Modeling of the Collapse of a Macroporous Material

Del Olmo, Daniel
ETSICCP-UPM

Serrano, Alcibíades
ETSICCP-UPM

ABSTRACT: The macroporous materials are a mix of solid particles, joined together with bridges of materials that may be the same or different of the solid particles. For example, volcanic rocks like volcanic agglomerates. In this way, it is interesting to trying to explain how the collapse of these materials takes place. With the great improvement of the numerical methods and the power of computers it has been possible to carry out a discrete analysis instead of a continuum one, like would had happened with the classical theories of continuum. This article shows the first steps taken in this path of modeling the collapse of macroporous materials in a discrete way.

1 INTRODUCTION

This article is trying to show the first steps in modeling a macroporous material as a discrete media and not as a continuum. That is to say, the macroporous is modeled as a conjunction of particles joined together by a breakable material and not as a homogeneous continuum material.

In this research, the macroporous material had been modeled as a series of spheres particles that are joined with a material of some stiffness with the capacity of breaking. The model created in this way is loaded similarly to a consolidated triaxial test. In the model, it is observed how the contacts get broke and how the collapse of the macroporous material takes place.

The research and the article have been structured in the following way:

- A numerical macroporous material was created.
- The equations of behavior of the material were defined.
- A series of calculus were run, according to the equations and to a criterion of failure of the joining material.
- Finally some conclusions were obtained from the calculus in order to improve the model and take it to the next stage of development.

2 CREATING THE MACROPORUOS MEDIUM

To make the mechanical study of the three dimensional macroporous medium, it is necessary to generate it numerically. To achieve this, the following hypotheses were assumed:

1) The granular material must be contained in a container of prefixed shape and dimensions. The chosen shape was a one meter mat cell. This cell was chosen because from an operative point of view, it is necessary that the boundary conditions are separated enough to obtain results that are not boundary affected. And in addition, a cubic shaped container simplifies the numerical process of filling it with particles.

2) For simplicity in the generation of the material and attending to geometrical criterions only, the particles are spherical shape. In this way, when one sphere is placed over another, the programmed process is simplified, compared with other possible shapes of the particles as ellipsoids, that would have made the process much more complicated for not being a constant radius shape.

3) When the spheres fall into the cell, they roll over the others until a stable position is reached. A position is considered to be stable whenever a sphere is in contact with three other elements or with the bottom of the cell. Any wall or any particle already inside the cell is considered to be one of the three necessary elements to obtain such stable position in addition to the bottom of the cell.

4) The particles or spheres had a continuous grain size distribution. This curve can be seen in Figure 1. In the generation of the medium, with this predetermined distribution, a certain probability of size is established. In this way, the probability of size is directly proportional to the percentage retained by the size of each fraction, according to the curve.

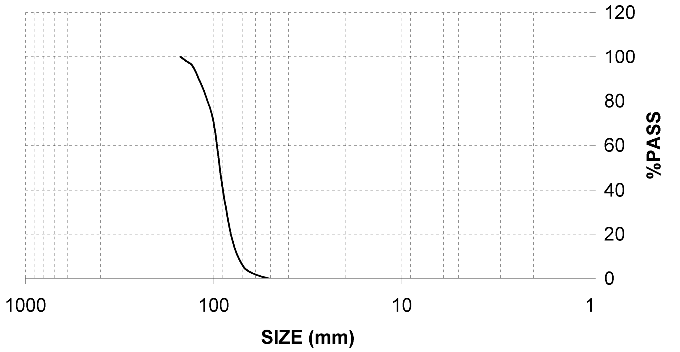

Figure 1. Grain size distribution for the particles that formed the macroporous material.

5) After this process, a cubic cell filled with particles is obtained. These particles are in contact but a joining material is filling those contacts, keeping the particles joined and cohesioned. In order to introduce this joining material a gap is needed between the particles in contact. To do this, a random uniform distribution between two values was used to reduce the diameter of the particles accordingly to it.

6) An elastic and breakable joining material was inserted in the previously existing particle to particle contacts. The joining material was assimilated to square section prisms with a section side (B) and a length (H) that was determined in the fifth step. The B dimension is randomly created using a random uniform distribution between 2 and 6 times the length. Doing this, a 2 to 6 B/H ratio is obtained. This is necessary because a beam model is not wanted but a wall-beam model instead. By doing this, it is obtained a joining tablet material in each previously existing contact.

After all this process, a macroporous medium is obtained. In Figures 2 and 3 it is shown some examples of the macroporous created. The particles are in white and the joining material in red. The joining material is not in its real dimensions, it has only been included to observe which particles are in contact and which are not.

