a jet of water inside a resting fluid tank. This mass of water is ejected through a nozzle of diameter $D = 2.54 \text{ cm}$. The piston strokes are limited to a ratio $L/D = 15$, with maximum accelerations of 2.5 m/s^2 . Based on a PIV system, vorticity and flow velocity field measurements are obtained for an area of $11.23 \times 8 \text{ cm}^2$ with a spatial resolution of $0.23 \times 0.23 \text{ cm}^2$. According to the authors the vortex ring circulation (Γ) depends on the piston stroke history $L = \int_t^0 u_p(t)dt$ where $u_p(t)$ is its instantaneous piston velocity. In the same way as Morton [144] they identify the vorticity flow on the surface by $U_m/2$ where U_m is the velocity at the nozzle exit. Based on the visualizations, the authors define a value known as the formation time $\bar{U}_p t/D$ or stroke ratio $(L/D)_{lim}$ at which the vortex no longer absorbs vorticity or fluid mass, and is shown as concentrated vorticity at the trailing edge of the vortex ring. When this happens, the vortex ring is disconnected from the trailing region, which is known as pinch-off process. According to the authors, this process is not instantaneous and takes approximately $t \sim 2L/\bar{U}_p$ to complete. The maximum circulation values found in the experiments were in the range of formation numbers between 3.6 and

4.5. What is particularly interesting in this study is the existence of a formation number which, depending on its history, determines the vortical ring structure, its formation and its disconnection. Particularly interesting is a series of concluding remarks related to the ability to increase circulation after the disconnection process and to circulation reduction phenomena by ejection of vorticity into the wake (although the authors do not specify whether this is due to viscous annihilation, diffusion or other mechanisms). In a later work, Mohseni and Gharib [152] developed a prediction model of this formation number based on integral quantities of the vortex ring motion such as energy E , momentum I and circulation Γ , so that the formation of the vortex ring is the end of a relaxational process. Using numerical simulations, Mohseni et al. [153] studied the formation of vortical rings as a consequence of the application of long-term non-conservative forces in the fluid domain. In particular, normalized values of the energy, circulation and pinch-off time parameters are calculated ($\tilde{E} = E/\Gamma^{3/2}I^{1/2}$, $\tilde{\Gamma} = \Gamma/I^{1/3}U_{v2/3}$ and $t^* = t_{po}U_v/D$ respectively). Under these normalization the variability is reduced when compared to normalization based on the generation mechanism. Thus it was found that normalized energy and circulation values of 0.3 and 2.0 are obtained respectively for the pinch-off process of the main vortical ring. According to the authors, when the generation process is not able to provide energy and circulation at a rate comparable to that of the normalized circulation and energy associated with the translation of the vortical ring, this pinch-off occurs. This dynamic interpretation suggests that the pinch-off process will depend on the generation conditions and on the vortical structure itself.

Jeon and Gharib [154] extend the study of the vortex formation concept for flow around bluff-body. In particular, two experimental studies are carried out: the first one employs a towing tank facility (GALCIT X-Y), which consists of a $4.5 \times 1.0 \times 0.5$ m long, wide and deep tank, respectively, where a cylinder is dragged by means of an electric actuator, and the second one employs a water tunnel, with a square section of 45×55 cm² where a cylinder is moved following a sine law. The vorticity is studied from a PIV system. The authors conclude that for the case of the starting cylinder there are analogies with the work of [151], so that the formation time seems to signal the transition from a symmetric structure of vortices, anchored to the cylinder wake and accompanying it, and the transition to the vortex street pattern. The observations show that the separated boundary layer begins to coil into symmetric vortical structures extending into the wake. These structures fed by the vorticity and produced in the boundary layer continue to grow until the asymmetric limit is reached. The authors found that for cylinders reaching the same velocity with different acceleration ramps, the symmetry limit was $4D$. This, according to the authors, suggests that a similar saturation process occurs in the wake of the cylinder leading to an apparent shared morphology and growth rate as that of the vortex ring, but with an extra vorticity flow source from the acceleration [144]. These saturation behaviours were also found for plates starting from rest by Ringuette et al. [155]. In this paper the impact of tip vortexes on the unsteady force-generation of low-aspect-ratio plates was studied. According to their results, the peak drag force corresponds to the interaction between the tip and the edge vortices. The structures are similar to those found in previous studies, including the pinch-off mechanisms, although they found lower formation times than in previous studies.

In a very interesting experimental setup, Sattari et al. [156] studied the formation of vortexes as a consequence of shear layers at the outlet of a 0.15 m thick flat plate. A dielectric barrier discharge plasma actuator is used for the formation of the incident flat jet on the plate, so that the particles are accelerated by the electromagnetic field close to the wall favoring vortex formation. PIV displays are used in regions of interest (ROI), one of $20 \times 12 \text{ mm}^2$ at the exit and another, that includes the plasma actuator, of $80 \times 50 \text{ mm}^2$. Control of the shear layer is thus actuator-dependent. Of particular interest are the correlation techniques applied and the filters used to study the vortex core. The authors find that the vortex growth does not seem to be related to inviscid entrainment, but mainly to the vorticity mass flow at the shear layer. As in previous works, they found a limit, related to the competition of the shear layer to remain aligned with the streamlines and the vortex induced shear layer strain.

Another experiment related to vortical structures generated by square discs and plates uniformly accelerated from rest is that of Fernando and Rival [157]. Tests are developed in a towing tank facility where they make use of force measurements, PIV and dye visualizations to determine the impact of geometry on the formation of vortical structures. The authors found that for plate aspect ratios ($AR = a/b$) greater than one, an unstable elliptical vortex ring is generated behind the plate, while for $AR = 1$ a stable, plate-attached vortical structure is instead formed. As a result, it appears that the parameter that dominates the instantaneous drag forces and the initial growth of the vortical structure, and subsequent pinch-off, is the aspect ratio. Thus, both the hydraulic diameter and the perimeter (which feeds the shear layer) determine the evolution of the vortical structure and therefore its progression towards steady state drag. In the case of circular plates of $AR = 1$ the vortical structure remains close to the plate and therefore has a significant impact on the drag. Not so in the case of the rectangular plate where it convects away from the edge. Thus the singular result is that the vortex rings formed in the wake show two behaviours: a first one related with the vortex-growth stage where a steady structure is formed (which is linked to the drag reduction of the plate); and a last one related with the loss of coherence by diffusion, which translates into an increase of drag.

Finally, it is worth highlighting the work by Lorite-Díez et al. [158], which shows the applicability of this type of basic studies. In this work, the structure of the wake of D-shaped bluff bodies with three different rear geometries that are accelerated in a fluid at rest is studied. In particular, numerical and experimental studies are performed in towing tank facilities for different constant normalized acceleration profiles from 0.08 to 1 until a stationary Reynolds number of $Re = 2000$ is reached. The results divide the non-stationary problem into three phases: a first phase in which, as a consequence of the acceleration, the primary eddies accumulate in the rear region of the body, increasing the length of the recirculation region and its circulation magnitude; a second phase in which the recirculation length decreases, as a consequence of the interaction of the shear layers; and finally a last phase characterized by the destabilization of the structure, which begins to alternate the vortex shedding. Thus, the higher the acceleration, the faster the transition between phases and

the higher the vorticity of the primary eddies residing near the rear base for longer periods of time. Finally, by means of DNS, the shape of the body is optimized achieving a reduction of the energy consumed in the acceleration of around 85% compared to the original bluff body, with applications in terms of control and stability.

It is clear from the above that the importance of experiments and numerical simulations for bluff bodies lies in the fact that the wake region structure for stationary and non-stationary problems is largely unknown, and that the problem of wake instability at moderate Reynolds is also a priority. Particularly focusing on the impact of the flow field on the non-stationary dynamics of the spheres, improvements in experimental and numerical techniques have been applied to relate this flow field topology to the measured hydrodynamic forces, and although Telionis and Tsahalis [138] stated that: *If the computing machines continue to develop at the present rate, it will be soon possible to solve numerically the complete mathematical models and all approximate solutions will then be of little value*, so experiments become necessary in order to evaluate the numerical simulations and to understand the underlying physics of these complex fluid mechanics problems. The following section aims to summarize the most relevant studies in this respect.

2.2.2 Experimental studies for unsteady sphere's dynamics

Regarding experimental approaches, one of first published studies is the one by Odar and Hamilton [118]. Their test rig involved a reciprocating mechanism that imparted an oscillatory rectilinear motion to a sphere. The working fluid was oil, and forces exerted on the sphere were measured using transducers and strain gauges. The maximum Re was 62. This could be one of the first journal articles in which the acceleration number was defined and used in the analysis. The main conclusion of the study was that the added mass and history terms in the sphere dynamic equation depended on the acceleration number but not on the Reynolds number. The accelerating motion of a vertical falling sphere was studied by Ferreira and Chhabra [159]. This analytical work was limited to the case in which added mass and Basset forces were negligible (sphere density much larger than fluid density). In the end, the authors generalized their results in the form of a C_D (drag coefficient) relation to be included in the sphere dynamics equation of motion. Kendoush et al. [160] performed an experimental analysis of the added mass of two spheres accelerating in a still fluid with Re varying from 10 to 10^3 . Their experimental facility involved a tube 2.0 m long and 0.3 m in diameter where spheres (diameters ranging from 0.0094 to 0.015 m) were allowed to accelerate in a controlled way by using a pulley and a drive weight mechanism. In the case of a single sphere (the reference case), the measured added mass coefficient agreed well with theoretical analysis. However, a small deviation, possibly caused by the onset of a small separation zone, was detected as Re increased. The authors estimated that, for these conditions, the history force was much smaller, by several orders of magnitude, than the added mass force.

Concerning lateral instabilities of accelerating spheres in free fall, it is worth referring to the works of Viets [161] and Horowitz and Williamson [162] (later expanded to a larger

Reynolds number range by the same authors, Horowitz and Williamson [163]). Viets [161] conducted experiments in a $2.4 \times 0.3 \times 0.3 \text{ m}^3$ water tower. The author concentrated on analyzing fluctuating lift during free-fall acceleration. The heavier-than-water spheres were table tennis balls filled with different materials (like in the present manuscript). Re associated with the terminal velocity varied between 10^3 and 10^4 . Figures 3 and 4, [161] where a schematics of vortex shedding was presented, showed what it is now called a “vortex ring” topology, however non-axisymmetric. In the article, the authors called this structure a “toroidal shape wake”. Because of its relevance to the present manuscript, it is appropriate to quote what the author writes in the conclusions section: “another interesting of the result is the rapid decrease in vertical velocity after the vortex wake has been shed. This probably occurs because the sphere must build a new recirculating wake to replace the one which was shed. To accomplish this, some fluid must be accelerated so that it travels with the sphere. The energy required is obtained from the kinetic energy of the falling sphere, and thus, its vertical velocity decreases, corresponding to an increased drag”. Horowitz and Williamson [162] considered the dynamics of spheres and their vortex wakes as they rise or fall freely through a fluid. Their experiments were performed in the Re number range from 400 to 500. They found that there is a solid-to-fluid density threshold $m^* = 0.36$, above which the spheres do not vibrate. Below that value, the spheres underwent large in-plane oscillations. In addition, for some particular cases, the authors identified a novel four-vortex ring per cycle wake topology. The experiments were performed in a glass water tank whose dimensions were $0.2 \times 0.2 \times 0.9 \text{ m}^3$. In a subsequent work, Horowitz and Williamson [163] expanded the Re number range from 100 to 30000 where vibrating sphere motion was identified for all m^* above Re 500. Other recent articles related to the issue of sphere drag are those of Chae et al. [164] (new control device to reduce sphere drag), Chen and Li [165] (sphere drag coefficient in cylindrical pipes), Duan et al. [166] (fluid–particle drag in gas-solid suspensions), and Kamoliddinov et al. [167] (drag of horizontally pulled floating spheres).

Regarding the relation between acceleration and drag, Fernando et al. [119] have studied the time-dependent drag forces and wake topology of spheres (0.23 m in diameter) undergoing a prescribed accelerating trajectory. The experiments were performed in a horizontal $1 \times 1 \times 15 \text{ m}^3$ water towing-tank. The spheres were towed mechanically and, therefore, their velocity time profiles could be controlled. The authors found that the unsteady separation line in the sphere wake strongly affects vortex ring formation and growth, and, therefore, body drag. The authors studied, also, how acceleration influences the pressure gradient field around the sphere, and how flow separation position is related to the onset of the adverse pressure gradient. In a related article, Fernando et al. [120] explored whether inviscid added-mass theory may account for the force on an accelerated body with separated wake flow. In this case, the authors used a flat disk instead of a sphere (to make the problem independent of the location of the separation point) and concluded that the inviscid added-mass formulation is not adequate to predict time-dependent forces in flow past an accelerating body. In a third article by the same research group, Galler et al. [123] a new model (they called it “energized mass”), was proposed to predict time-dependent forces in flow past an accelerating body. In a sense, the authors presented an alternative added-mass model (that is formally valid for potential flows, only) for the case of separated

flows. Comparison between the new model and experimental results was reasonably successful, which suggest the convenience of further testing of the energized mass concept. Jetly et al. [121] have studied how vapor layers on the surface of a heated sphere in free fall (Leidenfrost effect) may decrease sphere drag significantly. Their experiments were performed in two tower tanks 2.0 and 3.0 *m* tall, respectively. The authors found drag coefficients as low as 0.04 in these conditions. Cross [122] studied a variation of the canonical problem. In particular, the author released spheres in the air above a water tank at different heights and studied, among others, cavity formation in the range of *Re* from 2×10^4 to 2.5×10^5 .

2.3 Droplet breakup in aeronautical applications

Droplet breakup as a consequence of airflow is of great interest not only for industrial applications but also in the field of aeronautics. The natural presence of water droplets in the atmosphere and their interaction with Aircraft or Spacecraft surfaces has a significant impact on both aircraft control and aircraft structures. The latter is particularly relevant in aircraft re-entry or supersonic flight, Reinecke et al. [168]. When a supersonic flying vehicle crosses a region in the atmosphere filled with droplets, these droplets impact against its surface with an energy of impact $E_{imp} \propto 2/3\pi R_d^3 U_A^2$, where U_A is the speed of the aircraft, causing damage and erosion Hobbs et al. [169]. However, as a consequence of the air flow and according to what was previously exposed in the section 2.1.2, these droplets can break up so that the presence of the flowing field reduces the damage by reducing the volume of the droplet. On the other hand, when the droplets are atomized in sufficiently small sizes (of the order of 1 μm) they can follow the streamlines avoiding the impact on their surface, Ranger and Nicholls [39]. According to the experimental observations of Waldman et al. [170] on the stagnation region of a blunt-nosed body and on its conical region far downstream of the nose installed inside shock tube facilities at Mach numbers between 3 and 11, most of the droplets would dissipate in the shock layers and would be carried downstream without reaching the surface. Therefore, this is one of the most important protection mechanisms for aircraft surfaces against droplet impact. However, this is highly dependent on the flight conditions and the configuration of the vehicle, so understanding the breakup mode of the different droplets according to the authors is particularly relevant. The application of composite materials in aircraft structures and aircraft engines, while generating little damage, can lead to significant erosion. A very interesting review of the research related to the erosion due to droplet impact phenomena and the different techniques used to simulate its impact on the aeronautical industry can be found in Gohardani [171].

In addition, droplets significantly affect the aerodynamic performance of aircraft. A systematic review of their degradation as a consequence of rainfall can be found in Cao et al. [172]. As a general rule, degradation can be attributed mainly to two reasons: Firstly under heavy rain conditions the impact of many drops per unit time on the front of the wings and the front of the fuselage generates a torque applied to the aircraft's centre of mass. Although the droplets are small in size, the total mass of the droplets coupled with the

distance from the point of impact to the centre of mass can generate a large torque on the aircraft. This causes the nose to pitch down by changing the centre of pressures of the aerodynamic surfaces. On the other hand, the accumulation of droplets on the wing surface can generate a non-homogeneous water film which increases the mass of the wing and leads to a roughening of its surface. As a consequence, a premature boundary layer transition can also occur, which contributes to the degradation of aerodynamic performance. In addition, a deceleration of the boundary layer can also be generated, which favors premature detachment and airfoil stall. This film is strongly influenced by the wettability of the surface, so that the smoother and wettable the surface, the less degradation of the aerodynamic performance will be found.

In relation to the impact that rain droplets has on airfoils, the experimental study by Hansman and Craig [173] in wind tunnel facilities to estimate the degradation of the aerodynamic performance of Wortmann airfoils FX67-K170, NACA 0012, and NACA 64-210 in severe rain conditions has shown significant degradations. In particular, they were exposed to Reynolds numbers of 3.1×10^5 and rain intensities of 1000 mm/h with droplet diameters between 0.1 and 0.9 mm . Two significant effects were found, firstly a premature transition was generated near the leading edge, generating a degradation for all the airfoils tested, finding a degradation at low angles of attack of 25% for the FX67-K170, 15% for the NACA 0012 and 5% for the NACA 64-210. In the case of the latter, a surprising result was found for high angles of attack where an increase in lift was generated, which the authors attributed to a leading-edge phenomenon that is causing a performance enhancement. In addition, with deflected slats, this water film can plug the gap between the slat and the main section leading to premature separation of the boundary layer. These results are extended in Benyin et al. [174] with numerical simulation on the DHC-6 Twin Otter wing and horizontal tailplane, showing premature boundary layer detachments in the tail section for heavy rain conditions.

Another atmospheric factor that most significantly influences the flight characteristics of aircraft is icing, a detailed description of it can be found in Cao et al. [175]. Of recent and growing interest are the problems associated with the appearance of icing due to the presence of large, super-cooled droplets known as SLD (Super-cooled Large Droplets). These droplets are larger than the size contemplated in Appendix C of JAR/FAR 25 regulations and have been the cause of accidents such as the well-known accident of the ATR 72 at Roselawn in 1994. In general, one of the most important adverse phenomena associated with ice formation is the anticipation of the dynamic stall phenomenon, Thomas et al. [176], which is the development of massive boundary layer detachments that drastically decrease lift. Thus, according to Cao et al. [175] the presence of SLD has a major impact on the growth of runback and/or ridge ice accretions behind ice protection systems. These types of accretions pose a great threat due to the potential loss of aerodynamic performance on stabilizing and control surfaces, figure 2.7. The authors found results with lift losses of up to 80%. In addition to the above, icing processes are complex and depend on both flight conditions and droplet characteristics. Another difficulty encountered is that experimental wind tunnel facilities for icing are complex and most research is carried

out using simulated-ice-shape models placed in wind tunnels or flight tests, from which to obtain the ice severity parameter (η_{ice}). This is why the use of numerical simulations has increased significantly as a way to address these problems. However, this requires solving equations to characterize the dynamics and the droplet population, which again shows the interest in studying reduced order models to include breakup phenomena and droplet unsteady dynamics. Derived directly from the investigation of an airplane crash of flight 3272, involving an Embraer EMB-120T model, Reehorst et al. [177] studied the ice accretion on airfoils equal to those of the aircraft: a NACA 23012 and a NACA 23018 airfoil, corresponding to the tip and root respectively, inside the IRT (Ice Research Tunnel). The degradation was of the order of 60% compared to that of the clean airfoil. Based on the results, the interaction between SLDs and the formation of ice structures is complex and requires further study. Furthermore, in relation to the recommendations for aircraft operation, the authors state that even a small amount of ice can compromise the aircraft's safety. To characterize the aircraft safety in icing conditions, Gurbacki and Bragg [178] developed an icing-effects model that is applied individually to each of the aircraft's flight characteristics (performance, stability, or control derivative). The model is expressed from the equation:

$$C_{(A)_{iced}} = \left(1 + \eta_{ice}\kappa'_{C_A}\right) C_{(A)} \quad (2.31)$$

Where $C_{(A)_{iced}}$ is the aircraft aerodynamic derivatives modified by ice growth, $C_{(A)}$ is its value under ordinary conditions and κ'_{C_A} is the coefficient icing factor constant. This model, as Lampton and Valasek [179] point out, is still being refined and many variables are unknown, but its simplicity makes it very easily integrable into software packages for studying the dynamic response of aircraft to icing. Thus they studied such response in light aircraft finding for the aircraft flying at 600 *m*, a reduction of the rate of climb of 15 *m/min* with an increase of the angle of attack of 0.7 *deg* for the case of 100% iced configuration is found. For the case of distributed icing, 70% iced wings and 40% iced tail a reduction of the rate of climb of 9 *m/min* with an increase of the angle of attack of 0.6 *deg* is found, relative to 60 *m/min* in clean conditions. Thus the authors' simulations for these conditions have shown that the differences in the values of the aerodynamic derivatives affecting the control of the light aircraft can vary between 5% and 25% when icing is present.

Authors such as Tan and Papadakis [180], state the need to understand in greater depth the increase in drag due to the droplet deformation, as well as its break-up, holding this lack of basic knowledge responsible for the lack of prediction accuracy of the icing codes. These ideas are supported by other authors, Hospers and Hoeijmakers [181], who confirm the importance of these phenomena in the distribution of water when droplets reach the airfoil. The interest in SLD dynamics and the resulting ice accretion has led to a great deal of research related to droplet deformation and breakup ([182], [183], [63], [64], [62], [116], [82]), on the droplet strike and splash on surfaces and the freezing of surfaces. The aim is to better understand the fundamental processes of droplet dynamics, deformation, breakup and subsequent ice formation under continuously accelerated flow conditions, which will help to improve aerodynamic surface designs and to manufacture better coatings that mitigate the problem of aerodynamic efficiency loss.

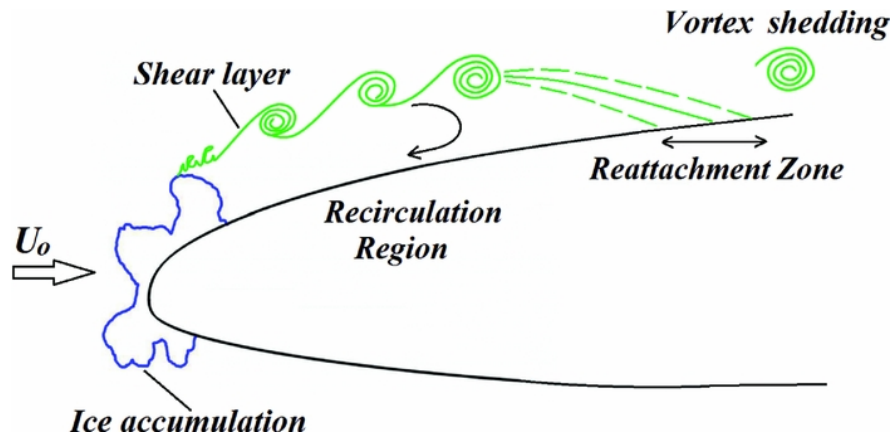


Figure 2.7: Sketch of the recirculation bubble production mechanism as a result of ice accretion horn growth at the leading edge of an airfoil. The reattachment point moves towards the trailing edge as the angle of attack increases. Figure from [184]M.D. Manshadi and M.K. Esfeh. 'Experimental investigation of flowfield over an iced aerofoil'. In: *The Aeronautical Journal* 120.1227 (2016), 735–756. ISSN: 2059-6464. DOI: [10.1017/aer.2016.29](https://doi.org/10.1017/aer.2016.29), reprinted with permissions from Cambridge University Press

Chapter 3

Breakup criterion for droplets exposed to the unsteady flow of an incoming aerodynamic surface

3.1 Introduction

The study of droplet deformation and breakup is a subject of interest both from the point of view of fundamental fluid mechanics and at the engineering application level due to the large number of processes in which it is involved. Thus, as can be seen in the section 2.1 great contributions have been made at the experimental, numerical and theoretical levels in relation to this subject, and there is a growing interest in the basic physics modeling of droplet related phenomena. This methodology, based on the characterization of phenomenological models resulting from experimental studies has demonstrated its relevance for industrial design and manufacturing purpose and its performance given its affordable computational cost. In relation to the processes of droplet deformation and breakup in flows of aeronautical concern, it is relevant the technical report Tan et al. [185] where the effect on the aerodynamic forces of droplets near the leading edge of airfoils is studied in a numerical approach. A Fluent simulation relied on the TAB model to study the deformation and breakage of the droplets. One of the conclusions of the study was the lack of experimental information for problems with similar velocity gradients.

In order to carry out a systematic study of the deformation and breakup of droplets approaching the leading edge of an airfoil, a collaboration framework was created between the National Aeronautics and Space Administration (NASA) and the "Instituto Nacional de Técnica Aeroespacial" (INTA). Subsequently, the study was extended on the basis of the collaboration between the "Escuela Técnica Superior de Ingeniería Aeronáutica y del Espacio" (ETSIAE) and INTA through the DFLOW project, which aims to obtain general models for the deformation and breakup of droplets in aeronautical flows on the basis of experimental and theoretical studies. As a result of both collaborative frameworks, a large

number of contributions have been made in the field [182], [183], [63], [64], [62], [116], [82] and [117] among others. The fact is that droplet breakup in the airfoil vicinity significantly degrades its performance in severe weather conditions, as shown in the literature review by Cao et al. [186], i.e the experiments of Hansman and Craig [187] shown a degradation of 15% in the aerodynamic performance of a NACA 64-210 airfoil in rain conditions with a water content of $30 \text{ gr}/\text{m}^3$. The impact of droplet deformation and breakup on aerodynamic performance is discussed briefly in section 2.3.

On the other hand, as was shown in the section 2.1.2, there are two canonical problems, namely the study of droplet breakup and deformation in air flows as a consequence of free fall and in conditions of constant velocity flow due to the passage of a shock wave. The literature has shown that the deformation history of the droplet and its dynamics significantly affect the breakup dynamics of the droplet. The developed models suffer from a certain lack of generality so that flows for different applications require particular studies. In the context of droplets approaching the airfoil close to stagnation line, we find that from the point of view of the droplet, the flow acquires a continuous and significant acceleration, which differs from the cases usually studied for industrial applications. Then, it is of interest to develop models that depending on the governing parameters, allow for the prediction of droplet trajectories and eventual breakup of droplets in the vicinity of airfoils. Its study, however, requires the use of a facility specifically designed for the problem in question, and it is in this NASA-INTA collaboration that the development of the rotating arm facility arose (Vargas and Feo [182], Feo et al. [183], Veras-Alba et al. [188]). The studies developed in this facility have focused on subsonic flight conditions and droplet sizes linked to heavy rainfall in situations that compromise flight safety (typical of intermediate flight regimes between landing/take-off and cruise) in which the ambient water droplets are cooled to temperatures in the order of -10 C , termed Supercooled Large Droplets (SLD).

As a result of this facility, it has been found that under the conditions of continuously accelerated flow, the droplet shows deformations above those expected in Shock-Tube type facilities and even its specific breakup modes. The analysis of the results has made it possible to develop a model of the droplet deformation, [82], of the droplet trajectory [116], and a semi-empirical breakup criterion for such flows of Aeronautical interest [62]. Focusing on the breakup phenomenon, it has been found that for the test conditions the droplets mainly breakup in the bag and multimode regimes. Thus, the aforementioned criterion is based on the analysis of the ratio between the characteristic time of deformation and the characteristic time of variation of the flow velocity, thus forecasting the onset of breakage with errors of $\pm 10\%$.

The present study is based on the developed models and experimental observations to formulate a break-up criterion based on the application linear stability analysis of hydrodynamic instabilities on its surface, previously found in Shock-Tube type installations. Taking into account the observations indicated in Garcia-Magariño et al. [62] a breakup

criterion based on RTI is proposed, considering the criticality limits exposed and the failure modes linked to it. In this regard, the proposed approach is based on the work of Joseph et al. [29] where from the RTI analysis the stability limit is found on the basis of the acceleration of the light phase over the dense phase considering a droplet-bounded moving reference frame (Chandrasekhar [15] and Leal [189]). Two important issues that have to do with the approaches aforementioned to be described in the following sections are: a) in their study, [29] did not account for droplet global deformation, and b) they choose the limiting instability wavelength as the droplet initial diameter. Both assumptions differ significantly from our case where the droplet is exposed to a continuously increasing slip velocity and significant deformations prior to breakage. On the other hand, other authors have found similar results in the RTI analysis of the bag and multimode rupture process i.e. [32], [57]. The final goal of this work is, once the breakup criterion has been defined, to couple it to the model previously developed for the deformation and droplet dynamics exposed to a flow in the surroundings of the stagnation line of an incoming airfoil.

The chapter is divided as follows: firstly, in section 3.2 the experimental setup is described, including the droplet generator and data acquisition systems. Then the test procedure is specified in section 3.3, showing the droplet ranges analyzed, the data analysis tools and the characterization of the base flow field are both included. The section 3.4 describes the droplet deformation and trajectory models on which the breakup criterion will be coupled. In section 3.5 the derivation of the proposed criterion is presented and its definite expression defined. The next section 3.6 provides a series of representative cases on which the proposed breakup criterion is evaluated, followed by an analysis of the results. Subsequently, an application case is presented for a case of aeronautical concern, section 3.7. Finally, the section 3.8 presents the conclusions of the overall study.

3.2 Description of the experimental setup

The experimental work of this study was carried out in the rotating arm installation that belongs to the experimental aerodynamics department of the INTA. The experimental setup consists mainly in four fundamental elements; the rotating arm device, the aerodynamic model installed on it, the droplet generator system and the image acquisition system. A sketch of the experimental setup is shown in figure 3.1. The set-up is controlled from a control room protected by an enclosure of anti-projection glass, adjacent to the room in the centre of which the rotating arm is placed. The figure 3.2 shows an overview of the rotating arm facility. This section is divided into detailed descriptions of the rotating arm facility, the aerodynamic models used (section 3.2.2), a description of the system used for droplet generation (section 3.2.3) and a description of the deformation and breakage acquisition system (section 3.2.4).

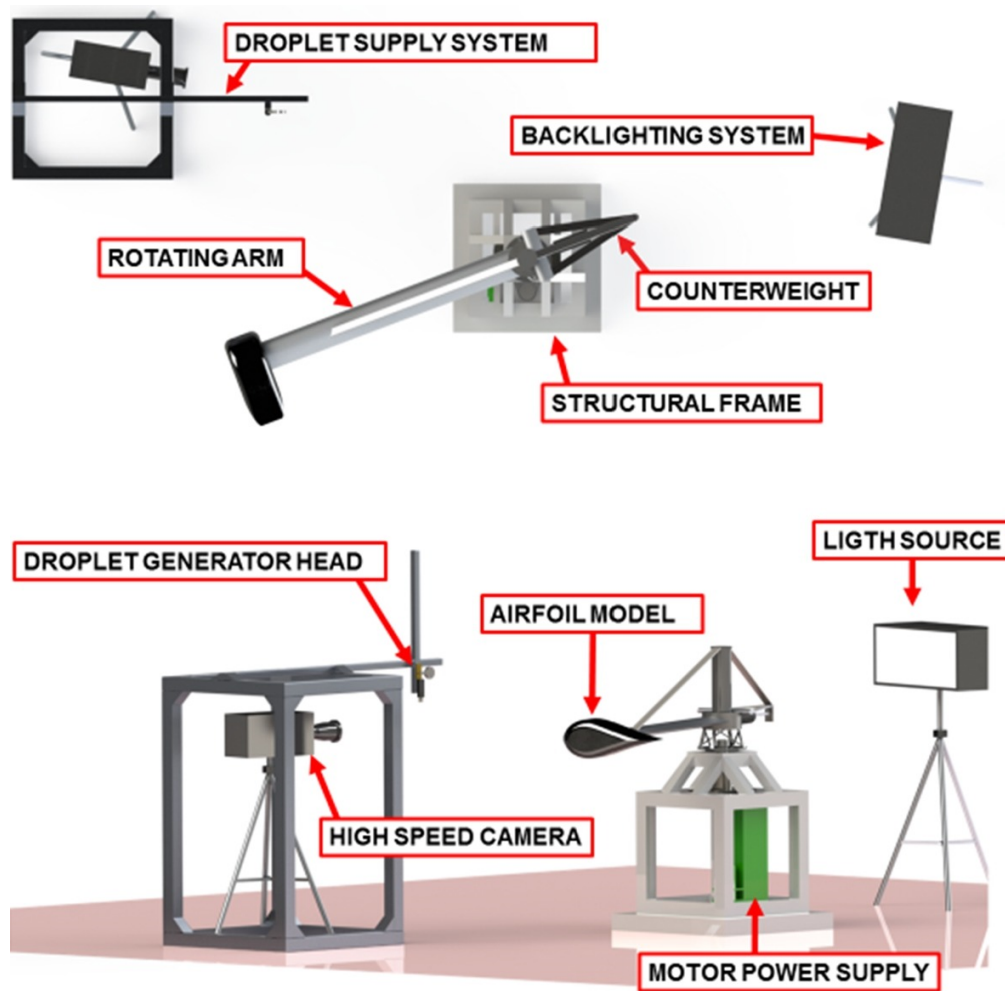


Figure 3.1: Sketch of the rotating arm facility. Top: view from above. Bottom: lateral view

3.2.1 Rotating arm facility

The rotating arm device consists of a supporting frame, an anchor base, the motor assembly and the arm-counterweight assembly. In relation to the support structure (figure 3.3b), this is made up of a welded set of hollow steel tubes with a square section of 86 mm. The structure is made up of two differentiated parts; a first 0.86 m cubic structure where the entire motor, transmission, power and control unit is located, and another upper trapezoidal structure with a height of 0.28 m, which supports the rotation axis through a set of bearings and which includes a 12 slip ring with channels to supply power and transmit measurement signals from the instrumentation that is attached to the arm. The structure is welded at the bottom to a steel plate anchored to a concrete block by means of four damping supports that absorb vibrations and support the resistive torque in the event of an emergency stop, constituting its anchoring base. These vibrations are monitored by accelerometers which, in the event of exceeding a threshold value, cut off the power supply to the motor and activate an emergency brake, stopping the rotation.

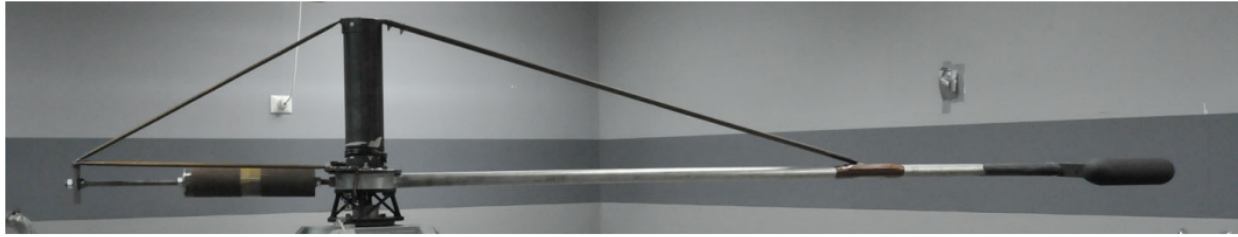
CHAPTER 3. BREAKUP CRITERION FOR DROPLETS EXPOSED TO THE UNSTEADY FLOW OF AN INCOMING AERODYNAMIC SURFACE

The arm-counterweight assembly is aimed at carrying the test model on its end, increasing its tangential velocity (U_m) to 90 m/s , which for the ambient conditions is equivalent to 0.26 Mach , with the smallest wake possible. A current image of the arm is shown in the figure 3.3a. This wake is a consequence of the passage of the model during its previous revolutions and its intensity has been studied through PIV (section 3.3.2). Thanks to the setup modifications [183] it has been reduced to the point where it can be considered negligible, with values below $0.05 \cdot U_m$ at a distance equivalent to the chord (c) from the test models leading edge. The assembly is connected to the driven shaft through a cylindrical steel matrix element which forms its centre of rotation. The arm is made of aluminium with a section in the form of a slender, symmetrical airfoil with a chord of 0.09 m . The arm has an extension of $R_{arm} = 2.16\text{ m}$ in length with respect to its axis of rotation, at the end of which the aerodynamic models to be tested are fixed, and another end consisting of a counterbalance system, whose distribution and position can be modified, with the function of balancing the rotating assembly. As a consequence of the high rotation speeds of more than 90 m/s , the arm is stiffened by means of struts to avoid vibrations at the ends and to stiffen the assembly.

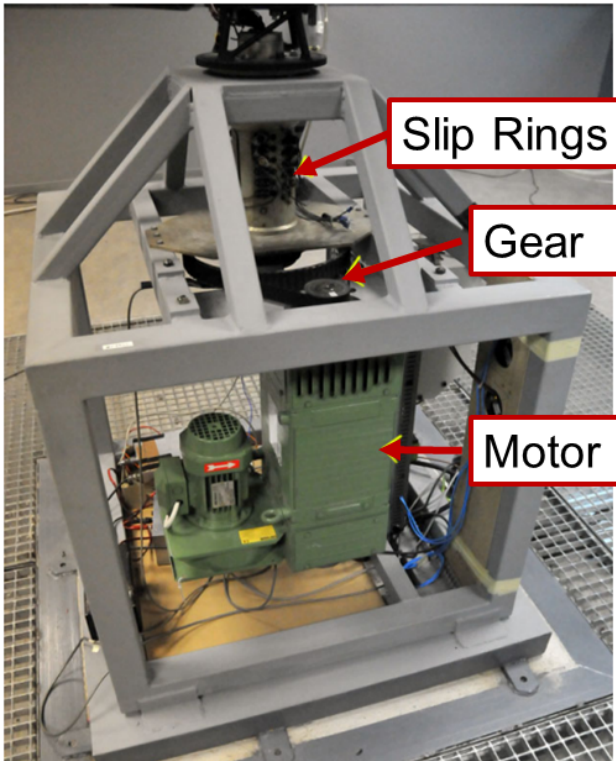
In order to rotate the arm at the speed required for the tests, a 5.0 kW DC motor is connected on its output shaft to a toothed belt transmission system that acts as a reduction gear. The motor-transmission assembly is capable of rotating the driven shaft at a maximum speed of 400 rpm when the largest airfoil is installed at the end of the arm. The control system is able to control its speed from the angular position measured by an optical encoder, from the derivative of which we obtain the angular speed at the output of the driven shaft ω_e in rad/s . Thus, the tangential velocity used as the reference at which the aerodynamic model moves for the tests, U_m , is obtained directly from the product of $\omega_e \cdot R_{arm}$, with an uncertainty of $\pm 1\text{ m/s}$.



