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Control of multivariable delayed systems with colored noise using incremental state formulation

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ABSTRACT
In this work, a novel approach based on incremental state models is proposed for the modelling and control of multivariable delayed systems with colored noise. One of the key features of the new approach is that the proposed incremental state model compared with the non-incremental one, solves the problem of computing the target state in a natural way, since for a desired output, a zero incremental state can be taken as an objective. Moreover, the control action in an incremental form is equivalent to introduce an integral action, thereby cancelling the steady state errors, even in presence of disturbances. An optimal Linear Quadratic Regulator (LQR) control, based on the incremental model, is proposed and a Kalman filter is used to obtain the estimated state. Thus, another advantage of the proposed method is minimizing the output variance. Illustrative examples are chosen to evaluate the effectiveness of the proposed method.

KEYWORDS
Incremental state models; colored noise; multivariable delayed systems; linear quadratic regulator; Kalman filter

1. Introduction

Nowadays, in the research literature, the Model Predictive Control (MPC) is usually formulated in the state space. In most practical problems, the states of the system are not directly accessible and have to be estimated. The quality of the estimated states has important bearings on the overall performance of a model predictive controller. The use of state space models for Generalized Predictive Control (GPC) was proposed in (Ordys & Clarke, 1993).

The MPC interpretation by state space models (Lee, Morari, & Garcia, 1994) allows a proper generalization to more complex cases, as multivariable systems in presence of measurement noise. The authors in (Li, Lim, & Fisher, 1989; Navratil, Lira, & Fisher, 1989) showed how to apply the step response model in state space form, and how to develop an MPC algorithm by the optimal stochastic control theory. In (Ricker, 1990), it is shown how an MPC traditional algorithm is developed by state space models.

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The authors in (D. Clarke, Mohtadi, & Tuffs, 1987; D. W. Clarke, Mohtadi, & Tuffs, 1987) developed a GPC theory, based on the Controlled Auto-Regressive Integrated Moving Average (CARIMA) model, which includes an integral term in the noise model. Observer polynomials have been included in the GPC algorithm (D. Clarke, 1991) and the weighting and restriction of the future input increments.

The application of control loop performance assessment and benchmarking techniques to industrial process control loops is considered in (Uduehi, Ordys, & Grimble, 2002). A state space approach is utilized as the state space description of systems, providing an uniform method for handing SISO, MIMO and process time delays. The main contribution in (Uduehi et al., 2002) is to extend the idea of MIMO benchmark to include GPC controller as a reference, whilst keeping the simplicity and ease of assessing the variances of system inputs and outputs.

The analysis of the stochastic properties of the GPC algorithm has been considered in (Blachuta & Ordys, 1987; Ordys, 1993). This analysis included the computation of the optimal variance of the process output and control output obtainable by using GPC controllers.

The design of LQG stochastic optimal tracking and regulating systems is considered in (Grimble & Hearns, 1998) for systems with different time delays. The state equation based controllers for both tracking and regulating systems include a Kalman predictor and state-estimate feedback. A form of the separation principle holds for linear systems containing transport delays on inputs and outputs. The process and measuring system noise terms may be correlated and be colored or white.

In (Moir & Grimble, 1984), the authors proposed an algorithm for self-tuning optimal fixed-lag smoothing or filtering for discrete-time multivariable processes. The solution involves spectral factorization of polynomial matrices and assumes knowledge of the process parameters and the noise statistics. The assumption is then made that the signal-generating process and noise statistics are unknown. The model parameters of the process can be estimated using an extended Kalman filter or, alternatively, an extended recursive least squares.

Although the minimal state space formulation of control design is a most natural and convenient approach for its use with computers, particularly in relation to multivariable systems, it has one major limitation; namely, the state vector is not normally available for direct measurement. As a result, much research effort has been applied to the development of state observer techniques, most notably the Kalman filter (Luenberger, 1971). The state observer is employed to generate a surrogate state vector, which converges asymptotically to the true state vector and so it can be used in the implementation of state feedback control. Unfortunately, this not only adds complexity to the design, but the controller becomes more dependent on the estimated model of the system, with the associated disadvantage of decreasing robustness (Bitmead, Gevers, & Wertz, 1990).

In (Al-Hadithi, Jiménez, & Perez-Oria, 2015), a novel approach based on incremental state models has been proposed for the modelling of multivariable nonlinear delayed systems expressed by a Takagi-Sugeno fuzzy model. One of the key features of the approach is that the proposed incremental state model compared with the no incremental one, solves the problem of computing the target state in a natural way, since for a desired output vector, a zero incremental state can be taken as an objective. Moreover, the control action in an incremental form is equivalent to introduce an integral action, thereby cancelling the steady state errors. Among other advantages of incremental state models are the disappearance of the affine terms. Then, a fuzzy based linear quadratic regulator is designed. Furthermore, a new optimal observer for multivariable fuzzy systems is developed, because not all states of the system are fully available or measured.

In addition, in (Adánez, Al-Hadithi, & Jiménez, 2018) a comparison between incremental state model and traditional discrete state model is developed for the linear multivariable optimal control of a wind turbine. It has been proven that for an optimal control based on traditional discrete state model and LQR algorithm, the calculation of reference state and reference input must be performed, and moreover, the resultant control shows steady state errors. On the other hand, it has been demonstrated that using incremental state model for optimal control, reference state and reference input come naturally without any calculation and the controlled system has zero steady state error.

In this new approach with incremental state models, we propose an extension of the previous works developed by the authors in (Adánez et al., 2018; Al-Hadithi et al., 2015) for linear multivariable delayed systems with colored noise. Thus, the proposed method presents the same advantages of the previous works and also deals with the problem of colored noise, which forces the reformulation of incremental state model to the model presented in this work.