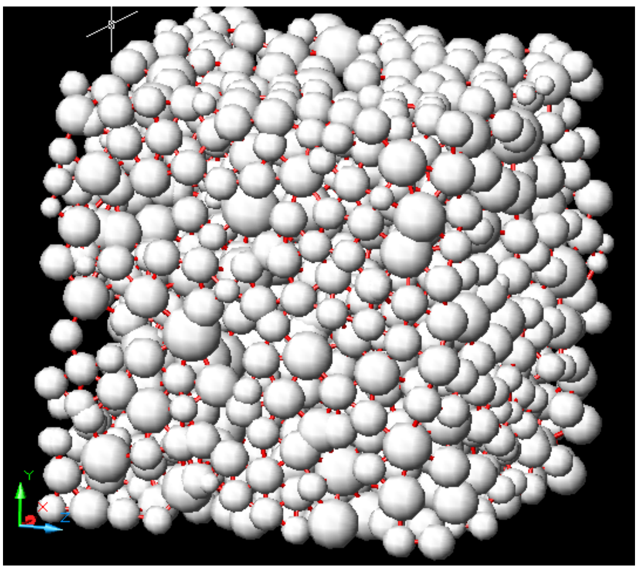

Figure 2. Example of a numerical macroporous material of 1300 particles.

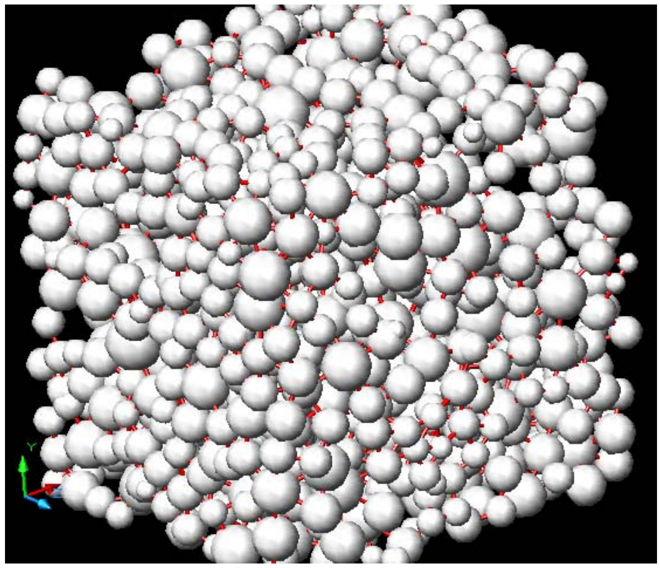

Figure 3. Example of a numerical macroporous material of 1300 particles.

3 EQUATIONS OF THE MODEL

In this point, the equations that command the behavior of the macroporous material are shown. In general, the macroporous can be under any kind of load in each of its boundaries. But this research is focused in modeling a consolidated triaxial test.

From the point of view of the strength, the response of the medium is conditioned by the joining material and its collapse. To model this material, it has been assumed that a wall-beam, similar to a tablet shaped material, under axial and shear stresses is a good enough approximation. The parameters considered for this material has been the following ones:

$$E = 600\,MPa \quad (1)$$

$$\nu = 0.2 \quad (2)$$

$$\sigma_c = 1\,MPa \quad (3)$$

Where: E is the Young´s modulus, ν is the Poisson´s ratio and σ_c is the unconfined compressive strength.

With this defined properties, a finite element method (FEM) calculus was carried out. In this FEM the joining material was modeled as a tablet loaded with axial and shears forces. The tablet was considered to be broken whenever any fiber of the material reaches the Tresca condition:

$$\sigma_c \geq \sqrt{\sigma_N^2 + 3\tau^2} \quad (4)$$

Where: σ_n is the normal stress and τ is the shear stress. In this way, the failure diagrams (pair of values N;Q, that produces the failure of the tablet) were defined. Each contact will have its own diagram depending on the B/H ratio.

In addition, the normal stiffness (Kn) and the shear stiffness (Kt) were calculated by dividing the applied force against the obtained displacement. The variation of the normal and shear stiffness against the B/H ratio is shown in Figures 4 and 5.

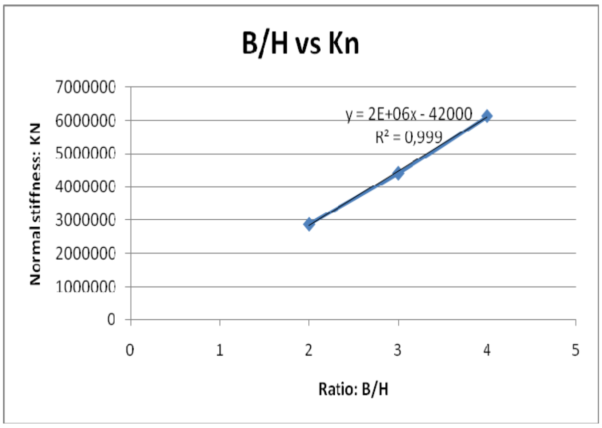

Figure 4. Relation between B/H ratio and normal stiffness Kn.