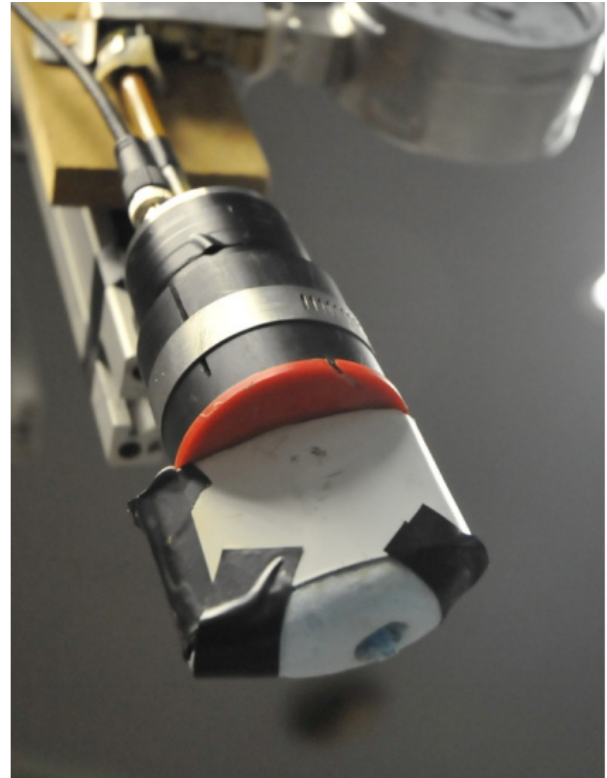
Figure 3.2: General view of the rotating arm facility, at the left of image control room can be seen



(a) Rotating arm detail



(b) Support structure detail



(c) Droplet generator detail

Figure 3.3: (a) Detail of the arm-counterweight assembly with stiffening structure consisting of struts and model installed at its end. (b) Detail of the support structure, including transmission and supports of the carried shaft. (c) Detail of the droplet generator including aerodynamic shape to reduce the interaction of the air flow in the room with the generated droplets

3.2.2 Airfoil models

Regarding the aerodynamic models, these are made of Styrofoam, a low-density material chosen to reduce centrifugal forces on the structure. The airfoil is manufactured using a hot wire cutter and is reinforced at the anchoring points with light alloys. Another region is reinforced at the leading edge, to which a metal sheet is added to prevent that the repeated impacts of the droplets against the airfoil could end up degrading its shape. Specifically, two self-similar symmetric bluff body airfoil models have been analysed based on the

design described in Vargas and Feo [182] of the DBKUP 002 aerodynamic model. The models, referred to as M1, M2 and M3 have a leading edge radius, chord, and thickness of 0.103 *m*, 0.690 *m* and 0.276 *m* respectively for model M1, 0.070 *m*, 0.468 *m* and 0.187 *m* for model M2 and 0.030 *m*, 0.199 *m* and 0.080 *m* for model M3, table 3.1. The dimensionless coordinates from the chord are given in the table 3.2 and a view of the airfoil's upper half cross-section, with dimensionless coordinates is shown in figure 3.4. The models have a span of 0.2 *m* and are closed at their ends by the revolution around the chord line of the described curve, this decreases the trailing edge effects so that the flow around the profile is mainly two-dimensional. Finally, the profile is painted matt black to reduce the effect of reflections of the lighting systems on its surface.

Table 3.1: Summary of the main geometric of the airfoil Models

Model	Chord c	Leading edge radius (R_c)	Thickness (T_c)
M1	0.690 <i>m</i>	0.103 <i>m</i>	0.276 <i>m</i>
M2	0.468 <i>m</i>	0.070 <i>m</i>	0.187 <i>m</i>
M3	0.199 <i>m</i>	0.030 <i>m</i>	0.080 <i>m</i>

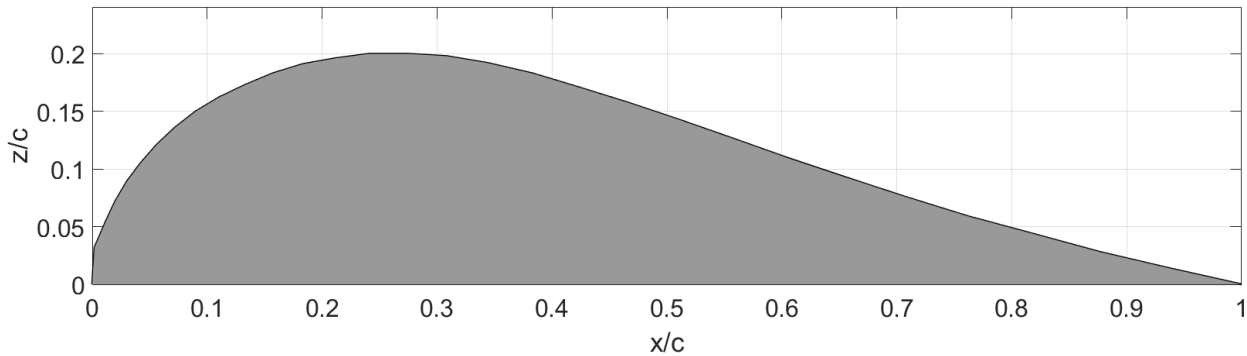


Figure 3.4: Upper half of the airfoil cross-section shape in dimensionless coordinates

3.2.3 Droplet generation system

One of the most critical points of the test is the generation of spherical droplets, with prescribed and repeatable volumes for each case. For this purpose, a TSI MDG-10 mono-dispersed droplet generator is used, which is installed on an aluminium frame structure that raises it above the plane of rotation of the arm, at a distance h_d . See the detail of the element in the figure 3.3c. The system generates a set of droplets from a liquid jet that exits from a nozzle into the ambient gas. To form this liquid jet the kinetic energy of the liquid flow must be greater than the surface energy derived from the surface tension, i.e. $We_l = \rho_a d_j U_j^2 / \sigma > 2$ where d_j is the diameter of the nozzle and U_j is the jet exit velocity. The breaking principle is based on the generation of capillary instabilities in the liquid

Table 3.2: Airfoil dimensionless coordinates

x/c	x/c	x/c	z/c
0.000	0.000	0.274	0.200
0.002	0.032	0.308	0.198
0.006	0.041	0.344	0.192
0.012	0.055	0.383	0.183
0.020	0.072	0.423	0.171
0.030	0.089	0.465	0.158
0.042	0.105	0.510	0.143
0.056	0.121	0.556	0.127
0.072	0.136	0.604	0.110
0.090	0.150	0.655	0.093
0.110	0.162	0.707	0.076
0.133	0.173	0.762	0.059
0.157	0.183	0.818	0.044
0.183	0.191	0.877	0.028
0.211	0.196	0.937	0.014
0.241	0.200	1.000	0.000

jet by means of periodic perturbations through a piezoelectric transducer that contracts and expands the nozzle following a sinusoidal voltage. The figure 3.5 shows a schematic of how the droplet is generated. The principle of breakup is based on the seminal work of Lord Rayleigh [190] in the inviscid case, which was later extended by Weber [191] and Chandrasekhar [15] by considering the viscosity in the limit of linear analysis. In this linear, inviscid limit, Lord Rayleigh states that the perturbation is unstable when the wavelength exceeds its circumference and is therefore determined exclusively by the radius of the jet as $\lambda_{max} = \sqrt{2\pi}d_j$. Rayleigh argues that it is the most unstable mode that breaks the droplet, which considering the case of viscous flows is equivalent to:

$$\lambda_{max}^2 = 2\pi^2 d_j^2 \left(1 + \frac{3\mu_d}{\sqrt{\rho_d d_j \sigma}} \right) \quad (3.1)$$

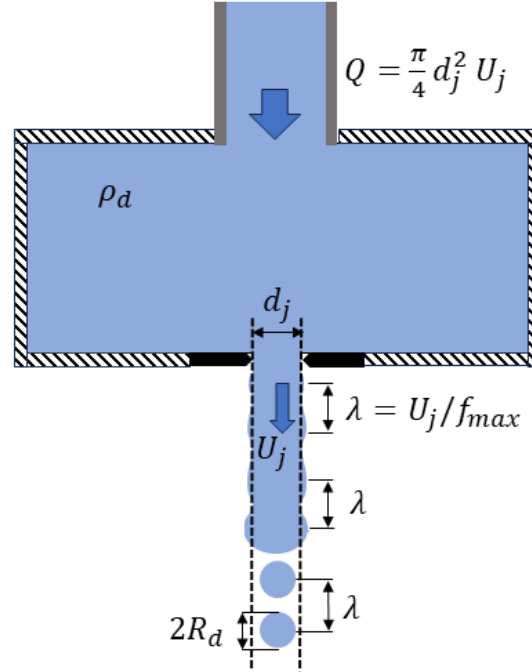


Figure 3.5: Sketch of monodisperse droplet generator

This expression in the non-viscous limit returns the Rayleigh solution. The wavelength for the same nozzle diameter determines the volume of the droplet produced, $V_d = \pi d_j^2 \lambda_{max} / 4$. However, when the disturbance is a consequence of a sinusoidal input, as in our case, the frequency of the disturbance can be related to the liquid free jet velocity by the quotient $f_{max} = U_j / \lambda_{max}$. The expression for the expected droplet radius under these simplifications takes the form:

$$R_d = \frac{1}{2} \sqrt[3]{\frac{3 d_j^2 U_j}{2 f_{max}}} \quad (3.2)$$

It should be noted that only a narrow range of frequencies and jet velocities linked to the Rayleigh wavelength generates droplets of uniform diameter [192]. Thus, for the tests contained in this study, droplets of diameters between $400 \mu m$ and $1300 \mu m$ are generated by modifying the jet exit diameter by means of calibrated holes, the liquid flow rate and the oscillation frequency. Then the system is calibrated from imaging systems where the drop volume is calculated, more details in section 3.3.1. It must be taken into account that the actual breakup process is not linear, being one of the most common deviations the formation of a ligament between the drops that when breaking generates drops of smaller diameter known as satellite drops. The latter join the parent drops, inducing a slight loss of repeatability in the initial diameter. The system includes at its end an aerodynamic protection profile that minimizes the interaction of the disturbed flow field in the room with the droplet generation process.

3.2.4 Data acquisition system

In relation to the image system, a high-speed camera has been used to characterize the deformation and displacement of droplets. For this purpose, a Photron SA-5 high-speed camera equipped with a 12 bit CMOS sensor with a pixel size of $20\ \mu\text{m}$ is used, capable of working up to 775000 fps with a resolution of 128×24 pixels and working at full resolution of 1024×1024 pixels up to 7500 fps. For this experimental study, the camera is set to operate at 75000 frames per second with a maximum resolution of 192×312 pixels, this being the compromise between the deformation and breakup process frequency and the physical size of the area of interest (AOI) necessary to observe it. The camera is placed at approximately $1.47\ \text{m}$ from the plane that contains the drops. In relation to the optics, a $200\ \text{mm}$ Nikkor Micro lens is used. To avoid blurring phenomena, the shutter speed is set to work at $1\ \mu\text{s}$. The shadowgraph imaging technique is used due to its ability to highlight the interface between two media of different refractive index, the image shows a bright background and a dark interface as a consequence of the refracted light. For this purpose, a continuous illumination system consisting of a $2000\ \text{W}$ xenon lamp is placed behind the image plane, which, generates a high quality white diffuse light. Photron's proprietary FASTCAM software is used for the image capture. In relation to the calibration of the imaging system, an AP-G100 standard is used, thus correcting aberrations and obtaining a relation $14.20\ \text{px/mm}$. Due to limited storage space and insufficient data transmission speed, the camera recording is initiated by a trigger, which is linked to the angular position of the rotating arm that is enabled when the command to record is given. In this way the images are captured and then sent via an Ethernet bus to the PC where they are stored.

3.3 Test procedure and data analysis

The test procedure is as follows; firstly, prior to the test, the camera is aligned using an XY positioning table and a tilting table, then the backlighting system is adjusted so that the image plane concur with the symmetry plane of the profile and the falling path of the droplet. To do this, the leading edge is placed below the droplet generator and a laser pointer is placed on the leading edge using a tool designed for this purpose. Once the laser beam appears focused and contained in the image plane, it is calibrated, from which the magnification of the system is obtained and which will be used to obtain the px/mm ratio. The personnel then move to the adjacent room from where the test is initiated. The arm is accelerated to the desired speed and the droplet injection process is started for at least three complete revolutions. Once completed, the rotating arm stops and the recorded film is analyzed to verify that the droplets are in focus and with sufficient image contrast for the analysis. When this information has been verified, it is moved on to the next test point, repeating the process.

A total of 4902 droplets have been analyzed during the test campaigns that feed data to this and other works, with the airfoil models described in the table 3.1, traveling at velocities of around $50\ \text{m/s}$, $60\ \text{m/s}$, $70\ \text{m/s}$, $80\ \text{m/s}$ and $90\ \text{m/s}$. The tests were carried out in still air at an ambient temperature of 20°C , droplets were formed by distilled water, with diameters

(D_d) between $200 \mu m$ and $1300 \mu m$ which implies Ohnesorge numbers in the range between 0.0034 and 0.0084, indicating that surface tension forces are dominant over viscous forces in reducing the deformation. Furthermore, if we evaluate a Weber number based on the airfoil velocity, it ranges between 8.85 and 169. Finally the Reynolds numbers based on droplet diameter and maximum airfoil velocity are valued between 679 and 7270. The following section (3.3.1) describes how the data from these campaigns are analyzed and how the base flow field was characterized for the study (section 3.3.2).

3.3.1 Data analysis of the droplet trajectories, deformation and breakup

For the image processing and analysis, the SITEA tool developed in Matlab environment is used (Sor et al. [193]), which includes the following stages:

1. Droplet tracking: a user-selected droplet is individually tracked by determining a region of interest (ROI).
2. Droplet detection: the system performs a binarization taking advantage of the high contrast generated by the shadowgraph technique and the homogeneity of the background, by means of the Otsu's method [194] which determines the global threshold for binarization. Subsequently, a removal of elements not attached to the droplet is carried out by means of a filtering criterion of elements of smaller areas than expected. Finally, due to the characteristics of the droplet image, it is necessary to fill the centre of the droplet where no reflection is generated. After all these steps the drop appears as a single element.
3. Analysis of the parameters of the droplet: the binary image is used to calculate the centroid of the droplet, assuming that it is deformed as an oblate spheroid. Its area in pixels and the size of its semi-major axis, R_{max} and minor axis, R_{min} , are also measured. The equivalent radius is calculated as follows:

$$R_d = M_0 \sqrt[3]{R_{max}^2 R_{min}} \quad (3.3)$$

where M_0 is the magnification in mm/px . The value of this equivalent radius is constant until the onset of breakup. To characterise this instant, we use the minimum radius criterion [183], by which the breakup in the cases of bag and bag and stamen occurs when, assuming that the droplet deforms as an oblate spheroid, the semi-minor axis reaches a minimum and then grows. From that point on, the oblate spheroid assumption is no longer applicable and the droplet is considered to have started its fragmentation process. Additionally, in order to avoid characterizing the noise of the analysis method as breakup, it is only considered that this minimum corresponds to breakup if the final value of the semi-axis before the impact is 20% higher than the minimum detected.

To determine the uncertainty of the method, it is assumed that the mass of the droplet is conserved and that its equivalent radius must remain constant during the process. Five representative cases with droplets radii between $596 \mu m$ and $207 \mu m$ were used, finding that for the smaller droplets the uncertainty was larger with a value of $\pm 16\%$ and for the

larger ones around $\pm 5\%$. It will therefore be possible to characterize the quality of the data on the basis of the fluctuations of this measure for the droplet tracked.

3.3.2 Characterization of base flow

As seen in the literature review, it is necessary to know the flow to which the droplet is exposed in order to study its impact on the deformation and breakup processes. For this purpose, it will be assumed that the flow field remains practically unchanged by the presence of droplets, so it will be possible to characterize the air flow in the vicinity of the airfoil leading edge by using the Particle Image Velocimetry (PIV) technique. This means that, implicitly, a one way interaction approach was considered. This technique measures in a non-intrusive way the flow field by measuring the displacement of particle images between two consecutive photographs. Thus, the image is divided into interrogation windows and their displacement (Δx_{PIV}) is obtained by cross-correlation methods. In this case we study the two components contained in the Horizontal Plane (HP) and Vertical Plane (VP) that pass through the symmetry planes of the profile, figure 3.6, in such a way that they contain the central stagnation line. The particles must be small enough to follow the flow without significantly disturbing it or altering its properties and at the same time being able to be illuminated so that they can be recorded by the camera. In this case, the room was seeded with atomized particles of olive oil of average diameter $1 \mu m$. Regarding the illumination system, this usually comes from thin light-sheet generated by two high power lasers that deliver two consecutive pulses of energy, one per laser, with a time delay Δt_{PIV} between them, generating a pair of exposures in the same image or a pair of independent images synchronized with the aperture of the camera's diaphragm. Two TSI PIV sets were used for each measurement plane consisting of a pulsed ND:Yag laser $190 mJ$ with a laser thickness $\Delta Z_L = 1 mm$ aligned by means of a set of mirrors, and a Power Plus 4MP camera. Image synchronization, capture and analysis is done by the proprietary TSI Insight 3G software. Up to three different sets of lenses were used depending on the Field of View (FOV); a AF-S VR Micro Nikkor $105 mm f/2.8 G IF-ED$ Nano Crystal Coat, a AF Nikkor $80 - 200 mm f/2.8 D IF-ED$, and a Nikkor $50 mm f/1.4$. Thus, the characterization of the base flow field is based on a test campaign for which the M1, M2 and M3 models were used, rotating at different velocities up to a maximum of $92 m/s$. The acquisition of each pair of images is synchronized so that the HP and VP take the normalized FOV shown in the figure 3.7. For this purpose, a time lag is estimated and a trigger is placed to take measurements at a given distance from the leading edge. The sampling frequencies are in the range of $4.75 Hz$ and $6.5 Hz$. The time between pulses and the size of the interrogation window are also calculated according to the valid detection criteria defined in Adrian and Westerweel [195] below:

$$N_e = N_I \cdot F_i \cdot F_O \cdot F_\Delta \geq 7 \quad (3.4)$$

This classical criterion determines a minimum effective number that meets the conditions necessary for the formation of a reliable and well defined correlation peak to derive the displacement of the group of particle images inside an interrogation window, of dimension $D_I \cdot D_I$. From the equation (3.4), N_I is the average number of particles in the interrogation window defined as $N_I = N_{ppp} D_I^2$, where N_{ppp} is the number of particles per pixel. The

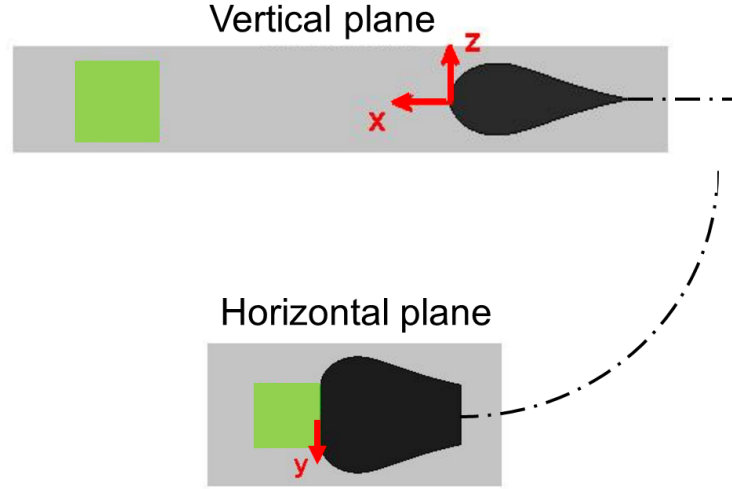


Figure 3.6: Sketch of PIV planes and moving reference frame. Upper figure: Vertical plane (VP). Bottom figure: Horizontal plane (HP). Based on the figure from [63] A. García-Magariño, S. Sor and A. Velazquez. ‘Experimental characterization of water droplet deformation and breakup in the vicinity of a moving airfoil’. In: *Aerospace Science and Technology* 45 (2015), pp. 490–500. ISSN: 1270-9638. DOI: [10.1016/j.ast.2015.06.025](https://doi.org/10.1016/j.ast.2015.06.025).

other factors that appear are the following; the loss of correlation factor for in-plane motion F_i , the loss of correlation factor for out-of-plane motion F_O and the loss of correlation factor for velocity gradients F_Δ . Their expressions are defined below:

$$F_i = 1 - \frac{u\Delta t_{PIV}}{L_I}, \quad F_O = 1 - \frac{w\Delta t_{PIV}}{\Delta Z_L}, \quad F_\Delta = e^{-2a_{PIV}^2/3d_i^2} \quad (3.5)$$

Being u an estimate of the expected in-plane velocity modulus, w the expected velocity component perpendicular to the plane, L_I the physical length of the interrogation window, $a_{PIV} = M_0\Delta u\Delta t_{PIV}$ and d_i the size of the particle image. As stated in Scharnowski et al. [196] it is not enough with the criterion of equation (3.4) but the factors must independently fulfill $F_i \geq 0.65$, $F_O \geq 0.7$ and $F_\Delta \geq 0.3$. For the latter to be satisfied, the tests are configured by adjusting them to the model velocity so that the distance between pulses Δt_{PIV} is ranged between $1.1 \mu s$ and $200 \mu s$. In all cases, interrogation windows of 64×64 pixels are used. The leading edge of the airfoil is the origin of the reference frame and the flow field is analysed up to distances of the order of the upstream model chord. Since this exceeds the size of the FOV, it is necessary to reconstruct the flow field based on multiple region analysis. Each of these analysis is made up of a total of up to 100 pairs of images, one per revolution, from which the ensemble average flow field of that region is obtained. More details about the experimental PIV procedure used to characterize the flow field could be found in references [82], [116], [64] and [197].

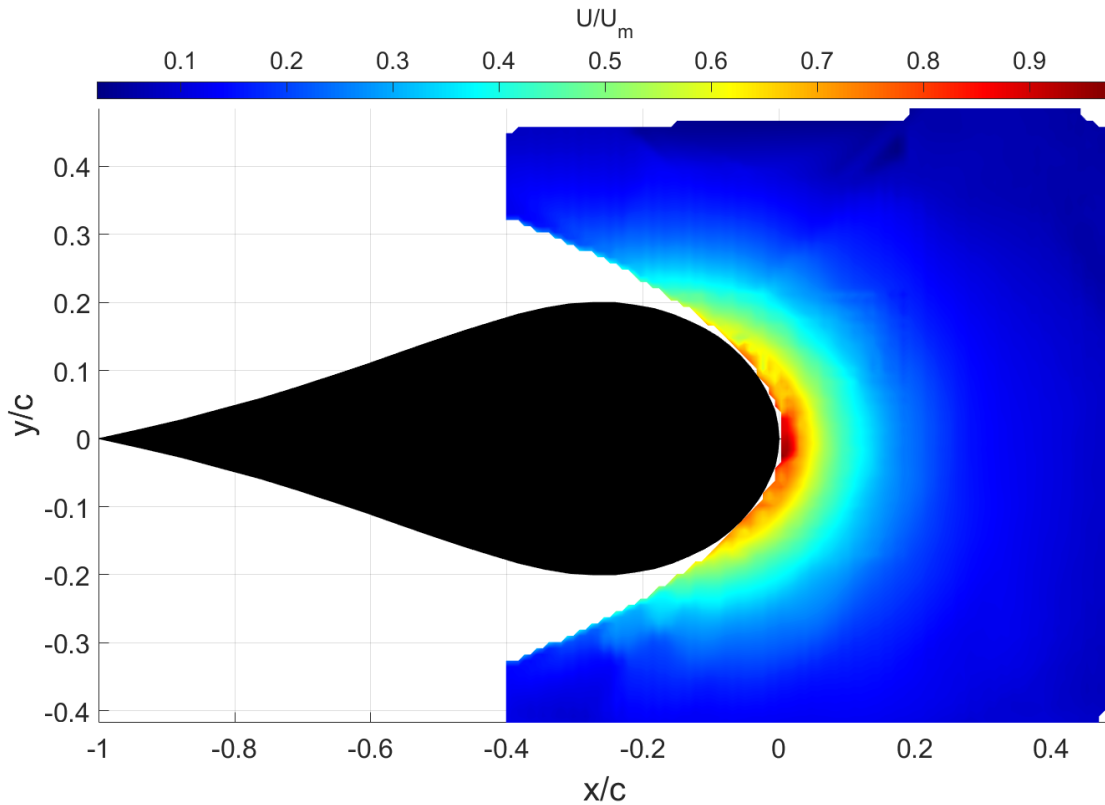


Figure 3.7: Flow field obtained via PIV for VP with the velocity modulus normalized by the airfoil velocity, with coordinates normalized with its chord, corresponding to the M1 airfoil model.

As can be seen in the figure 3.7 one of the technical limitations of the method is that as a consequence of the reflections on the airfoil, the velocity field in regions close to the airfoil could not be analyzed. This is a limitation mainly in regions where higher velocity gradients are expected. On the other hand, the resolution of the PIV is limited to 32×32 interrogation windows per FOV. To complete the unanalyzed regions and improve the spatial resolution of the flow field, CFD simulations are performed in Ansys Fluent. Two simulations are carried out, an inviscid 2D flow simulation and a unsteady 3D simulation where the airfoil rotates within a domain at rest. The objective of this simulation is twofold: on the one hand, it allows to improve the resolution of the flow field, allowing to check certain hypotheses and to provide complementary information in relation to the base flow, including turbulence intensity estimation. On the other hand, it serves as a basis for future work, where the droplet travels further away from the stagnation line, see chapter 7. Focusing on the 3D simulation, the M1 model case is chosen rotating 2.3 m from the center of the profile with respect to the centre of a cylindrical volume with a height of 5.3 m and a diameter 14 m at a tangential velocity of 92 m/s at the center of the airfoil. This model has been chosen because it has the lowest span-chord ratio and the highest R_c , which leads directly to a larger wake and greater tip effects. An unstructured mesh of tetrahedral elements is used globally, applying a mesh refinement based on two volumes, one that surrounds the model and another that is applied at the trailing edge

of the airfoil. In the near region of the airfoil, a mesh is generated for the boundary layer with a $y^+ > 30$ consisting of 7 layers with a growth rate of 1.2 for the application of the enhanced wall function. To capture the transient character and the effect of the model rotation, a sliding mesh configuration will be applied on a smaller cylindrical volume including the body and the refinement volumes. The air inside the volume is at rest at the beginning and will be treated as incompressible with density 1.225 kg/m^3 and dynamic viscosity $\mu = 1.789e - 05$. A k- ϵ model with standard enhanced wall function will be used in combination with correction terms for low Re number, turbulence production limitation and curvature correction, the model constants are those of the developer's standard model. The upper region of the room cylinder is configured as pressure outlet and the rest as stationary walls, being the airfoil the only moving surface, with roughness constant 0.5 and standard model. First order discretization schemes have been chosen for the turbulent quantities and second order for the remaining ones. The transient formulation will be of the bounded second order implicit type. The ensemble is rotated, monitoring the time average of the drag coefficient and the lift coefficient until the former becomes constant and the latter converges to zero.

3.4 Droplet trajectory and deformation models

This section describes the model used to characterize the deformation and trajectory of droplets exposed to the passage of an airfoil in the vicinity of the stagnation line. This section is divided into two subsections. First the deformation model is described. This deformation model is, basically, apart from some minor modifications, the one described by the authors in reference [116]. The model is summarized here for the sake of completion. The second subsection addresses the modeling of base air flow velocity along the stagnation line, section 3.4.2.

3.4.1 Full deformation and trajectory model

The following sketch, see figure 3.8, describes the situation that is accounted for by the model. The experimentally verified hypotheses on which the deformation model is based are as follow:

- The model is valid only in regions close to the leading edge and around the stagnation line of the airfoil, so that the flow is considered to be unidimensional and no shear effects appear in the flow field. The use of this hypothesis is reasonable if it is taken into account that the droplets under study move around the stagnation line and that the ratio between the terminal velocity and the velocity of the airfoil moves at the slowest velocity under study, U_t/U_m , is 0.045 for the smallest droplets under consideration and 0.082 for the larger ones. Terminal velocity has been assessed assuming spherical geometry ($C_D \approx 0.5$), by means of the balance between weight and aerodynamic forces leads to the expression:

$$U_t = \sqrt{\frac{8}{3} \frac{\rho_d}{\rho_{air}} \frac{R_d}{C_D} g} \quad (3.6)$$

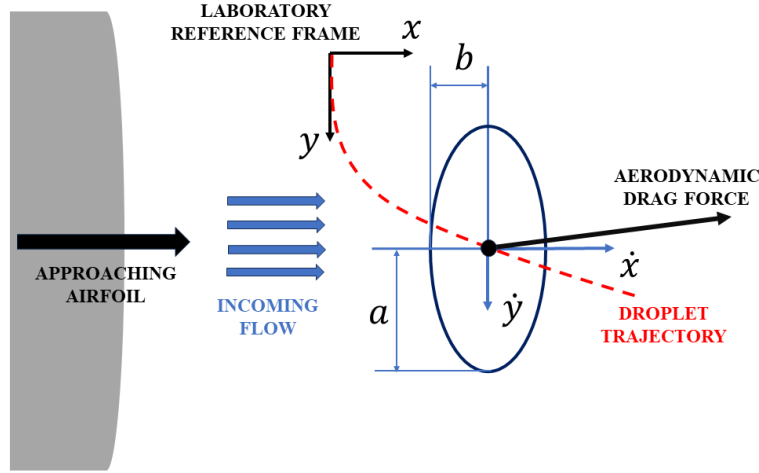


Figure 3.8: Sketch showing the basic model setup

- The initial condition of the droplet assumes spherical geometry before being exposed to the airstream. This hypothesis is consistent with previous observations, [182], [183], [63], [82]. On the other hand, it is possible to justify the use of this hypothesis also on the basis of the Weber number associated with the terminal velocity of the droplet. An example of a typical initially free-falling unperturbed droplet can be found in figure 3.12a. Thus, values of $We = 0.139$ and $We = 0.451$ are found for the smaller and larger diameter droplets respectively. Whereas $Oh \leq 0.1$, following the work of Hsiang and Faeth [42] and Hsiang and Faeth [43] it can be observed from the space $Oh - We$, figure 2.3, that the situation under study correspond to regions with deviations from their spherical shape below 5%. It is further considered that there is no perturbation at the initial instant that moves it away from this condition, or equivalently $dR_d/dt = 0$.
- The model will be valid under conditions where the volume of the droplet remains constant, corresponding to the deformation process and the onset of breakup. The volume will be that of the initial spherical droplet:

$$V_{droplet} = 4/3\pi R_d^3 \quad (3.7)$$

- The drop is deformed as an oblate spheroid, being its major and minor semi-axes a and b respectively. This, together with the hypothesis of volume conservation, allows us to relate the major and minor semi-axes through the expression $b = R_d^3/a^2$, which allows us to express the deformation as a function of a single variable. This assumption, which also underlies the DRD model, [82] has been widely used in previous models ([66],[68],[74],[112]) to capture the deformation process. For more information on models based on the same hypothesis, see the section 2.1.3.
- The model is formed by an equation linked to the deformation of the drop and another two related to the horizontal and vertical motion. The first is based on the work of

Sor and García-Magariño [82], where the DRD model is defined and takes the final expression appearing in the equation (2.22). This involves setting the deformation rate of the droplet dependent on the external pressure forces, the area forces due to surface tension and the deformation opposing forces due to viscosity. The viscous dissipation effects will be considered negligible with respect to the first two forces, so this term will not be considered in the analysis. This hypothesis of negligible viscous force is close to the experimental observations when evaluated on the basis of the measurements of the different cases studied and compared with the rest of the forces involved in the model. On the other hand, the equations that model the horizontal and vertical movement of the droplet for the flow under study is developed in Sor et al. [116]. The latter involves modelling the non-stationary drag coefficient on the basis of two terms, one stationary and one non-stationary as described in the equation (2.28). The first term captures the deformation effect on the droplet acceleration based on an interpolation between the drag coefficient of a sphere and the drag coefficient of the flat plate.

- Another hypothesis used is that the droplet velocity is negligible compared to that of the incoming airfoil, so the slip velocity $V_{sx} = V_s - dx/dt \simeq V_s$, where V_s is the air velocity at a given location in the flow field. Regarding the vertical slip velocity and assuming the hypothesis of unidirectional flow we have $V_{sy} = V_y - dy/dt \simeq -dy/dt$.

Considering these starting assumptions and based on the model described in [116] the dimensionless model equations are given below:

$$\frac{d^2\eta}{d\tau^2} = \Pi_1 \mathbb{V}_{sx}^2 \alpha^2 \left[\left(C_{D_{sphere}}^{(R_d/a)^3} \cdot C_{D_{disk}}^{1-(R_d/a)^3} \right) + \left(\Pi_2 \frac{1}{\alpha^2 \mathbb{V}_{sx}^2} \frac{d\mathbb{V}_{sx}}{d\tau} \right) \right] \quad (3.8)$$

$$\frac{d^2\zeta}{d\tau^2} = -\Pi_1 \alpha^2 \mathbb{V}_{sx} \mathbb{V}_{sy} \left(C_{D_{sphere}}^{(R_d/a)^3} \cdot C_{D_{disk}}^{1-(R_d/a)^3} \right) + \Pi_3 \quad (3.9)$$

$$\frac{d^2\alpha}{d\tau^2} = -\Pi_4 \mathbb{F}(\alpha) + \frac{16}{3} \Pi_1 C_P \mathbb{V}_{sx}^2 \quad (3.10)$$

Thus, the equation (3.8) models the dynamics of the droplet along the x axis, the equation (3.9) the dynamics of the droplet on the y axis and finally the equation (3.10) the deformation of the droplet, all of them under the aforementioned hypotheses. Dimensionless variables and functions are defined as follows:

$$\eta = \frac{x}{R_d}, \quad \zeta = \frac{y}{R_d}, \quad \alpha = \frac{a}{R_d}, \quad \tau = \frac{tU_m}{R_c}, \quad (3.11)$$

$$\mathbb{V}_{sx} = \frac{V_{sx}}{U_m}, \quad \mathbb{V}_{sy} = \frac{V_{sy}}{U_m}, \quad \mathbb{A}_d = \frac{A_d}{R_d^2}, \quad \mathbb{F}(\alpha) = \frac{F(a)}{R_d} \quad (3.12)$$

Where x , y and t are the spatial coordinates and time respectively, a is the largest semi-axis of the oblate spheroid (see figure 3.8). U_m is the velocity of the incoming airfoil. R_d

and R_c are the initial droplet radius and airfoil leading edge radius respectively. $\mathbb{V}_{sx}$ and $\mathbb{V}_{sy}$ are the x and y components of the slip velocity, (air velocity minus droplet velocity). Under the hypothesis of negligible droplet velocity, $\mathbb{V}_{sx}$ and $\mathbb{V}_{sy}$ can be approximated by the air velocity components relative to the droplet. A_d is the droplet surface area and $F(a)$ is dA_d/da , that is an analytical function. In particular, for an oblate spheroid, the dimensionless surface area A_d is defined as:

$$\mathbb{A}_d = \frac{A_d}{R_d^2} = 2\pi\alpha^2 + \frac{\pi}{\alpha(\alpha^6 - 1)^{1/2}} \ln \frac{1+e}{1-e} \quad (3.13)$$

$$e = \frac{(\alpha^6 - 1)^{1/2}}{\alpha^3} \quad (3.14)$$

The dimensionless parameters that appear in equations (3.8), (3.9) and (3.10) are,

$$\Pi_1 = \frac{3\rho_{air}}{8\rho_d} \left(\frac{R_c}{R_d} \right)^2 \quad (3.15)$$

$$\Pi_2 = \frac{kR_d}{R_c} \quad (3.16)$$

$$\Pi_3 = \frac{gR_c}{U_m^2} \left(\frac{R_c}{R_d} \right) \quad (3.17)$$

$$\Pi_4 = \frac{16}{3\pi} \frac{\sigma}{\rho_d} \left(\frac{R_c}{U_m R_d} \right)^2 \frac{1}{R_d} \quad (3.18)$$

where ρ_{air} and ρ_d are the air and water density respectively, g is the gravity constant, and σ the water surface tension. k is an experimental calibration coefficient that was found to be constant (equal to 9) that multiplies the unsteady drag term in equations (3.8). The terms $C_{D_{sphere}}$ and $C_{D_{disk}}$ are the steady drag of the sphere and disk respectively that are used to interpolate the steady drag term of the droplet. In our case it is decided to use the correlation for the drag coefficient of the rigid sphere defined in White [198] as:

$$C_{D_{sphere}} = \frac{24}{Re_d} + \frac{6}{1 + Re^{0.5}} + 0.4 \quad (3.19)$$

Where Re is the Reynolds number based on the initial droplet diameter. This expression is valid for the Reynolds numbers found in the test. In the case of the disc the drag coefficient is modeled as constant and equal to 1.17 since it corresponds to regions where it has been significantly deformed and the Reynolds number is significantly above 10^2 , so the error of such consideration is small. The initial conditions needed to integrate equations (3.8), (3.9) and (3.10) are:

$$\eta(0) = 0, \quad \left(\frac{d\eta}{d\tau} \right)_{\tau=0} = 0, \quad (3.20)$$

$$\zeta(0) = 0, \quad \left(\frac{d\zeta}{d\tau} \right)_{\tau=0} = \frac{R_c}{R_d} \frac{V_{yd0}}{U_m}, \quad (3.21)$$

$$\alpha(0) = 1, \quad \left(\frac{d\alpha}{d\tau} \right)_{\tau=0} = 0, \quad (3.22)$$

Being $V_{y_{d0}}$ the free falling droplet velocity when reaching the stagnation line. Integration of equations (3.8), (3.9) and (3.10), together with the initial conditions (3.20), (3.21) and (3.22), yields the (x, y) trajectories of the droplet and its deformation. Mass conservation of the deformed droplet after the initial spherical shape implies that deformation is described using just one parameter a (see figure 3.8).

3.4.2 Base air flow velocity

To characterize the base flow, we first used the results obtained using the PIV technique described in the section section 3.3.2. This was done because as it has been aforementioned, accurate air flow characterization is crucial as it affects critically the droplet acceleration which, in turn, governs droplet breakup (as it will be described in the next subsection). However, for the practical solution and analysis of equations (3.8), (3.9) and (3.10), it is far more useful to have an analytical description of the flow field along the stagnation streamline. The fact that the selected airfoil shape was rather blunt (typical of control surfaces in modern aircraft) suggested that the stagnation streamline could be assimilated to the potential flow past a cylinder with an appropriate coefficient; and this was indeed the case. In particular, it was found that the following analytical expression for the stagnation streamline flow closely matches the air flow actually being measured:

$$V_s = U_m \frac{R_c^n}{(R_c + r_0 - U_m t)^n} \quad (3.23)$$

where $n = 1.8$ ($n = 2$ in the case of the potential flow around a circular cylinder) and r_0 is the initial distance from the droplet to the airfoil leading edge. It should be noted that under the assumptions expressed $V_{sx} = V_s$. Defining a new dimensionless parameter that characterizes the initial distance of the drop to the profile as,

$$\Pi_5 = \frac{r_0}{R_c} \quad (3.24)$$

adimensionalizing equation (3.23) and including the defined parameter leaves the expression of the dimensionless velocity as a function of τ .

$$\frac{V_s}{U_m} = \frac{1}{(1 + \Pi_5 - \tau)^n} \quad (3.25)$$

The figure 3.9 shows the comparison between the PIV experimentally measured airflow velocity curves along the stagnation streamline, the results are obtained by performing

numerical flow simulation using Fluent, and the curve obtained using the approximate relation (3.23). The curve corresponding to the PIV is obtained by polynomial fitting of the velocity obtained during the PIV tests for the characterization of the base flow field specially for the stagnation line of the airfoil. It can be seen from the figure that the Eulerian simulation of the two-dimensional problem fits the experimental results quite well, suggesting that in the stagnation line environment this approximation is reasonable.