Moreover, in this work, the proposed method is compared with the traditional GPC model formulation (D. Clarke et al., 1987; D. W. Clarke et al., 1987), which also considers the colored noise problem and includes an integral action in the noise term. In this work, it is demonstrated that the noise term which includes an integral action is not a model of a real system, since a stochastic process modelled with integral action would mean that the system has a noise of infinite variance. Thus, as the control law in the GPC algorithm is formulated in incremental form, it presents zero steady state error in front of step disturbances as in the proposed method but, the output noise is not completely compensated because a modelling error is introduced in the colored noise.

The rest of the paper is organized as follows. A description of the discrete model under study is presented in section 2. The modelling of multivariable systems with colored noise using state models is explained in section 3. In section 4, the modelling of multivariable systems with colored noise using the proposed incremental state models is developed. In section 5, a control algorithm for multivariable systems with colored noise using the incremental state model is analyzed, obtaining zero steady state error. An illustrative example is added. A comparison with multivariable MPC with colored noise is made in section 6. An illustrative example is included.

2. Discrete model of the system

Discrete models, using difference equations, allow the modelling of continuous systems with multiple delays. It is well known that the time delay is often inherent in dynamic systems, which can be an important source of instability and degradation in the control performance. In particular, neglecting the presence of time delays in the measurable states may jeopardize or result in catastrophic failures for operations.

Given a multivariable discrete system with time delays and colored noise, with p inputs and q outputs, where $q \leq p$, we have identified each output subsystem parameters by the well-known least squares method,

$$
\begin{aligned}
&y_i(k) + a_{i1}y_i(k-1) + a_{i2}y_i(k-2) + \cdots + a_{in_i}y_i(k-n_i) = \\
&\sum_{j=1}^{p} [b_{ij1}u_j(k-1) + b_{ij2}u_j(k-2) + \cdots + b_{ijn_i}u_j(k-n_i)]
\end{aligned} \tag{1}
$$

whereby the variables are increments around an equilibrium point and some of the parameters a_{in_i}, b_{i1n_i}, ..., b_{ipn_i} are nonzero ones. If there is a pure time delay of d_{ij} samples between input j and output i, it is verified that $b_{ij1} = b_{ij2} = \cdots = b_{ijd_{ij}} = 0$.

In case of systems with colored noise, what is obtained with the least squares method is a model of the deterministic part of the system as follows:

$$
y_{ui}(k) = \sum_{j=1}^{p} \frac{B_{ij}(q^{-1})}{A_i(q^{-1})} u_j(k) \tag{2}
$$

The output variables, when the input variables are null, are the noises which affects the system, i.e, its stochastic part. This noise can be modelled, according to Wold theorem (Anderson, 2011; Wold, 1939), as the output of a linear system whose transfer function can be estimated using spectral factorization (Moir & Grimble, 1984; Uduehi et al., 2002) and whose input is a white noise $w_i(k)$ with variance $var(w_i)$ as follows:

$$
y_i(k) - y_{ui}(k) = \frac{C_i(q^{-1})}{D_i(q^{-1})} w_i(k) \tag{3}
$$

fulfilling that $C_i(q^{-1})$ and $D_i(q^{-1})$ are monic polynomials and of the same degree.

The complete model is:

$$
y_i(k) = \sum_{j=1}^{p} \frac{B_{ij}(q^{-1})}{A_i(q^{-1})} u_j(k) + \frac{C_i(q^{-1})}{D_i(q^{-1})} w_i(k) \quad 1 \le i \le q \tag{4}
$$

Note that, since the colored noise is modelled according to Wold theorem (Anderson, 2011; Wold, 1939), a white noise variable $w_i(k)$ has been added to the model, but this is a virtual variable necessary for all formulations with colored noise. This variable is not an accessible or manipulable signal, so it cannot be measured or controlled, and this fact has been considered in the proposed control method.

3. Modelling of multivariable systems with colored noise using state models

We start from the complete model given in (4), assuming that:

$$
\begin{aligned}
deg(A_i(q^{-1})) &= deg(B_{ij}(q^{-1})) = n_i \\
deg(C_i(q^{-1})) &= deg(D_i(q^{-1})) = m_i
\end{aligned}
$$

For each output variable, we define:

$$y_{ui}(k) = \sum_{j=1}^{p} \frac{B_{ij}(q^{-1})}{A_i(q^{-1})} u_j(k)$$

With the following state space representation:

$$\begin{aligned}
& x_{ui}(k) \in \Re_i^n \\
& x_{ui}(k+1) = \begin{bmatrix} -a_{i1} & 1 & \cdots & 0 \\ -a_{i2} & 0 & \ddots & 0 \\ \vdots & \vdots & \ddots & 1 \\ -a_{in_i} & 0 & \cdots & 0 \end{bmatrix} x_{ui}(k) + \begin{bmatrix} b_{i11} & b_{i21} & \cdots & b_{ip1} \\ b_{i12} & b_{i22} & \cdots & b_{ip2} \\ \vdots & \vdots & \ddots & \vdots \\ b_{i1n_i} & b_{i2n_i} & \cdots & b_{ipn_i} \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \\ \vdots \\ u_p(k) \end{bmatrix} \\
& y_{ui}(k) = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} x_{ui}(k)
\end{aligned}$$

which can be rewritten as:

$$\begin{aligned}
x_{ui}(k+1) &= A_{ui} x_{ui}(k) + B_{ui} u(k) \\
y_{ui}(k) &= C_{ui} x_{ui}(k)
\end{aligned}$$

where

$$\begin{aligned}
& A_{ui} \in \Re^{n_i \times n_i} \quad B_{ui} \in \Re^{n_i \times p} \\
& C_{ui} \in \Re^{1 \times n_i}
\end{aligned}$$

