ADER schemes for linear advection-diffusion problems with nonlinear source term using several flux reconstructions.

R. Castedo, A. Hidalgo

Dpto. de Matemática Aplicada y Métodos Informáticos, E.T.S.I. Minas, Univ. Politécnica de Madrid, Madrid, Spain

ricardo.castedo@upm.es, arturo.hidalgo@upm.es

Abstract

The aim of this work is to compare the behaviour of the use of different flux reconstruction techniques when solving advection-diffusion problems with nonlinear source terms in fully saturated porous media. High order schemes are developed within the finite volume ADER framework (see for example [4]) together with a suitable spatial reconstruction method. For the sake of efficiency, advective and diffusive part are decoupled, using a different reconstruction technique for each case.

The advective part is solved by means of WENO spatial reconstruction (see for example [1]) together with different intercell numerical fluxes such as GFORCE, FORCE, WAF with a limiter function [2] and Rusanov ones. On the other hand, the diffusive part and the nonlinear source term are solved using conservative centred spatial reconstruction, with a sixth degree polynomial. In order to get the intercell flux reconstruction for the diffusive part, a simple arithmetic mean of right and left fluxes is used. Nevertheless, to solve the source term is involved a Gaussian quadrature.

A comparison of the use of different numerical fluxes for several selected test cases - which involve advective, diffusive and nonlinear source term - is shown. In order to test its accuracy a convergence rates study has been carried out, considering smooth initial solutions, getting up to sixth order of accuracy.

Finally, other results have been obtained for problems involving non-smooth solutions, such as sharp advancing fronts (see reference [1, 3]). Although, as expected, accuracy orders are not fulfilled due to the sharp gradients, the error norms are quite small. For these problems, it is analyzed and compared the behavior of the different intercell flux reconstructions considered in this work. Especially good results are achieved for the second order TVD WAF scheme with limiter function.

Keywords: Finite Volumes; High-order; ADER; WENO; Orders of accuracy.

References

[1] V.A. Titarev and E.F. Toro, *Finite-Volume WENO schemes for three-dimensional conservation laws*, Journal of Computational Physics, Volume 201, Issue 1, 20 November 2004, Pages 238-260.

[2] V.A. Titarev and E.F. Toro, *WENO schemes based on upwind and centred TVD fluxes*, Computers and Fluids, Volume 34, 2005, pag, 705-720.

[3] E.F. Toro and A. Hidalgo, *ADER finite volume schemes for nonlinear reaction-diffusion equations*, Applied Numerical Mathematics, Volume 59, Issue 1, January 2009, Pages 73-100.

[4] E.F. Toro, *Riemann solvers and numerical methods for fluid dynamics: a practical introduction*, Springer, 2009.