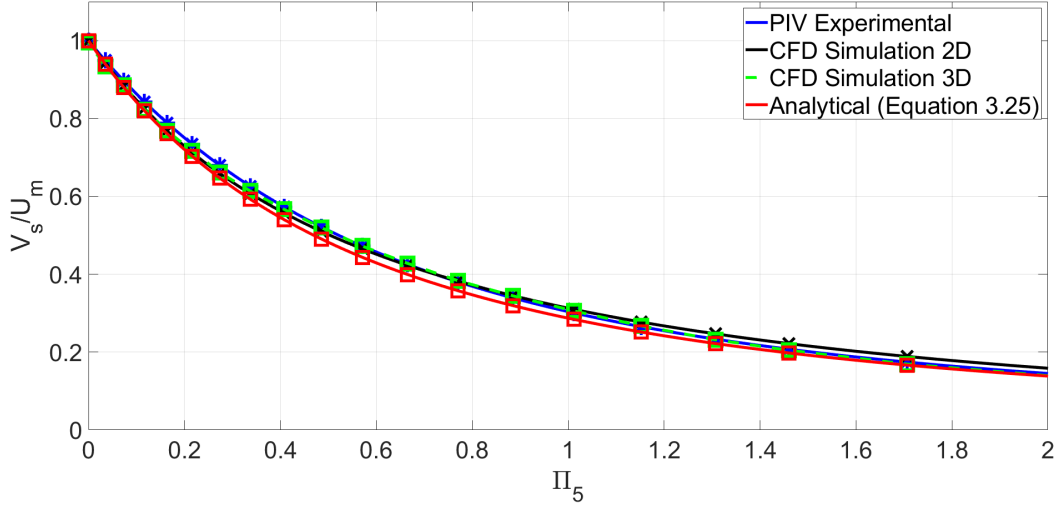


Figure 3.9: Incoming flow characterization. Blue line: PIV measurement, black line: 2D numerical simulation, green line: 3D numerical simulation, red line: approximate relation(equation (3.23))

3.5 Breakup model

In this section, the theory of hydrodynamic stability analysis for the Rayleigh-Taylor instability will be defined in detail, leading to the definition of a specific breakup criterion. The problem consists in determining the conditions under which a perturbation causes a series of fluid variables to grow in time. The analysis of this response to perturbations is defined as linear stability theory, and is based on the linearized Navier-Stokes equations, which requires that the base state be simple in order to arrive at an analytical solution. All this base theory is exposed in the section 3.5.1. Subsequently, based on the work of Joseph et al. [29], we define a series of hypotheses to define a breakup criterion expressed in the dimensionless variables of the problem, section 3.5.2. Finally, in the section 3.5.3, this criterion is coupled to the droplet deformation and trajectory model defined in the previous section.

3.5.1 Rayleigh-Taylor instability

In the following, we will describe the traditional development of the Rayleigh-Taylor instability based Leal [189], for a more exhaustive analysis of this type of instabilities please

refer to Chandrasekhar [15]. As discussed in section 2.1.1, this instability has its origin in the presence of two fluids with different densities superimposed one on top of the other (or accelerated with respect to each other). It is therefore an instability that arises at the interface between two different fluids.

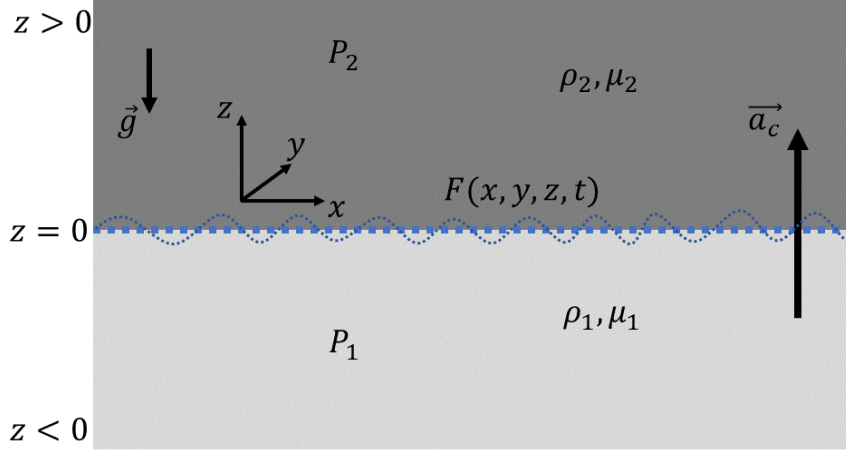


Figure 3.10: Sketch of Rayleigh-Taylor Instability

Lets assume two immiscible fluids designated by the subindex $i = 1, 2$ and separated by an interface. A Cartesian reference frame is used, as shown in the figure 3.10. The domain is infinite in x and y , as well as in $\pm z$. Both the density ρ_i and the dynamic viscosity μ_i are constant for each fluid. The analysis considers the fluids to be initially static with respect to the reference system attached to the interface, so that any deviation from this state is a consequence of a perturbation on its fluid variables. The interface is also considered to be initially flat, defining the perturbed interface through a scalar function $F(x, y, z, t) \equiv 0$. This way of expressing the interface allows us to express the unit vector normal to the interface as $\pm \nabla F / |\nabla F|$, which will be particularly useful when applying the normal stress boundary conditions, equation (2.3). The velocities as well as the interface will therefore be formed by the following perturbation magnitudes:

$$u_i = \epsilon \delta u_i, \quad v_i = \epsilon \delta v_i, \quad w_i = \epsilon \delta w_i, \quad z = \epsilon \delta z(x, y, t) \quad (3.26)$$

Where u_i, v_i, w_i are the components of the flow velocity (considered small) for each fluid, ϵ is the magnitude of the perturbed variable and the symbol δ before the variables indicates that we are referring to the perturbation variable. In relation to the pressure, considering g is the magnitude of the gravitational acceleration which will have component $\vec{g} = -g\vec{i}_z$, being $\vec{i}_z$ the director vector in z , the hydrostatic pressure for each fluid takes the following expression:

$$p_1 = -\rho_1 g z + \epsilon \delta p_1, \quad p_2 = -\rho_2 g z + \epsilon \delta p_2 \quad (3.27)$$

We derive the equations of motion from the Navier-Stokes equations of the linearized perturbed variables, neglecting all those terms of order $O(\epsilon^2)$. Hence:

$$\rho_i \frac{\partial \delta u_i}{\partial t} = -\frac{\partial(\delta p_i)}{\partial x} + \mu_i \nabla^2 \delta u_i \quad (3.28)$$

$$\rho_i \frac{\partial \delta v_i}{\partial t} = -\frac{\partial(\delta p_i)}{\partial x} + \mu_i \nabla^2 \delta v_i \quad (3.29)$$

$$\rho_i \frac{\partial \delta w_i}{\partial t} = -\frac{\partial(\delta p_i)}{\partial x} + \mu_i \nabla^2 \delta w_i \quad (3.30)$$

$$\frac{\partial \delta u_i}{\partial x} + \frac{\partial \delta v_i}{\partial y} + \frac{\partial \delta w_i}{\partial z} = 0 \quad (3.31)$$

As we have seen, the domain is infinite and there are no restrictions on its shape in x and y , but the perturbations must satisfy the boundary conditions at the interface $z = \epsilon \delta z(x, y, t)$. Considering that the perturbation can be analyzed on the assumption that it is static in the ground state and that the velocity only corresponds to that of its perturbation. If we analyze these perturbation variables in it's normal modes, their dependence on x, y y t takes the former expressions:

$$\delta u_i(x, y, z, t) = u_i(z) \exp(ik_x x + ik_y y) \exp(nt) \quad (3.32)$$

$$\delta v_i(x, y, z, t) = v_i(z) \exp(ik_x x + ik_y y) \exp(nt) \quad (3.33)$$

$$\delta w_i(x, y, z, t) = w_i(z) \exp(ik_x x + ik_y y) \exp(nt) \quad (3.34)$$

$$\delta p_i(x, y, z, t) = \delta p_i(z) \exp(ik_x x + ik_y y) \exp(nt) \quad (3.35)$$

The stability analysis will be based on studying the sign of the real part of n . Substituting the equations (3.28-(3.31)) in (3.32-(3.35)), multiplying the result of the substitution by ik_x and ik_y and then combining them into a single equation by using the continuity equation we have the following general equation:

$$\left[-\rho_i n + \mu_i (D^2 - k^2) \right] (D^2 - k^2) w_i = 0 \quad (3.36)$$

Where $k^2 \equiv k_x^2 + k_y^2$ and $D = d/dz$. It remains to define the general boundary conditions that will apply. First, for $z \rightarrow \pm\infty$ the magnitude of perturbations tends to zero.

$$u_1, v_1, w_1, p_1 \rightarrow 0 \quad \text{for } z \rightarrow -\infty \quad u_2, v_2, w_2, p_2 \rightarrow 0 \quad \text{for } z \rightarrow \infty \quad (3.37)$$

In addition, we will have the no-slip boundary conditions for all the points belonging to the interface $F(x, y, z, t) \equiv 0$. Which given the linear stability problem for small perturbations can be directly approximated for $z = 0$ as:

$$u_1(0) = u_2(0) \text{ and } v_1(0) = v_2(0) \quad (3.38)$$

CHAPTER 3. BREAKUP CRITERION FOR DROPLETS EXPOSED TO THE UNSTEADY FLOW OF AN INCOMING AERODYNAMIC SURFACE

In relation to the kinematic boundary conditions, the disturbance at the interface can be considered to be expressible with functions of the form:

$$\delta z(x, y, t) = Z \exp(ik_x x + ik_y y) \exp(nt) \quad (3.39)$$

Where Z is a constant to be determined. Applying the expression for the kinematic boundary conditions, equation (2.1), and applying equation (3.35) at $z = 0$ the kinematic boundary conditions are expressed as:

$$w_1(0) = w_2(0) = nZ \quad (3.40)$$

It remains to define the stress balances on the interface. In relation to the normal component we apply the expressions of the perturbations, equations (3.32) to (3.35) and (3.39) at $z = 0$ on the equation (2.3), which would give rise to the following equilibrium condition:

$$p_2(0) - p_1(0) + (\rho_1 - \rho_2) gZ + 2(\mu_1 Dw_1 - \mu_2 Dw_2)|_{z=0} + \sigma n^2 Z = 0 \quad (3.41)$$

Finally, the tangential stress balance, using the equation (2.3), can be expressed in its linearized form as:

$$\mu_1 (ik_x w_1 + Du_1)|_{z=0} = \mu_2 (ik_x w_2 + Du_2)|_{z=0} \quad (3.42)$$

$$\mu_1 (ik_y w_1 + Dv_1)|_{z=0} = \mu_2 (ik_y w_2 + Dv_2)|_{z=0} \quad (3.43)$$

Now we consider the stability analysis in the inviscid limit, that is $\mu_1 \sim \mu_2 \sim 0$, under these conditions the no-slip boundary conditions and the equilibrium tangential stresses cannot be applied. On the other hand it is possible to simplify equation (3.36) and equation (3.41) respectively as:

$$(D^2 - k^2) w_i = 0 \quad (3.44)$$

$$p_2(0) - p_1(0) + (\rho_1 - \rho_2) gZ + \sigma n^2 Z = 0 \quad (3.45)$$

The solution of equation (3.44) after applying the kinematic boundary conditions described in equation (3.40) are for each fluid:

$$w_1 = nZ \exp(kz) \quad \text{for } z < 0 \quad (3.46)$$

$$w_2 = nZ \exp(-kz) \quad \text{for } z > 0 \quad (3.47)$$

Using these expressions in the continuity equation, equation (3.31), substituting in equation (3.45) and undertaking a series of transformations we arrive at the equation for the growth-rate coefficient:

$$n^2 = \frac{((\rho_2 - \rho_1)g - \sigma k^2)k}{\rho_2 + \rho_1} \quad (3.48)$$

The stability limit equals $n = 0$ which applying the appropriate simplifications in equation (3.48) sets the critical wave number, k_c , as:

$$k_c^2 = \frac{g(\rho_2 - \rho_1)}{\sigma} \quad (3.49)$$

This result is revisited in Taylor [16] which extends the analysis to pairs of fluids undergoing acceleration. In order to do this the equations are rewritten in the moving reference frame which turns out to be equivalent to adding a quantity $\vec{a}_c$, being this the acceleration of the whole system, to the body force per unit of mass $\vec{g}$, the total balance of body forces per unit of mass would be $\vec{g}_{mr} = \vec{g} - \vec{a}_c$. If we particularize in our problem $|\vec{g}| \ll \vec{a}_c$ and considering that the acceleration is directed in the direction from the lighter fluid 1 to the denser fluid 2 it is enough to substitute g by a_c in the equation (3.48) and equation (3.49). Also considering that $\rho_2 \gg \rho_1$ it can be simplified as:

$$n = \left[\frac{(\rho_2 a_c - \sigma \kappa^2) \kappa}{\rho_2} \right]^{1/2} \quad (3.50)$$

$$\kappa_c = \left[\frac{\rho_2 a_c}{\sigma} \right]^{1/2} = \frac{2\pi}{\lambda_c} \quad (3.51)$$

Where λ_c is the critical wavelength. If the variables n and κ are adimensionalized as $n' = n\kappa_c$ and $\kappa' = \kappa t_c$, where $t_c = (\sigma/a_c^3 \rho_2)^{1/4}$. It is possible to express equation (3.50) in its dimensionless form:

$$n' = \sqrt{\kappa' - \kappa'^3} \quad (3.52)$$

This equation allows us to obtain the maximal growth rate from its derivative which is simply $n' = 2/3/\sqrt{3}$ for the critical wave number $\kappa' = 1/\sqrt{3}$ for the non-viscous case. We plot n' as a function of κ' in figure figure 3.11.

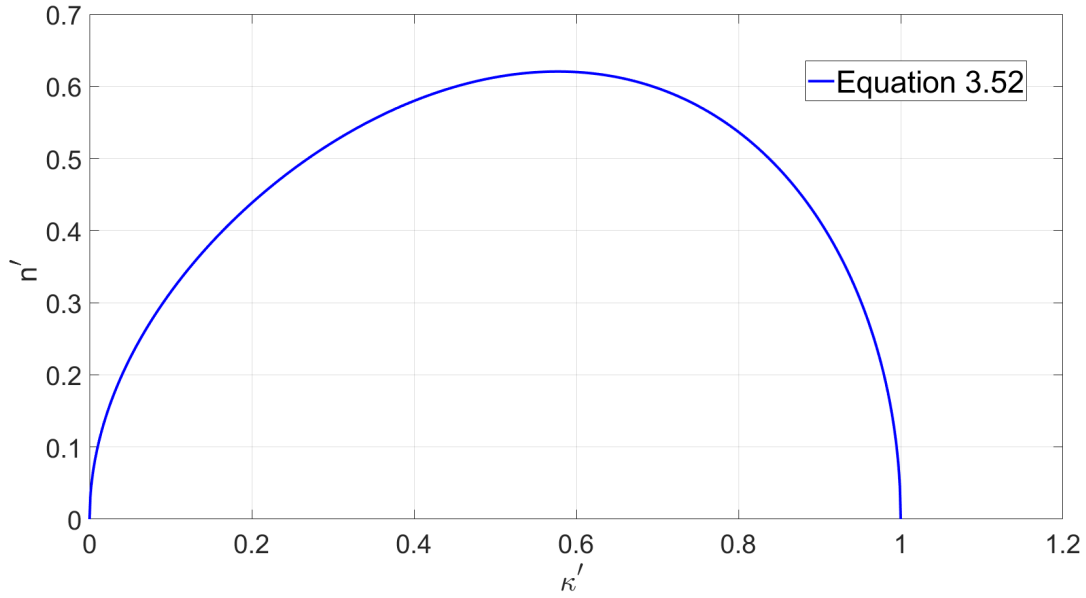


Figure 3.11: Plot of the dimensionless growth rate for RTI, n' as a function of the dimensionless wave number κ' as expressed in the equation (3.52)

Finally, in relation to the application of the instability analysis for non-viscous fluids, it should be noted that as stated in Leal [189]: “The only change from including the viscosity of the fluids is that the growth rates are reduced relative to those for inviscid fluids”. This indicates that the critical wave number derived from the analysis, the onset of instability in the non-viscous limit is valid independently of the actual impact of the viscosity on the system. For more information on how this result is reached, see Chandrasekhar [15].

3.5.2 Breakup model: hypothesis and derivation

Following a equivalent approach, Joseph et al. [29] performed a stability analysis on the problem of an instability at the interface between two overlapping fluids where the acceleration of the whole is directed from the lighter fluid towards the denser one. In their analysis, they calculated the growth rate using the analysis from the point of view of viscous potential flow, previously used in Joseph and Liao [46]. Under this perspective, the velocity perturbation is defined from the potential $\vec{u} = \nabla\Phi$. Its application modifies the growth rate which is equivalent to that of the Chandrasekhar [15] analysis in the limit of viscous potential flows, but does not change the instability border which remains as in the expression equation (3.50) for $n = 0$ and independent of the viscosity. As extracted from equation (3.51), if $\lambda < \lambda_c$ the heavy fluid is stable. Now, Joseph et al. [29] made an important assumption. In particular, when dealing with droplets, they assumed that λ_c is the droplet diameter D_d so that if $D_d < \lambda_c$ the droplet is stable (no breakup occurs). Notice, also, that the analysis of Joseph et al. [29] did no account for droplet deformation and considered a constant velocity incoming flow, which differs significantly from our case. Now, a set of main hypotheses are made in the present work:

- The breakup criterion is valid for droplets that breakup under the RTP regime. It is therefore assumed that the droplets always break under RTI and that in no case does KHI arise. In our study, the expected breakup mechanisms are of the bag and multimode type, so we will be in the RTP regime. A typical snapshot of the expected breakup mechanism is shown in figure 3.12d.
- The droplet acceleration in equation (3.51) is computed from the droplet trajectory and deformation model, equation (3.8-3.10).
- The interface of the deformed droplet can be represented as a flat surface, so that it is possible to model the problem on the basis of the stability analysis described above. This assumption is reasonable if one takes into account the images extracted from the test campaign, figure 3.12b , or the results for uniform flow from Hsiang and Faeth [43].
- The critical wavelength in equation (3.51) that signals the onset of breakup is the hydraulic diameter of the deformed droplet. The hydraulic diameter of a droplet is defined as:

$$\phi_d = 6 \frac{V_{droplet}}{A_d} \quad (3.53)$$

where $V_{droplet}$ and A_d are the droplet volume and surface area respectively. This expression of the hydraulic diameter is also known as volume to surface ratio and is equivalent to the diameter of the sphere with the same volume to area ratio. It was first described in the thesis of Sauter [199] for which it also takes the name of Sauter diameter. This equivalent diameter is commonly used in processes where the active surface area is decisive, such as in reactants or combustion processes [200]. Since the volume of the deforming droplet remains constant until breakup, it is possible to define it as a function of the initial radius, equation (3.7). Combining equations (3.53) and (3.7), the hydraulic diameter is written as:

$$\phi_d = 6 \frac{4/3\pi R_d^3}{A_d/R_d^2} = \frac{8\pi R_d}{\mathbb{A}_d} \quad (3.54)$$

This hydraulic diameter is taken as the critical wavelength that defines the onset of instability (breakup) of the deformed droplet. $\mathbb{A}_d$ is the dimensionless surface area of an oblate spheroid previously defined in equations (3.13)-(3.14). Naming $\mathbb{A}_d^*$ as the $\mathbb{A}_d$ value at the onset of instability and replacing it in relation (3.51) the following relation is obtained:

$$\frac{8\pi R_d}{\mathbb{A}_d^*} = 2\pi \left(\frac{\sigma}{\rho_d \ddot{x}^*} \right)^{1/2} \quad (3.55)$$

where $\ddot{x}^*$ is the droplet horizontal acceleration at the instant of breakup. Rendering relation (3.55) dimensionless using the scaling described in (3.11)-(3.12) the outcome is:

$$\frac{2^4}{(3\pi)^{1/2}} \frac{\ddot{\eta}^{*1/2}}{\mathbb{A}_d^*} \frac{1}{\Pi_4^{1/2}} = 1 \quad (3.56)$$

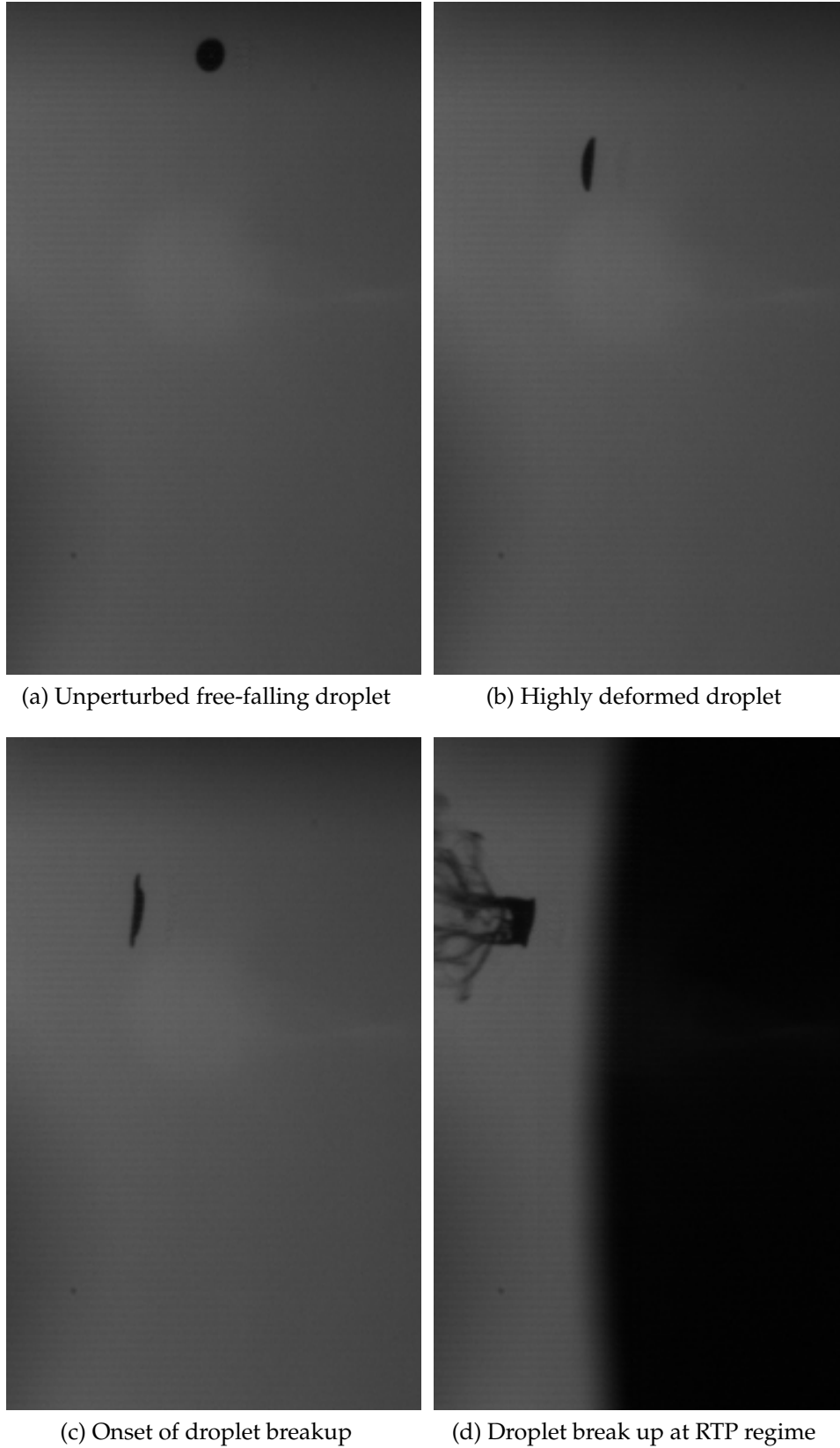


Figure 3.12: Different stages of the breakup process for $U_m = 91 \text{ m/s}$ and M2 airfoil.

3.5.3 Coupling deformation and breakup model

This relation (3.56) is the breakup criterion actually proposed in this study. Also Π_4 is the dimensionless parameter defined in relation (3.18). Then, the procedure to predict the onset of breakup is as follows:

- Droplet trajectory and deformation equations (3.8)-(3.10) are solved in time starting with initial conditions (3.20)-(3.22). The initial position is also taken into account so the air velocity is expressed as the equation equation (3.25).
- Ψ (see relation (3.57)) is continuously monitored during integration of the dynamics equations.

$$\frac{2^4}{(3\pi)^{1/2}} \frac{\ddot{\eta}^{*1/2}}{\mathbb{A}_d^*} \frac{1}{\Pi_4^{1/2}} = \Psi \quad (3.57)$$

- The onset of breakup is defined at the instant when $\Psi = 1$. This instant defines $\ddot{\eta}^*$ and α^* that are the droplet acceleration and droplet deformation at onset of breakup

Notice that, from an experimental standpoint, the onset of breakup is defined as the instant in which the deformed droplet deviates from the oblate spheroid shape. This is illustrated in figure 3.12c where a frame is presented in which the protrusion around the droplet meridian marks the departure from the oblate spheroid shape. For large deformations, the droplet hydraulic diameter defined in relation (3.54) acquires a limiting closed form that simplifies the analysis. This is shown in Annexes where a simpler form of the breakup criterion (3.56) is developed in the limit of large deformations.

3.6 Results

The breakup model formulated in equation (3.56) has been tested in a series of 42 experimental test where the onset of breakup was targeted. These cases are considered representative of the total droplet analysis described in the section 3.3. In particular, the cases of the representative tests chosen involve models M1 and M2, table 3.1, with leading edge radius: 0.103 m , and 0.070 m , three airfoil velocities: 91 m/s , 81 m/s , and 71 m/s , and seven droplet radiuses: ranging from about 300 μm to 600 μm . This makes a total number of $2 \times 3 \times 7 = 42$ cases. The tables 3.3 and 3.4 shows the physical magnitudes of each case as well as the values of the dimensionless parameters of the corresponding case, according to equations (3.15) to (3.18) and (3.24). Finally, the experimentally measured value for the parameter Ψ for each case is shown. In order to characterize the latter, a procedure similar to the one described in the section 3.5.3 is followed, see:

- Firstly, the initial horizontal distance of the drop to the model (r_0) and its instant of rupture (t_{bk}), defined through the $db/dt = 0$ criterion, are obtained from the data analyzed for each case. Using also the relevant physical magnitudes of each case, the parameters Π_1 , Π_2 , Π_3 , Π_4 and Π_5 are determined.
- From these parameters, the numerical integration of the system of equations that constitutes the deformation model and the dynamic equation of the drop is carried

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out, from which both the trajectory and the acceleration are extracted. The integration is carried out using the ODE45 function in Matlab.

- The parameter Ψ vs τ is monitored, obtained by numerical integration of the equation (3.57), until the experimental dimensionless time of breakup $\tau_{bk} = t_{bk}U_m/R_c$, which corresponds to a value $\Psi(\tau_{bk})$.

Table 3.3: Measured Ψ values at the onset of the droplet breakup for M1 airfoil representative cases. The theoretical model predicts $\Psi = 1$.

Case	R_c (m)	U_m (m/s)	R_d (μ m)	Π_1	Π_2	Π_3	Π_4	Π_5	Ψ
1	0.103	91	597	13.74	0.0522	0.0211	0.39	1.95	1.10
2	0.103	91	550	16.19	0.0481	0.0229	0.46	2.30	1.11
3	0.103	91	525	17.77	0.0459	0.0239	0.50	2.21	1.10
4	0.103	91	450	24.19	0.0393	0.0279	0.69	2.13	1.00
5	0.103	91	396	31.23	0.0346	0.0317	0.89	2.15	0.98
6	0.103	91	359	38.00	0.0314	0.0350	1.08	1.74	0.95
7	0.103	91	302	53.70	0.0264	0.0416	1.52	1.29	0.88
8	0.103	81	580	14.56	0.0507	0.0273	0.46	1.82	1.10
9	0.103	81	549	16.26	0.0480	0.0289	0.52	1.90	1.04
10	0.103	81	485	20.82	0.0424	0.0327	0.66	1.32	1.04
11	0.103	81	450	24.19	0.0393	0.0353	0.77	2.25	1.03
12	0.103	81	427	26.86	0.0373	0.0371	0.86	2.35	1.03
13	0.103	81	348	40.44	0.0304	0.0456	1.29	1.23	0.94
14	0.103	81	300	54.42	0.0262	0.0529	1.74	1.05	0.93
15	0.103	71	580	14.56	0.0507	0.0356	0.53	1.90	1.05
16	0.103	71	551	16.13	0.0481	0.0375	0.59	1.76	1.06
17	0.103	71	504	19.28	0.0440	0.0410	0.70	1.70	1.00
18	0.103	71	447	24.51	0.0391	0.0462	0.89	1.99	0.99
19	0.103	71	402	30.31	0.0351	0.0514	1.10	1.09	0.97
20	0.103	71	357	38.43	0.0312	0.0578	1.40	2.36	0.92
21	0.103	71	304	53.00	0.0266	0.0679	1.93	2.00	0.88

Remember that, according to the breakup analytical criterion, Ψ should be 1 at the onset of breakup. Then experimental values of $\Psi(\tau_{bk})$ different than 1 in figures 3.13 and 3.14 give a direct indication of the discrepancies between experimental and theoretical results. When analyzing the data provided in tables 3.3 and 3.4, the following aspects could be noticed:

- The average measured value of Ψ for cases from 1 to 21 ($R_c = 0.103\text{ m}$) is 1.01, its standard deviation is 0.071, and the maximum and minimum Ψ values are 1.11 and 0.88 respectively. Then, it could be said that measured and predicted values of Ψ agree closely.
- The average Ψ value of cases from 22 to 42 ($R_c = 0.070\text{ m}$) is 1.08 and its standard deviation is 0.075. The maximum and minimum values Ψ are 1.20 and 0.94 respectively. These figures show larger discrepancies than the previous ones (Cases 1 to 21). The reason could be that the airfoil in Cases 1 to 21 has a leading edge radius of 0.103 m , while in Cases 22 to 42 the leading edge radius was 0.070 m . That is: the smaller the airfoil leading edge radius the smaller the region around the stagnation streamline where the flow could be considered one-dimensional, thereby weakening the consistency of the model hypothesis. That is: it is reasonable to expect that smaller airfoil leading edge radius lead to larger discrepancies.
- Within the measurements uncertainty, the breakup model accuracy is not affected by the velocity of the incoming airfoil, i.e.: it is not affected by scale changes in the continuously accelerated flow that impinges on the droplet. This means that the drag term that has been implemented in the droplet trajectory equations (made up of a steady term plus an unsteady one) reasonably models the actual drag effects.
- Typically, the largest discrepancies between model and experimental results occur when the largest droplets are considered. In this regard, it was observed that the larger droplets emitted from the droplet generator had a slight tendency to deviate from the perfect spherical form. Also, some of them were undergoing small shape oscillations as they were falling down through the air and approached the measurement window in the airfoil path. Typically, these droplets had a tendency to break at a time when this deformation was, still, not the one predicted by the model. Given the form of equation (3.57), this translates into a breakup value of Ψ larger than 1.

The next question to be addressed is the repeatability of the results. To this end, 9 other equivalent cases to Case 2 were chosen (the largest airfoil leading edge radius, the highest velocity, and a large droplet). Also in relation to Case 42 (the smallest airfoil, at the lowest velocity, with the smallest droplet) 13 repetitions carried out under equivalent conditions were chosen. The results obtained are presented in Table 3. It is to be noticed that these tests could not be repeated exactly with regard to the initial droplet radius. The reason is that it is not possible to control the droplet generator in such a way so as to produce exactly similar droplets. In the original Case 2 the droplet radius was $550\text{ }\mu\text{m}$. In the repeatability tests, droplet sizes ranged from $526\text{ }\mu\text{m}$ up to $560\text{ }\mu\text{m}$. In the original case 42, the droplet radius was $331\text{ }\mu\text{m}$. In the repeatability tests, droplet radius varied between $330\text{ }\mu\text{m}$ and $359\text{ }\mu\text{m}$.

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Table 3.4: Counterpart of table 3.3 for the M2 airfoil representative cases

Case	R_c (m)	U_m (m/s)	R_d (μ m)	Π_1	Π_2	Π_3	Π_4	Π_5	Ψ
22	0.070	91	567	7.04	0.0729	0.0102	0.29	3.03	1.18
23	0.070	91	558	7.27	0.0717	0.0104	0.30	2.41	1.10
24	0.070	91	522	8.30	0.0671	0.0111	0.35	2.55	1.14
25	0.070	91	451	11.12	0.0580	0.0129	0.46	2.87	1.06
26	0.070	91	403	13.93	0.0518	0.0144	0.58	2.98	1.03
27	0.070	91	347	18.79	0.0446	0.0167	0.78	2.33	1.02
28	0.070	91	303	24.64	0.0390	0.0192	1.03	1.83	0.95
29	0.070	81	591	6.48	0.0760	0.0124	0.30	1.83	1.20
30	0.070	81	522	8.30	0.0671	0.0140	0.39	2.99	1.13
31	0.070	81	507	8.80	0.0652	0.0145	0.41	3.05	1.20
32	0.070	81	450	11.17	0.0579	0.0163	0.52	1.61	1.09
33	0.070	81	397	14.35	0.0510	0.0185	0.67	2.98	1.05
34	0.070	81	347	18.79	0.0446	0.0211	0.88	1.63	1.04
35	0.070	81	296	25.82	0.0381	0.0248	1.21	1.50	0.94
36	0.070	71	614	6.00	0.0789	0.0155	0.32	1.82	1.11
37	0.070	71	553	7.40	0.0711	0.0172	0.40	1.81	1.16
38	0.070	71	514	8.56	0.0661	0.0186	0.46	3.14	1.12
39	0.070	71	444	11.48	0.0571	0.0215	0.61	1.33	1.07
40	0.070	71	410	13.46	0.0527	0.0233	0.72	2.63	1.08
41	0.070	71	347	18.79	0.0446	0.0275	1.01	1.73	1.01
42	0.070	71	331	20.65	0.0426	0.0288	1.11	1.79	1.00

Table 3.5: Results of the repeatability tests.

Case	R_c (m)	U_m (m/s)	R_d (μm)	Ψ	Case	R_c (m)	U_m (m/s)	R_d (μm)	Ψ
2A	0.103	91	546	1.10	42A	0.070	71	359	1.05
2B	0.103	91	534	1.06	42B	0.070	71	336	0.96
2C	0.103	91	526	1.08	42C	0.070	71	333	0.95
2D	0.103	91	560	1.12	42D	0.070	71	335	0.98
2E	0.103	91	546	1.10	42E	0.070	71	335	0.87
2F	0.103	91	546	1.06	42F	0.070	71	342	0.99
2G	0.103	91	532	1.04	42G	0.070	71	357	1.00
2H	0.103	91	536	1.12	42H	0.070	71	347	1.01
2I	0.103	91	560	1.09	42I	0.070	71	340	0.96
					42J	0.070	71	339	1.00
					42K	0.070	71	337	0.85
					42L	0.070	71	330	1.00
					42M	0.070	71	342	1.00

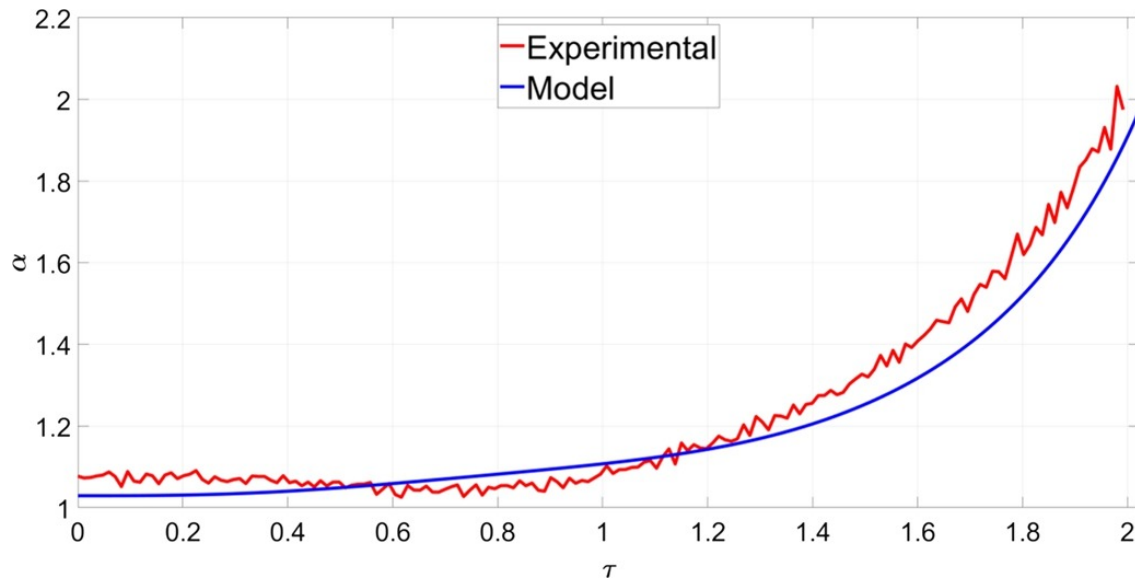


Figure 3.13: Comparison between measured and predicted α values of the deforming droplet as a function on dimensionless time τ for Case 2 until breakup dimensionless time τ_{bk}

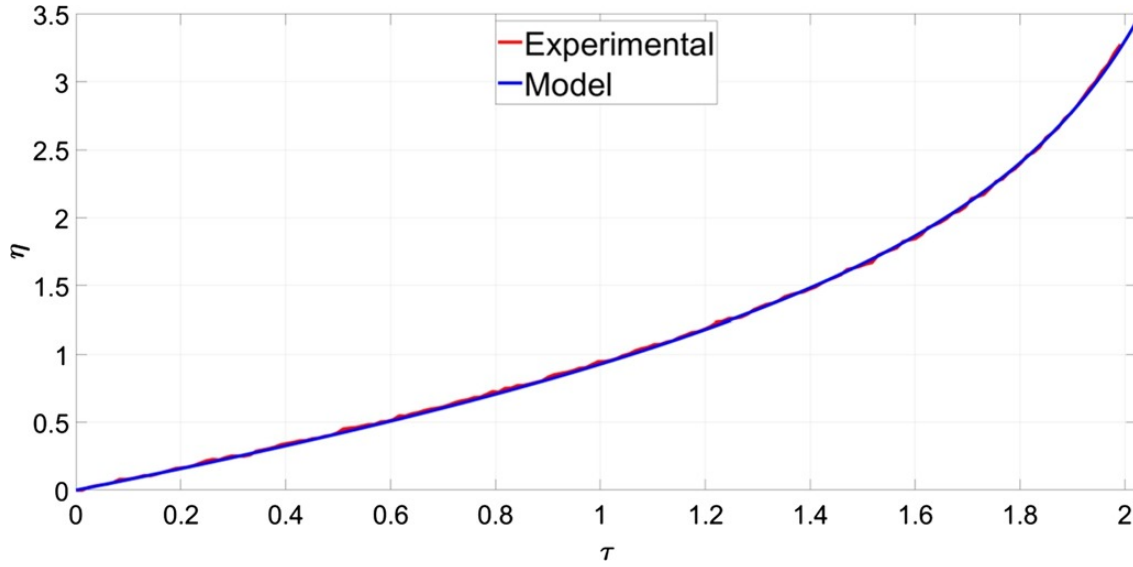


Figure 3.14: Comparison between measured and predicted η values of the deforming droplet as a function on dimensionless time τ for Case 2 until breakup dimensionless time τ_{bk}

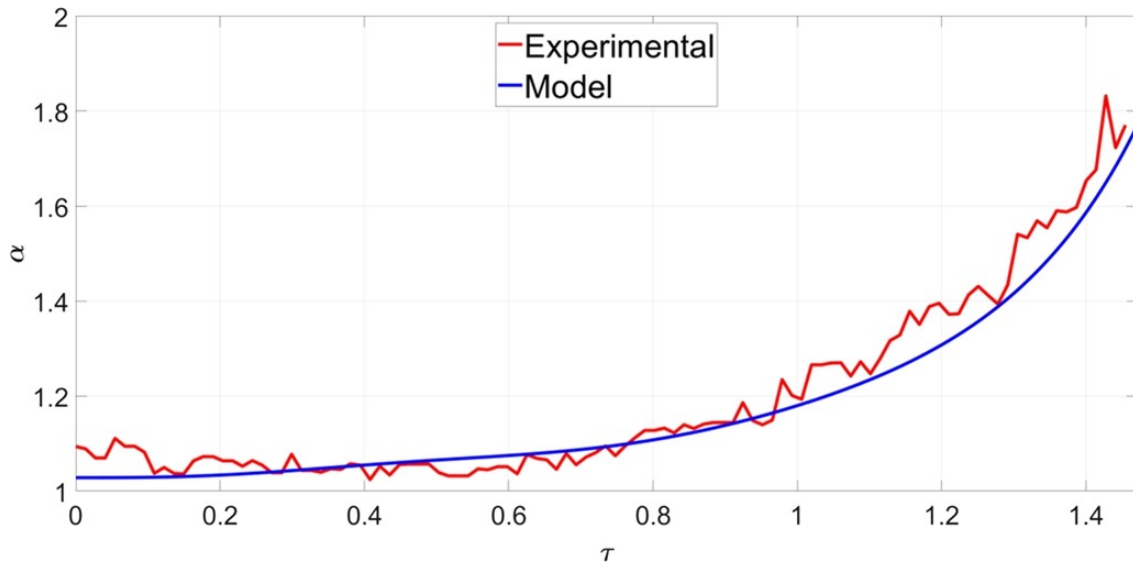


Figure 3.15: Counterpart of figure 3.13 for Case 42

The following aspects could be observed in table 3.5:

- In Cases 2A to 2I, the average Ψ at the breakup was 1.09, the standard deviation was 0.028, and the maximum and minimum values were 1.12 and 1.04. So, basically, all repeatability test fit in a band of $\pm 3\%$ around their mean.
- In Cases 42A to 42M, the average Ψ at the breakup was 0.98, the standard deviation was 0.056 and the maximum and minimum values were 1.05 and 0.85. So the repeatability test fit in a band of about $\pm 13\%$ around their mean. Again, this larger band

could be caused by the fact that Cases 42A to 42M are further away from the model hypothesis that Cases 2A to 2I.