Grouping variables, the deterministic part of the multivariable system can be modelled as:

$$\begin{aligned}
\begin{bmatrix} x_{u1}(k+1) \\ x_{u2}(k+1) \\ \vdots \\ x_{uq}(k+1) \end{bmatrix} &= \begin{bmatrix} A_{u1} & 0 & \cdots & 0 \\ 0 & A_{u2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_{uq} \end{bmatrix} \begin{bmatrix} x_{u1}(k) \\ x_{u2}(k) \\ \vdots \\ x_{uq}(k) \end{bmatrix} + \begin{bmatrix} B_{u1} \\ B_{u2} \\ \vdots \\ B_{uq} \end{bmatrix} u(k) \\
y_u(k) &= \begin{bmatrix} C_{u1} & 0 & \cdots & 0 \\ 0 & C_{u2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_{uq} \end{bmatrix} \begin{bmatrix} x_{u1}(k) \\ x_{u2}(k) \\ \vdots \\ x_{uq}(k) \end{bmatrix}
\end{aligned}$$

which can be rewritten as:

$$\begin{aligned}
x_u(k+1) &= A_u x_u(k) + B_u u(k) \\
y_u(k) &= C_u x_u(k)
\end{aligned} \tag{5}$$

where

$$n = \sum_{i=1}^{q} n_i$$
$$A_u \in \Re^{n \times n} \quad B_u \in \Re^{n \times p}$$
$$C_u \in \Re^{q \times n} \quad x_u(k) \in \Re^n$$

In the stochastic part, since C_i and D_i are polynomials of the same degree, we divide:

$$\frac{C_i(q^{-1})}{D_i(q^{-1})} = 1 + \frac{E_i(q^{-1})}{D_i(q^{-1})} \tag{6}$$

And we take:

$$y_{wi}(k) = \frac{E_i(q^{-1})}{D_i(q^{-1})} w_i(k)$$

Obtaining the multivariable stochastic part, in an equivalent way as the multivariable deterministic part:

$$\begin{aligned} x_w(k+1) &= A_w x_w(k) + B_w w(k) \\ y_w(k) &= C_w x_w(k) \end{aligned} \tag{7}$$

where

$$m = \sum_{i=1}^{q} m_i$$
$$A_w \in \Re^{m \times m} \quad B_w \in \Re^{m \times q}$$
$$C_w \in \Re^{q \times m} \quad x_w(k) \in \Re^m$$

The multivariable complete system can be modelled as:

$$\begin{aligned} \begin{bmatrix} x_u(k+1) \\ x_w(k+1) \end{bmatrix} &= \begin{bmatrix} A_u & 0 \\ 0 & A_w \end{bmatrix} \begin{bmatrix} x_u(k) \\ x_w(k) \end{bmatrix} + \begin{bmatrix} B_u \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ B_w \end{bmatrix} w(k) \\ y(k) = y_u(k) + y_w(k) + w(k) &= \begin{bmatrix} C_u & C_w \end{bmatrix} \begin{bmatrix} x_u(k) \\ x_w(k) \end{bmatrix} + w(k) \end{aligned} \tag{8}$$

which becomes the basic system:

$$\begin{aligned} x_b(k+1) &= A_b x_b(k) + B_b u(k) + v_b(k) \\ y(k) &= C_b x_b(k) + w(k) \end{aligned} \tag{9}$$

where

$$
\begin{array}{rl}
x_b(k) \in \Re^{n+m} & v_b(k) \in \Re^{n+m} \\
A_b \in \Re^{(n+m)\times(n+m)} & B_b \in \Re^{(n+m)\times p} \\
C_b \in \Re^{q\times(n+m)} & w(k) \in \Re^{q}
\end{array}
$$

The parameters of the deterministic subsystem (A_u, B_u, C_u), are obtained from an input-output identification. Therefore, if the degrees n_i are not oversized, it becomes controllable and observable. On the other hand, the parameters of the stochastic subsystem (A_w, B_w, C_w), are obtained from an identification by spectral estimation. Thus, if the degrees m_i are not oversized, it becomes observable but not controllable, since it is disconnected from the input $u(k)$.

This model is not suitable to control the system because, firstly, it would be necessary calculating the reference state from the reference output and, secondly, the controlled system would have steady state errors.

4. Modelling of multivariable systems with colored noise using incremental state models

In order to solve these two problems; the calculation of the reference state and the presence of steady state errors, we propose a new incremental state model formulation for colored noise systems. Applying the deterministic state model (5) for one previous step, it can be fulfilled that:

$$
\begin{aligned}
x_u(k) &= A_u x_u(k-1) + B_u u(k-1) \\
y_u(k-1) &= C_u x_u(k-1)
\end{aligned}
\tag{10}
$$

Subtracting (10) from (5), we get:

$$
\begin{aligned}
x_u(k+1) - x_u(k) &= A_u(x_u(k) - x_u(k-1)) + B_u(u(k) - u(k-1)) \\
y_u(k) - y_u(k-1) &= C_u(x_u(k) - x_u(k-1))
\end{aligned}
\tag{11}
$$

Let us define:

$$
\begin{aligned}
\Delta x_u(k) &= x_u(k) - x_u(k-1) \\
\Delta y_u(k) &= y_u(k) - y_u(k-1) \\
\Delta u(k) &= u(k) - u(k-1)
\end{aligned}
\tag{12}
$$

Substituting (12) into (11):

$$
\begin{aligned}
\Delta x_u(k+1) &= A_u \Delta x_u(k) + B_u \Delta u(k) \\
\Delta y_u(k) &= C_u \Delta x_u(k)
\end{aligned}
\tag{13}
$$