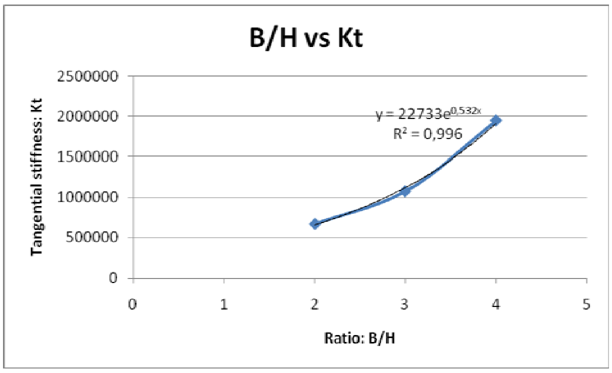

Figure 5. Relation between B/H ratio and the shear stiffness Kt.

In order to introduce the laws of behaviour in the matrix system, the stiffness formulation for one contact enounced by Kishino (1989) and Tsutsumi and Kaneko (2008) was considered. Being the stiffness expressed as:

$N = K_N \cdot (\delta_{I,x} - \delta_{J,x})$ (5)

$t = K_t \cdot (\delta_{I,y} - \delta_{J,y})$ (6)

$T = K_T \cdot (\delta_{I,z} - \delta_{J,z})$ (7)

Where:

{N, t, T} are the forces in local axis, corresponding to the axial force and the two tangential forces respectively, in the union material.

$\{K_N, K_t, K_T\}$ are non-linear variables that relate the relative displacements between two particles in contact with the mobilized force in each direction.

$\{\delta_{I,x}, \delta_{I,y}, \delta_{I,z}\}$ is the displacement of the sphere I in the "j" directions of the local axis {x, y, z}.

$\{\delta_{J,x}, \delta_{J,y}, \delta_{J,z}\}$ is the displacement of the sphere J in the "j" directions of the local axis {x, y, z}.

The global stiffness matrix of the system is the addition of the local stiffness of each contact, projected from the local axis to the global ones. Using the Newtons equations of the static:

$\Sigma F_X = 0$ (8)

$\Sigma F_Y = 0$ (9)

$\Sigma F_Z = 0$ (10)

Where: {X, Y, Z} are the global axis. Using these expressions for each particle in the macroporous material it is obtained the following system:

$[F] \cdot \{\delta\} = \{I\}$ (11)

Where:

[F] is the stiffness matrix of the system.

$\{\delta\}$ is the displacement vector of the particles or spheres.

{I} is the vector that contains the load in the three global axis {X, Y, Z}.

The procedure of calculus is the following:

1. - Once the macroporous has been defined geometrically, a normal stiffness (K_N) and a shear stiffness (K_t) are assigned to each existing contact (Fig. 4-5).

2. - The local stiffness is projected to the global axis to obtain the matrix [F], using the expressions (5) to (10).

3. - The vector {I} is built considering the forces applied on the boundaries of the macroporous material.

4. - The matrix system of equations (11) is solved and the displacements of each particle are obtained. Once the displacements are known, they are translated to forces in every joining material of the macroporous.

5. - Every contact is checked one by one in order to break those that have exceeded the maximum allowable force by the failure diagrams, (N, Q, pair of values that produces the reach of the Tresca criterion in the tablet joining material) obtained in the FEM calculus. The contacts that had broken are removed from the macroporous by deleting its contribution to the stiffness matrix [F].

By increasing the load progressively further displacements and inner forces are calculated to check how many contacts had broken. In addition, the position (dip and dip direction) of the broken contacts are measured.

This process is repeated until the macroporous cannot reach the equilibrium with the external load. When this situation happens it is said that the collapsed of the macroporous material had taken place.

4 FIRST RESULTS

The model is actually in an early stage of development and needs some improvements. But the first results obtained with this methodology are promising. For example, without having reached the collapse, we have been able to observe the following:

1. - In the early steps of loading (point A in Figure 7), the first contacts to break had been the ones with a low dip angle independently of the dip direction. That is to say, the more vertical contacts are the firsts to break as can be seen in Figure 6.