Comparison of results for Case 2 in table 3.3 are presented in figures 3.13 and 3.14 respectively. In particular, figure 3.13 shows the comparison between the measured and predicted evolution of α (droplet shape) as a function of the dimensionless time τ . The serrated form of the experimentally measured α evolution has to do with the method (already described in section 3.3.1) to translate a pattern of pixelated grey shading into an equivalent spheroid shape. figure 3.14 shows the comparison between measured and predicted evolution of the droplet trajectory. Both comparisons shown in figures 3.13 and 3.14 show a reasonable agreement between experiments and models. The same type of comparison is presented in figures 3.15 and 3.16 for Case 42 (table 3.4). In this case, the local discrepancies are larger but the global comparison in terms of the value of Ψ at the onset of breakup is still reasonable.

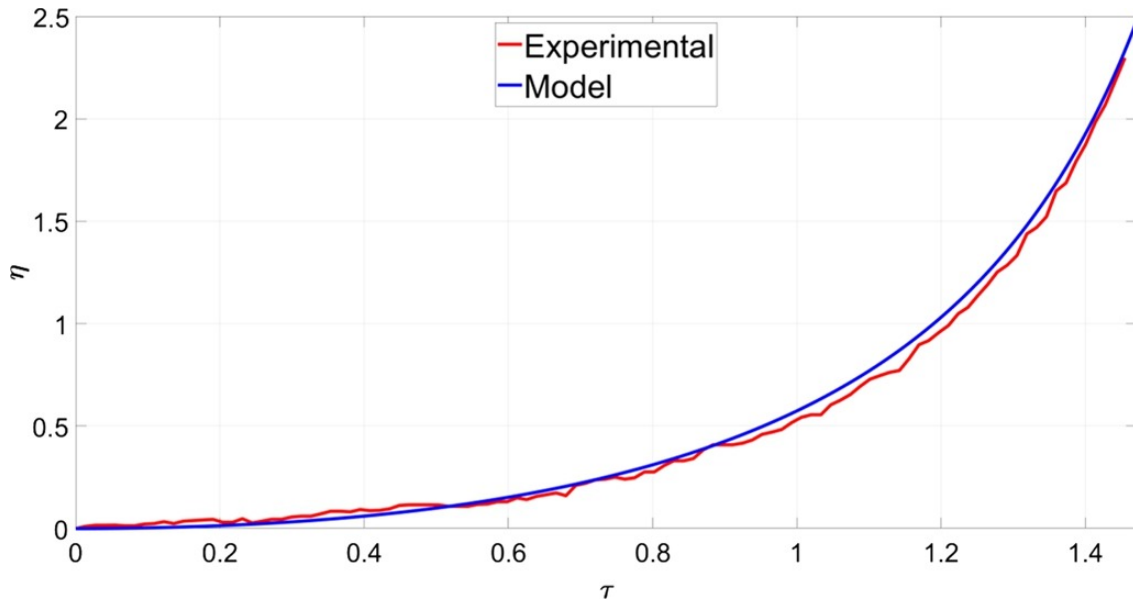


Figure 3.16: Counterpart of figure 3.14 for Case 42

3.7 Application case

The application case presented hereafter characterizes the breakup pattern of a series of water droplets placed at different horizontal positions away from the incoming airfoil. These initial positions are defined by a new dimensionless parameter Π_5 . Regarding the vertical direction, all droplets are placed initially at a distance of $0.1 R_c$ above the stagnation streamline. Computation is carried out either until the droplet falls at a distance of $0.1 R_c$ below the stagnation streamline, or breaks, or is hit by the airfoil. figure 3.17 illustrates the basic configuration chosen for this application case. To keep the illustration lighter, only a few droplets have been plotted. However, the number of actually computed droplets was 100 per case.

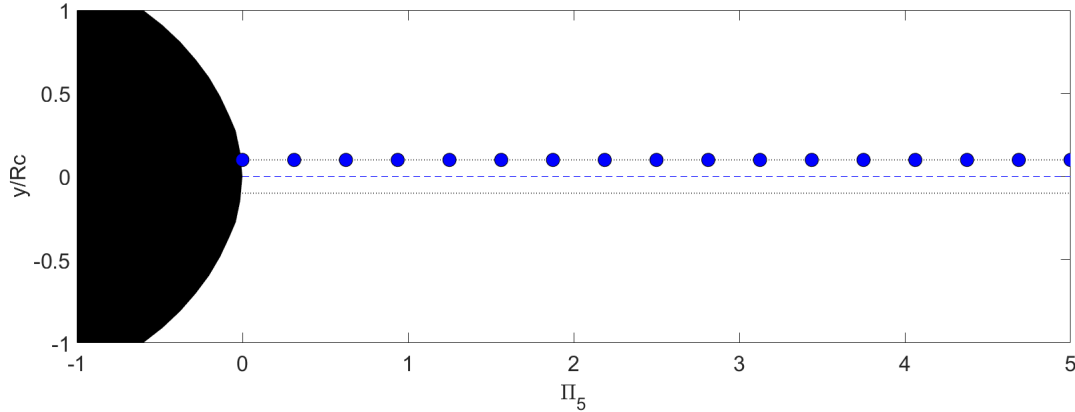


Figure 3.17: Sketch of the configuration chosen for the application case.

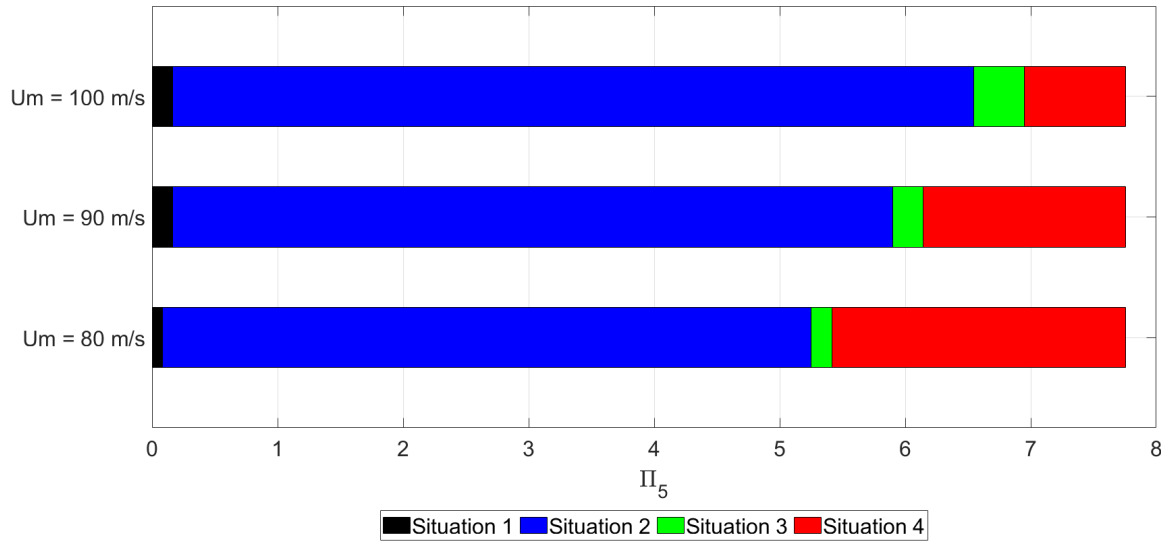


Figure 3.18: Predicted $500 \mu m$ radius droplet situation for three different airfoil velocities, equivalent to M1 shape, as a function of Π_5

The residence time t_r of a droplet inside the $\pm 0.1R_c$ vertical region is $t_r = 0.2R_c/U_t$, where U_t is the free fall terminal velocity of the droplet (it is assumed that droplets reach the computational region at their free fall terminal velocity) computed by means of equation (3.6). Integration of the model is carried out from $t = 0$ up to $t = t_r$ unless the droplet reaches breakup conditions, or the airfoil reaches the droplet. That is, the following situations may occur during integration:

- Situation 1: The residence time t_r of the droplet in the $\pm 0.1R_c$ vertical region is larger than the time needed for the airfoil to reach the droplet. In this case, the droplet impacts the airfoil in the shape of an oblate spheroid.
- Situation 2: the droplet reaches the critical conditions for the onset of breakup before it is reached by the airfoil. Two situations can arise as a consequence of the above. The

first is that the droplet initiates the breakup process but does not have enough time to complete it, so that it is impacted without having generated secondary droplets. The second option is that the droplet breaks before it is hit by the airfoil. However, assuming that the centroid of the secondary droplets distribution moves with the same velocity of the original droplet, the airfoil reaches the cloud of secondary droplets before it leaves the $\pm 0.1R_c$ integration region. In this case, a cloud of secondary droplets impacts the airfoil.

- Situation 3: The droplet presumably breaks before reaching the residence time t_r and leaves the $\pm 0.1R_c$ integration region without been hit by the airfoil.
- Situation 4: The droplet enters and leaves the $\pm 0.1R_c$ integration region without either breaking or being hit by the airfoil.

Figure 3.18 shows the outcome of three different cases: droplets having an initial radius 0.5 mm being approached by an airfoil with $R_c = 0.103\text{ m}$ at three different velocities: 100 m/s , 90 m/s , and 80 m/s . The Π_5 parameter (dimensionless initial distance between droplet and airfoil) was varied between 0 and 8.

It could be observed that the proposed method allows for a quantitative prediction of the fraction of droplets that go through one of the four above described situations. For the droplets size and airfoil velocities being considered in this application case, it is found that most of the falling droplets located within a distance of approximately 6 leading edge airfoil radiuses reach breakup condition and impact the airfoil afterwards. The method could easily be adapted to the case where a certain distribution of droplet sizes is used instead of a series of droplets of the same size. The computed fraction of droplets that impact the airfoil (either as a deformed oblate spheroid or as a cloud of secondary droplets) could be used to estimate the deterioration of the airfoil aerodynamics performance.

3.8 Conclusions

A droplet trajectory and deformation model has been coupled to a droplet breakup model. The objective has been to predict the onset of droplet breakup when an incoming airfoil is approaching. The breakup model is based on an instability analysis that computes the characteristic parameters associated to the growth of disturbances. The main hypothesis that has been made in the model is that the critical wavelength that signals the onset of droplet breakup is the hydraulic diameter of the deformed droplet (assuming that it deforms as an oblate spheroid). This has led to a single analytical relation that is monitored during time integration of the trajectory and deformation droplet dynamics equations. When this relation reaches a certain value, the droplet is assumed to be in breakup conditions.

The theoretical model predictions have been tested experimentally under a variety of conditions in a rotating arm facility. In particular, two airfoil leading edge radiuses: 0.103 m , and 0.070 m , three airfoil velocities: 91 m/s , 81 m/s , and 71 m/s , and seven different droplet radiuses: ranging from about $300\text{ }\mu\text{m}$ to $600\text{ }\mu\text{m}$, were considered. For the airfoil with the

largest leading edge radius (closer to the model hypothesis) it was found that experimental and model discrepancies in the breakup parameter were of the order of $\pm 10\%$. For the airfoil with the smaller leading edge radius (thereby further away from the hypothesis of the model) discrepancies were of the order of $\pm 20\%$. Uncertainty of the experimental setup was estimated to be of the order of $\pm 10\%$. Repeatability tests were carried out for some representative cases and it was found that, depending on the case, repeated cases fitted within a band of the order of $\pm 3\%$ to $\pm 13\%$ around their mean.

All this means that the proposed model could be used with reasonable confidence when trying to predict what situation is to be encountered by an airfoil that approaches a distribution of droplets (for instance, in the fashion of falling rain). This has been illustrated by presenting an example in which the model predicts what is the fraction of droplets that go through one of the following situations: 1) the spheroidal droplet impacts the airfoil, 2) the droplet reach breakup condition and possibly breaks up and the ensuing cloud of secondary droplets impact the airfoil afterwards, 3) the droplet possibly breaks up but it is not impinged by the airfoil, and 4) the droplet neither breaks nor it is impinged by the airfoil. Note that situations 1 and 2 have a significant influence on the aerodynamics behavior of the airfoil.

Because of the reasonable agreement that has been found between the theoretical model and the experiments, it could be worth to evolve the model to account for droplets that are not necessarily restricted to be around the airfoil stagnation streamline region. In this case, it would be necessary to account for the shear in the incoming airflow which might lead to rotation of the droplets. This would, possibly, require the introduction of another droplet equation for the rotation dynamics and some changes in the breakup model. If this is to be achieved, it would be possible predict airfoil aerodynamics performance under rain conditions accounting for the whole region around the airfoil.

Chapter 4

Experimental study on the impulsively started motion of a freely decelerating sphere

4.1 Introduction

One of the main conclusions of the chapter 3 is the impact of acceleration on the droplet breakup, revealing the importance of acceleration in determining the breakup onset in the form of the breakup criterion, equation (3.56). Thus, characterizing the acceleration accurately becomes critical, given its impact on the droplet deformation and the complex coupling of deformation and break-up. The character of this coupling means that regardless of the flow, we are faced with an inherently non-stationary problem, where the establishment of the flow around the droplet influences the pressure on its surface and therefore its kinematics, deformation and subsequent breakup. Considering the flow of interest in our study, where the base flow presents a continuous acceleration, but at the same time considering the great variety of flows of technical interest, the need of understanding the underlying physics of these non-stationary flow establishment problems arises.

Regarding previous works, although the flow over a sphere is a well-established problem in fluid mechanics, a problem that continues to be extensively studied is the case of such bodies accelerating or decelerating impulsively. The study of this canonical form under non-stationary motions goes back to the beginning of the century, with the study of Boltze [132], until today: Fernando et al. [119, 120], Jetly et al. [121], Cross [122] and Galler et al. [123]. In relation to the establishment of the flow over impulsively accelerated blunt bodies, the evolution of the separation point sets the transition from the potential flow solution to a more complex topology. The study of the evolution of this separation point in the initial instants after impulsive acceleration has traditionally been studied analytically from inner and outer boundary layer expansions, for example the works of Wang [201] and Collins and Dennis [137] for the case of the cylinder and Dennis and Walker [135] for the sphere. In the

latter, they obtain an analytical solution showing that the separation occurs, as a function of the Reynolds number, for values of the separation time parameter $t_0 \cdot U_\infty / R$ between 0.3 and 0.4. For decelerating spheres Kim et al. [202] numerically solve the Navier-Stokes equations for the non-stationary axisymmetric flow around a sphere for $5 < Re < 200$. The conclusion is that although the sphere decelerates and thus its instantaneous Reynolds number decreases, there is a recirculating eddy structure in its rear section that increases, with a size significantly larger than that of its stationary solution. For both accelerating and decelerating conditions Wakaba and Balachandar [150] numerically study the results of a sphere moving at Re around 50 varying its velocity impulsively, revealing that at these instants the potential flow is superimposed on the previous flow field. As a conclusion, although at moderate Reynolds numbers, the potential solution around the sphere is shown to be the starting point for analyzing these start-up processes.

When analyzing the separation point, one of the main problems is that the vast majority of these studies use the infinitely thin laminar boundary layer approximation. Under these conditions the vorticity would be limited to an extremely thin region around the sphere that allows modeling the pressure distribution along the sphere on the basis of the potential solution. This limits the analyses to the initial instants, so that in order to study its evolution (and in particular under conditions of sustained acceleration/deceleration in time) it is necessary to make use of experimental studies. Roughly speaking, these studies are usually divided into forced motion studies (Odar and Hamilton [118], Kendoush et al. [160], Fernando et al. [119, 120] and Kamoliddinov et al. [167]); and free motion studies (Viets [161] and Horowitz and Williamson [162, 163]). From their results it has been drawn that one of the most relevant features to be analyzed is the flow field topology, showing that most of them contain a toroidal ring-shaped structure downstream. From the qualitative analysis of these structures, their dimensions and their evolution over time, it becomes clear how strongly they influence the hydrodynamic forces experienced by the body in non-stationary motion.

In this aspect, the recent studies by Fernando et al. [119, 120] for continuously accelerated spheres in towing tank type facilities are noteworthy. Their analysis of the detachment point and the topology of the flow field allow comparing the results with those of the present study, revealing that the evolution of the same body has a significantly different character depending on its dynamics. Finally, it is interesting the work of Galler et al. [123] where a new model of added mass applied to flows with massive detachment, called energized mass, is applied on experimental results of accelerating spheres. In all of them, the application of Time Resolved Particle Image Velocimetry (TR-PIV) visualization techniques has been shown to be very effective in the qualitative evaluation of the structures and, in some cases, even the derivatives of fluid quantities (such as vorticity).

In the study presented in this chapter, fluid-filled spheres will be used to modify the density ratio m^* and a new experimental setup will be used to impart impulsive accelerations to allow the sphere to decelerate freely. Unlike other studies where the movement of the

sphere was imposed by means of a prescribed law of motion forced from an actuator, in our case the kinematics of the sphere will be the consequence of the interaction between the flow field and the sphere's own dynamics, and therefore the evolution of both will be intrinsically coupled. This experimental configuration is novel compared with other configurations found in the literature, combining an impulsive acceleration followed by a free deceleration within a fluid at rest.

The chapter is structured as follows: firstly, in the section 4.2, the experimental setup is described, including the test samples and the acquisition systems used for the study of the sphere dynamics. Subsequently, the section 4.3 describes the test procedure and the image tools developed for the tracking of the sphere. At the end, test matrix is defined and an analysis of the uncertainty and repeatability of the system is made. Results of the sphere's kinematics are presented in section 4.4. In section 4.5 the setup and preprocessing tools for the TR-PIV analysis is described which has allowed to get insight into the flow topology. TR-PIV visualizations and an analysis of both the evolution of the fluid structures and the separation point onset are the subject of section 4.6. Results obtained and data reduction are discussed in section 4.7. Finally, conclusions are outlined in section 4.8.

4.2 Description of the experimental setup

The problem consisted of studying the motion and the flow field topology for a sphere moving in a fluid at rest. In relation to the type of motion to be studied, this is characterized by being initiated impulsively, so that the sphere subsequently freely reduces its velocity. The system therefore has two main requirements; firstly, it must be capable of delivering this linear momentum almost instantaneously, generating a unidirectional movement in the sphere and therefore it must not transmit any torque that could lead to its rotation. On the other hand, this momentum transfer must be carried out in such a way that the flow field is only disturbed under the action of the motion of the sphere. For this purpose, an experimental installation is designed, inspired by the drop towers used in impact absorption tests, which transmits momentum to the sphere by means of, as will be seen later, a collision. This installation is located in the Reciprocating Engines Lab, belonging to the "Departamento de Mecánica de Fluidos y Propulsión Aeroespacial" of the ETSIAE, UPM. Specifically, it is installed in a room of the laboratory with darkening screens used to carry out tests with PIV techniques.

4.2.1 Water tank, actuator system and main structure

The equipment is composed by three main elements: the still water tank, the actuator system and the main structure. An actual view of the setup can be found in figure 4.1. Regarding the water tank, it is made of 20 mm thick laminated glass with external dimensions of 1.0 m × 1.0 m × 1.0 m which includes a set of reinforcements in its upper part to withstand the hydrostatic pressure, being its useful volume of 0.9 m³. To guarantee its leveling, prevent irregular ground contact and dampen vibrations that may be transmitted

through the floor, the volume rests on two rigid extruded polystyrene styrofoam (XPS) sheets 80 mm thick that settle the volume.

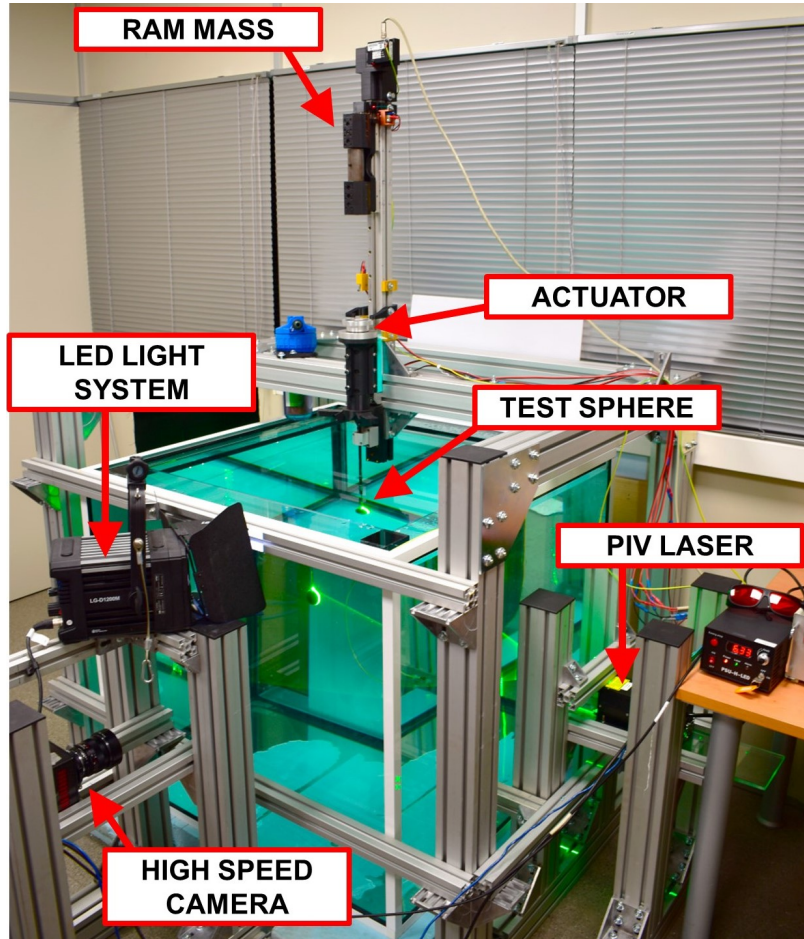


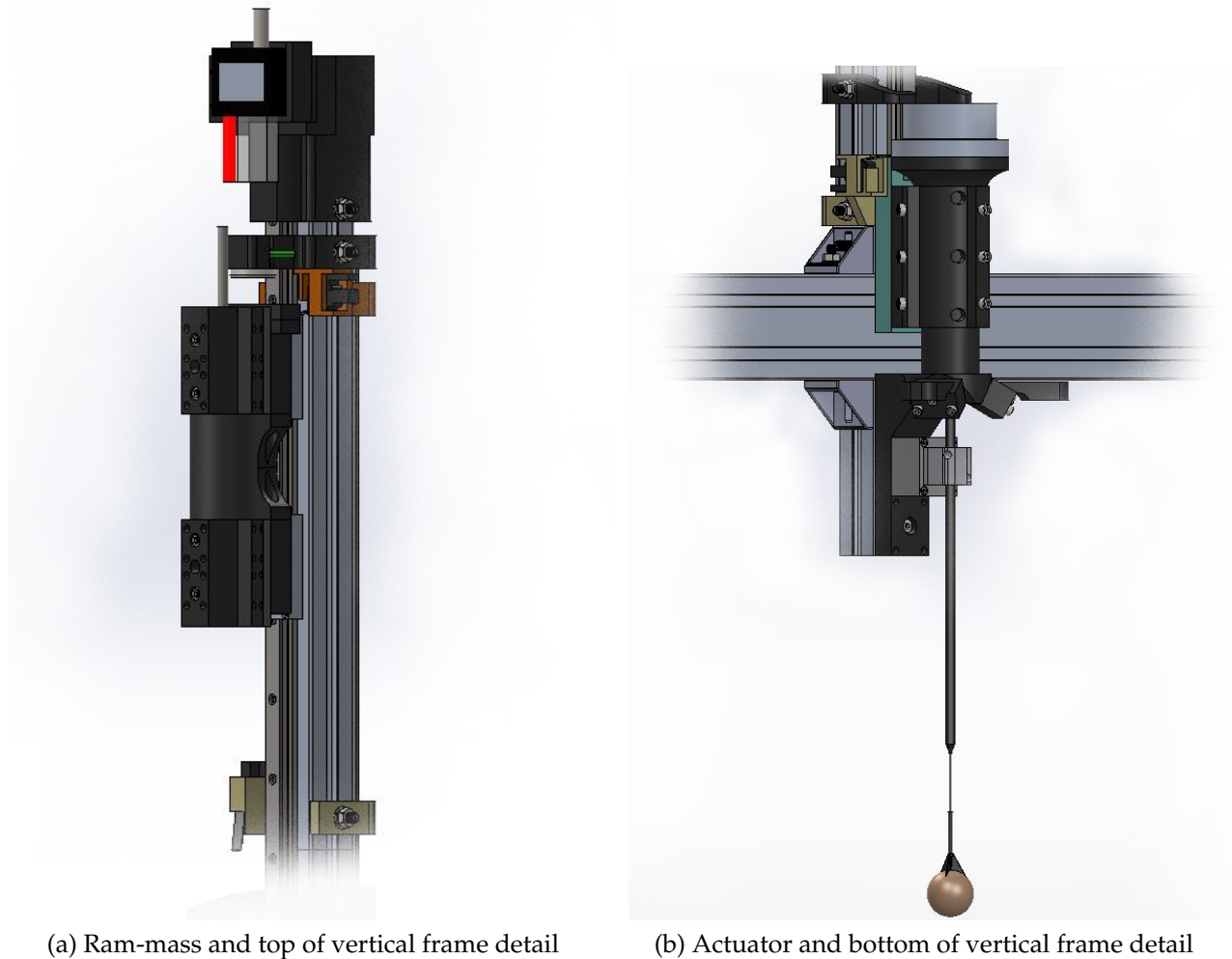
Figure 4.1: General view of the setup. The main elements that compose it are referenced in the image above.

The actuator system is composed by a set of mobile elements whose purpose is to deliver an impulsive and vertical movement to the test sphere. It is made up of two subsystems: the ram mass and the actuator system. The purpose of the ram mass is to transform the potential gravitational energy into kinetic energy. Specifically, it is an assembly with a mass of $m_{ram} = 1.69 \text{ kg}$ which is allowed to fall from a height (h_{drop}) of up to 0.27 m with respect to a retention system which stops its movement. At part assembly level (figure 4.2a), the ram mass is comprised of two ball recirculation slide blocks joined together by means of a set of parts manufactured in PLA by 3D printing, on which a cylindrical iron mass is fixed. The use of two recirculation blocks avoids pitching movements on the assembly. At the upper end, an iron plate is integrated which, in combination with an electromagnet that is fixed in the main structure, prevents it from falling until the start of the test sequence. Thus the ram mass sub-system moves during its free fall on a precision linear guide which is mounted on a vertical structure.

CHAPTER 4. EXPERIMENTAL STUDY ON THE IMPULSIVELY STARTED MOTION OF A FREELY DECELERATING SPHERE

The second subsystem, the actuator system (figure 4.2b), has a significantly lower mass, 0.489 kg , and has two main purposes; the first is to keep the test sphere still and submerged at its end until the moment of impact; the second is to transmit to the sphere the momentum caused by the impact of the ram mass, converting it into a unidirectional and vertical movement. In this way, once the actuator system with the sphere at its lower end is reached by the ram mass as it falls, it moves 2 mm in the vertical axis until it reaches a retention system, ejecting the ball at the instant of the first impact. To guide its displacement in the vertical axis, a combination of a sliding bearing, fixed to the structure at its lower end, and a self-lubricating material made bushing, installed at its upper end, is used, thus avoiding the bending of the assembly as it has two contact points. In relation to the constructive elements of the actuator system, this is mainly made up of an aluminium plate, an extension rod and a centring-support cap. The main function of the aluminium plate is to receive the impact of the ram mass and to act as a flat surface against the retention plate. By means of a clamping bush, the extension rod is fixed to the aluminium plate. Extension rod is made of stainless steel and divided into two sections; a first section of length 550 mm and diameter of 8 mm ; and a second section of smaller diameter (2 mm) and 50 mm length at the end of which the centring-support cap is mounted. The difference in diameters is fundamentally designed to achieve a high stiffness while maintaining a small volume of water displaced in the 2 mm distance of travel, so the interaction with the fluid at rest is minimized. Finally, it is necessary to describe the centring-support cap, whose function is to support the test sample, preventing it from rising due to buoyancy, and to center it at the moment of impact, ensuring that the impulsive movement is in the vertical direction. It consists in a 3D printed piece with three slender support legs so that their influence on the flow is minimal. Total length of the actuator is 675 mm , so sphere can be submerged 280 mm below water surface. To avoid light reflections, bottom part of the assembly is painted matt black.

In order to guide the displacement and stop the motion of the ram mass and the actuator system, both elements are installed on a structural assembly consisting of a base frame and a vertical frame. The former is made from $90\text{ mm} \times 90\text{ mm}$ square section anodized aluminum extruded profiles with slots for T-nuts fastening. The profiles use 90° aluminum brackets as jointing elements and steel plates are used to stiffen the assembly. A gantry structure of overall dimensions $1200\text{ mm} \times 1380\text{ mm} \times 1380\text{ mm}$ ($H \times D \times W$) is thus formed, which front beam is positioned in such a way that the actuating system transfers the momentum at the center of the water volume. The structure has four support legs resting on anti-vibration pads made of neoprene. The structure incorporates a series of $45\text{ mm} \times 45\text{ mm}$ profiles as references to assure that the structure, the actuator system and the acquisition systems are located at the desired distance and plane.



(a) Ram-mass and top of vertical frame detail

(b) Actuator and bottom of vertical frame detail

Figure 4.2: (a) Detail of the ram mass, triggers and laser position sensor that integrate the upper part of the vertical frame. (b) Detail of the actuator system, retention plate, guiding system and fastening of the vertical frame to the main structure.

Regarding the second element of the structural assembly, the vertical frame, it consists mainly of an aluminum profile 550 *mm* long and 45 *mm* × 45 *mm* section that is installed in the middle of the front beam by means of brackets and 3-D printed auxiliary fastening block. To increase its stiffness, a steel profile is installed on its rear face. At the top of the vertical frame, a 3-D printed head is installed on which an Acuity AP-522 Laser displacement measurement system is integrated to measure the displacement of the ram mass and, from its derivative, its speed. This sensor provides a voltage output of 0 – 10 *V* with a maximum range of 500 *mm*. On the previously mentioned auxiliary mounting plate an assembly is installed that incorporates the retention frame, in the form of a hollow cylinder in which the actuator system moves and that integrates all the sliding guides for it. The upper part of the retention frame consists of an aluminum disk that can be adjusted by means of calibrated thickness sheets to ensure that the contact between its surface and the lower face of the actuator system aluminum plate is in the same plane. This eliminates any pitching movement of the actuator system when stopped that could deviate the test

sphere from its vertical trajectory. A set of discs made of neoprene and butyl are installed on the disc, which act as dumpers and prevent the ram-mass and actuator assembly from bouncing after the collision with the retention plate.

Due to the characteristics of the test, the operation of the system requires a series of auxiliary elements that, installed in the vertical frame, act sequentially as the test develops. Specifically in order of actuation it would be as follows:

- A height adjustable system that supports the ram mass until it is released. It is a piece manufactured by 3D printing in PLA, which is fixed by T-nuts to the vertical frame and in whose center is located an electromagnet. This, when powered by electric current exerts an attractive force on the iron plate fixed on the ram mass holding it. In this way, when the power supply is interrupted by manual operation of a switch, the ram mass falls by gravity, initiating the test sequence.
- A normally closed (NC) micro limit switch, installed in a 3D printed auxiliary fixture, is actuated at the initial position of the ram mass. When this switch loses contact with the ram mass, it sends a signal that triggers the acquisition of ram mass positioning data acquired by the laser sensor.
- A normally open (NO) micro limit switch acts as a trigger to start the acquisition of images by the high speed camera and is installed on a auxiliary support, that is adjustable in height. It is placed, depending on the ram-mass expected speed prior to the collision, at distances between 5 and 10 *mm* from the actuator system holder to prevent recording delays that could cause the loss of information in the initial instants of the spheres movement.
- A permanent magnetic holder made in PLA that holds the actuator system 2 *mm* away from the retention plate by means of two pairs of neodymium cylindrical magnets of 5 *mm* diameter and 1 *mm* thickness. In this way one of the pairs is adhered with cyanoacrylate on the magnetic holder and the other one on the upper part of the aluminum plate of the actuator system, avoiding its fall until the moment of the impact. Like the rest of the elements it is fixed by means of T-nuts to the vertical frame.

For the acquisition of the ram-mass position measured by the laser sensor and of the different events resulting from the actuation of pushbuttons or microswitches, the acquisition system National Instruments NI-7826 is used, which is programmed through the specific MATLAB toolbox. The CAD software SolidWorks 2018 was used for the design of the mechanical components and ANYCUBIC Delta Kossel Linear Plus was used for the 3D printing of the components. The remaining mechanical elements were manufactured by the department's workshop staff and resources. The main elements of the experimental setup are summarized in a schematic in the figure 4.3.

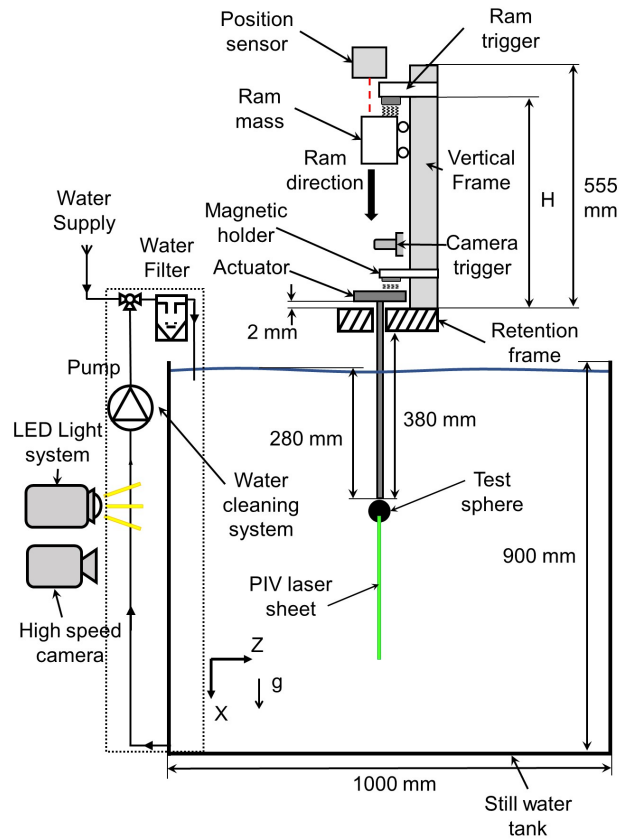


Figure 4.3: General view of the setup. The main elements that compose it are referenced in the image above.

4.2.2 Test sample spheres

Table tennis balls, made mainly of acrylonitrile butadiene styrene, with a diameter of $D = 40 \pm 0.5 \text{ mm}$, filled with liquids, are used as a sample of spherical geometry. The balls are filled with three materials of different densities, 95 octane petrol, diesel fuel, and distilled water, all at a temperature of 20°C . For this temperature the approximate densities are respectively 732 kg/m^3 , 840 kg/m^3 and 998 kg/m^3 . For its filling the following process is carried out: first the sphere is fixed with clamps and two punctures are made in its upper end, very close to each other, using $0.60 \times 30 \text{ mm}$ hypodermic needles. Then, by means of a syringe, the material is injected through one of the holes, allowing the air inside the sphere to escape through the other. Once it overflows for the first time, a large part of the pressure of the jaws is released, allowing air to enter and, at the same time, extracting the needle while keep injecting material. The overflowing remnant from the holes is cleaned using a gauze with solvent and once evaporated, a mixture of cyanoacrylate with $5 \mu\text{m}$ fiberglass particles is used to seal both holes. After the application of the sealing mixture, the pressure of the jaws is completely released so that, by pressure difference, the sealant is introduced into the holes. After 2 hr the generated protrusion is eroded with fine-grit sandpaper until it is smooth. The sample is weighed in a precision balance and is waited

two days to see that its mass has not changed, pressing it to detect leaks. Once validated, matt black acrylic paint is applied by airbrush so that the thickness of the coating is below the manufacturing tolerance previously indicated. To characterize the different samples, the solid-to-fluid density ratios parameter is used, defined as:

$$m_i^* = \frac{m_i}{\rho_w \pi D^3 / 6} \quad (4.1)$$

Where m_i is the mass measured on the precision balance, ρ_w is the reference density of the medium, that is considered as 1000 kg/m^3 . It is worth mentioning that several samples had to be manufactured as a consequence of their use, leaks and mainly in the case of petrol-filled samples due to their capability to attack acrylonitrile butadiene styrene. Table 4.1 summarizes the relevant information related to the samples.

Table 4.1: Summary of the main characteristics of the deceleration test samples

Test sample	Filling material	Produced samples	m_i (kg)	m_i^*
m_1^*	95 octane petrol	6	0.0261 ± 0.0004	0.779 ± 0.012
m_2^*	Diesel fuel	3	0.0295 ± 0.0002	0.880 ± 0.006
m_3^*	Distilled water	2	0.0314 ± 0.0002	0.937 ± 0.006

Regarding the initial position of the sphere, as previously mentioned, it is submerged at 0.28 m , equivalent to $7 \cdot D$, below the water surface. Initially, the distances between the sphere and bottom and lateral tank walls were 620 mm ($15.5 \cdot D$) and 500 mm ($12.5 \cdot D$), respectively. It was estimated that these distances minimized possible interferences between solid walls, free water surface, and sphere motion. A crude estimate of the wake width, w_{wake} , is $w_{wake} \sim C_d \cdot D$. A sphere typical drag coefficient $C_d = 0.5$ yields a wake influence length of about $5 \cdot D$ that is smaller than the minimum distance ($10 \cdot D$) between sphere and tank walls during the experiments. On the other hand, the blockage ratio (defined as sphere cross-sectional area ($\pi D^2/4$) vs tank cross-sectional area ($1.0 \times 1.0 \text{ m}^2$)) was less than 0.01. As a reference, the blockage ratio in the experiment of [119] was 0.05. The minimum distance between sphere and walls was about 10 diameters that is larger than in the experiments of Fernando et al. [119] (four diameters). A time span of 2 hr was initially allowed between each experiment to let water become still again, finding that time spans of 30 min were enough when studying the sphere kinematics and the initial flow field in the TR-PIV tests.