In the same way, we transform the stochastic subsystem described in (7) to:

$$
\begin{aligned}
\Delta x_w(k+1) &= A_w \Delta x_w(k) + B_w \Delta w(k) \\
\Delta y_w(k) &= C_w \Delta x_w(k)
\end{aligned}
\tag{14}
$$

where

$$\begin{aligned} \Delta x_w(k) &= x_w(k) - x_w(k-1) \\ \Delta y_w(k) &= y_w(k) - y_w(k-1) \\ \Delta w(k) &= w(k) - w(k-1) \end{aligned} \tag{15}$$

To formulate the incremental model (14) as function of $w(k)$, we consider $\Delta w(k)$ as the output of a dynamic system. Then, in order to model it, we define a new state variable $s_w(k) \in \Re^q$ which fulfils:

$$\begin{aligned} s_w(k+1) &= -w(k) \\ \Delta w(k) &= s_w(k) + w(k) \end{aligned} \tag{16}$$

In these formulations (13) and (14), we need to relate the output $y(k)$ and the new state variables above defined. To obtain this, we define a new state variable $s_y(k) \in \Re^q$ as follows:

$$s_y(k) = y(k) - w(k) = y_u(k) + y_w(k) \tag{17}$$

Equation (17) in a previous sample is:

$$s_y(k-1) = y_u(k-1) + y_w(k-1) \tag{18}$$

Subtracting (18) from (17), we get:

$$s_y(k) - s_y(k-1) = y_u(k) - y_u(k-1) + y_w(k) - y_w(k-1) = \Delta y_u(k) + \Delta y_w(k)$$

And thus:

$$\begin{aligned} s_y(k+1) &= s_y(k) + \Delta y_u(k+1) + \Delta y_w(k+1) = \\ &= s_y(k) + C_u \Delta x_u(k+1) + C_w \Delta x_w(k+1) = \\ &= s_y(k) + C_u A_u \Delta x_u(k) + C_u B_u \Delta u(k) + \\ &+ C_w A_w \Delta x_w(k) + C_w B_w s_w(k) + C_w B_w w(k) \end{aligned} \tag{19}$$

The incremental state model finally becomes:

$$\begin{aligned} \begin{bmatrix} s_y(k+1) \\ \Delta x_u(k+1) \\ \Delta x_w(k+1) \\ s_w(k+1) \end{bmatrix} &= \begin{bmatrix} I & C_u A_u & C_w A_w & C_w B_w \\ 0 & A_u & 0 & 0 \\ 0 & 0 & A_w & B_w \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s_y(k) \\ \Delta x_u(k) \\ \Delta x_w(k) \\ s_w(k) \end{bmatrix} + \\ &+ \begin{bmatrix} C_u B_u \\ B_u \\ 0 \\ 0 \end{bmatrix} \Delta u(k) + \begin{bmatrix} C_w B_w \\ 0 \\ B_w \\ -I \end{bmatrix} w(k) \\ y(k) &= \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s_y(k) \\ \Delta x_u(k) \\ \Delta x_w(k) \\ s_w(k) \end{bmatrix} + w(k) \end{aligned} \tag{20}$$

That is,

$$\begin{aligned} x(k+1) &= Ax(k) + B\Delta u(k) + v(k) \\ y(k) &= Cx(k) + w(k) \end{aligned} \tag{21}$$

where

$$\begin{array}{rr} x(k) \in \Re^{n+m+2q} & v(k) \in \Re^{n+m+2q} \\ A \in \Re^{(n+m+2q)\times(n+m+2q)} & B \in \Re^{(n+m+2q)\times p} \\ C \in \Re^{q\times(n+m+2q)} & \end{array}$$

In this system, the subsystems defined by $s_y(k)$ and $\Delta x_u(k)$ are controllable, but they could be not observable. The increment of the stochastic part $\Delta x_w(k)$ is observable but not controllable. However, $s_w(k)$ is not controllable and not observable, then, the system is not observable. In addition, the stochastic subsystems, which are not controllable ones, have been modelled by spectral factorization and thus, the stochastic parts result in stable subsystems.

Thus, this model is suitable to control the output of the system, but not to observe the state of the system.

5. Control algorithm for multivariable systems with colored noise using incremental state model

In order to obtain a state estimation, we cannot use the Kalman filter in the incremental state, since it is not observable. However, we can use the Kalman filter in the basic non incremental model given by (9).

Taking into consideration that if $R_2 = E\left\{w \cdot w^T\right\}$ then:

$$\begin{aligned} R_1 &= E\left\{v_b \cdot v_b^T\right\} = E\left\{\begin{bmatrix} 0 \\ B_w \end{bmatrix} w \cdot w^T \begin{bmatrix} 0 & B_w^T \end{bmatrix}\right\} = \begin{bmatrix} 0 \\ B_w \end{bmatrix} R_2 \begin{bmatrix} 0 & B_w^T \end{bmatrix} \\ R_{12} &= E\left\{v_b \cdot w^T\right\} = E\left\{\begin{bmatrix} 0 \\ B_w \end{bmatrix} w \cdot w^T\right\} = \begin{bmatrix} 0 \\ B_w \end{bmatrix} R_2 \end{aligned} \tag{22}$$

Using Kalman filter algorithm, we get the observer gain matrix $L \in \Re^{(n+m)\times q}$ and the filter gain matrix $M \in \Re^{(n+m)\times q}$. At each step, the observation error is calculated as follows:

$$\varepsilon(k) = y(k) - C_b x_b(k|k-1) \tag{23}$$

So, the estimation becomes:

$$x_b(k|k) = x_b(k|k-1) + M\varepsilon(k) \tag{24}$$

Finally, the estimation of the new state is done as follows:

$$x_b(k+1|k) = A_b x_b(k|k-1) + B_b u(k) + L\varepsilon(k) \tag{25}$$

From the estimation of:

$$x_b(k|k) = \begin{bmatrix} x_u(k|k) \\ x_w(k|k) \end{bmatrix}$$

we can estimate:

$$\begin{aligned} \Delta x_u(k|k) &= x_u(k|k) - x_u(k-1|k-1) \\ \Delta x_w(k|k) &= x_w(k|k) - x_w(k-1|k-1) \end{aligned} \tag{26}$$