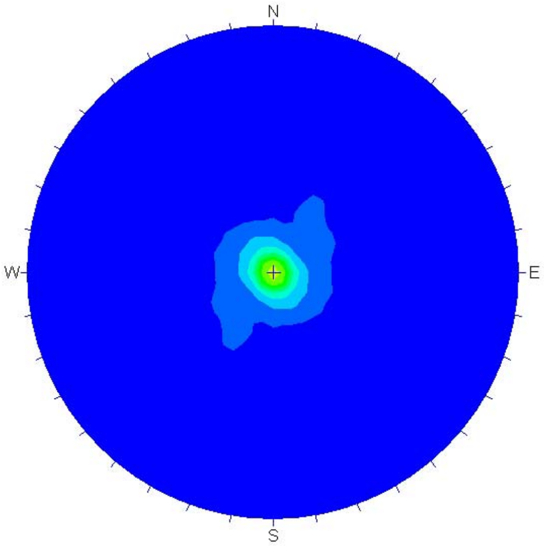

Figure 6. Contour plot of concentration of poles that has broken on the early steps of loading.

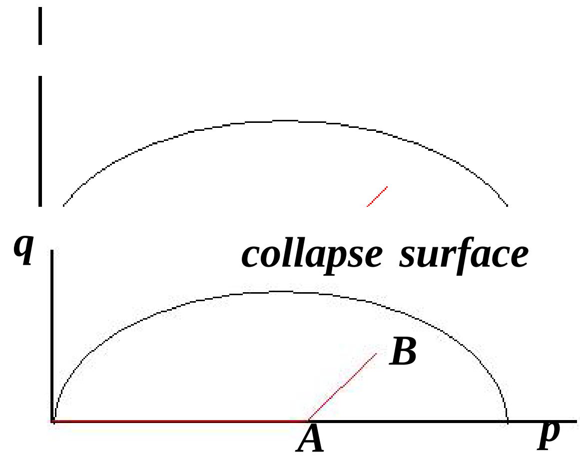

Figure 7. Example of the trajectory of tensions during a collapse calculi.

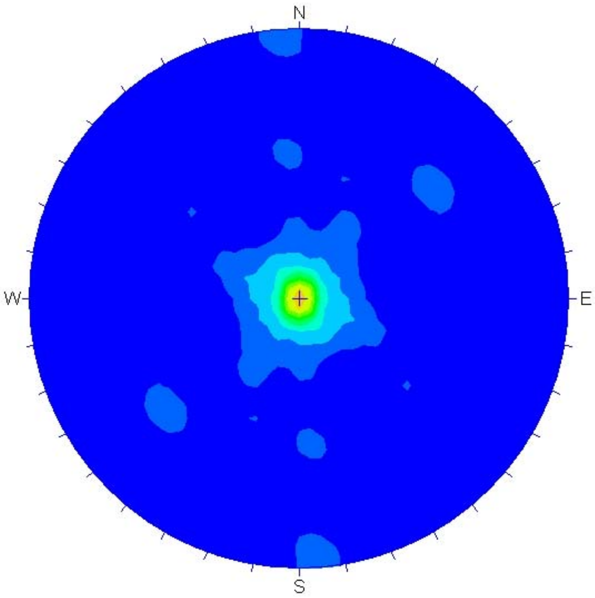

Figure 8. Contour plot of concentration of poles that has broken on the advanced steps of loading.

It is thought that in more advanced steps of loading; the concentration of poles among 45º of dip angle will generalize in every dip direction, leading to the collapsed of the macroporous.

5 CONCLUSIONS

In this research, the first steps on how to model the collapse of a macroporous material had been shown. The model is in an early stage of development and this article is only trying to enounce one of the possible ways to model this kind of fragile break. The main ideas and conclusions obtained with the proposal formulation are the following:

- A macroporous material had been numerically generated by using random functions. In the contacts a breakable joining material had been inserted.
- It has been presented a general vision on how to set out the system of equations that govern the phenomenon.
- Finally, the first results that have been obtained from the calculus, applying these theories had been shown.

6 REFERENCES

Del Olmo, D. 2009. Elaboración de un modelo discreto de partículas en 3D para estudio tensodeformacional de materiales gruesos. *Doctoral Thesis. E.T.S.I.C.C.P. – U.P.M.*

Kishino, Y. 1989. Computer Analysis of Dissipation Mechanism in Granular Media, Powders and Grains. A.*A. Balkema, Rotterdam (1989) pp. 323-330.*

Perucho, A. (2004) “Estudio de deformabilidad de escolleras”. *Doctoral Thesis E.T.S.I.C.C.P.- U.P.M.*

Serrano, A.A. 1976. Aglomerados volcánicos en las Islas Canarias. *Mem. Simposio de Rocas Blandas: Tomo II. pp. 47-53.*

Tsutsumi, S. & Kaneko, K. 2008. Constitutive response of idealized granular media under the principal stress axes rotation. *International Journal of Plasticity.*

Uriel, S. & Serrano, A.A. 1973. Geotechnical Properties of two collapsible volcanic Soils of low bulk density at the site of two dams in Canary Island (Spain). *8^{th} Congreso de I.S.S.M.F.E. Moscú, vol I, pp: 257-264.*