4.2.3 Data adquisition system for sphere dynamics setup

To acquire the trajectory of the spheres, a high-speed camera has been used, which is mounted on a polycarbonate plate adjustable in inclination by means of wedges that is fixed to an aluminum profile structure resting on neoprene anti-vibration pads. The assembly can be moved along the aluminum profile guides installed on the main structure.

Specifically, the high-speed camera consists of a Photrom FASTCAM Mini AX100 type 540-K-M-8GB, capable of acquiring up to 540000 *fps* with a resolution of 128×16 pixels and images at a full resolution of 1024×1024 pixels when operating up to 4000 *fps*. The camera is equipped with a CMOS sensor with pixel sizes of $20 \mu m$, high light sensitivity (ISO 40000 in monochrome) and high quantum efficiency (QE) of up to 46% for red light of 630 *nm*. This makes the camera functional for TR-PIV testing, as described in the section 4.5.1. Regarding its storage capacity prior to data transmission, it can acquire up to 5457 images at full resolution. For our setup the camera is equipped with a Carl Zeiss Makro-Planar 2/50 ZF lens which for the sphere trajectory tests is configured with a focal aperture $f/16$. The camera is set to acquire images at a frequency of 500 *fps* thus allowing to work at maximum resolution. The magnification of the system is $M_0 = 2.902 \text{ px/mm}$, the method for obtaining it is detailed in section 4.3.1. For this magnification the area of interest (AOI) has physical magnitudes of $0.352 \times 0.352 \text{ m}^2$. Finally, to avoid blurring of the sphere's image that may affect the image processing tool, the shutter speed is set to $1/5000 \text{ s}$. The images are stored in *.tif* format in 16-bit greyscale.

Regarding the illumination system, a LED system has been used (LG-D1200M, 120 W light power and light temperature of 5600 K) which is placed on the camera support structure and rotated at 45° in the horizontal plane with respect to the front face of the tank in order to avoid reflections on the laminated glass. A white screen with a glossy finish is placed in the background, so that in combination with the matte black paint of the sphere a high contrast, similar to that achieved from the shadowgraph techniques, is provided allowing to accurately measure the position of the center of the sphere (X_{sp}, Y_{sp}) and its radius R_{sp} by means of the image procesing tool, section 4.3.1.

4.3 Test procedure and data analysis

The test procedure is as follows: firstly, before the tests are carried out, the condition of the spherical samples is checked to look for inner leakages, by using a precision scale and comparing it with the original weight. On the other hand, to exclude the presence of significant surface deformations, a caliper is used to assess the sphericity of the samples. Then, ram mass and actuator system are placed in its initial position. Afterwards the LED system is switched on at full power and the ambient lights are switched off to avoid fluctuations in light intensity. With the LED light on and the sphere at its starting position, the focus of the camera is adjusted. After the time required for the water to be still has elapsed, the ram mass is released and the position data acquisition is started. As the ram mass continues to fall, the system triggers the camera to start recording images. Immediately after, the actuator system is struck, generating a collision that propels the sphere. Actuator and ram-mass collide against the retention plate and the energy is dissipated from the structure to the ground. In the meantime the camera continues to store images until the buffer is filled and acquisition of the laser position measurement system is stopped. The images are then checked, verifying that the movement is vertical and mostly unidirectional. If considered valid, the dataset is stored according to the following designation: tested mass (M), the initial velocity (V) and the corresponding repetition (R) for subsequent

pre-processing. Once this data is stored, it is moved on to the next test point, repeating the process.

The following section will describe the tool developed for the analysis of the acquired images, the methodology for image calibration and the characterization of its accuracy and precision. Subsequently, in the section 4.3.2, it is described how the test matrix has been designed and the initial conditions of the set of tested cases.

4.3.1 Data analysis of the spheres trajectory and its derivatives

For the measurement of the sphere trajectory, an image tool has been developed that includes the preprocessing in MATLAB and the analysis of the image set, composed of N_{im} frames, related to a run. Hereafter, the X-axis refers to the vertical direction and the Y-axis to the horizontal direction. For more information on some of the transformations, morphological operations and segmentation methods applied, please consult González and Woods [203]. Specifically, the tool consists of the following steps:

1. First, a reference image (I_r) composed of the first and last frame of the set is generated for each run. Considering the image resolution $m \times n$, an image (I_r) is generated with the pixel values of the rows $m/2$ to m corresponding to the first frame of the set and those pixel values of the rows 1 to $m/2 - 1$ of the last frame of the set. As a result we have the figure 4.4b.
2. A Moving Area of Interest (MAOI) region is generated, which limits the area of the original frame (4.4a) to be analyzed (I) in which to apply the image analysis. For this purpose, a square region is calculated, centered on a point (X_{AOI}, Y_{AOI}) , whose location is estimated based on the position and displacement of the previous frame. The dimensions of the region will be $3 \cdot R_{sp}$, calculated through the sphere position analysis of the previous frame. The cropping (C) is applied to both the original frame ($I_c = C(I)$) and the reference image frame ($I_{rc} = C(I_r)$).
3. Then, the transformation T_n is applied to obtain the complement image of I_c and I_{rc} and with both resulting images a background subtraction is applied: $I_{bs} = T_n(I_c) - T_n(I_{rc})$.
4. A transformation, T_γ consisting of a contrast enhancement through a gamma correction of the form $s = r^{0.5}$, is applied to I_{bs} . The result is I_γ .
5. We remove the white noise of I_γ by means of a morphological opening ($\circ$) with disk-like structuring element, B_d of 3 px . The result is $I_{op} = I_\gamma \circ B_d$ represented in the figure 4.4c.
6. On I_{op} an image segmentation method is applied through edge detection by the Canny [204] algorithm. The standard deviation of the Gaussian filter is $\sigma_{canny} = 1.2$ and the low and high normalized intensity thresholds are respectively 0.3 and 0.75. The result is a binary image formed by a set of N_{pt} pixels of coordinates (X_i, Y_i) that compose that edge.

7. Then we proceed to determine the initial guess of the position, of the center and of the sphere's radius that best fits on the extracted points. The first estimation will be:

$$\begin{aligned} X_{sp0} &= \frac{\max(X_i) - \min(X_i)}{2}, & Y_{sp0} &= \frac{\max(Y_i) - \min(Y_i)}{2}, \\ R_{sp0} &= \frac{(\max(X_i) - X_{sp0}) + (\max(Y_i) - Y_{sp0})}{2} \end{aligned} \quad (4.2)$$

8. We use the first estimation of the center and radius as the initial guess for the nonlinear least squares problem solved by means of an in-house implemented Newton-Raphson method over the total N_{pt} points of the edge with the error function e_i based on the equation of the circumference of the form:

$$e_i = R_{sp} - \sqrt{(X_{sp} - X_i)^2 + (Y_{sp} - Y_i)^2} \quad (4.3)$$

Where the parameters to optimize X_{sp} , Y_{sp} , R_{sp} are respectively the coordinates of the sphere center and the radius. The circumference resulting from the optimization of these parameters is represented in the figure 4.4d. We repeat the process with all the frames to build the data matrix.

To evaluate the accuracy and precision of the image processing tool, a special fixture is used to place the sphere in different vertical positions in the image field for measurement purposes. Once camera is in its definitive position, focus is adjusted so that edges are sharp and a set of $N_{cal} = 587$ images are taken at 27 different positions until the vertical distance of the AOI is complete. Since the position is known, a measure of the error in determining the position of the sphere on the X and Y axis is obtained, represented in the figure 4.5a together with its uncertainties $\pm 0.074 \text{ px}$ and $\pm 0.133 \text{ px}$ respectively. On the other hand, the magnification is obtained from the measurement of R_{sp} for each image as:

$$M_0 = \frac{2 \cdot \sum_{i=1}^{N_{cal}} R_{sp}[\text{px}]}{N_{cal} \cdot D[\text{mm}]} = \frac{2R_{mean}[\text{px}]}{D[\text{mm}]} \quad (4.4)$$

Where R_{mean} is the mean value in pixels of the sphere's radius and is equal to $58.032 \pm 0.196 \text{ px}$ and plotted next to the R_{sp} measurements in figure 4.5b. The magnification will therefore be $M_0 = 2.902 \text{ px/mm}$.

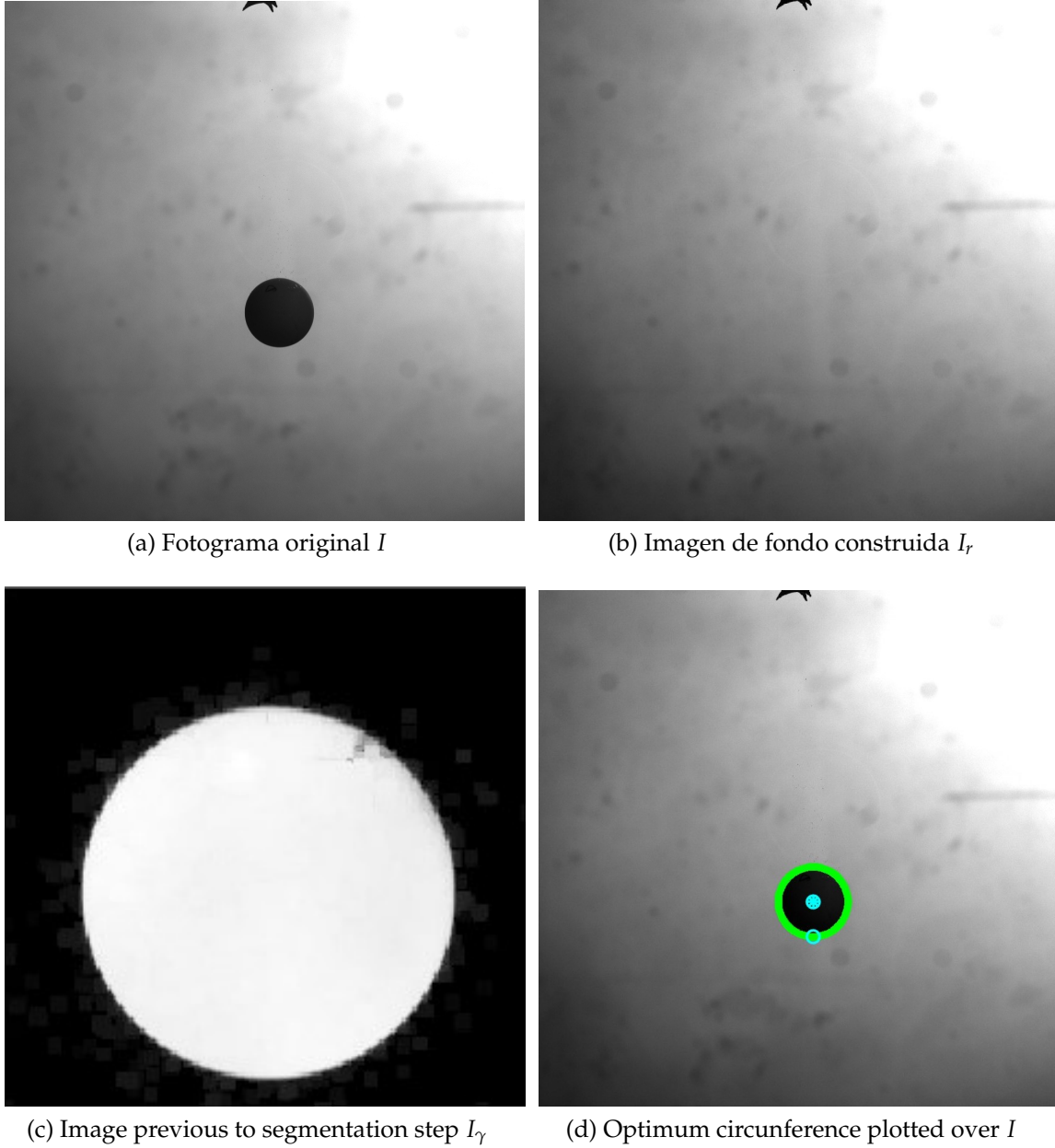
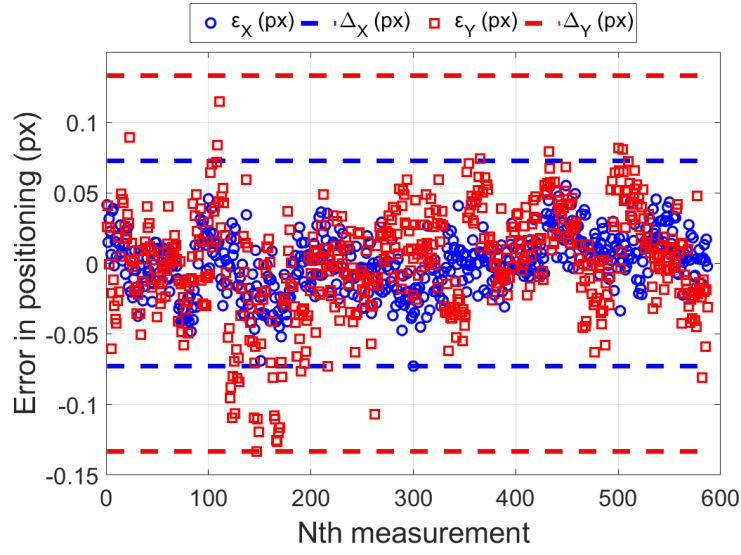
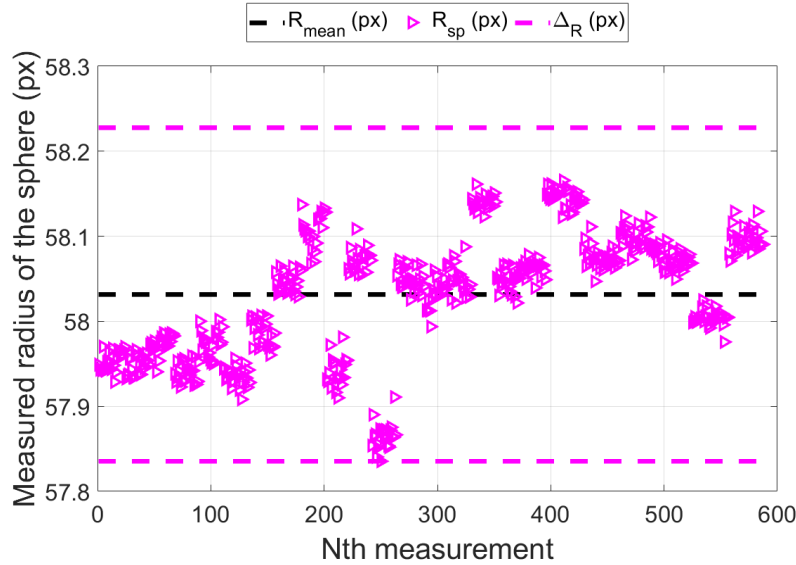


Figure 4.4: Different images obtained during processing are shown. Figure (a) shows the original image. In figure (b) the result of the reference image for background subtraction I_r is shown. In (c) the image inside the MAOI is represented after performing the transformation T_n , the background subtraction and then the morphological opening on T_γ . Finally, in (d) we can see the optimum circumference (green) with the centre $\otimes$ (cyan) and the lower radius point $R_{sp} \circ$ (cyan).



(a) Error in sphere positioning



(b) Measured sphere radius and uncertainty bands

Figure 4.5: (a) Position error analysis of the imaging tool developed on a total of 587 images. ϵ_X refers to the error of the measurement on the X axis, ϵ_Y to the error on the Y axis and Δ_X and Δ_Y to the uncertainty for the position measurement on x and y respectively. (b) Mean value and uncertainty in the radius characterisation for the same images, the mean value R_{mean} will be used for the determination of the system magnification. The measurements for each frame, ϵ_R , and the uncertainty bands Δ_R are plotted.

4.3.2 Test matrix for the sphere dynamics analysis

The design of the setup limits the displacement of the free falling ram mass up to the actuator to $h_{drop} = 0.268 \text{ m}$. To estimate the ultimate velocity, the law of conservation of mechanical energy is used, which depending on the height of the releasing device will be $U_{drop} = \sqrt{2gh_{drop}}$, being g the acceleration of gravity. For the maximum height, the pre-impact velocity will be $U_{dmax} = 2.29 \text{ m/s}$. Given the way it works, when the actuator is struck the impact is transferred so that the sphere is propelled as a consequence of the collision. As in almost all collisions, the momentum will in general be conserved, while the kinetic energy before and after the collision will not necessarily remain the same. The degree of conservation of kinetic energy will be defined by the coefficient of restitution CR Serway [205], which for an inelastic collision between two masses, the latter being at rest and being the velocity of the former U_{drop} , is defined as:

$$CR = \left(1 + \frac{m_i}{m_{ram}}\right) \cdot \left(\frac{U_0 - U_{drop}}{U_{drop}}\right) \simeq \frac{U_0}{U_{drop}} - 1 \quad (4.5)$$

Where U_0 is the velocity of the sphere. In the approximation it has been taken into account that $m_i \ll m_{ram}$. It should be noted that the coefficient of restitution CR will capture the effects of the non-conservation of kinetic energy due to the dissipative effect of the collision with the actuator, as well as other effects not accounted for. Given that the mass of the actuator is of the order of 20 times that of the sphere (table 4.1), the actuator velocity is supposed to be independent of the test sphere tested. Specifically, the joint mass of the actuator and the sample m_1^* is 0.515 kg , while for m_3^* it is 0.520 kg . The difference is therefore of the order of $0.9\% < 1\%$, so it is reasonable to assume that the initial velocity depends mainly on the ram mass velocity at the moment of impact and therefore so does CR . Specifically, for the maximum height, the average velocity of the sphere is $\hat{U}_{01} = 3.78 \text{ m/s}$, see table 4.3. For this velocity the coefficient of restitution calculated according to equation (4.5) takes the value $CR = 0.651$.

Based on the maximum speed of the actuator, five equidistant speeds (U_{dk} with $k = 1...5$) are chosen for the calculation of the position of the electromagnet release device: U_{dmax} , $7/8 \cdot U_{dmax}$, $3/4 \cdot U_{dmax}$, $5/8 \cdot U_{dmax}$ and $1/2 \cdot U_{dmax}$. An additional velocity is added, $U_{d6} = 1/4 \cdot U_{dmax}$. The results of the ram-mass repeatability test carried out for three characteristic heights in different trials are shown in the table 4.2. They show the distance traveled by the ram-mass prior to impact obtained from the laser position sensor, the velocity at collision and their mean values. The theoretical value obtained by applying the law of conservation of mechanical energy is also shown.

In order to estimate the velocity of the sphere after the collision, the following equation is used:

$$U_{0k} = \frac{CR \cdot m_{ram} \cdot U_{dk} + m_{ram} U_{dk}}{m_{ram} + m_i} \simeq 1.65 U_{dk} \quad (4.6)$$

Thus, the estimated velocities of the spheres based on the simplification of equation (4.6) will be respectively: $U_{01} = 3.78 \text{ m/s}$, $U_{02} = 3.31 \text{ m/s}$, $U_{03} = 2.84 \text{ m/s}$, $U_{04} = 2.36 \text{ m/s}$, $U_{05} = 1.89 \text{ m/s}$ and $U_{06} = 0.95 \text{ m/s}$. The Reynolds numbers are between 1.51×10^5 and 3.79×10^4 . Thus the Reynolds numbers are below the critical value, $Re_D = 2.5 \times 10^5$ approximately, where the boundary layer on the sphere becomes turbulent for the steady-state sphere and above $Re = 1.0 \times 10^3$, where the steady-state drag starts to increase. The results of this research may not be extrapolable outside these limits, but it allows us to focus on regions where, in steady-state, we find a drag coefficient practically constant with the Reynolds number.

Table 4.2: Ram mass repeatability test.

U_0	m^*	h_{drop}^{ms}			U_{drop}^{ms}			$\hat{h}_{drop}^{ms}$	$\hat{U}_{drop}^{ms}$	$U_{drop} = \sqrt{2g\hat{h}_{drop}^{ms}}$
		R1	R2	R3	R1	R2	R3			
	m_1^*	0.270	0.270	0.270	2.216	2.215	2.219			
U_{01}	m_2^*	0.269	0.270	0.270	2.225	2.219	2.227	0.270 ± 0.001	2.221 ± 0.006	2.301 ± 0.004
	m_3^*	0.270	0.270	0.270	2.216	2.227	2.225			
	m_1^*	0.152	0.151	0.150	1.671	1.671	1.673			
U_{03}	m_2^*	0.152	0.152	0.151	1.68	1.674	1.666	0.151 ± 0.001	1.671 ± 0.009	1.724 ± 0.006
	m_3^*	0.152	0.152	0.151	1.669	1.671	1.666			
	m_1^*	0.070	0.071	0.071	1.132	1.142	1.143			
U_{05}	m_2^*	0.070	0.070	0.070	1.146	1.145	1.148	0.070 ± 0.001	1.144 ± 0.012	1.174 ± 0.008
	m_3^*	0.070	0.070	0.070	1.146	1.147	1.145			

The summary of the 45 cases that were tested experimentally is presented in table 4.3. They involved as previously described in 4.2.2 three values of m^* (0.78, 0.88, and 0.94), six velocities (3.78, 3.29, 2.94, 2.54, 2.03, and 0.91 m/s), and three repetitions per velocity and m^* . The uncertainty bands (far right column) spanned between $\pm 2\%$ and $\pm 9\%$ around the mean. In this table 4.3, U_0 , $\hat{U}_0$, and R1,R2 and R3 stand for initial velocity, average initial velocity, and repetition key, respectively. The Reynolds numbers associated with the initial velocities, based on the sphere diameter, were 1.51×10^5 , 1.31×10^5 , 1.17×10^5 , 1.01×10^5 , 8.1×10^4 , and 3.6×10^4 , respectively. Theoretical values of equation (4.6) are also shown. Differences with the experimental values may originate from the precision of the measuring scale for locating the releasing device ($\pm 2\text{mm}$), coupled with possible slippages of the fixture when setting the ram mass. Considering the uncertainty bands, the repeatability of the experiment in terms of kinetic energy transferred is between 0.04% and 0.81%, which considering drop tower rigs is quite reasonable.

Table 4.3: Text matrix. Cases tested.

m^*	U_0 (m/s)	Repetitions			$\hat{U}_0$ (m/s)	U_0 (Eq. 4.6)
		R1	R2	R3		
m_1^*	U_{01}	3.78	3.87	3.77		
m_2^*	U_{01}	3.86	3.72	3.95	$3.78 \pm 6\%$	3.78
m_3^*	U_{01}	3.79	3.74	3.54		
m_1^*	U_{02}	3.53	3.52	3.45		
m_2^*	U_{02}	3.16	3.31	3.30	$3.29 \pm 7\%$	3.31
m_3^*	U_{02}	3.09	3.10	3.13		
m_1^*	U_{03}	3.15	3.11	3.13		
m_2^*	U_{03}	3.05	3.08	2.76	$2.94 \pm 9\%$	2.84
m_3^*	U_{03}	2.73	2.77	2.67		
m_1^*	U_{04}	2.57	2.58	2.58		
m_2^*	U_{04}	2.51	2.49	2.47	$2.54 \pm 2\%$	2.36
m_3^*	U_{04}	2.57	2.53	2.54		
m_1^*	U_{05}	2.01	1.98	2.05		
m_2^*	U_{05}	2.16	2.05	2.06	$2.03 \pm 6\%$	1.89
m_3^*	U_{05}	2.01	2.00	1.98		
m_1^*	U_{06}	0.89	0.90	0.87		
m_2^*	U_{06}	0.93	0.92	0.91	$0.91 \pm 3\%$	0.95
m_3^*	U_{06}	0.89	0.95	0.95		

4.4 Results trajectories of the spheres

As indicated in the section section 4.3.1, the trajectory of the spheres is obtained from the position of the centre of the sphere from the developed imaging tool. The camera captures all the instants of the movement including a small time window before the collision. One of the questions that arises is the point that will be considered as the initial instant $t = 0$. In our case the sphere is initially subjected to an impulsive acceleration that increases its velocity almost instantaneously. This translates into an initial positive acceleration, according to the coordinate system represented in figure 4.9, until the sphere reaches a maximum velocity and from that moment onwards it decays continuously until it stops, at which point the sphere is accelerated by buoyancy and ascends. We will consider this starting point $t = 0$ corresponding to the first instant when $dx(t)/dt = 0$. It should be noted that for the calculation of velocity, U , and acceleration, A , central finite differences were applied.

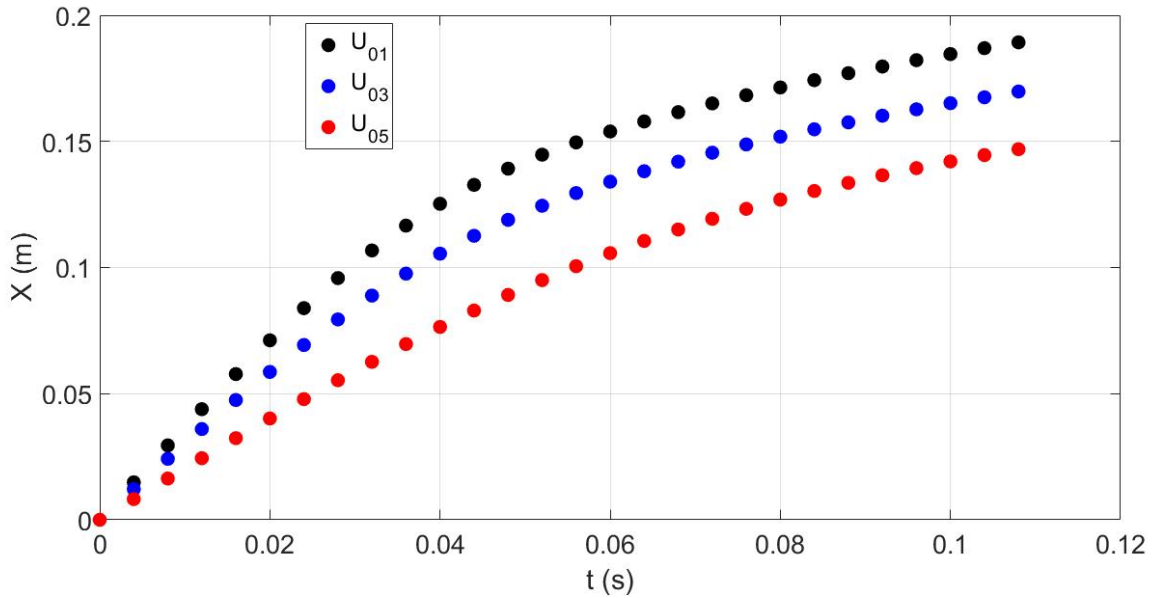


Figure 4.6: Sphere's position as a function of time for m_2^* and three initial velocities: U_{01} , U_{03} and U_{05} .

First, it is illustrative to present time evolution of spheres position figure 4.6, velocity figure 4.7, and acceleration figure 4.8 for some representative trajectories. The uncertainty band of the sphere's initial velocity ranges from 2% up to 9%, see Table I. The spread of m^* is of the order of 10% around the mean. This implies that the investigated range of m^* is too narrow to be of significance. The cases that have been selected are those corresponding to m_2^* and U_{01} , U_{03} , and U_{05} (see table 4.3). For the sake of clarity in the figures, not all measured points are drawn. While position and velocity curves figures 4.6 and 4.7 were smooth, some small ripples could be observed in the acceleration profiles in figure 4.8. Their origin may be ascribed to the inherently larger errors associated with the computation of second-order finite differences.

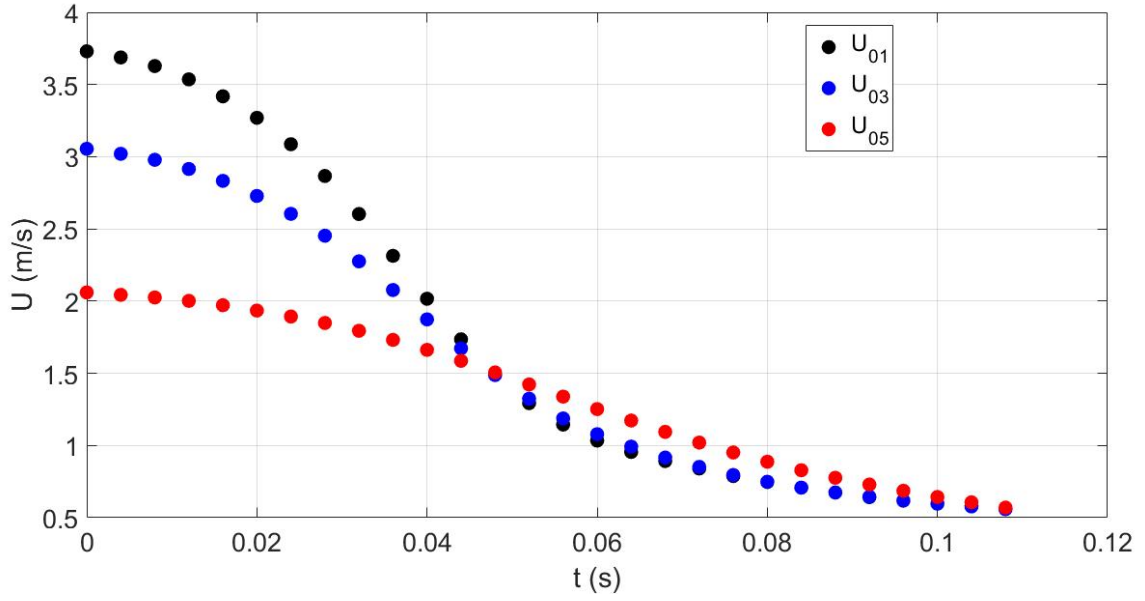


Figure 4.7: Counterpart of figure 4.6. Sphere's velocity as a function of time for m_2^* and three initial velocities: U_{01} , U_{03} and U_{05} .

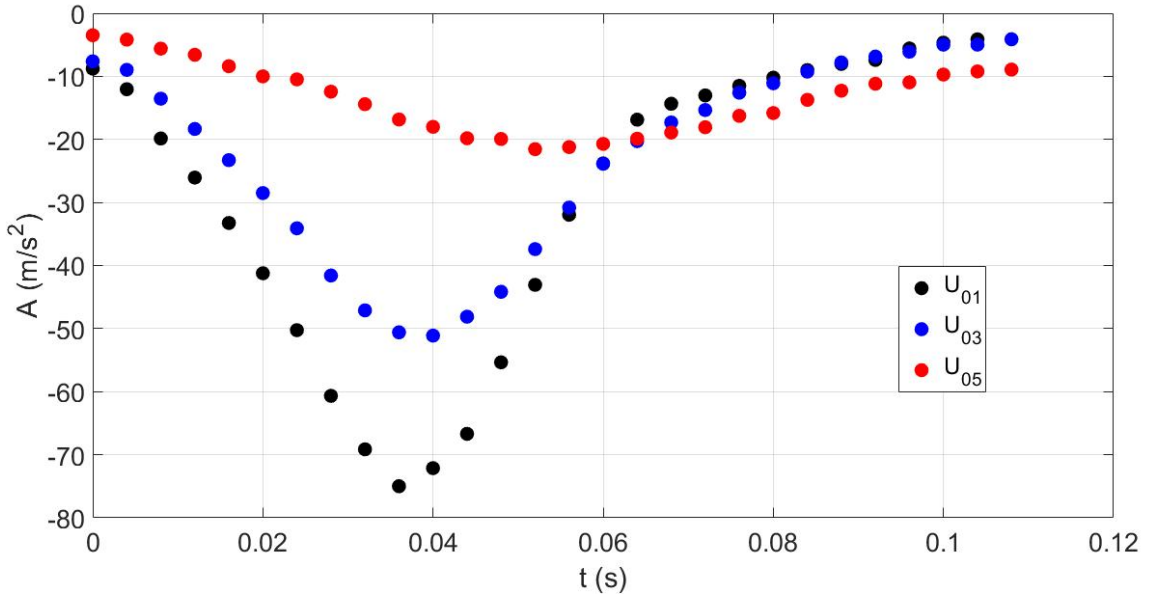


Figure 4.8: Counterpart of figure 4.6. Sphere's acceleration as a function of time for m_2^* and three initial velocities: U_{01} , U_{03} and U_{05} .

The results presented in figure 4.7 allow for the estimation of the sphere response time (defined as the time needed for the sphere to reach a velocity equal to two-thirds of the initial velocity). In particular, this response time for trajectories U_{01} , U_{03} , and U_{05} was 0.047, 0.052, and 0.083 s, respectively. Acceleration results shown in figure 4.8 allow

for a comparison between the terms that appear in the sphere dynamics equation. In dimensional variables, this equation could be written as:

$$m_s \frac{d^2 x}{dt^2} = -F_H + (m_s - m_w) g \quad (4.7)$$

where m_s is the sphere mass, x the sphere displacement, t the time, F_H the hydrodynamic force, m_w the mass of an equivalent sphere of water, and g the gravity acceleration, see figure 4.9. Term $(m_s - m_w) g$ represents the buoyancy force. Defining the dimensionless variables $\eta = x/D$ and $\tau = tU_0/D$, where D and U_0 are the sphere's radius and initial velocity, respectively, equation (4.7) reads as:

$$\frac{d^2 \eta}{d\tau^2} = -\frac{F_H D}{m_s U_0^2} + \left(\frac{m^* - 1}{m^*} \right) \frac{g D}{U_0^2} \quad (4.8)$$

Renaming,

$$\frac{F_H D}{m_s U_0^2} = P_1, \quad \left(\frac{m^* - 1}{m^*} \right) \frac{g D}{U_0^2} = P_2 \quad (4.9)$$

yields

$$\frac{d^2 \eta}{d\tau^2} = -P_1 + P_2 \quad (4.10)$$

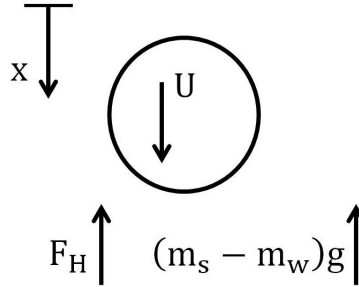


Figure 4.9: Sketch showing the terms in the sphere's dynamic equation (equation (4.7)).

Then, since $d^2 \eta / d\tau^2$ can be obtained from the experimental measurements (see figure 4.8) and P_2 is constant for each prescribed set of m^* and U_0 , it is possible to evaluate the absolute value of the ratio of hydrodynamic force vs buoyancy force, $abs(P_1/P_2)$. This is shown in figure 4.10, where it could be observed that the hydrodynamic force dominates the buoyancy force for most of the measured trajectories by a large factor. A relevant aspect of the measured profiles is the fact that the absolute value of the sphere deceleration increases significantly along with time until a maximum is reached. Beyond that point, the absolute value of deceleration decreases ($P_1 \rightarrow 0$), while the sphere falls toward reaching the maximum depth before rising again. At that point, $d\eta/d\tau = 0$; and $d^2 \eta / d\tau^2 < 0$. A time shift in the position of the maximum value of $abs(P_1/P_2)$ in figure 4.10 is apparent for the case of the lower velocity U_{05} . This shift has been observed consistently for the lower

velocities regardless of m^* . Its value is larger than the uncertainty of the experimental measurement, so the effect is actually present. However, the authors were not able to find a plausible explanation for this effect.

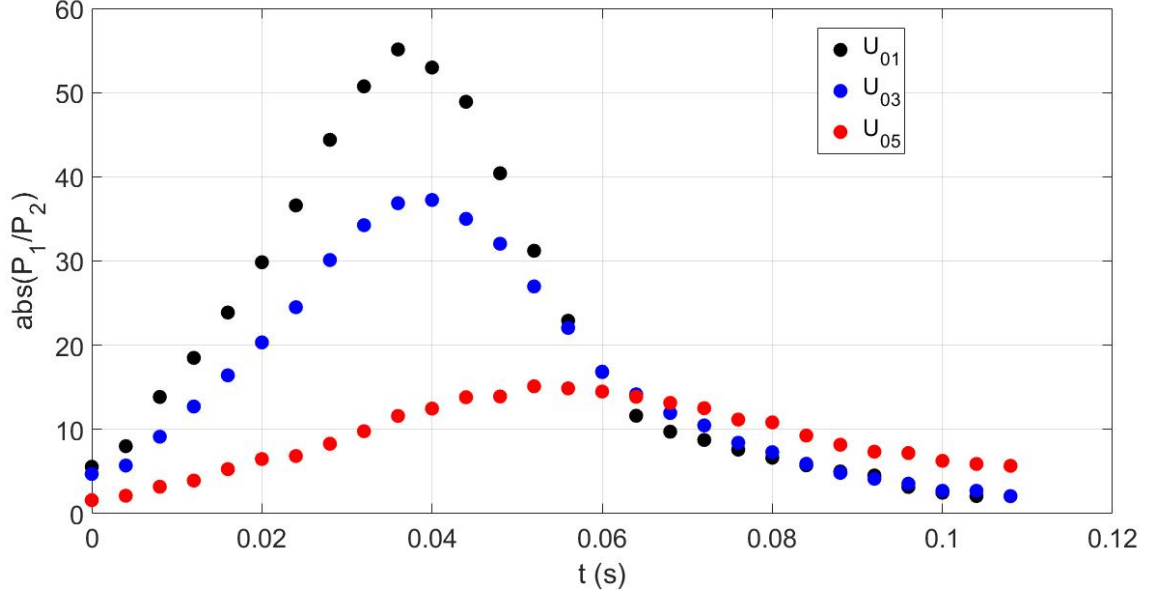


Figure 4.10: Sketch showing the terms in the sphere's dynamic equation. (equation (4.7))

Finally, it might be relevant for practical engineering applications to have information about the evolution of the drag coefficient, $C_D(t)$, for the different cases. This has been carried out by defining $C_D(t)$ in the conventional way in the sphere equation of motion:

$$m_s \frac{d^2x}{dt^2} = -\frac{1}{2} C_d(t) \rho_w \pi \frac{D^2}{4} \left(\frac{dx}{dt} \right)^2 - (m_w - m_s) g \quad (4.11)$$

It is of interest to compare it with the definition of the stationary coefficient of Brown and Lawler [206]:

$$C_{D_s}(Re_D) = \frac{24}{Re_D} \left(1 + 0.150 Re_D^{0.681} \right) + \frac{0.407}{1 + 8710/Re_D} \quad (4.12)$$

Where Re_D is the Reynolds number based on the sphere's diameter, which will be calculated based on its instantaneous velocity $U(t)$. The results obtained are presented in figure 4.11 that is the counterpart of figure 4.20 for the drag coefficient $C_D(t)$. It is observed that the stationary coefficient, equation (4.12), assumes a practically constant value $C_D = 0.46$, as expected according to the chosen initial conditions. Comparing it with the results, we observe that in the initial stages its value is less than the stationary value and then increases significantly until it reaches a maximum in the vicinity of $C_D \sim 1.2$, before decreasing. It follows that, since the interaction of the fluid with the sphere produces a behaviour significantly different from the stationary one, the fluid structures will also be significantly different [98] and [30]. The results partly contradict the conclusions regarding decelerating spherical particles of authors such as [98] and [100]. In the following sections

we will emphasise the fluid structure found in our experiments through TR-PIV analysis techniques.