And taking into account that $w(k)$ is a zero mean white noise, the best estimation for $s_y(k) = y(k) - w(k)$ becomes $y(k)$. Finally, considering $s_w(k) = -w(k-1)$ and, in addition to that, it is not observable, the best estimation is its mean value $E\{s_w(k)\} = E\{-w(k-1)\} = 0$. So, we have an optimal state estimation given by:

$$x(k|k) = \begin{bmatrix} y(k) \\ \Delta x_u(k|k) \\ \Delta x_w(k|k) \\ 0 \end{bmatrix} \tag{27}$$

If the goal is to achieve y_r, the incremental reference state becomes:

$$x_r = \begin{bmatrix} s_{y_r} \\ \Delta x_{ur} \\ \Delta x_{wr} \\ s_{wr} \end{bmatrix} = \begin{bmatrix} y_r \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

It should be noted that the reference state comes naturally, without the necessity of performing any calculations.

For an optimal control of the system, the cost function to be minimized, is defined as

$$J = E\left\{ \sum_{k=0}^{\infty} \left[(x_r - x(k))^T Q (x_r - x(k)) + \Delta u^T(k) R \Delta u(k) \right] \right\}$$

where $Q \in \Re^{(n+m+2q)\times(n+m+2q)}$ is a positive semidefinite symmetric matrix and $R \in \Re^{p\times p}$ is a positive definite one. The well-known solution that minimizes the above mentioned cost function is the LQR algorithm. The state feedback control can be obtained as follows:

$$\Delta u(k) = K(x_r - x(k|k)) \tag{28}$$

$$K \in \Re^{p\times(n+m+2q)}$$

and thus,

$$u(k) = u(k-1) + \Delta u(k) \tag{29}$$

Taking into consideration that the controllable subsystem is the one defined by:

$$\begin{bmatrix} s_y(k) \\ \Delta x_u(k) \end{bmatrix}$$

it seems reasonable to define the Q matrix weighting only this subsystem

$$Q = \begin{bmatrix} Q_c & 0 \\ 0 & 0 \end{bmatrix}$$

where $Q_c \in \Re^{(n+q)\times(n+q)}$ is a symmetric positive definite matrix.

It is interesting to observe that the mean value of the control action in steady state becomes:

$$E\{\Delta u(k)\} = K(x_r - E\{x(k|k)\}) =$$
$$= \begin{bmatrix} K_{s_y} & K_u & K_w & K_{s_w} \end{bmatrix} \left(\begin{bmatrix} y_r \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} E\{y(k)\} \\ E\{\Delta x_u(k|k)\} \\ E\{\Delta x_w(k|k)\} \\ 0 \end{bmatrix} \right)$$

Fulfilling that in steady state

$$E\{\Delta u(k)\} = 0$$
$$E\{\Delta x_u(k|k)\} = 0$$
$$E\{\Delta x_w(k|k)\} = 0$$

since they represent estimations of incremental variables in steady state. Substituting,

$$0 = \begin{bmatrix} K_{s_y} & K_u & K_w & K_{s_w} \end{bmatrix} \left(\begin{bmatrix} y_r \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} E\{y(k)\} \\ 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

it accomplishes that if K_{s_y} is a full rank matrix, then:

$$E\{y(k)\} = y_r$$

This means that the system has zero steady state error. It should be noted that if the matrices Q_c and R of LQR algorithm are positive definite ones and, as the deterministic part of the system, $s_y(k)$ and $\Delta x_u(k)$, is controllable, then K_{s_y} is a full rank matrix.

Observe that, the proposed algorithm has two phases. The first phase is the offline one. In this phase, the main steps may be listed as follows:

- From the identified system (4), obtain the system matrices of the basic model (8) and the proposed incremental model (20).
- Calculate the observer gain matrix L and the filter gain matrix M by Kalman filter using the basic model matrices (8).
- Calculate the control matrix K by LQR using the proposed incremental model matrices (20).

The second phase is the online one. In this phase, the main steps are as follows:

- Calculate the filtered state $x_b(k|k)$ using the matrix M and equations (23) and (24).
- Calculate the incremental state $x(k|k)$ using equations (26) and (27) .
- Calculate the control action $\Delta u(k)$ and $u(k)$ using the matrix K and equations (28) and (29).
- Calculate the predicted state $x_b(k+1|k)$ using the matrix L and equation (25).

Summarizing, this algorithm is optimal in terms of the estimated state, since Kalman filter is used, and it is also optimal in terms of the control action, because the LQR algorithm is used.

We are obtaining the control action increment as a function of the output error, which is equivalent to introducing an integral action. So, the resultant system does not only have zero steady state error in the presence of reference signal variations but it presents zero steady state error in front of step disturbances as well.