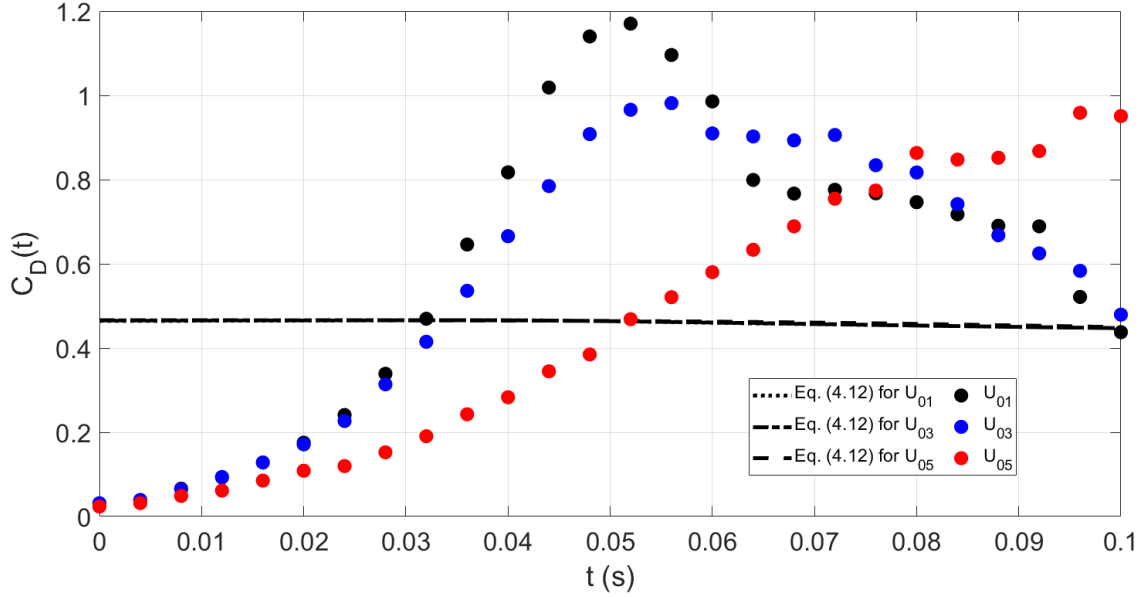


Figure 4.11: Evolution of drag coefficient with time and comparison with the application of equation (4.12) for steady drag coefficient [206].

4.5 Data adquisition system: TR-PIV setup

In this section we will first describe the elements that make up the TR-PIV acquisition system, section 4.5.1, in particular regarding the configuration of the camera, the characteristics of the laser system for generating particle light scattering and the tracer particles used. Next, in section 4.5.2, we will deal with the test procedure used to obtain the PIV images. Given the characteristics of our test, in the section section 4.5.3 we describe the image preprocessing tool developed for TR-PIV analysis. Finally, in section 4.5.4 we specify a criteria for the selection of image pairs to apply the correlation and its configuration using the MATLAB PIVlab tool.

4.5.1 TR-PIV setup

In order to study the flow field topology, the TR-PIV or Time Resolved particle image velocimetry technique is used, Beresh [207]. This type of configuration is used when acquiring high frequency information of the flow is a requirement and, broadly speaking, it has the same elements as the conventional PIV system, i.e. a recording camera, a coherent light source and tracer particles. Ultimately, the laser and the camera must act at such a temporal resolution that the evolution of the relevant fluid structures can be traced. To achieve this temporal resolution, high repetition high power pulsed laser in combination with high-resolution high speed digital cameras may be used, or the latter may be combined

with diode-pumped solid state laser light sources (DPSS) emitting in continuous wave (CW) (Bross et al. [208] and Ryerson and Schwenk [209]). The latter combination is the one used to follow the evolution of the fluid structures that develops around our decelerating sphere. Specifically, for the acquisition of the images, the same setup in terms of mount, lenses and camera, described in section 4.2.3, is used. Since the laser system delivers light continuously, the distance between consecutive images within the correlation image pair is controlled by the camera sampling rate $1/t_{sp}$ and the amount of light captured per frame is controlled by the exposure time t_e . The sequence of frame pairs to be correlated will therefore be:

$$[I(1), I(2)]_1, [I(2), I(3)]_2, \dots [I(j), I(j+1)]_j \quad (4.13)$$

Where the subscript $j = 1 \dots N - 1$ refers to the image pair sequence, and I is referred to an image of the set of N snapshots sequentially ordered and equidistantly separated by a time t_{sp} . Given the test sample displacements (section 4.4), the AOI must span vertical distances of the order of 0.20 m to which are added, at least, one diameter of the sphere 0.04 m in order to be able to track it at the initial and final instants. Thus, the magnification M_0 for our TR-PIV setup is $M_0 = 3725 \text{ px/m}$. The image magnification is obtained through the image tool and procedure described in section 4.3.1. Given the magnification and the size of the sphere, we considered for the estimation of the valid detection criteria, equation (3.4), the use of square interrogation windows with $D_I = 16 \text{ px}$ sides. For this interrogation window size, the sampling rate must be greater than 3725 fps in order to fulfill the in-plane loss of correlation criterion, $F_i > 0.75$. On the other hand, based on experimental observations, the exposure time is set to be $t_e = 1/5000 \text{ s}$, so blurring effects in particle images can be avoided. Since the loss of correlation factor for velocity gradients, F_Δ , increases with the sampling rate, it is set to an acquisition rate of 4500 fps , so that the camera resolution ($L_y \times L_x$) becomes 1024×992 pixels, which translates to a rectangular area ($l_y \times l_x$) of $273 \times 264 \text{ mm}^2$. For this resolution, the Dynamic Spatial Range (DSR) for single-pass correlation methods will be 64, which is a common value for cameras equipped with CMOS sensors.

Regarding the illumination system, a 4 W CW DPSS green light laser with wavelength $532 \pm 1 \text{ nm}$, CNI-MGL-N-532B-4000-5-LAB DPSS, is used. The system generates a beam with a waist diameter of $D_L = 3 \text{ mm}$, a quality factor of $M^2 = 3.0$ and a transverse mode close to TEM00, so its light intensity profile is close to Gaussian. To adjust the power, the system incorporates a PSU-H-LED source with the possibility of pulse modulation via TTL input. The system has a rms power stability lower than 5% which, given the characteristic time of our test (in the order of tenths of a second), guarantees a steady intensity. In order to generate a light sheet illuminating the AOI, we use a Powell lens with a fan angle of $\alpha_L = 30^\circ$. This type of lens generates an intensity pattern whereby the central regions have a nearly uniform profile that typically contains a factor $K_L = 0.80$ of the total energy contained in the beam, while the remaining 20% is concentrated in two lines at the ends under increased brightness appearance. A further effect of applying this type of lens is that as a consequence of the aperture of the sheet, the power per area unit decreases as we

move away from the source. This gradient can be calculated as follows:

$$\nabla (P_L / A_{yz}) = \frac{-K_L P_L \tan \alpha_L}{D_L \left(\frac{\pi}{4} D_L + x_L \tan \alpha_L \right)^2} \sim \frac{-K_L P_L}{D_L (x_L^2 \tan \alpha_L)} \quad \text{for } x_L \tan \alpha_L \gg D_L \quad (4.14)$$

Being x_L the distance to the laser, and the expression on the right an approximation for large angles and long distances from the laser lens. In our case we will use the maximum power that the system is capable of delivering (4 Watts), which will be supplied through a vertical laser sheet, so that the plane will cut the sphere through the meridian located in the plane perpendicular to the camera, in such a way that the entire circumference of the sphere is illuminated. As a result of the illumination from right to left, a shadow region will appear in the images as a consequence of the presence of the sphere, figure 4.12a.

Dantec Dynamics Hollow Glass Spheres (HGS) particles with a diameter of $d_p = 10 \mu m$ and density of $\rho_p = 1.1 \text{ kg/m}^3$ are used as tracer particles, with a response time for our test of $\tau_s \sim 6.84 \times 10^{-07} \text{ s}$, which is much shorter than the temporal resolution of our system. The number of particles per pixel is approximately $N_{ppp} = 0.2$, with particle image diameters in the range of $D_p = 1.0 \text{ px}$ to 1.5 px so their density in the image is high. Since the tap water has a large amount of impurities of significant size and high density, the water is filtered and recirculated to remove all impurities above $5 \mu m$ prior to seeding. The water temperature for all the tests was in the range of 15°C to 17°C .

4.5.2 TR-PIV test procedure

The test procedure is almost the same as described in section 4.3, but with a number of modifications related to the light source and the measuring elements (tracer particles). In the same way, first of all it is checked that the mass and sphericity of the samples have not changed. Once the checks have been carried out, the TR-PIV tests are performed, which follow the procedure below:

1. The water tank is completely filled with tap water filtered through the cartridge filter described above. Subsequently, the particle dilution is prepared by incorporating a mass m_{HGS} of tracer particles, measured on a precision scale $\pm 0.1 \text{ gm}$, into 200 ml of water and mixed using a magnetic stirrer. The following relationship is used to calculate the estimated mass:

$$m_{HGS} = \rho_p \frac{4\pi}{3} (d_p/2)^3 \cdot N_{ppp} \frac{L_X L_Y}{l_x l_y D_L} \cdot V_{tank} \simeq 0.6 \text{ gm} \quad (4.15)$$

Where it has been assumed that $V_{tank} = 1 \text{ m}^3$. The mixture is incorporated into the filled tank and spread by mechanical means for at least 10 min . A homogeneous distribution is verified from imaging.

2. The system is armed by placing the ram mass in its free fall position, held by the electromagnet and the actuator in its standby position supported by the permanent magnet, more details of the system in section 4.2.

3. The sphere is placed manually at the end of the actuator and is waited for 30 minutes for the water to become still again. Simultaneously, the laser is switched on so that its warm-up time elapses and thus ensuring constant delivery of maximum power during the test.
4. The location of the camera is checked and its alignment and focus adjusted if necessary. It should be noted that, unlike the sphere trajectory measurements, in this case the adjustments are made on the basis of laser sheet.
5. The ram mass free fall is manually triggered and the system collects all the frames of the test sequence. In case the result is considered acceptable, the images are dumped from the camera buffer into a solid state memory where they are stored in *.tif* format. The cases are stored following the previously described designation for subsequent pre-processing.

A total of 3 valid repetitions per case were performed for samples m_1^* , m_2^* and m_3^* at speeds U_{01} , U_{03} , U_{05} and a set of three repetitions for the special case U_{06} with sample m_3^* . Constituting a total of 30 cases. Since HGS are coated with surfactant, their coating can be significantly altered over time impacting on the TR-PIV analysis results, Hadad and Gurka [210]. For the latter reason the tank was emptied by means of a pump prior to the test days. This was also used to clean the water tank surfaces.

4.5.3 TR-PIV image pre-processing tool

Prior to the TR-PIV analysis, a series of operations must be carried out on the images in order to remove the brightness effects on the surface of the sphere, suppress the intensity gradient resulting and to intensify the image of particles with respect to other undesired objects. For this purpose, an image tool is developed in MATLAB for pre-processing images previously to the TR-PIV analysis, consisting of the following steps:

1. First, the image (I) originally stored in *.tif* format and in grayscale is loaded. Then, a rescaling is applied on the limits of its class $[1, 2^{16} - 1]$, figure 4.12a.
2. An adaptive contrast limited histogram equalization or CLAHE, Zuiderveld [211], is applied on I , with a division of 100 tiles, that correspond to sizes smaller than D_I , and limiting the contrast enhancement to a value of 0.002 in order to avoid the formation of artifacts in homogeneous regions. The result is the image I_h .
3. An average intensity subtraction is then applied to the equalized image. Outcome can be found in the figure 4.12b.

$$I_{ad} = I_h - \hat{I} \quad (4.16)$$

4. Given the analysis region, it is necessary to eliminate the laser reflection phenomena on the surface of the sphere and the intensity gradients due to laser fan angle described in equation (4.14). The control over the illumination and reflectance is performed by the application in the frequency domain, (u_f, v_f) , of an in-house developed tool for homomorphic filtering, based on the homomorphic function:

$$H_{emph}(u_f, v_f) = \alpha_{hmf} + \beta_{hmf} \left[1 - e^{-D^2(u_f, v_f)/D_0^2} \right] \quad (4.17)$$

Being $D_0 = 5\sqrt{2}$ and $D(u_f, v_f)$ the distance between a point (u_f, v_f) of the frequency domain and the centre of the frequency rectangle, and $\alpha_{hmf} = 0.5$ and $\beta_{hmf} = 2.0$ a set of parameters chosen specifically for this application. As a result of filter application in the frequency domain on the image I_{ad} , a simultaneous contrast enhancement and dynamic range compression is generated and the result is the image I_{hmf} . For the fast fourier transform (FFT) and the inverse (IFFT), the MATLAB functions *fft2* and *ifft2* are used. The application of high pass filters has been successfully applied by other authors to remove glare on moving surfaces, while retaining particle images, Sciacchitano and Scarano [212].

5. Finally, a highboost filter, González and Woods [203], is applied on I_{hmf} to emphasize the high frequency components of the image (particle intensity peaks) without interfering with the low frequency components (sphere surface, blurred surfaces and background). To do this, we first apply a Gaussian filter in the frequency domain with a standard deviation of $\sigma_{boost} = 3$ and a square structural element of size 3×3 obtaining the blurred image I_{lp} , that is subtracted from the original resulting in a mask. The mask multiplied by a highboost factor $k_{HB} = 10$ is added to the original one. Highboost factor is chosen so no halo is generated because of the addition of negative intensity values. The result is I_{boost} which will be the image to be used in the PIV analysis, figure 4.12c.

Due to the existence of shadow regions and the presence of solid surfaces, the pixels associated with them must be excluded from the TR-PIV analysis. For this purpose, an image tool has been developed that generates binary images Mk_i that can be loaded as masks on the PIV analysis tool. The developed tool is based on the one previously described in the section 4.3.1 for trajectory analysis, but makes use of the edge generated by the reflection of the laser plane on the sphere for the location of the optimal centre (x_{piv}, y_{piv}) and the characterisation of the radius R_{piv} . From these values an equivalent sphere is generated which constitutes one of the elements of the mask. It remains to determine the shaded pixels. For this purpose, the Hough transform is applied to obtain the ordinate at the origin and slopes of the two straight lines that appear at the top and bottom right of the image as a consequence of the opening of the laser sheet. At the intersection of the two lines we obtain the theoretical origin of the laser (x_{focus}, y_{focus}) from which we obtain the tangent lines to the circle with origin at (x_{piv}, y_{piv}) and radius R_{piv} . The perimeters contained in the shaded regions are filled with 1 values until the binary mask represented in the figure 4.12d is obtained.

From the information of the position of the sphere's centre, (x_{piv}, y_{piv}) , it is possible to obtain its velocity in order to perform the change to the sphere's moving reference system. Unlike the trajectory measurements obtained in section 4.2.3, as a consequence of the illumination system and the sampling frequency, the noise level of the derivatives increases significantly, making it unsuitable for direct usage. Therefore, the need arises to fit the

velocity data to a rational model of the form:

$$U(t) = \frac{p_1 t^2 + p_2 t + p_3}{t^3 + q_1 t^2 + q_2 t + q_3} \quad (4.18)$$

Where the terms p and q are fitting coefficients. The following figure 4.13 illustrates the fitting result for one of the tested cases.

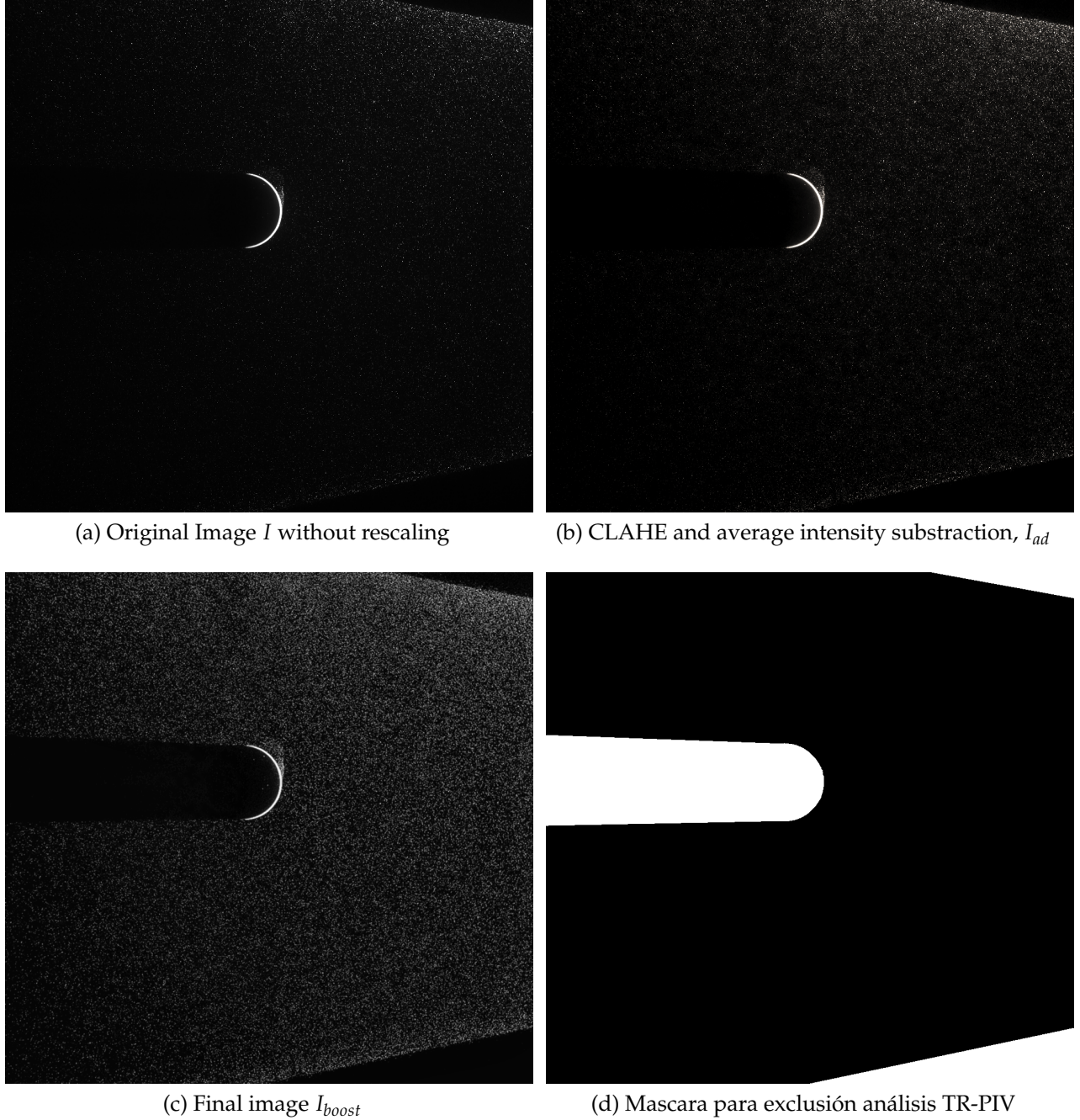


Figure 4.12: Different stages of the image pre-processing tool. Snapshots of the m_3^* , U_{06} case

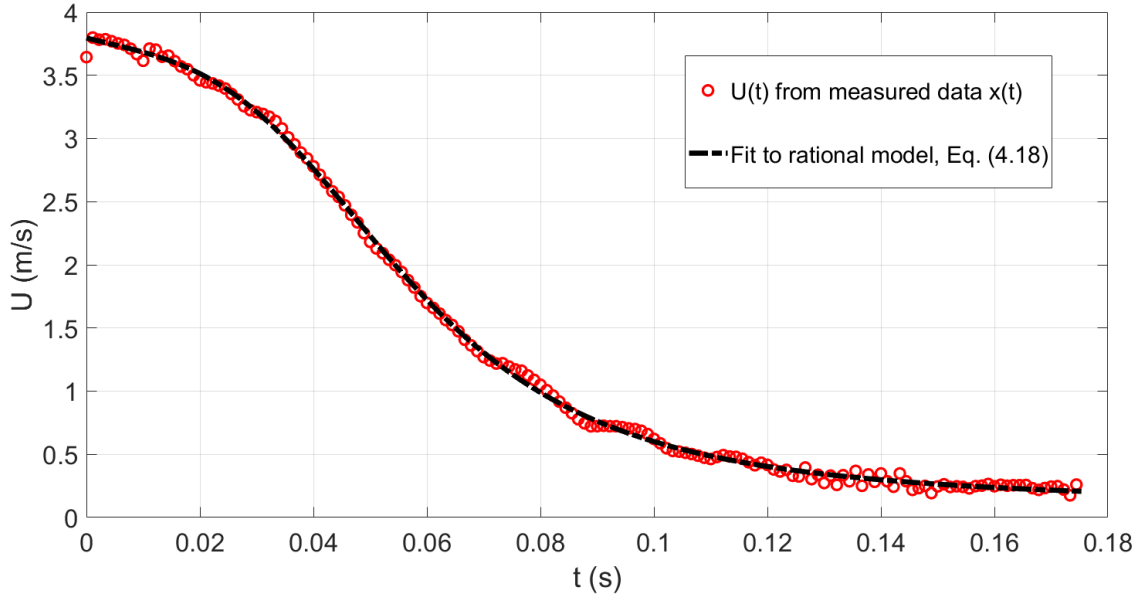


Figure 4.13: Rational model fitting of the velocity extracted from the trajectory for TR-PIV tests, case m_3^* with initial velocity U_{01}

4.5.4 Cross-correlation analysis

One of the issues arising from the non-stationary character of the flow field under study is the impact on the correlation of the ratio of maximum and minimum velocities resolvable via PIV or Dynamic Velocity Range (DRV). Thus the DRV for a PIV analysis based on single-pass correlation, represented by the superscript (s) , is represented by the following relation:

$$DRV^{(s)} = \frac{0.25D_I}{\sigma_{\Delta x}^{(s)}} = \frac{U_{max}}{\sigma_U^{(s)}} \quad (4.19)$$

Where $\sigma_{\Delta x}$ and σ_U are the minimum resolvable displacement and velocity respectively, and U_{max} is the maximum velocity according to the criterion defined by $F_i > 0.75$, which is calculated from the expression:

$$U_{max} = \frac{0.25D_I}{t_{sm}M_0} \quad (4.20)$$

In very well-adjusted experiments with CCD cameras, it is common to consider $\sigma_{\Delta x}^{(s)} = 0.05 \text{ px}$ (Kaehler et al. [213], Westerweel [214]). However, this minimum velocity or minimum resolvable displacement for single-pass correlation is affected by a large number of parameters, see D_I , the particle diameter in pixels D_p , the number of particles per pixel N_{ppp} , the background noise of the image σ_n , flow curvature effects, velocity gradients or proximity to reflective surfaces among others (Raffel et al. [215])). On the other hand, the use of multipass correlation methods combined with window deformation techniques, super-index (m) , reduces this minimum resolvable displacement by an order $\sigma_{\Delta x}^{(s)} / \sigma_{\Delta x}^{(m)} \simeq 10$. In our case, given the use of a CMOS sensor camera, which are characterized by lower

Table 4.4: Sub-sampling for TR-PIV analysis

Frame distance (K_j)	Time separation (t_s)	U_{max} (m/s)	σ_U (m/s)
1	2.22×10^{-04}	4.832	0.121
2	4.44×10^{-04}	2.416	0.060
3	6.67×10^{-04}	1.611	0.040
4	8.89×10^{-04}	1.208	0.030
5	0.0011	0.966	0.024
6	0.0013	0.805	0.020
7	0.0015	0.690	0.017
8	0.0017	0.604	0.015

resolution, significantly larger physical pixel sizes s_{px} and higher background noise levels, as well as the complexity of the flow under study, $\sigma_{\Delta x} \simeq 0.1 \text{ px}$ will be assumed, which is a conservative criterion, (Stanislas et al. [216], Hain and Kaehler [217] and Persoons and O'Donovan [218]). The estimated dynamic velocity range will therefore be $DR_V = 40$.

In the problem under study the sphere moves in a fluid at rest, in a non-stationary way, until it stops. On the other hand, complex fluid structures related to the development of the wake and vortex structures are to be expected. Considering the above, the expected range of velocities is very wide, so that as we approach the DR_V the average quality of the vectors decays as a consequence of the low displacements. In particular, this loss of quality will be greater in inner regions of the vortical structures or at instants of the motion where the sphere reduces its velocity significantly. Therefore, based on the Multiple Pulse Separation (MPS) methodology ([218]), it is modified the approach described in the expression 4.13 in such a way that the distance between pairs takes the form:

$$[I(1), I(1 + K_1)]_1, [I(2), I(2 + K_2)]_2, \dots [I(j), I(j + K_j)]_j \quad (4.21)$$

This is therefore a sub-sampling within the sequence of frames that requires an estimation criterion to value the distance between image pairs K_j . This criterion will be based on the fact that, except for the final instants, the maximum velocity can be estimated based on the instantaneous velocity of the sphere $U(t_i)$ for frame i , considering the ground reference system. Using the expression for the loss of correlation factor for in-plane motion equation (4.20) and equation (4.19) we obtain the minimum resolvable velocity as $\sigma_U = U_{max}/DR_V$. The table 4.4 shows the ranges of the maximum velocity, the minimum resolvable velocity and the time between frames as a function of the distance between frames for the recording frame rate.

The open-source PIVlab software (Thielicke and Stamhuis [219] and Stamhuis and Thielicke [220]) was used for TR-PIV cross-correlation analysis under the configuration Time-Resolved, that analyses several image pair uploaded as showed in 4.13. The FFT multi-pass window deformation correlation technique was used. TR-PIV images were partitioned via a three-pass approach into interrogation windows of 64×64 pixels for the first pass, 32×32 pixels for the second, and 16×16 pixels for the last, all of them with an overlap of 50% and a 2×3 Gaussian sub-pixel estimation. The global vector grid resolution was 2.1 mm that is much smaller than the characteristic size of the observed vortical structures, whose size was of the order of the sphere's diameter, 40 mm . In post-processing, the data are validated based on a local median filtering (WesterWeel [221])) in combination with a local median velocity standard deviation filter. The parameters of both validation methods are set as recommended by the developer of the analysis tool. Figure 4.14 shows an image directly extracted from the PIVlab software representing in orange the vectors removed and subsequently interpolated with respect to those obtained directly from the analysis represented in green, for the R_3 repetition of the m_3^* case, $U_{06} = 0.91 \text{ m/s}$. As expected, worse quality is achieved in the regions close to the sphere surface where large shear strain appear and large vorticities are present.

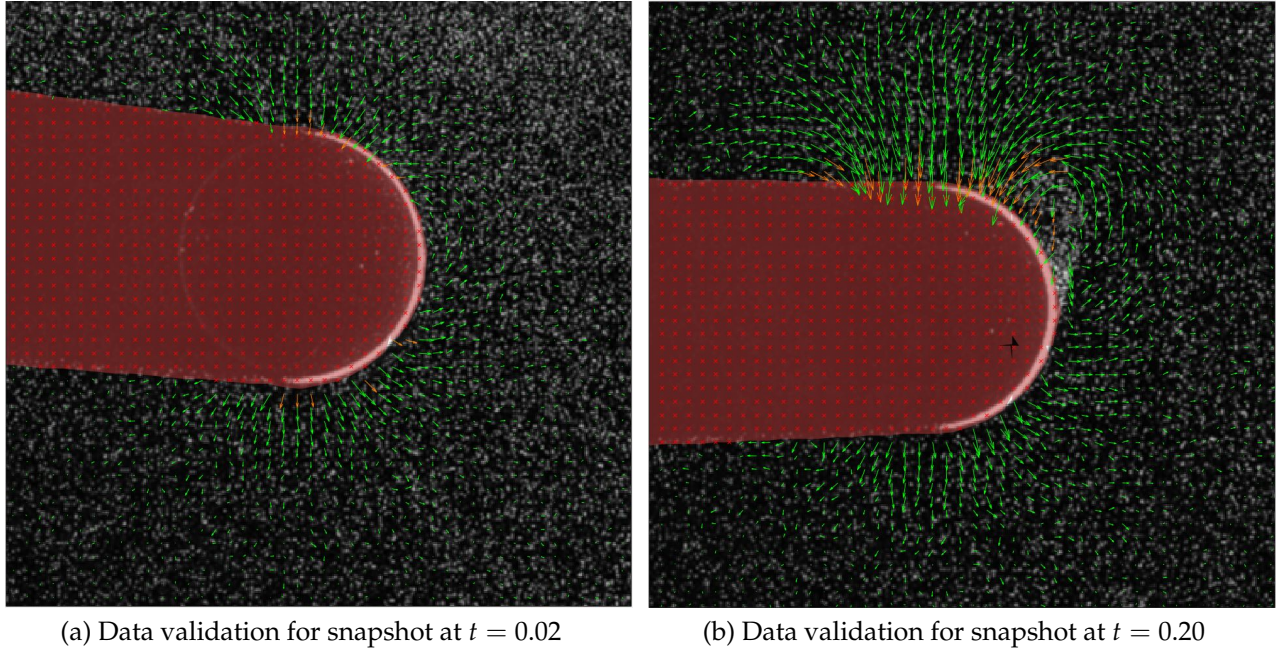


Figure 4.14: Result of the post-processing carried out using PIVlab in a ground fixed reference system. Interpolated vectors are shown in orange and unscreened vectors in green. Please note that the vector length are not in the same scale

4.6 Results: Flow topology via TR-PIV

In this section the results from the TR-PIV analysis will be presented. Firstly, the analysis of the time evolution of the flow structures is presented, comparing it with previous observations of the case of a continuously accelerated sphere. Then, in the section 4.6.2, the loss of potential flow character referred to the initial separation instant will be analysed. For this purpose we will first use the analysis of Fernando et al. [119] for the case of an continuously accelerating sphere from an initial velocity U_0 . Then we will evaluate the initial instants after the start-up process using a perturbation expansion on the basis of the infinitely thin laminar boundary layer theory for $Re \rightarrow \infty$.

4.6.1 Qualitative analysis of flow topology via TR-PIV

For illustrative purposes, a sequence of TR-PIV snapshots for the case m_3^* with U_{01} is presented in the figure 4.15. In the figure, the instantaneous values of the velocity vector have been used to draw the streamlines. For the calculation of these streamlines, the MATLAB function *mmstream2* is used, which calculates the streamlines, including the closed ones, both upstream and downstream from some user-selected starting points, which in our case will be those at a distance of $0.65D$ from the centre of the sphere, those on its surface and those at the ends of the image. In relation to the velocity magnitude field shown, the velocity modulus normalized by the initial velocity of the case ($|U_j(x, y)|/U_{0i}$) is represented, where j is the snapshot index.

By observing the figure 4.15 it is possible to estimate the evolution of the separation point on the sphere with time. The separation point (figure 4.16a) is defined as the point at which the boundary layer leaves the wall so that, as a result of a near-wall flow reversal, a strong thickening of the boundary layer is generated, resulting in a velocity component perpendicular to the wall V of the order of U_δ , Schlichting and Gersten [129]. Where U_δ is the average velocity inside the boundary layer. This is reflected in the existence of streamlines leaving the wall at a given angle (figure 4.16b). When the thickness of the boundary layer (δ), with an order of magnitude of $O(Re^{-\alpha})$, being $\alpha \sim 1/2$ for the laminar case, is small when compared to the size of the separation region perpendicular to the outer flow, we are dealing with a massive separation (figure 4.16c). Under these conditions the pressure distribution changes drastically as compared to the inviscid case and a region of much larger thickness than δ with high vorticity, $\omega \neq 0$, is generated. Considering our experimental cases, at the initial instants of the motion the Reynolds numbers will be in the neighbourhood of $10^3 - 10^5$, so laminar boundary thickness can be estimated from the Blasius solution $\delta \sim 5.0 \cdot x / \sqrt{Re_x}$ (where $x = \pi R$), which corresponds to 2.50 mm and 0.79 mm in our case. Given that our spatial resolution is 2 mm and considering the intrinsic difficulties of applying TR-PIV techniques in the close-wall vicinity, it will only be possible to observe separations that are above this threshold and therefore significantly larger than δ .

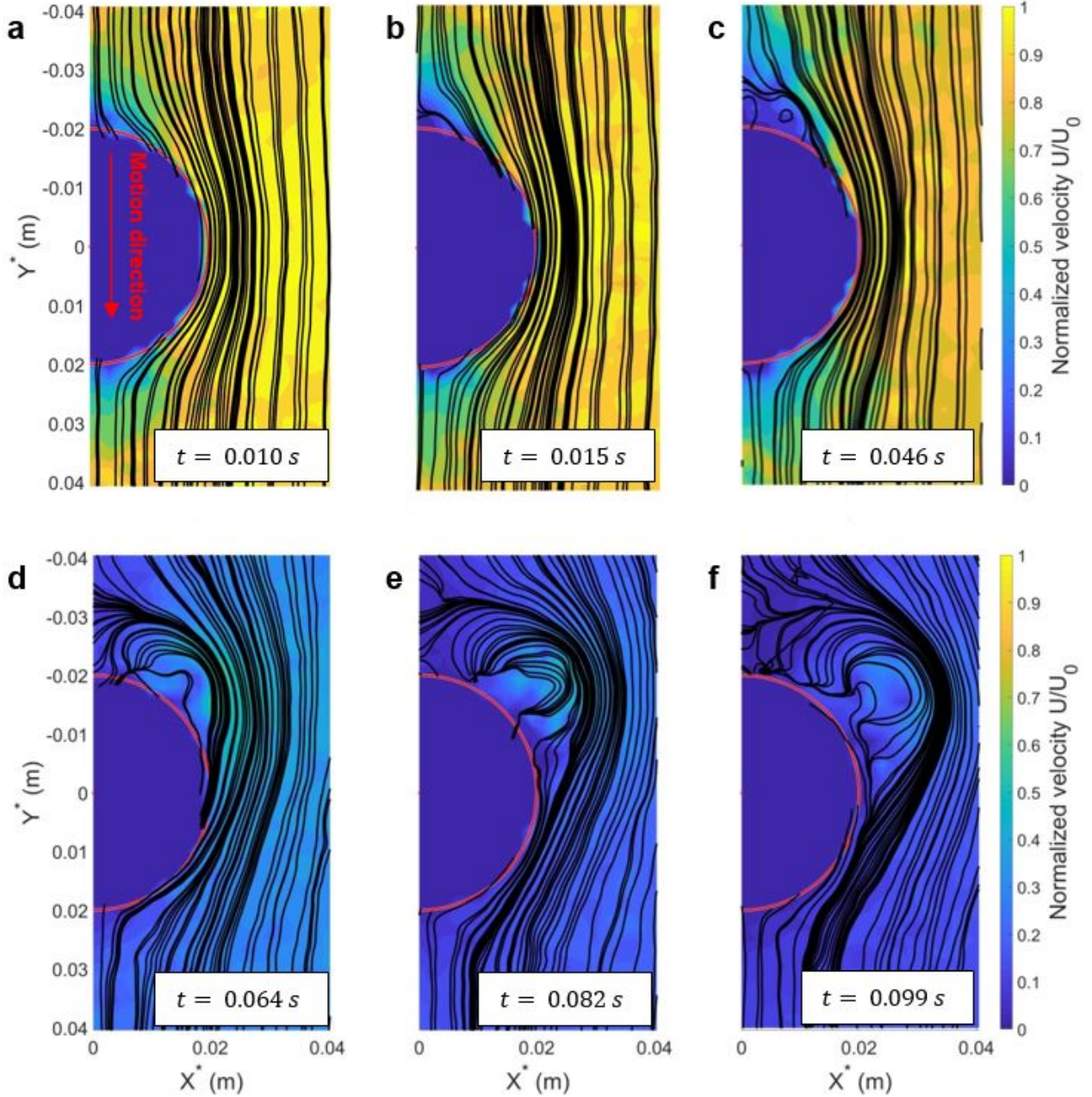


Figure 4.15: Selected PIV snapshots for the case m_3^* and $U_0 = U_{01}$. Where U is the flow velocity. Subplots (a)-(f) correspond to times 0.010, 0.015, 0.028, 0.046, 0.082 and 0.099 s, respectively. The flow field is transformed into the resting frame of the sphere.

Considering the non-stationarity of the case under study, in the first instants of an impulsively accelerated sphere we find an infinitely thin region around its surface with non-zero vorticity ($\omega \neq 0$). Under these conditions the flow around the sphere is equivalent to the potential flow solution. It could be observed in figure figure 4.15 that the streak-line pattern starts to lose forward and backward symmetry (down and up in the figure) as early as in the first snapshot as shown in figure 4.15 (a). Since the potential flow solution is symmetric,

this is an indication that at that time ($t = 0.015$ s), the flow has already lost its initial potential character. This loss of potential character is directly related to the increase of the region with non-zero vorticity ($\omega \neq 0$) as a consequence of its production, which generates a recirculation region downstream in the form of a vortical structure. The increase of this region occurs until the production of vorticity is balanced with its diffusion and convection, or until it cannot accept any more vorticity, reaching maximum vortex circulation (Gharib et al. [151]). When the latter occurs, the vortical structure is disconnected from the source of vorticity generation in what is known as the pich-off process (Gharib et al. [151] and Fernando and Rival [222]). According to Morton [144], vorticity generation is an inviscid and instantaneous process and the generation rate is directly related to the acceleration of the wall boundary and to the existence of a tangential pressure gradient. In particular, he establishes that for an impulsively accelerated flat plate in an incompressible fluid (First Stokes problem) the vorticity generation rate takes the form:

$$\frac{dk_\omega}{dt} = -\frac{1}{\rho} \frac{\partial P}{\partial s} - \frac{dU(t)}{dt} \quad (4.22)$$

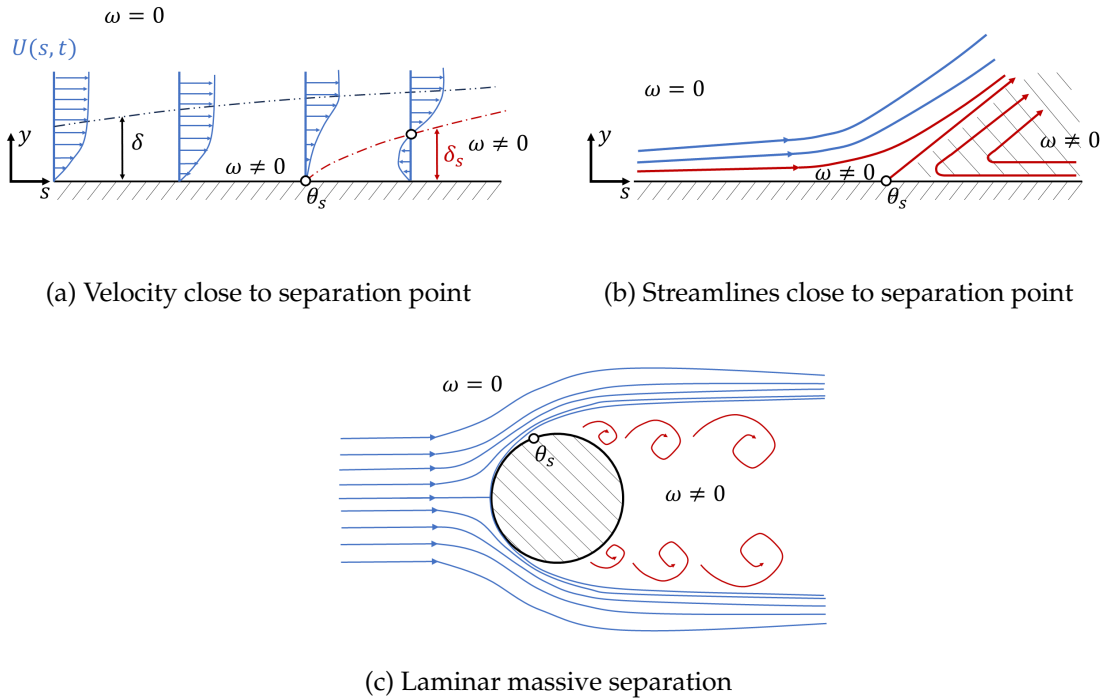


Figure 4.16: Figure (a) Velocity distribution and boundary layer thickness (δ) close to the separation point θ_s for an outer velocity $U(s, t)$, for $D \gg \delta$. (b) Streamlines close to the separation point. (c) Sketch of flow past a sphere under laminar massive separation.

Where k_ω is the gross generation. Thus in our case, we would have an immediate generation as a consequence of the initial impulse, where the vorticity at $t = 0$ is produced impulsively with a gross generation per unit of surface area proportional to $-U_0$. On the other hand as the sphere continuously decelerates we would have two generation contributions; one consequence of the acceleration where the generation rate is proportional to $-dU/dt$; and another as a result of the variation of pressure due to the sphere's curvature $-\partial P/\partial s$. It is important to note that the signs determine the sense of the vorticity vector $\vec{\omega}$. The produced vorticity is immediately redistributed by viscous diffusion increasing the flow region where $\omega \neq 0$. For fast generation cases, only viscous diffusion remains after the momentum ceases, while for slow generation cases, diffusion and generation coexist, making it difficult to distinguish between the two effects.

A direct consequence of the increase in size of this recirculation region is that it significantly changes the shape of the outer region $\omega = 0$, which is why the potential solution of the sphere is no longer valid and the problem loses its initial symmetry, figure 4.16c. Then, this lack of symmetry leads to a vortical structure which can be observed in figure 4.15 (b) and figure 4.15 (c). This vortical region grows [figure 4.15 (d) and figure 4.15 (e)], and at $t = 0.099$ s, figure 4.15 (f), its size in the 2D plane is of the order of the sphere's diameter (0.04 m). This vortical structure can be identified as a vortex ring. It is worth comparing this figure 4.15 with figure 8 of Fernando et al. [119]. In the present decelerating case, the vortex ring is much larger relative to the sphere diameter (vortex ring smaller diameter $\sim 40\%$ of the sphere diameter) than in the case in which the sphere accelerates (vortex ring smaller diameter $\sim 10\%$ of the sphere diameter, Fernando et al. [119]). The reason could be that as the sphere decelerates the wake overcomes the sphere. And it is that as reflected in the work Sattari et al. [223], for an isolated vortex the growth rate of the vortical structure, and therefore its size, is related to the transport of vorticity-containing fluid mass to the wake. At the same time, according to Fernando et al. [119] the vortex dynamics vary significantly as a consequence of the mutual vortex-body interaction, being in the present case of study of greater significance as it is freely decelerating. Regardless of the aforementioned, a practical consequence is that the separation point moves forward up to a point located between 60° and 70° from the forward stagnation point.

In relation to the presence of this vortex ring attached to the sphere in time, it is interesting to compare our experimental observations with the results obtained by Fernando et al. [119] for the case of a sphere under constant acceleration. Focusing on the subcritical conditions $Re = 50000$ the authors found that for normalized distances of $x(t)/D = 1.5$ the formation of the vortex ring had been practically suppressed (totally suppressed for the supercritical case) and the sphere evolves towards the steady state practically at $x(t)/D = 3$. In our case, as can be seen from the streamlines in figure figure 4.15 (f) the situation differs from the steady state and a vortex ring structure is still present for traveled distances of the order of $x(t)/D = 4$. It is also observed that for these distances traveled by the sphere, the vortex ring structure retains its symmetry and its relative position with respect to the sphere, which seems to indicate that there has not been a disconnection from its vorticity source or pinch-off. The latter departs from the experimental observations of Fernando

et al. [119], where the pich-off was generated at normalized distances of $x(t)/D > 3$ and from the observations of Jeon and Gharib [154] for the cylinder, where for $x(t)/D > 4$ the vortical structures have already reached its vortex formation time and the first vortex shed is generated.

To assess the dependency of the observed flow pattern sequence along with the initial velocity, a new case (with three repetitions) was tested. In this case, the initial velocity was $U_{06} = 0.91 \text{ m/s} \pm 3\%$ (less than half the minimum velocity in table 4.3). Figure figure 4.17 shows the representative sequence of snapshots. It could be observed that the sequence of events is the same as it was for U_{01} (see figure 4.15) but delayed in time (recall that $U_{01} = 3.78 \text{ m/s}$ and $U_{06} = 0.91 \text{ m/s}$). For this later case, the initial Re was 36000 that is well into the subcritical regime. Qualitatively, the size of the vortex ring was similar, too, in both cases. So, the conclusion is that within the range of velocities considered, the flow topology sequence leading to the large vortex ring formation is always similar.

4.6.2 Analysis on the onset of separation point after impulsive start-up

In relation to the analysis of the separation point proposed by Fernando et al. [119], the authors considered a potential flow solution for the velocity field and, in combination with the x -component of the unsteady Euler equation, generated an analytical expression for unsteady pressure over the surface of the sphere that they used to analyze the effect of acceleration qualitatively. In this paper, the authors estimated the separation point based on the favorable pressure gradient $\partial P/\partial s > 0$. Thus, the distribution of $\partial P/\partial s$ for the non-stationary potential flow on the sphere was given by the authors as follows:

$$-\frac{1}{\rho} \frac{\partial P}{\partial s} = \frac{3}{2} \ddot{x}(t) \sin(\theta_s) + \frac{9}{4R} \sin(\theta_s) \cos(\theta_s) [U_0 + \ddot{x}(t)t]^2 \quad (4.23)$$

The first adverse pressure point occurs when $\partial P/\partial s = 0$ and from this point it will be possible to clear θ_s . If we substitute the approximation $U_0 + \ddot{x}(t)t$ by $\dot{x}(t)$, the equation results in following expression:

$$\theta_s = \cos^{-1} \left(\frac{-2R\ddot{x}(t)}{3[\dot{x}(t)]^2} \right) \quad (4.24)$$

However, in the present decelerating case, said approach did not work. Applying Fernando et al. [119] to the results obtained from our trajectories, we find that the results do not agree with the experimental observations, predicting at the initial instant a separation point close to $\theta_s = \pi/2$. On the other hand, when considering the negative acceleration (deceleration), $\partial P/\partial s$ takes the opposite sign in some cases, resulting in unrealistic solutions. It should be taken into account that the model presented by the authors applied to constant positive accelerations, so that the stationary solution only appeared at the final moments. On the other hand, in the present case the attached vortex ring structure is so large (of the order of the sphere itself) that the hypothesis of a potential velocity field on the sphere surface is no longer applicable.

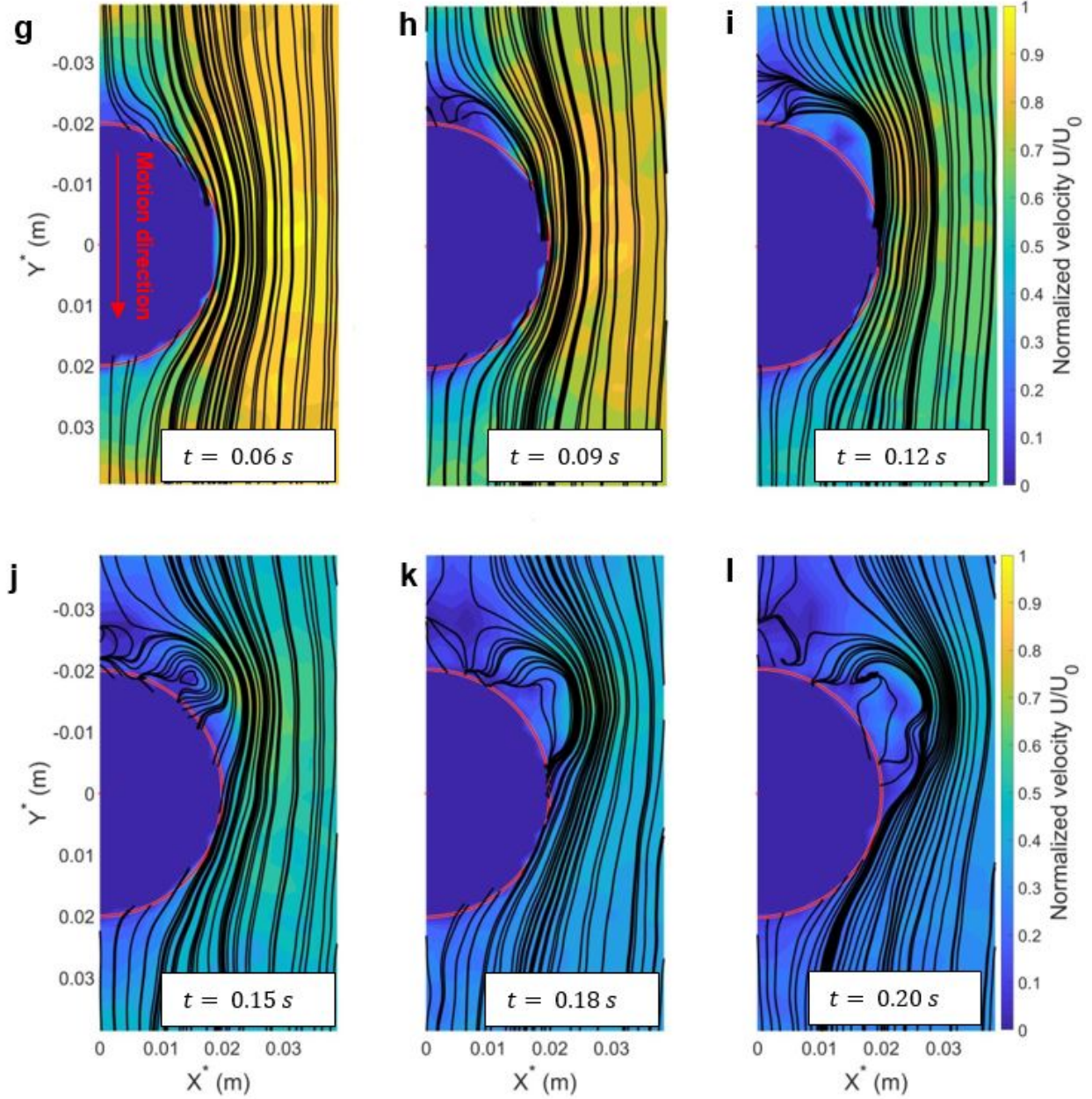


Figure 4.17: Counterpart of figure 4.15 for m_3^* and $U_{06} = 0.91 \text{ m/s}$. Subplots (a)-(f) correspond to times 0.06, 0.09, 0.12, 0.15, 0.18 and 0.20 s, respectively. The flow field is transformed into the resting frame of the sphere.

If we focus on this initial loss of potential character, it is interesting to estimate the evolution of the flow around the impulsively accelerated sphere as a consequence of the start-up process. Using a sphere-bounded moving reference frame, at the instant $t \rightarrow 0$ the flow can be considered inviscid and the development of the boundary layer will depend on the time from instant $t = 0$. Considering these initial instants it is reasonable to assume that the boundary layer is limited to a very thin region of the order $O(aRe^{-\alpha})$. Thus, at these initial instants it is possible to appeal to the laminar boundary layer theory by which it

is possible to approximate the solution to the asymptotic limit $Re \rightarrow \infty$ (Leal [189]). As previously mentioned this approximation is satisfied for conditions where the vorticity containing region, $\omega \neq 0$, is restricted to very thin regions and therefore the outer region is still approximate to the potential flow around a sphere $U_{pot}(s)$, being s the distance inside the boundary layer so $s = 0$ is the front stagnation point of the sphere or $\theta = 0$. Following Schlichting and Gersten [129], the velocity in the outer region for a start-up process can be approximated as:

$$U(s, t) = U_{pot}(s) \left[1 - e^{-t^*} \right] \quad (4.25)$$

Being $t^* = t/t_R$, and $t_R \rightarrow 0$ in the limit of an impulsively accelerated wall. For these limiting cases the characteristic time takes the form:

$$t_R^* = \frac{\nu t_R}{l^2} \quad (4.26)$$

Where ν is the kinematic viscosity of the fluid and l is the characteristic length to be taken as R . Subsequently, the Schlichting and Gersten [129] notation is followed for the dimensionless boundary layer variables, denoted by the superscript *. The transformation variables for the boundary layer equations are defined as:

$$\bar{y} = y^* \sqrt{\frac{R^2}{t_R \nu}}, \quad \bar{v} = v^* \sqrt{\frac{R^2}{t_R \nu}}, \quad for \quad t_R^* \rightarrow 0 \quad (4.27)$$

Considering incompressible flow and constant fluid properties, the momentum equation in x-direction will be:

$$\frac{\partial u^*}{\partial t^*} + t_R^* \left(u^* \frac{\partial u^*}{\partial s^*} + \bar{v} \frac{\partial u^*}{\partial \bar{y}} \right) = \frac{\partial U^*(s^*, t)}{\partial t^*} + t_R^* U^*(s^*, t) \frac{\partial U^*(s^*, t)}{\partial s^*} + \frac{\partial^2 u^*}{\partial \bar{y}^2} \quad (4.28)$$

Since $t_R^* \rightarrow 0$ we can apply the perturbation series of the form:

$$u^*(s, \bar{y}, t^*) = u_0(s, \bar{y}, t^*) + t_R^* u_1(s, \bar{y}, t^*) + \dots \quad (4.29)$$

From this point on, it will be considered as a two-term expansion series, $u^* = u_0 + t_R^* u_1$, whose boundary conditions will be:

$$\bar{y} = 0 \quad u_0 = 0, \quad u_1 = 0 \quad (4.30)$$

$$\bar{y} = \infty \quad u_0 = U^*(s^*, t), \quad u_1 = 0 \quad (4.31)$$

Considering the velocity expression defined in equation (4.25) coupled with the boundary conditions equation (4.31) it is possible to define the exact solution u_0 as:

$$\frac{\partial u_0}{\partial t^*} - \frac{\partial^2 u_0}{\partial \bar{y}^2} = 0 \quad (4.32)$$

This equation presents the general solution of form $u_0/U^*(s^*) = f_0'(y/2\sqrt{t\nu})$ which substituted into the equation (4.32) leads us to solve the differential equation $f_0''' + 2(y/2\sqrt{t\nu})f_0'' =$

0 for the boundary conditions $f'_0(0) = 0$ and $f'_0(\infty) = 1$. Locally the exact solution is equivalent to the first Stokes problem and will take the form:

$$\frac{u_0}{U^*(s^*)} = f'_0(y/2\sqrt{tv}) = \frac{2}{\sqrt{\pi}} \int_0^\gamma e^{-\gamma^2} d\gamma \quad (4.33)$$

Where $\gamma = y/2\sqrt{tv}$. If the first-order perturbative correction is included, considering the axylsymmetric problem, the expansion takes the form:

$$\frac{u_0}{U^*(s^*)} = f'_0(\gamma) + \frac{dU^*(s^*)}{ds^*} f'_1(\gamma) \quad (4.34)$$

On the other hand, in Schlichting and Gersten [129] it is indicated that the position of the separation point for non-stationary boundary layers cannot be specified on the basis of the criterion $d\tau_w = 0$ alone, but that the separation point will be the one at which a singularity ($d\tau_w/ds \rightarrow 0$) occurs. This separation point can be determined on the basis of the MRS criterion (Moore [140], Rott [141] and Sears [142]), by which the separation point must occur on the zero vorticity line bisecting a recirculation region so it must be satisfied that $u = 0$ and $\partial u/\partial y = 0$. If we derive with respect to y and after a series of simplifications we arrive at:

$$1.128 + t_0 \left(1.607 \frac{dU_{pot}}{ds} \right) = 0 \quad (4.35)$$

That considering the case of the sphere in the conditions under study, $U_{pot} = 3/2U_{0i} \sin(s/r)$, would be as follows:

$$1 + 1.573t_0 \frac{3}{2} \frac{U_0}{R} \cos(\theta_s) = 0 \quad (4.36)$$

The first instant when $\tau_w = 0$ corresponds to $\theta_s = \pi$ and therefore $t_0 U_0/R = 0.4238$ can be cleared. According to the separation criterion for unsteady boundary layers previously described, the separation will occur at times $t > t_0$. For stationary conditions the separation point over the sphere expressed in degrees in laminar regime is estimated to be $\theta_s = 110^\circ$ (Schlichting and Gersten [129]). Applying a similar approach to that of Wang [224] and Cowley [225] for the impulsively accelerated cylinder it is possible to estimate the instant when stationary separation point is reached by substituting this in the equation (4.36), corresponding to $t_s = 1.239 \cdot R/U_0$. It is possible to represent from equation (4.36) the evolution of θ_s as a function of $t_s U_0/R$. Substituting both for the case $U_0 = U_{01} = 3.78 \text{ m/s}$, we obtain $t_0 = 0.002 \text{ s}$ and $t_s = 0.007 \text{ s}$. In the same way, considering the work of Dennis and Walker [135], where the authors perform a higher order expansion perturbation, it is possible to use the results obtained by them numerically to establish the instant t_0 for finite Reynolds numbers. In this work, the authors consider this instant t_0 as the instant at which separation occurs. According to the authors, the dimensionless separation time $T_s = t_s U_0/R$, that in our case is 0.0396 for $Re > 100000$. In the present case, $U_0 = U_{01} = 3.78 \text{ m/s}$, so $t_s = 0.002 \text{ s}$. Then, the sequence of TR-PIV frames corresponding to the case m_3^* and U_{01} was plotted and the potential solution subtracted at each point in the flow field obtained via TR-PIV. The idea is to identify the first frame in which the asymmetry was present, thereby indicating a breakaway from the potential solution. This was done by subtracting the potential flow solution from the TR-PIV flow field and comparing the

differences qualitatively. This first snapshot is presented in figure 4.18, and it correspond to $t_{sf} = 0.013$ s.

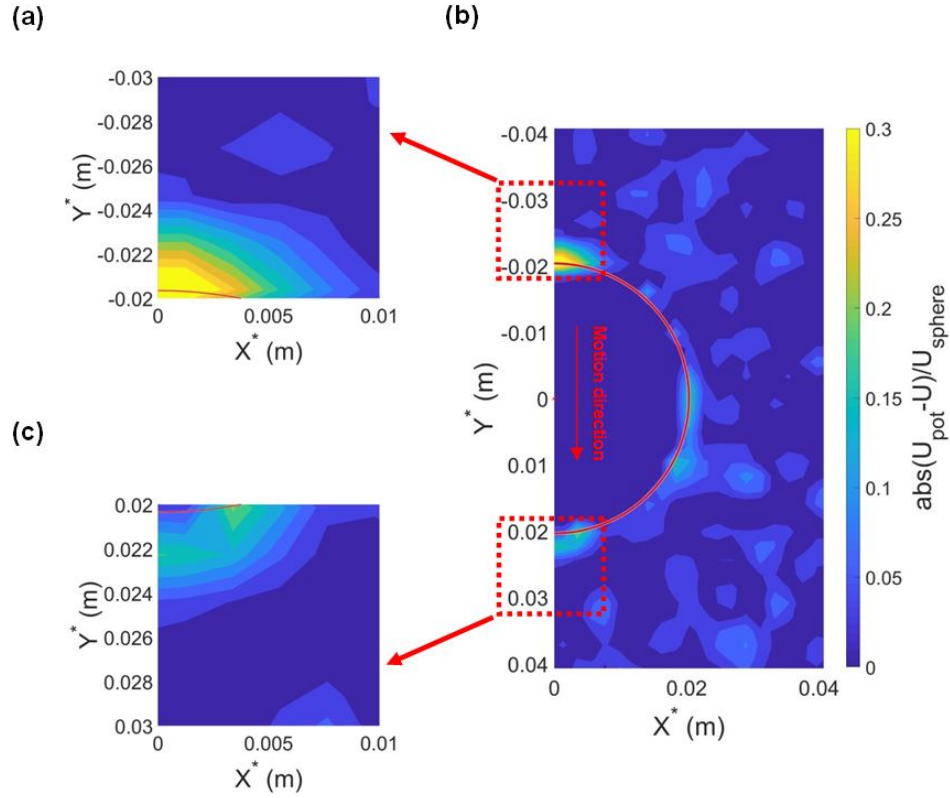


Figure 4.18: TR-PIV snapshot at $t = 0.013$ s (m_3^* and U_{01}). The potential flow solution has been subtracted from the TR-PIV flow field. (a) Close-up view of the backward stagnation region, (b) velocity field around the sphere, and (c) close-up view of the forward stagnation region. U_{sphere} is the sphere velocity in the laboratory frame.

The discrepancy between analytical and experimental results ($t_{sf} = 0.013$ s and $t_s = 0.002$ s according to Dennis and Walker [135] and $t_s = 0.007$ s according to the estimation based on Wang [224] and Cowley [225]) may seem large, but few aspects need to be accounted for. First, on the analytical side, the analysis of Dennis and Walker and the analysis of the condition of vanishing wall shear stress were based on the condition that the movement was a start-up process, so that after an impulsive acceleration the sphere remained at constant velocity U_0 . In our case, when the sphere has moved of the order of one diameter, the velocity has decayed of the order of 7%. On the other hand, both analyses required that $t_R^* \rightarrow 0$ and therefore it is only valid for the initial instants. Secondly, on the experimental side we recall that the TR-PIV spatial resolution is about 2 mm (5% of the sphere diameter), so separation can be identified provided that the size of the recirculation region, where vorticity is non zero, is larger than the threshold. Then, given these experimental and analytical shortcomings, what could be said, only, are that considering the difficulty of detecting very all deviations from the potential flow solution, the present experimental result at most does not contradict the predicted value.

4.7 Results:Data Reduction

The objective of this section is to generalize the results obtained in section 4.4 as much as feasible. This generalization will be based on a modified acceleration parameter (Ac).

From the Fluid Mechanics standpoint, the conventional Ac represents a balance between the local flow derivative, $\partial/\partial t$, and the convective derivative, $\vec{v} \cdot \vec{\nabla}$, in the Navier-Stokes equations. Therefore, Ac is a measure of unsteadiness. In the present case, the local derivative is of order $\dot{U}(t)$, and the convective derivative is of the order of $U^2(t)/L_{ref}$, being L_{ref} a reference length of the problem and $U(t)$ the sphere's velocity. There are, however, some differences regarding the case of a sphere that accelerates/decelerates from a constant non-zero velocity, and the case in which the sphere accelerates/decelerates impulsively from a zero-velocity condition. In the former case, the wake is already developed when the acceleration (or deceleration) phase begins; ideally, the wake length goes to infinity, and the characteristic length of the problem is taken to be the sphere's diameter. In the latter case, the sphere starts from rest, and the wake grows along with the sphere's motion. In this case, the characteristic length of the flow topology is taken to be the wake length, similar to distance covered by the sphere, $x(t)$. This hypothesis has, nevertheless, a limitation because the characteristic length cannot grow indefinitely. As it will be shown below, the hypothesis, for the range of parameters that were considered in the study, holds up to a maximum wake length of the order of four sphere diameters (the maximum depth reached by the sphere for the largest initial velocity was of the order of seven diameters).

Then, the parameter defined herein for data reduction purposes is $Ac = x(t)\dot{U}(t)/U(t)^2$. However, two aspects should be considered about the limiting behavior of Ac . First, $Ac \rightarrow 0$ for $t \rightarrow 0$, which means that information about unsteadiness is lost in this limit ($Ac \rightarrow 0$ for $t \rightarrow 0$ in the conventional definition). Second, both the modified and conventional Ac are not well suited to deal with times at which the sphere ($m^* < 1$) reaches zero velocity at the end of the downward trajectory. At this point, $U = 0$, $\dot{U} \neq 0$, and $Ac \rightarrow \infty$. For all these reasons, parameter $x(t)\dot{U}(t)/U(t)^2$ should not be properly called acceleration parameter, but data reduction parameter. In any case, to keep nomenclature simple, it will still be called the acceleration parameter (albeit modified) in the text.

It could be observed that Ac involves the computation of second derivative, $d^2x(t)/dt^2$, divided by the square of a first derivative $(dx(t)/dt)^2$. This term might be difficult to obtain accurately via finite differences out of the sphere position measured by image tool, $x(t)$. To verify that actual computation of Ac is reasonably accurate, the repetitions described in table 4.3 have been used to assess its repeatability. Taking into account the results of the calibration of the image system described in the section 4.3.1, where it was determined that the error of the tool $\Delta_x = \pm 0.073 \text{ px}$, when considering the magnification $M_0 = 2.902 \text{ px/mm}$ is equivalent to 0.026 mm and it is obtained that the relative error of the position measurement $\epsilon_{x_r} = 0.026/x(t)$. Considering the clock time error of the camera $\Delta t = 1 \times 10^{-6}$ and since the timelapse between frames was $t_{sm} = 2 \times 10^{-3} \text{ s}$, the

relative error associated with time was $\epsilon_{t_r} = 5 \times 10^{-4}$. The velocity, $\dot{x}(t)$, and acceleration, $\ddot{x}(t)$, extracted from the trajectory and calculated by centred differences, table 4.3, are used. Therefore, the relative errors in the velocity will be of the order $\epsilon_{\dot{x}_r} = 2\epsilon_{x_r} + \epsilon_{t_r} \simeq 2\epsilon_{x_r}$ (where the error associated with the camera's clock time has been neglected). On the other hand, the error relative to the acceleration will be of the order $\epsilon_{\ddot{x}_r} \simeq 4\epsilon_{x_r}$. Considering the definition of the Acceleration number Ac , its relative error can be directly estimated from the errors in position, ϵ_{x_r} , velocity error, $\epsilon_{\dot{x}_r}$, and acceleration error, $\epsilon_{\ddot{x}_r}$, being its approximate value $\epsilon_{Ac} \sim 7\epsilon_{x_r}$. This relative error is largest during the initial instants of the displacement and decreases rapidly as the sphere moves. For example, if the sphere has moved a distance of 4 mm or $D/10$, the relative error in Ac will be $\epsilon_{Ac} \sim 0.0475$.

The value of the parameter Ac for the cases m_3^* and U_{01} with their three respective repetitions R1, R2 and R3, table 4.3, is illustrated in figure 4.19. In the same, the points of Ac corresponding to the instants around the deceleration peak are represented, which for the present case will be considered approximately $t < 0.05$ s. On the other hand, the parameters Ac_1 , Ac_3 and Ac_5 for the cases m_3^* and U_{01} , U_{03} and U_{05} (table 4.3) are represented in the figure 4.20. Direct observation of the curves leads us to ask whether there is a relationship between the curves that allows them to collapse into a single curve. For this purpose the results of the figure 4.20 are used and represented in two figures; a first one where Ac_3 vs Ac_1 is represented, figure 4.21; and a second one where Ac_5 vs Ac_3 is represented, figure 4.22. One relevant aspect highlighted from the figures is that a linear relation (fitting red straight lines) become apparent. This linear relation holds up to $t = 0.05$ s and from this instant onwards the experimental data tend to deviate from these linear relations. Although this relation is small in origin, it follows that the linear fitting may extend slightly beyond $t = 0.05$ s, probably around the deceleration peak.

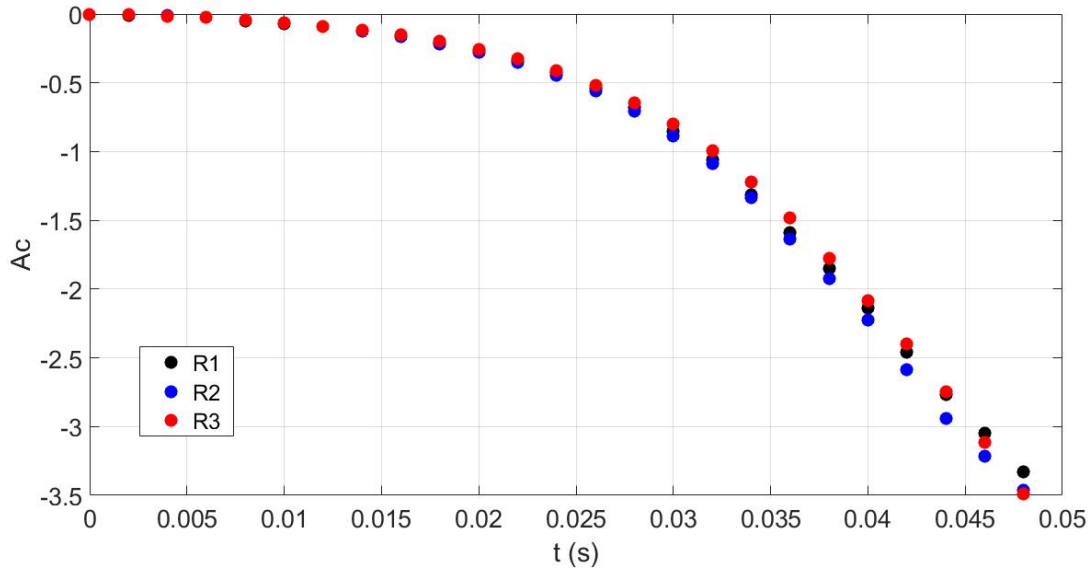


Figure 4.19: Ac vs time until $t = 0.05$ s of case m_3^* and U_{01} , the repetitions R1,R2 and R3 are represented.

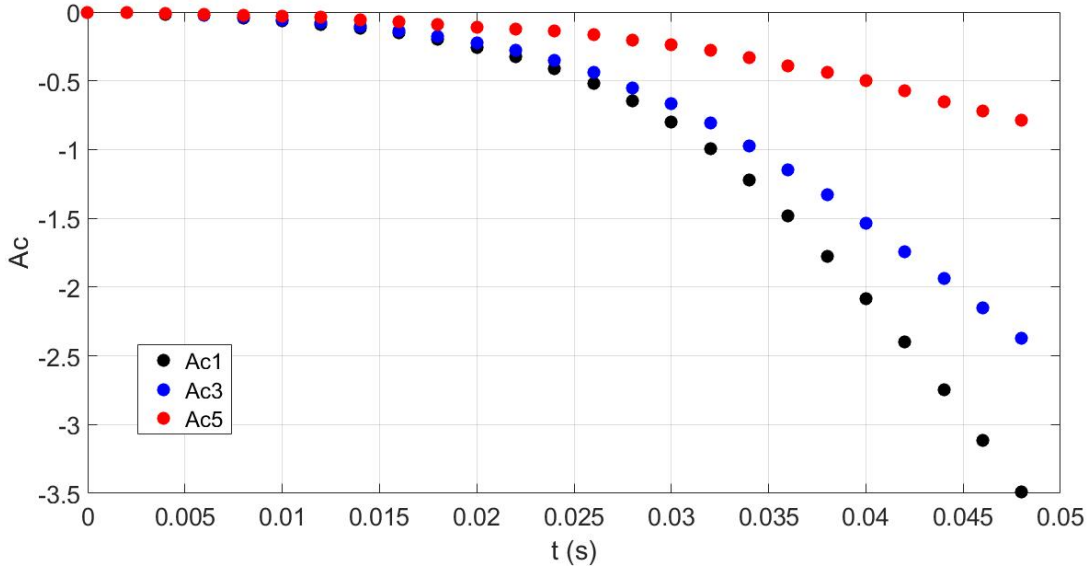


Figure 4.20: Ac as a function of time until $t = 0.05$ s of case m_3^* and U_{01} , U_{03} and U_{05} .

Then after relations have been found to exist between the experimental modified acceleration parameters, the relevant question for data reduction purposes is whether the slopes of these relations can be associated with some physical characteristic of the trajectories. Inspection of the results obtained shows that modified acceleration parameter of different trajectories can be linked to each other by a parameter that is the ratio of the square of their initial velocities (this correspondence should be considered data reduction relation rather than a scaling law). Then, the slopes of the linear relations presented in figure 4.21 and figure 4.22 are approximated as:

$$Ac_3 = \left(\frac{U_{03}}{U_{01}} \right)^2 Ac_1, \quad (4.37)$$

$$Ac_5 = \left(\frac{U_{05}}{U_{03}} \right)^2 Ac_3, \quad (4.38)$$

Then, the following data reduction law (found to be valid for all other m^* and U_0 cases) can be formulated:

$$\frac{x_i(t)}{U_i^2(t)} \frac{dU_i(t)}{dt} = \left(\frac{U_{0i}}{U_{0j}} \right)^2 \frac{x_j(t)}{U_j^2(t)} \frac{dU_j(t)}{dt} \quad (4.39)$$

This relation links the acceleration parameter of trajectory i to the acceleration parameter of trajectory j via the square ratio of their respective initial velocities U_{0i} and U_{0j} . The practical implication is that, for a given m^* , equation (4.39) allows for the computational prediction of all trajectories at whatever initial velocity provided that one simple sphere trajectory is measured experimentally. Suppose, for instance, that the trajectory associated

with m_3^* and U_{02} has been measured experimentally, or in other words $x_2(t)$, $U_2(t)$ and $dU_2(t)/dt$ are known. Then defining $\Phi(t)$ as:

$$\Phi(t) = \frac{x_2(t)}{U_2^2(t)} \frac{dU_2(t)}{dt} \quad (4.40)$$

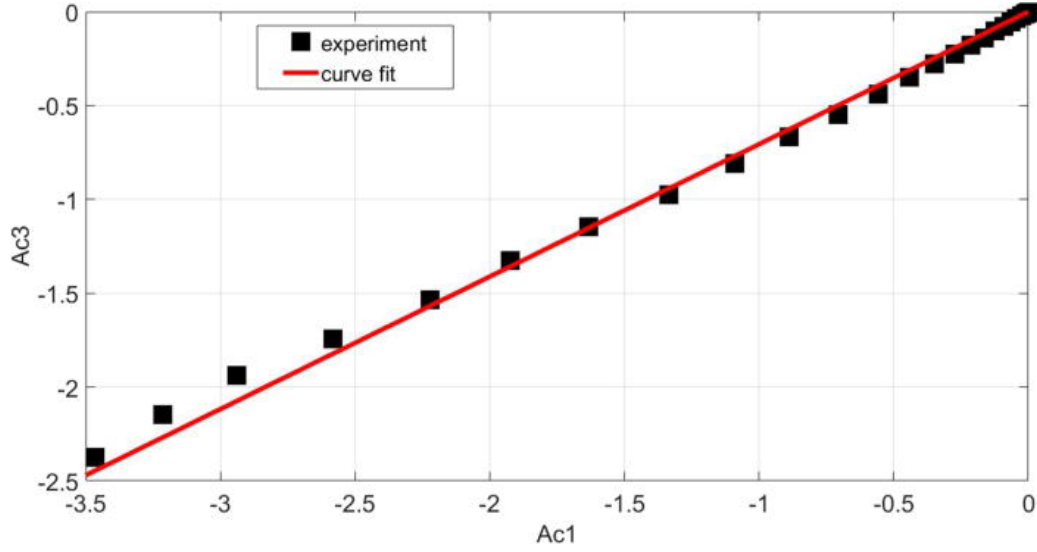


Figure 4.21: Ac_3 vs Ac_1 for m_3^*

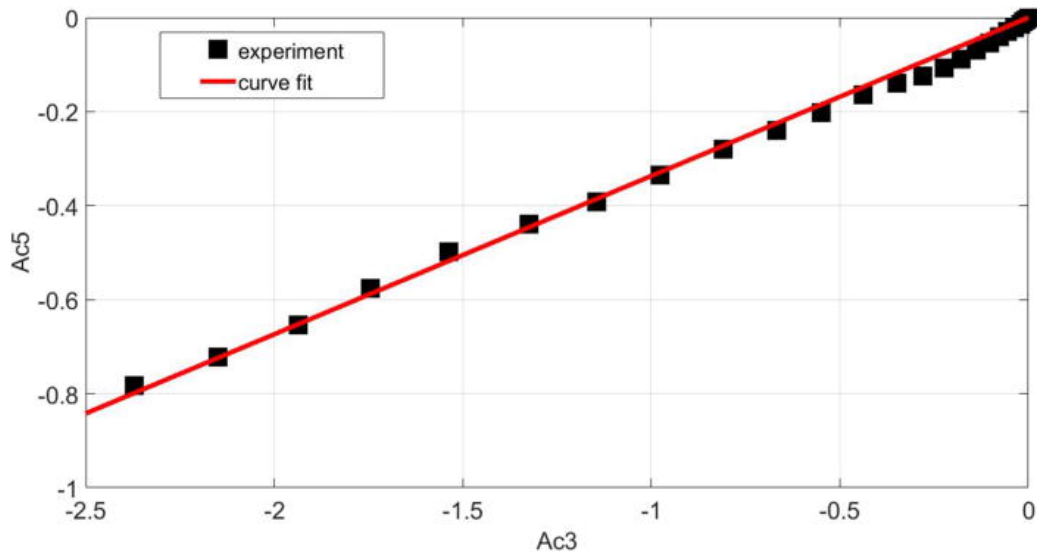


Figure 4.22: Ac_5 vs Ac_3 for m_3^*

The trajectory corresponding to, say, U_{04} can be obtained solving the following ordinary differential equation and initial conditions:

$$\Phi(t) = \left(\frac{U_{02}}{U_{04}} \right)^2 \frac{x_4(t)}{\left(\frac{dx_4(t)}{dt} \right)^2} \frac{d^2 x_4(t)}{dt^2}, \quad (4.41)$$

$$x_4(0) = 0 \quad \left. \frac{dx_4(t)}{dt} \right|_{t=0} = U_{04} \quad (4.42)$$

The linear relation for this specific pair of acceleration parameters was assumed to hold, approximately, until $t = 0.07$ s, that correspond to $Ac \sim -3.5$ for trajectory U_{02} . This assumption, as explained above, extends the typical curve fitting range illustrated in figures figures 4.21 and 4.22. At this time instant, $t = 0.07$ s, the sphere has covered 0.15 m that is of order four times the diameter of the sphere ($4 \cdot D$), and its velocity has decreased from 3.29 m/s to 0.9 m/s. Since, in this case, the maximum depth reached by the sphere was 0.26 m, relation 4.39 held for about 60% of the trajectory. This analysis has been repeated for other combinations of the modified acceleration parameters, and it has been found that relation 4.39 holds, typically, up to distance of three to four sphere diameters depending on the case. This represents between 40% and 60% of the full trajectory. Another way of presenting relation 4.39 is as follows:

$$\Psi(t) \equiv \frac{\frac{x_i(t)}{U_{0i}^2(t)} \frac{dU_i(t)}{dt}}{U_{0i}^2} = \frac{Ac_i(t)}{U_{0i}^2}, \quad \forall i \in 1, \dots, n \quad (4.43)$$

Where $\Psi(t)$ is a general function, and n identifies the trajectory. Figure 15 presents $\Psi(t)$ for $i = 1, 3, 5$, where it could be observed that $\Psi(t)$ has a general character up to $t = 0.05$ s. Beyond that point the curves $Ac_i(t)/U_{0i}^2$ starts to diverge. More specifically, the spread around the mean of $\Psi(t)$ at $t = 0.05$ s is $\pm 5\%$.