5.1. *Illustrative example of the proposed method*

Let us suppose the following system:

$$y_1(k) = \frac{0.5q^{-1} - 0.4q^{-2}}{1 - 1.6q^{-1} + 0.63q^{-2}} u_1(k) + \frac{0.1q^{-2}}{1 - 1.6q^{-1} + 0.63q^{-2}} u_2(k) + \frac{1 - 0.5q^{-1}}{1 - 0.6q^{-1}} w_1(k)$$
$$y_2(k) = \frac{0.5q^{-2} - 0.6q^{-3}}{1 - 1.5q^{-1} + 0.56q^{-2}} u_1(k) + \frac{0.2q^{-3}}{1 - 1.5q^{-1} + 0.56q^{-2}} u_2(k) + \frac{1 - 0.6q^{-1}}{1 - 0.9q^{-1}} w_2(k)$$

Moreover, we assume that:

$$\begin{aligned} var(w_1) &= 0.05 \\ var(w_2) &= 0.01 \\ cov(w_1, w_2) &= 0 \end{aligned}$$

And therefore,

$$R_2 = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.01 \end{bmatrix}$$

The state model of the deterministic part becomes:

$$x_u(k+1) = \begin{bmatrix} 1.6 & 1 & 0 & 0 & 0 \\ -0.63 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.5 & 1 & 0 \\ 0 & 0 & -0.56 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} x_u(k) + \begin{bmatrix} 0.5 & 0 \\ -0.4 & 0.1 \\ 0 & 0 \\ 0.5 & 0 \\ -0.6 & 0.2 \end{bmatrix} u(k)$$
$$y_u(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} x_u(k)$$

In order to obtain the stochastic model, we firstly calculate:

$$\frac{1-0.5q^{-1}}{1-0.6q^{-1}} = 1 + \frac{0.1q^{-1}}{1-0.6q^{-1}}$$
$$\frac{1-0.6q^{-1}}{1-0.9q^{-1}} = 1 + \frac{0.3q^{-1}}{1-0.9q^{-1}}$$

So, the state model of the stochastic part is:

$$x_w(k+1) = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.9 \end{bmatrix} x_w(k) + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix} w(k)$$
$$y_w(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_w(k)$$

The complete model becomes:

$$\begin{bmatrix} x_u(k+1) \\ x_w(k+1) \end{bmatrix} = \begin{bmatrix} 1.6 & 1 & 0 & 0 & 0 & 0 & 0 \\ -0.63 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.5 & 1 & 0 & 0 & 0 \\ 0 & 0 & -0.56 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.9 \end{bmatrix} \begin{bmatrix} x_u(k) \\ x_w(k) \end{bmatrix} +$$
$$+ \begin{bmatrix} 0.5 & 0 \\ -0.4 & 0.1 \\ 0 & 0 \\ 0.5 & 0 \\ -0.6 & 0.2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.1 & 0 \\ 0 & 0.3 \end{bmatrix} w(k)$$
$$y(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_u(k) \\ x_w(k) \end{bmatrix} + w(k)$$

This system is observable. The deterministic subsystem $x_u(k)$ is controllable and the stochastic $x_w(k)$ one is not controllable, since it is disconnected from the input.

Using this model, the estimation is obtained by Kalman filter where:

$$R_2 = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.01 \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0005 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.009 \end{bmatrix}$$

$$R_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.005 & 0 \\ 0 & 0.03 \end{bmatrix}$$

The filter and observer gain matrices are:

$$M = 10^{-4} \begin{bmatrix} 0.97 & -0.0091 \\ -0.68 & 0.0064 \\ 0.03 & -0.0003 \\ -0.02 & 0.0002 \\ 0 & 0 \\ -0.24 & 0.0023 \\ -0.04 & 0.0004 \end{bmatrix}$$

$$L = 10^{-4} \begin{bmatrix} 0.87 & -0.0082 \\ -0.61 & 0.0058 \\ 0.02 & -0.0002 \\ -0.02 & 0.0001 \\ 0 & 0 \\ 1000 & 0.0021 \\ -0.03 & 3000 \end{bmatrix}$$

Thus, the incremental model becomes:

$$\begin{bmatrix} s_y(k+1) \\ \Delta x_u(k+1) \\ \Delta x_w(k+1) \\ s_w(k+1) \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 0 & 1.6 & 1 & 0 & 0 & 0 & 0.6 & 0 & 0.1 & 0 \\ 0 & 1 & 0 & 0 & 1.5 & 1 & 0 & 0 & 0.9 & 0 & 0.3 \\ 0 & 0 & 1.6 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.63 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.5 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.56 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.6 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9 & 0 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s_y(k) \\ \Delta x_u(k) \\ \Delta x_w(k) \\ s_w(k) \end{bmatrix} +$$

$$+ \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \\ 0.5 & 0 \\ -0.4 & 0.1 \\ 0 & 0 \\ 0.5 & 0 \\ -0.6 & 0.2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \Delta u(k) + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.1 & 0 \\ 0 & 0.3 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} w(k)$$

$$y(k) = w(k) + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s_y(k) \\ \Delta x_u(k) \\ \Delta x_w(k) \\ s_w(k) \end{bmatrix}$$

Using the following control matrices:

$$Q_c = \begin{bmatrix} 0.1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The state feedback matrix is:

$$K = \begin{bmatrix} 0.0601 & 0.0355 & 1.3550 & 0.9870 & 0.8425 & \cdots \\ -0.0251 & 0.1985 & -1.2665 & -1.3707 & 4.6456 & \cdots \end{bmatrix}$$
$$\cdots \begin{matrix} 0.8062 & 0.6549 & 0.0830 & 0.1956 & 0.0138 & 0.0652 \\ 4.8010 & 4.5639 & -0.0429 & 1.2400 & -0.0071 & 0.4133 \end{matrix} \Bigg]$$

Figure 1 and figure 2 show the transient response with reference changes in 50 and 100 and in the presence of disturbances in 150 and 200. It can be clearly seen that the system has zero steady state error.

The variances of the system without control and zero input signal are:

$$var(y_1) = 0.0503$$
$$var(y_2) = 0.1421$$

The variances of the system with control and zero reference signal are:

$$var(y_1) = 0.0534$$
$$var(y_2) = 0.1283$$

6. Comparison with MPC with colored noise

It is well-known that the GPC theory does not allow for the presence of measurement noise (Grimble, 1992), other than by lumping it in with the disturbance model. Thus, this approach may not be the adequate one. The GPC algorithm does not account for the different variances of the disturbance signals entering the system, since it ignores the stochastic nature of the problem. Therefore, it seems inappropriate when

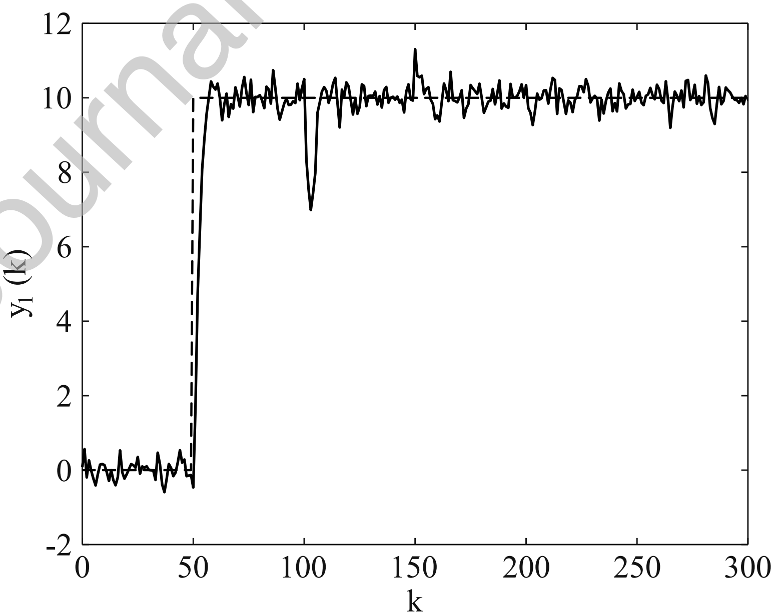

Figure 1. Transient response of $y_1(k)$ with reference changes and in the presence of disturbances.

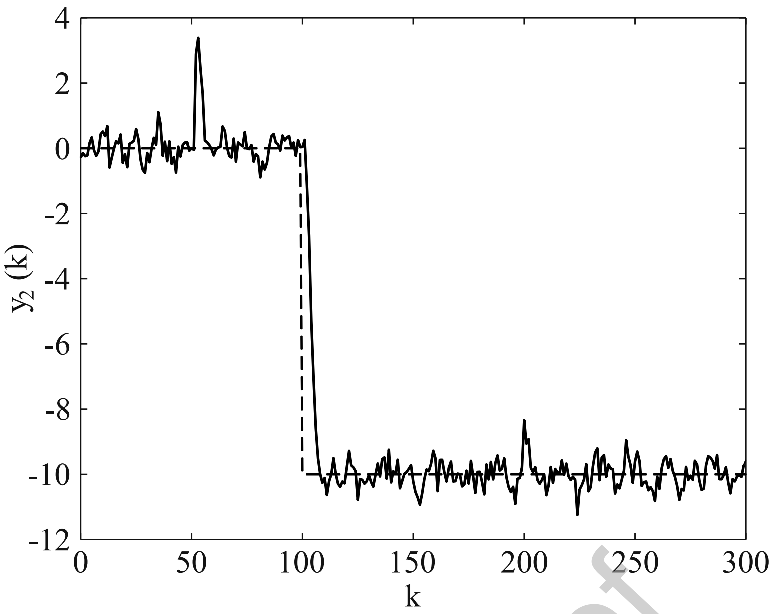

Figure 2. Transient response of $y_2(k)$ with reference changes and in the presence of disturbances.

the system includes substantial stochastic terms (as for example in wind turbine or marine control problems).

It should be noted that in the traditional GPC model formulation (D. Clarke et al., 1987; D. W. Clarke et al., 1987), it is assumed that, the noise term is represented with integral action in the system model given by (5), which can be useful for the formulation of the algorithm, although it is not a model of the real system. A stochastic process modelled with integral action would mean that the system has a noise of infinite variance.

The traditional formulation of the GPC given in (D. Clarke et al., 1987; D. W. Clarke & Mohtadi, 1989; D. W. Clarke et al., 1987), and its multivariable extension to Multivariable MPC, adds an integral term to this model

$$A_{ti}(q^{-1})y_i(k) = \sum_{j=1}^{p} B_{tij}(q^{-1})u_j(k) + \frac{C_{Gi}(q^{-1})}{1-q^{-1}}w_i(k)$$

If this model corresponds to a physical system, it means that the noise output of the system would be modelled as a stochastic process

$$g_i(k) = \frac{C_{Gi}(q^{-1})}{(1-q^{-1})A_{ti}(q^{-1})}w_i(k)$$

If the impulse response of this system is $\{g_{i0}, g_{i1}, g_{i2}, g_{i3}, \cdots\}$, then, using the final-value theorem of the Z transform properties (Åström & Wittenmark, 2013), if $A_{ti}(z^{-1})$ is stable, we get:

$$\lim_{k\to\infty} g_{ik} = \lim_{z\to 1}(1-z^{-1})G_i(z) = \lim_{z\to 1}(1-z^{-1})\frac{C_{Gi}(z^{-1})}{(1-z^{-1})A_{ti}(z^{-1})} = \frac{C_{Gi}(1)}{A_{ti}(1)} \neq 0$$

So, the variance of the noise becomes:

$$\sigma_{g_i}^2 = \sigma_{w_i}^2 \sum_{k=0}^{\infty} g_{ik}^2 = \infty$$

which does not correspond to a real system.