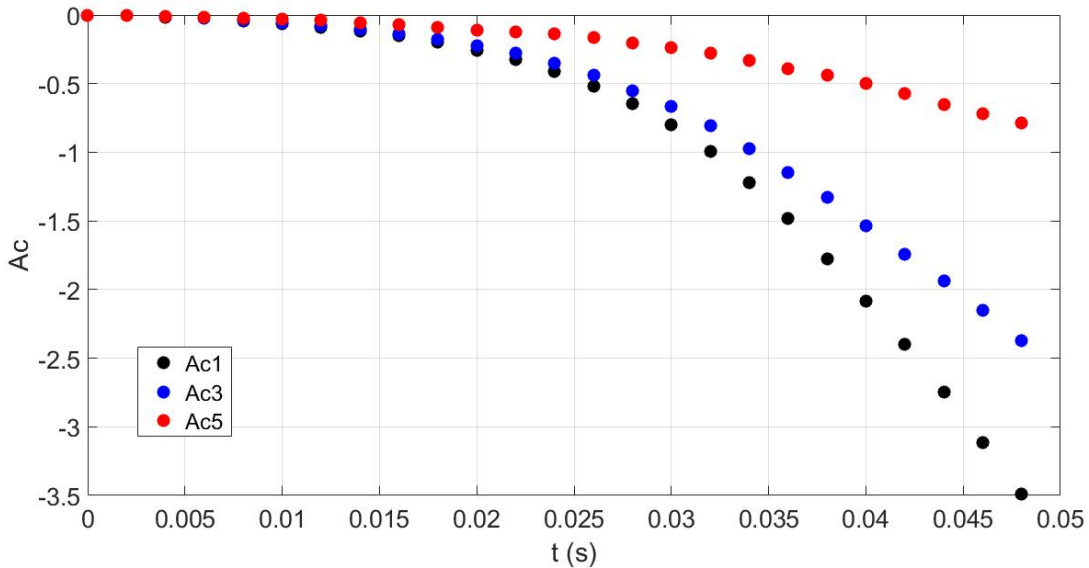


Figure 4.23: Counterpart of figure 4.20 for parameter $\Psi(t) = Ac_i(t)/U_{0i}^2 \quad \forall i \in 1, \dots, n$

To assess quantitatively the validity of equations (4.41) and (4.42), figure 4.24 shows the comparison between the actual experimental results (position and velocity) corresponding to the trajectory of case m_3^* and U_{04} , and thus obtained by solving numerically equations (4.41) and (4.42) using the experimental data of trajectory U_{02} . Integration has been carried out until limit $t = 0.07$ s that is slightly beyond the threshold value of $t = 0.05$ s that has been mentioned above. Figure 4.25 presents a more unfavorable case: actual experimental results of trajectory U_{05} vs those obtained by solving equation (4.41) using the experimental data in trajectory U_{01} . The agreement shown in both figures 4.24 and 4.25 between experimental measurements and model predictions seems to be reasonable. Other comparisons involving different sets of velocities have been performed and present a similar level of agreement.

Finally, a qualitative global flow field assessment of equation (4.39) was performed using the TR-PIV frames (for m_3^*). For experimental trajectory U_{01} , two different times were selected: $t_{a1} = 0.025$ s and $t_{b1} = 0.032$ s. For experimental trajectory U_{06} , the times chosen were as follows: $t_{a6} = 0.080$ s and $t_{b6} = 0.1$ s. The criterion was that at t_{a1} and t_{a6} modified acceleration parameter (Ac) was the same ($Ac = -0.5$). In addition, this was also the case for t_{b1} and t_{b6} ($Ac = -0.95$). For then, the idea was to compare TR-PIV flow fields for different times and trajectories in such a way that the corresponding modified acceleration parameters were the same. For the case $Ac = -0.5$, the vortex ring structure has not yet fully formed, while for the case $Ac = -0.95$, the vortex ring is already developing and moving forward. This qualitative comparison of TR-PIV snapshots is performed in figure 4.26. It could be observed that flow topologies for the same Ac but for different trajectories agree reasonably well. This supports the idea that, in a sense, Ac might be considered as a kind of similarity parameter for this particular problem of the freely decelerating sphere before and impulsively start-up process.

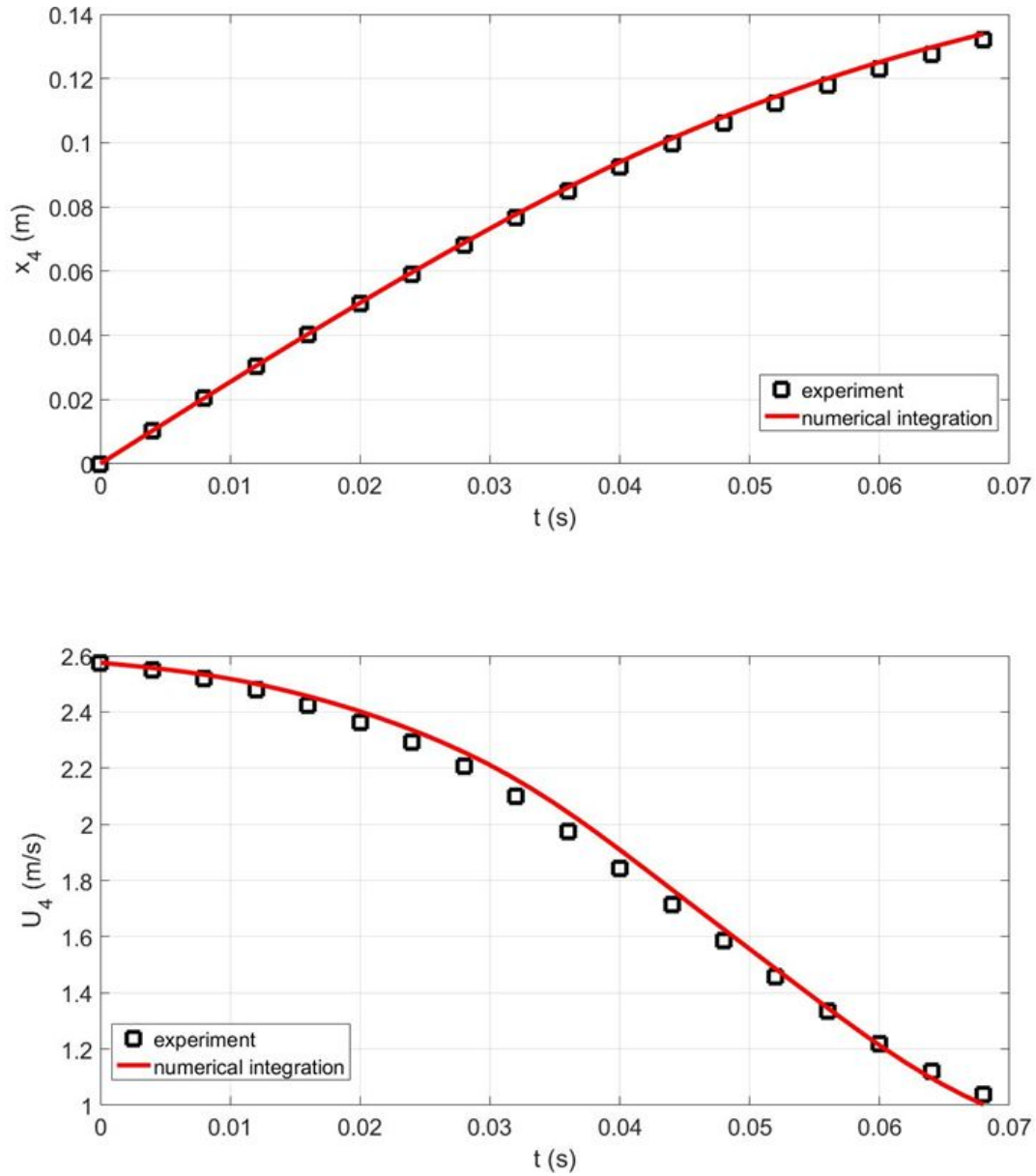


Figure 4.24: Comparison (for m_3^*) between experiments and model results (sphere position-top; sphere velocity-bottom) for trajectory U_{04} , and those obtained solving differential equation and initial condition equations (4.41) and (4.42). $\Psi(t)$ is obtained from U_{02}

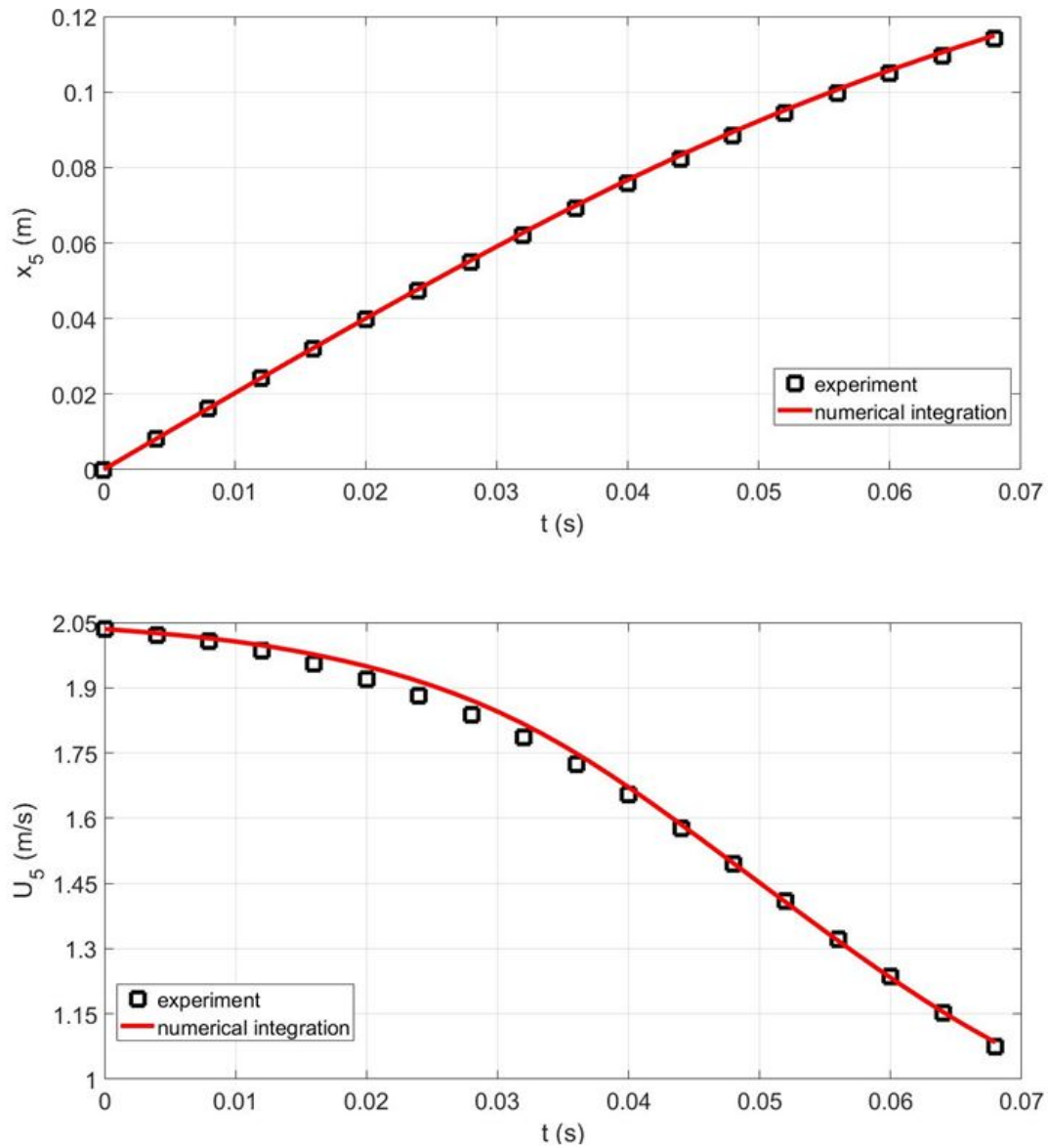


Figure 4.25: Comparison (for m_3^*) between experiments and model results (sphere position-top; sphere velocity-bottom). The case is the one in which trajectory U_{05} is computed using the experimental data of trajectory U_{01} .

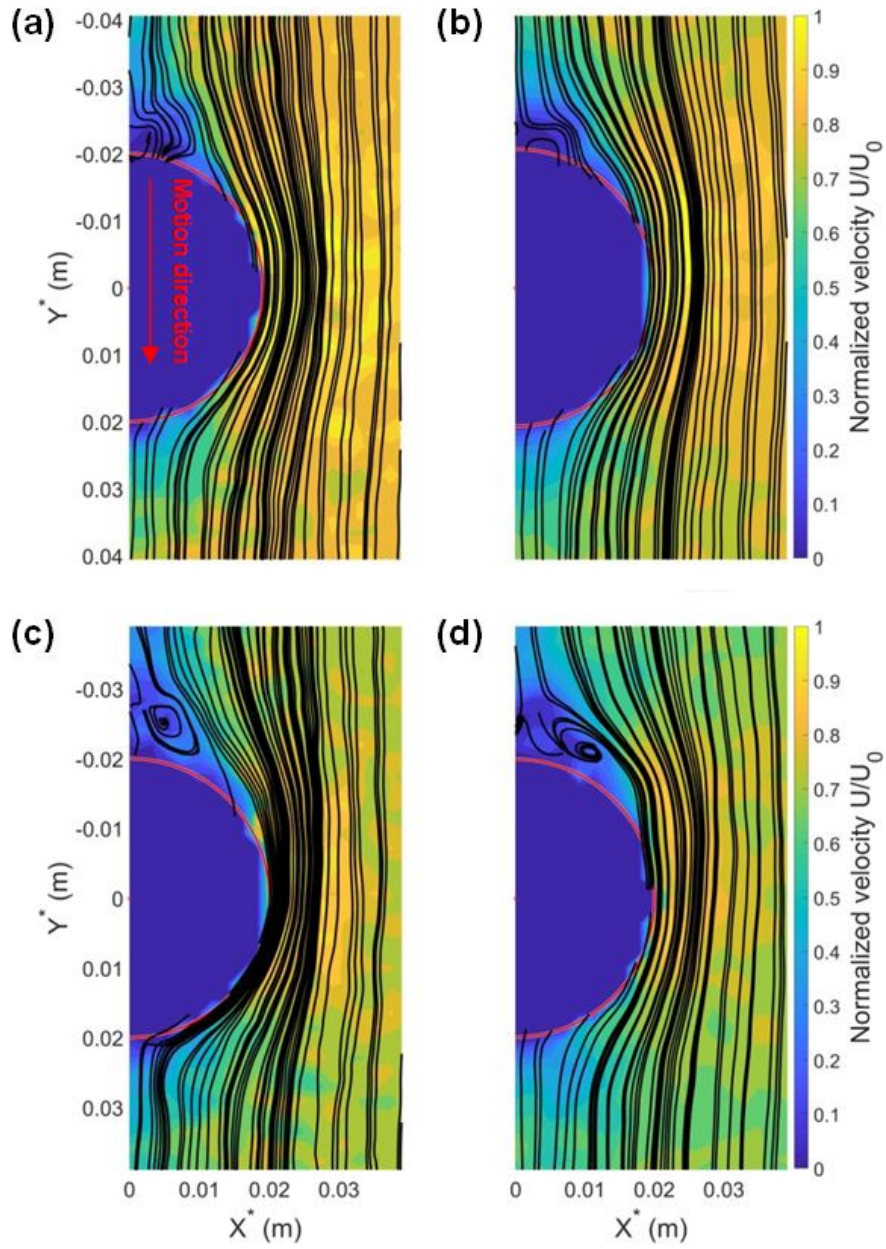


Figure 4.26: (a) and (c) TR-PIV snapshots of case m_3^* and U_{01} at times $t_{a1} = 0.025$ s and $t_{b1} = 0.032$ s. (b) and (d) TR-PIV snapshots of trajectory U_{06} at times $t_{a6} = 0.08$ s and $t_{b6} = 0.1$ s. Ac for (a) and (b), and (c) and (d) were the same (-0.5 and -0.95 , respectively). The flow field is referred into the sphere's moving reference frame.

4.8 Conclusions

An experimental study has been performed on the problem of an impulsively started close-to-neutral buoyancy freely decelerating sphere 0.04 *m* in diameter. The range of six initial velocities considered went from 0.91 up to 3.78 *m/s*. Initial *Re* (subcritical) varied from 3.6×10^4 up to 1.51×10^5 . Solid-to-fluid density ratio spanned from 0.78 to 0.94. An optical system allowed for direct measurement of the sphere position, $x(t)$. From this information, velocity and acceleration were computed via finite differences. Care was taken to remove noise out of the position measurements as much as feasible to compute reasonably accurate first and second derivative.

In these conditions, and within the range of tested parameters, it was found that the hydrodynamics drag force opposing the motion was much larger than the buoyancy force. This drag force increased as a function of time, meaning that its third derivative was positive for a sizable part of the trajectory. After reaching a maximum, the drag force decreased toward zero along with the velocity. The region of the trajectory where potential flow could be assumed to hold (no boundary layer separation present) was very small, which agrees with previous theoretical results based on asymptotic analysis.

TR-PIV analysis showed that as the sphere decelerates, the wake overcomes the sphere itself. This translates into the generation of a vortex ring structure that grows along the trajectory until it reaches a torus smaller diameter whose size is of the order of the sphere diameter. The vortex ring is generated at the aft part of the sphere and moves gradually forward. It appears from the flow visualizations that the vortex ring reaches, at some stage, an asymptotic limit where further growth is suppressed, and the ring does not move further relative to the sphere. All this is in contrast with cases reported in the literature dealing with accelerating spheres.

It has been found that the sphere dynamics can be reasonably modeled using a modified version of the conventional acceleration parameter $Ac = D\dot{U}(t)/U(t)^2$. In particular, it has been found that the appropriate spatial scale of the problem is not the sphere diameter but the developing wake length (approximately equal to the sphere position). Then, when the modified acceleration parameter is defined as $Ac = x(t)\dot{U}(t)/U(t)^2$, different trajectories with different initial velocities can be linked via their corresponding acceleration parameters and their initial velocities, so, in a sense, this modified *Ac* could be seen as a kind of similarity parameter. For example, for two different trajectories, say 1 and 2, with initial velocities U_{01} and U_{02} , the following relation holds $x_1(t)\dot{U}_1(t)/U_1(t)^2 = (U_{01}/U_{02})^2(x_2(t)\dot{U}_2(t)/U_2(t)^2)$. This has practical consequence: for any given m^* , all spheres trajectories can be computed out of the experimental measurement of a single trajectory only.

This computation involves solving a relation that has the form of a second-order ordinary differential equation with two initial conditions. This relation has been tested extensively

by means of computing a number of trajectories out of the experimental data of other different trajectories, and comparing the results obtained with experimental data of the test trajectories. For the range of parameters that have been tested, the hypothesis that the reference length is $x(t)$ holds for about $x(t) \sim 3 - 4D$ sphere diameters that represents about 40% to 60% of the full trajectory length. Another qualitative global flow comparison has been made by comparing TR-PIV snapshots of different trajectories with different initial velocities at different times, but in such way that the corresponding modified acceleration parameters were the same. Visual comparisons of the flow topologies proved to be satisfactory, so this suggests that the problem has an underlying mathematical structure that might be worth investigating in a future work.

Chapter 5

Conclusions

Throughout this thesis document, the problem of droplet breakup phenomena exposed to flows of aeronautical interest has been addressed, developing a breakup criterion based on the formation of Rayleigh-Taylor instabilities on their surface. From its development, the impact of acceleration on droplet breakup has been established, as it conditions both the onset of breakup and the deformation history of the droplet, the latter effect being coupled to the drag forces and therefore to the acceleration itself. Derived from this significance of the acceleration in the breakup onset and supported by the existing literature, an in-depth study of the non-stationary processes of the establishment of flow around a freely decelerating sphere has been motivated. This short chapter aims to summarise the conclusions drawn as a consequence of the work carried out in this thesis and which have enabled the main milestones set out in the section section 1.3 to be met.

In relation to the development of a breakup criterion based on the theoretical and experimental analysis of droplet breakup when exposed to continuously accelerated flows as a consequence of the passage of an airfoil, the main conclusions have been:

- The phenomenon of droplet breakup around the stagnation line of the flow generated by an airfoil has been studied on the basis of experimental results extracted from the INTA rotating arm facility. In these tests, a symmetric airfoil rotates at a 2.2 *m* radius and reaches a distilled water droplet generated by a mono-disperse generator. The tests provide data on the trajectory, deformation and subsequent breakage of the droplets using a high-speed camera in combination with a 2000 *W* xenon backlight system. Specifically, AOI of physical dimensions $13.52 \times 21.97 \text{ mm}^2$ with acquisition frequencies of 75000 *fps* are analysed. On the total of 4902 droplets tested 42 representative cases are analyzed with droplet diameters between 600 μm and 1200 μm which are reached by the flow generated by two symmetric profiles of radius 0.103 *m* and 0.070 *m* moving at tangential velocities of 91, 81 and 71 *m/s*.
- Semi-empirical models developed by [116] are used to predict the time evolution of the droplet position and its deformation as a function of the fluid properties of the medium and of the droplet, together with the initial conditions of the droplet radius

(R_d), airfoil leading edge radius (R_c) and airfoil velocity (U_m). These models are based on a number of experimentally verified hypotheses, being the most relevant ones: that the volume of the droplet remains constant; that the droplet deforms as an oblate spheroid; that the drag coefficient can be modeled as the sum of a stationary and a non-stationary coefficient (where the deformation is accounted in the former based on an interpolation between the $C_{D_{disk}}$ and $C_{D_{sphere}}$); and that the droplet velocity is negligible compared to that of the airstream, and therefore its component is mainly horizontal $V_{sx} = V_s$.

- The airflow velocity generated by the airfoil approach has been studied based on the characterisation of the flow field by means of PIV analysis of 100 pairs of images, divided into a grid of 32×32 interrogation windows of 64×64 px. This analysis has been complemented by two CFD simulations; one of them 2D using an inviscid model, and the other by a 3D simulation using a sliding mesh configuration and a $k-\epsilon$ model in combination with enhanced wall functions. As a result, it has been possible to model the droplet's perceived air from the expression $V_s/U_m = 1/(1 + \Pi_5 - \tau)^{1.8}$ applicable in the stagnation streamline and based on the potential flow past a cylinder.
- A breakup criterion has been proposed based on the previous work of Joseph et al. [29], which is based on the analysis of Rayleigh-Taylor instabilities on droplet surface, generated as a consequence of the acceleration of one phase over the other. For its application, a series of hypotheses have been considered, the most relevant being: that the breakup regime is mainly Rayleigh Taylor Piercing (RTP), so that no Kelvin Helmholtz Instabilities (KHI) are produced; that the acceleration is calculated on the basis of the previously mentioned semi-empirical model; that at the instant of breakup, the interface of the drop can be approximated to a flat surface; and that the critical wavelength that marks the onset of breakup is the volume to surface ratio. This criterion has been satisfactorily coupled to the previously proposed droplet deformation and trajectory model.
- As a result, it has been feasible to track the evolution of the breakup parameter Ψ (equation (3.57)) and to quantify its value at the instant of breakup. Considering that at this instant its value should be exactly 1, we have analysed the discrepancies of the breakup parameter with experimental results, finding that its maximum values are in the range of $\pm 10\%$ for the model with $R_c = 0.103$ m and $\pm 20\%$ for $R_c = 0.070$ m. In addition, its repeatability has been tested using the repetitions of cases 2 and 42, achieving values around the mean of between $\pm 3\%$ and $\pm 13\%$ respectively. Based on these results, it has been concluded that under conditions of lower radius of curvature, situations further away from the proposed one-dimensional model are encountered, given that the dimensions of the region around the stagnation streamline where it is fulfilled is smaller. Furthermore, it is observed that the accuracy of the model is independent of the profile velocity, which indicates that the modelling of the V_s and the drag coefficient proposed models the effects of drag reasonably well. Finally, the dependence of the errors on radius seems to be consistent with the tendency of the droplets to drift into non-spherical shapes after generation, which is far from the initially assumed conditions.

- Finally, a case study has been proposed to demonstrate the value of the model in estimating droplet breakup situations in the surroundings of an airfoil. For this purpose, a horizontal train of droplets is numerically integrated, each of them separated by a dimensionless distance $\Pi_5 = r_0/R_c$, and the breakup is evaluated in the regions of the airfoil in the vicinity of $\pm 0.1R_c$. The following situations are considered: the droplet reaches the region without initiating breakage; the droplet initiates breakage before being struck; the droplet initiates breakage but leaves the analysis region without colliding; and the droplet leaves the region without initiating breakup. This example shows the predictive value of estimating the conditions around the stagnation line for the study of the degradation of the aerodynamic performance of a given airfoil under given weather conditions.

In relation to the study of the problem of the non-stationary dynamics of a freely decelerating sphere in a fluid at rest, the main conclusions of the work in chapter 4 are summarised below:

- A new experimental setup has been proposed, not previously found in the literature for this type of studies, where based on the collision of a mass, a transfer of momentum is produced in the vertical direction that propels a spherical sample of 0.04 m in diameter inside a fluid at rest. The sphere is filled with different materials allowing to reach solid-to-fluid density ratios (m^*) of 0.78, 0.88 and 0.94. The setup consists of a high stiffness structure with a ram-mass that moves vertically until it reaches an actuator. As a consequence of the impact, the sphere accelerates impulsively in a vertical direction and then decelerates freely inside a water tank of 0.9 m^3 volume achieving a unidirectional movement. Specifically, a total of six velocities (3.78, 3.29, 2.94, 2.54, 2.03, 0.91 m/s) are tested, which imply Reynolds numbers between 3.6×10^4 and 1.51×10^5 , thus corresponding to subcritical conditions referred to the sphere moving in a stationary regime. The experimental setup allows to reach high repetitivities for the kinetic energy transferred to the test sample in the range of 0.04% to 0.81%.
- The trajectory of the spheres is analysed using a high-speed camera that captures images at 500 fps for a physical AOI of $0.35 \times 0.35\text{ m}^2$ with a resolution of 1024×1024 pixels. The camera is configured to avoid image blurring with a shutter speed of $1/5000\text{ s}$ and a combination of illumination at an angle of 45° , which combined with the contrast generated by the matte black painted surface of the samples moving on a white background, allows the image to be optimised for analysis. Specifically, in order to obtain the trajectory, an image tool has been developed which, from a series of transformations, morphological operators and segmentation by canny edge detector, gets the points that constitute the edge of the sphere. These points will be used to solve by means of the Newton-Raphson method the least squares problem that will give the optimal centre and radius for each snapshot. The calibration of the method allows obtaining the magnification, $M_0 = 2.902\text{ px/mm}$, and the errors in position and radius for the method, which are $\pm 0.074\text{ px}$ for the vertical position and $\pm 0.196\text{ px}$ for the radius.
- Thanks to the experimental setup and the image tool developed, it has been possible to characterize the trajectory of the spheres with a low noise level and, based on it, to

obtain their velocity and acceleration as a function of time. The results are consistent with other studies of accelerating spheres, where acceleration peaks are found as the flow establishes around the sphere from the initial instant. With these results it has been possible to separate from the total the contribution of the buoyancy force and the contribution of hydrodynamic forces, finding that the value of the latter is much larger than that of buoyancy. Normalizing the hydrodynamic forces in the form of the drag coefficient, its evolution over time has been examined and it has been found that at the origin $C_D \sim 0$. Subsequently, the drag coefficient progressively increases its value with time, exceeding the stationary value (up to values of the order of $C_D \sim 1.2$) before decreasing.

- A TR-PIV analysis has been performed in order to link the sphere's kinematics with the structure of the flow field. For this purpose, the previously described high-speed camera is used in combination with a 4 W DPSS CW laser, which delivers a 3.0 mm thick laser light sheet. The camera is set to acquire images at 4500 fps at AOI of $273 \times 264 \text{ mm}^2$, with a shutter speed of 1/5000 s. Hollow Glass Spheres (HGS) of 10 μm have been used for seeding, so that particle image densities per pixel are approximately $N_{ppp} = 0.2$. The image is divided into $D_I = 16 \text{ px}$ interrogation windows and the open source software PIVlab is used under the configuration of the FFT multi-pass window deformation correlation technique, allowing vector resolutions of 2.1 mm. In order to perform this analysis, a preprocessing tool had to be developed to improve the image of the particle close to the surface of the sphere, eliminating brightness and the intensity gradient resulting from the laser sheet opening. The developed tool also obtains trajectory data and generates masks for TR-PIV analysis in the shadow regions and on the surface of the sphere. Since the velocity is not constant, a subsampling procedure is used to adapt the minimum resolvable displacement to the kinematics of the sphere.
- We show the TR-PIV analysis for the cases of m_3^* and velocities $U_{01} = 3.78 \text{ m/s}$ and $U_{06} = 0.91 \text{ m/s}$ which reveals fluid structures significantly different from those of the accelerated case studied in Fernando et al. [119] In particular, it has been observed from the streamlines that the separation point evolves from the initial instants until reaching 60° and 70° with respect to the frontal stagnation point. Moreover, in contrast to the results of Fernando et al. [119] the vortical structure reaches significantly larger dimensions in the ring ($\sim 40\%$ of the sphere diameter), without detaching from its surface. It should be noted that in the case of Fernando et al. [119] the sphere reached the stationary solution for distances traveled of the order of $x(t)/D = 3$, while in the present case at distances of $x(t)/D = 4$ the vortex ring structure is still present (with dimensions around $\sim 40\%$ of the sphere diameter). In general the vortex ring reaches what appears to be an asymptotic limit and does not advance beyond these 60° and 70° in the direction of the frontal stagnation point. These observations have been discussed in the framework of previous work concerning vortex dynamics, vorticity production and vortex structures. Finally, the loss of the initial potential solution is analysed by tracing the evolution of the separation point. Firstly, the application of the model proposed by Fernando et al. [119] for a continuously accelerated sphere has been proved to be inaccurate as it does not follow the experimental findings of

our case. Finally, the evolution of this separation point has been analysed using a low order perturbation expansion series under the thin boundary layer approximation. The result, when compared with previous studies conducted in this approach, justifies the potential character disappearance, although it reveals the need to use another experimental setup to analyse the exact moment of separation of the boundary layer and its evolution.

- Trajectory analysis has revealed that the dynamics of the sphere can be modeled using a variation of the acceleration parameter in the form of data reduction parameter $Ac = x(t)\dot{U}(t)/U(t)^2$. This appears to be related to the length of the developed wake. When comparing this acceleration parameter between different cases, a relation appears based on their initial velocities, U_{0i} and U_{0j} , in the form of $x_i(t)\dot{U}_i(t)/U_i(t)^2 = (U_{0i}/U_{0j})^2(x_j(t)\dot{U}_j(t)/U_j(t)^2)$. Based on this relationship between parameters it has been possible to obtain, by numerically solving this second-order ordinary differential equation, the trajectories of different cases known a given master trajectory. This relation holds for about $x(t) \sim 3 - 4D$ sphere diameters that represents about 40% to 60% of the full trajectory length.
- Another particularly interesting qualitative result is that, when comparing the topologies of the flow field obtained via TR-PIV for two different cases. If we compare those frames linked to time instants with the same value of the acceleration parameter (Ac), the topology of the flow structures of both are similar. This strengthens the idea that Ac can be treated in some way as a similarity parameter for our freely decelerating sphere after an impulsive start-up. This may suggest an underlying physics that might be worth investigating in a future work.

Chapter 6

Published Articles

During the execution of this doctoral thesis, two articles were published that compile the methodology, results and analysis collected in chapters 3 and 4. Specifically and respectively:

- P. Lopez-Gavilan, A. Velazquez, A. García-Magariño and S. Sor. 'Breakup criterion for droplets exposed to the unsteady flow generated by an incoming aerodynamic surface'. In: *Aerospace Science and Technology* 98 (2020), p. 105687. DOI: [10.1016/j.ast.2020.105687](https://doi.org/10.1016/j.ast.2020.105687)
- P. Lopez-Gavilan, A. Barrero-Gil and A. Velazquez. 'Experimental study on the impulsively started motion of a close-to-neutral buoyancy freely decelerating sphere'. In: *Physics of Fluids* 35.4 (2023), p. 047106. DOI: [10.1063/5.0141322](https://doi.org/10.1063/5.0141322)

Additional results regarding the different breakup modes for ethyl-alcohol droplets using the methodology and experimental setup presented in the chapter 3 have been presented at the ILASS-Europe 2019, 29th Conference on Liquid Atomization and Spray Systems in Paris, France:

- S. Sor, A. García-Magariño, P. Lopez-Gavilan and A. Velazquez. 'Visualization of ethyl alcohol droplet breakup with large deformations in a continuously accelerated flow field'. In: *ILASS-Europe 2019, 29th Conference on Liquid Atomization and Spray Systems*. 2019

In addition, using the same experimental methodology of chapter 3 with different airfoil models, results have been obtained by evaluating the non-stationary term of the model presented in Sor et al. [116] and presented at the AIAA AVIATION 2023 Forum in San Diego, CA:

- A. Garcia-Magariño, S. Sor and P. Lopez-Gavilan. 'A Study of the Influence of the Acceleration Number in Droplet Trajectory Model in the Vicinity of Airfoils'. In: *AIAA AVIATION 2023 Forum*. 2023, p. 4093. DOI: [10.2514/6.2023-4093](https://doi.org/10.2514/6.2023-4093)

Finally, although not directly related to this thesis but derived from it, the application of the TR-PIV experimental techniques included in the chapter 4, has served as a motivation

to study the impact of light deficiency and particle blurring and its contribution on the loss of correlation. Concluding in the following publication:

- P. Lopez-Gavilan and A. Barrero-Gil. 'On the limits of Particle Image Velocimetry with continuous wave lasers'. In: *Experimental Thermal and Fluid Science* 144 (2023). ID: 271092, p. 110873. DOI: [10.1016/j.expthermflusci.2023.110873](https://doi.org/10.1016/j.expthermflusci.2023.110873)

Chapter 7

Future Work

Derived from the development of the work of this thesis, one of the questions that arises is the degree of generality of the theoretical model described in [116]. Related to this generalisation, during the test campaign in [197] it was observed that the droplets with trajectories close to the shoulder region presented a different deformation from that studied in the stagnation line. Thus, when the droplets approached this region, they showed an apparent roll in the direction of the incident stream. In addition, due to the angle of the incident stream and the non-uniformity of the particle, lift effects, detachment of the centre of pressure from the centre of mass and therefore rotation are to be expected. In this line, an extensive review of the existing literature on particle spin models, particle motion in shear flows and dynamics of rotating non-spherical particles has been carried out.

Subsequently, the complete evaluation of the data set of [197] was carried out, where it was concluded that the non-stationary effects of the drop insertion in the test field caused a dispersion in the initial conditions that made its evaluation difficult. Therefore, an experiment was designed and built to eliminate the deformation effect of the multiphase fluid. For this purpose, a set of oblate spheroids was fabricated, by means of resin printing with a high resolution printer, with different ratios between the semi-axes that are equivalent to the shapes usually found in the experiments with droplets carried out in the rotating arm facility.

In addition, a system for the insertion of these particles at different inclination angles has been designed, manufactured and installed in a test section of the miniature wind tunnel of the experimental aerodynamics department of INTA. Based on the preliminary results, three models have been proposed, one model based on a torque coefficient, constructed as the sum of a stationary and a non-stationary coefficient, another quasi-static model based on the potential theory and another one that models the torque coefficient based on the equation of aeroelastic motion as an underdamped harmonic oscillator related to the aerodynamic momentum.

After the tests, the models have been evaluated and reasonably good results have been found, although a difficulty has been observed regarding the ability of the experiment to generate repetitive initial conditions that allow the data to be grouped in order to draw definitive conclusions. The next objective is to carry out an experimental campaign with a new setup where the initial conditions are repetitive and to test the existing models in the literature for rigid solids against the models proposed by the authors. The results will be compiled in a scientific article for publication in high impact JCR journals.

In relation to the study of the free deceleration of the sphere after impulsive acceleration, future work is aimed at repeating the TR-PIV tests in the developed installation, prioritising smaller regions of interest, so that the flow field can be analysed with greater spatial resolution at regions closer to the surface of the sphere. The intention is to assess whether it is possible to deduce fluid quantities such as fluid kinetic energy and vorticity. An attempt will be made to link the results with the boundary layer separation angle and the dimensionless acceleration parameter developed. Furthermore, thanks to its versatility, it is proposed to test other types of axisymmetric shapes, in order to study the mathematical structure suggested by the shape of the developed data reduction parameter and its underlying physics with a view to a generalisation of the results.

In addition to the above, the naval hydrodynamics lab of the Centro de Experiencias Hidrodinámicas del Pardo belonging to the sub-directorate of naval systems of INTA (CEHIPAR-INTA) is being equipped with a towing tank type installation without free surface where, by means of an actuator, studies of non-stationary dynamics can be carried out. From this installation it would be possible to replicate the results obtained in the research by means of forced laws of motion through a combination of collision plus actuator. This will allow the use of larger samples which, by recreating the results obtained in the research, will allow the incorporation of instrumentation for measuring fluid magnitudes on their surface and accelerations, obtaining more information in relation to fluid-body interaction.

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Annexes

Breakup criterion in the limiting case of large deformation

For a sphere, equation (3.53) can be simplified as $\Phi_d = 6(4/3\pi R_d^3)/(4\pi R_d^2) = 2R_d$. The surface area of an spheroid in dimensional variables is:

$$A_d = \pi \left(2a^2 + \frac{b^2}{e} \ln \left(\frac{1+e}{1-e} \right) \right) \quad (1)$$

where

$$e = \left(1 - \left(\frac{b}{a} \right)^2 \right)^{1/2} \quad (2)$$

In the limit $b \rightarrow 0$, second term in the equation (1) tends to:

$$\lim_{b \rightarrow 0} \pi \frac{b^2}{e} \ln \left(\frac{1+e}{1-e} \right) \rightarrow 0 \cdot \infty \quad (3)$$

Se trata por tanto de un límite indeterminado que se resuelve sustituyendo $e = (1 - \epsilon_d)^2$ y aplicando L'Hopital. El límite de equation (3) pasa a ser:

$$\lim_{b \rightarrow 0} \pi \frac{b^2}{e} \ln \left(\frac{1+e}{1-e} \right) = \lim_{\epsilon_d \rightarrow 0} \pi a^2 \frac{\epsilon_d^2}{(1 - \epsilon_d)^{1/2}} \ln \left(\frac{1 + (1 - \epsilon_d)^{1/2}}{1 - (1 - \epsilon_d)^{1/2}} \right) \rightarrow 0 \quad (4)$$

Se puede por tanto simplificar la superficie del esferoide descrita en equation (1) por la expresión:

$$A_d \sim 2\pi a^2 \quad (5)$$

Y por tanto podemos expresar la equation (3.53) como:

$$\phi_d \sim 6 \frac{4/3\pi a^2 b}{2\pi a^2} = 4b \quad (6)$$

That is, the critical wavelength that defines the onset of instability (breakup) of a deformed droplet for high deformations ($b \rightarrow 0$ so $a \gg b$) is $4b$, or what is equivalent four times the shortest diameter of the oblate spheroid. Naming b^* the minor semi-axis value that defines

the onset of instability and replacing it in equation (3.51), after some transformations a new relation is obtained:

$$4b^* = 2\pi \left(\frac{\sigma}{\rho_d \ddot{x}^*} \right)^{1/2} \quad (7)$$

Where $\ddot{x}^*$ is the acceleration that fulfills breakup conditions. By using the scaling described in equations (3.11) and (3.12) the relation 7 is rendered dimensionless and expressed as:

$$\frac{2^3}{(3\pi^3)^{1/2}} \frac{\dot{\eta}^{*1/2}}{\alpha^{*2}} \frac{1}{\Pi_4^{1/2}} = 1 \quad (8)$$

In the experimental cases analyzed, the right hand side of equation (8) at the breakup onset is not one so we employ the variable Ψ in order to check the breakup criterion as follow:

$$\frac{2^3}{(3\pi^3)^{1/2}} \frac{\dot{\eta}^{*1/2}}{\alpha^{*2}} \frac{1}{\Pi_4^{1/2}} = \Psi \quad (9)$$

This relation equation (8) is the simplified breakup criterion for large deformations when $a \gg b$. This simplified criterion has, also, been applied to the 42 cases presented in tables 3.3 and 3.4. In general, it has been found that theoretical equation (8) tends to anticipate the onset of experimental breakup. In the case of the larger airfoil leading edge radius, Ψ varied between 0.98 and 1.18 (as compared to $\Psi = 1$), while in the case of the smaller leading edge radius, Ψ varied between 0.97 and 1.35.