To compare GPC with the proposed method, we transform the system model (4) as follows:

$$y_i(k) = \sum_{j=1}^{p} \frac{B_{ij}(q^{-1})D_i(q^{-1})}{A_i(q^{-1})D_i(q^{-1})} u_j(k) + \frac{A_i(q^{-1})C_i(q^{-1})}{A_i(q^{-1})D_i(q^{-1})} w_i(k)$$

$$y_i(k) = \sum_{j=1}^{p} \frac{B_{tij}(q^{-1})}{A_{ti}(q^{-1})} u_j(k) + \frac{C_{ti}(q^{-1})}{A_{ti}(q^{-1})} w_i(k)$$

This means:

$$A_{ti}(q^{-1})y_i(k) = \sum_{j=1}^{p} B_{tij}(q^{-1})u_j(k) + C_{ti}(q^{-1})w_i(k)$$

where $A_{ti}(q^{-1})$ and $C_{ti}(q^{-1})$ are monic polynomials.

$$deg(A_{ti}(q^{-1})) = deg(B_{tij}(q^{-1})) = deg(C_{ti}(q^{-1})) = n_i + m_i$$

To compare both methods, we choose a polynomial $C_{Gi}(q^{-1})$ so that:

$$\frac{C_{Gi}(q^{-1})}{1 - q^{-1}} \approx C_{ti}(q^{-1})$$

For example, we might choose a polynomial of the same degree

$$deg(C_{Gi}(q^{-1})) = n_i + m_i$$

which fulfills

$$\frac{C_{Gi}(q^{-1})}{1 - q^{-1}} = C_{ti}(q^{-1}) + \frac{q^{-(n_i+m_i+1)}C_r}{1 - q^{-1}} \tag{30}$$

where C_r is the rest of the division $C_{Gi}(q^{-1})/(1-q^{-1})$. It is necessary to check whether $C_{Gi}(q^{-1})$ would be stable as required by the GPC method.

As the control law in the GPC algorithm is formulated in incremental form, it presents zero steady state error in front of step disturbances as in the proposed method. However, the output noise is not completely compensated because a modelling error is introduced in the colored noise. Nevertheless, the proposed method in this work is optimal against noise both in the state estimation by Kalman filter, and in the state feedback control using LQR.

6.1. *Illustrative example*

In this section, we explain through a numerical example the proposed method developed in this work. Since the advantages of the proposed method over the GPC can be demonstrated in the monovariable case, a multivariable example becomes unnecessary. Therefore, we use the same example given in (Camacho & Alba, 2013).

Let us suppose that:

$$A(q^{-1}) = 1 - 0.8q^{-1}$$
$$B(q^{-1}) = 0.4q^{-1} + 0.6q^{-2}$$

It is assumed that:

$$C(q^{-1}) = 1 - 0.1q^{-1}$$
$$D(q^{-1}) = 1 - 0.9q^{-1}$$

Then

$$\frac{C(q^{-1})}{D(q^{-1})} = 1 + \frac{0.8q^{-1}}{1 - 0.9q^{-1}}$$

The basic non-incremental model becomes:

$$\begin{bmatrix} x_u(k+1) \\ x_w(k+1) \end{bmatrix} = \begin{bmatrix} 0.8 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.9 \end{bmatrix} \begin{bmatrix} x_u(k) \\ x_w(k) \end{bmatrix} + \begin{bmatrix} 0.4 \\ 0.6 \\ 0 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ 0 \\ 0.8 \end{bmatrix} w(k)$$
$$y(k) = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_u(k) \\ x_w(k) \end{bmatrix} + w(k)$$

Using this model, the estimation is obtained by Kalman filter where if $R_2 = E\left\{w \cdot w^T\right\} = 0.1$. Then:

$$R_1 = E\left\{v_b \cdot v_b^T\right\} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.064 \end{bmatrix}$$
$$R_{12} = E\left\{v_b \cdot w^T\right\} = \begin{bmatrix} 0 \\ 0 \\ 0.08 \end{bmatrix}$$

The filter gain matrix and the observer gain matrix are:

$$M = 10^{-3} \begin{bmatrix} -0.1084 \\ 0 \\ 0.1239 \end{bmatrix}$$
$$L = \begin{bmatrix} -0.0001 \\ 0 \\ 0.8001 \end{bmatrix}$$

The incremental model becomes:

$$\begin{bmatrix} s_y(k+1) \\ \Delta x_u(k+1) \\ \Delta x_w(k+1) \\ s_w(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0.8 & 1 & 0.9 & 0.8 \\ 0 & 0.8 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.9 & 0.8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s_y(k) \\ \Delta x_u(k) \\ \Delta x_w(k) \\ s_w(k) \end{bmatrix} +$$

$$+ \begin{bmatrix} 0.4 \\ 0.4 \\ 0.6 \\ 0 \\ 0 \end{bmatrix} \Delta u(k) + \begin{bmatrix} 0.8 \\ 0 \\ 0 \\ 0.8 \\ -1 \end{bmatrix} w(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s_y(k) \\ \Delta x_u(k) \\ \Delta x_w(k) \\ s_w(k) \end{bmatrix} + w(k)$$

Using the discrete LQR algorithm, with

$$Q = \begin{bmatrix} 10I & 0 \\ 0 & 0 \end{bmatrix}$$

and $R = 1$, we obtain:

$$K = \begin{bmatrix} 0.6006 & 0.9690 & 1.2112 & 1.0898 & 0.9604 \end{bmatrix}$$

Figure 3 represents the transient response of the controlled system using the proposed method, with reference changes in 50 and in the presence of a disturbance in 100. It can be observed that there is no steady state error.

Finally, the variance of the output noise in the system without control with zero input and without disturbances (see Fig. 4) is $var(y) = 0.4242$. Figure 5 shows the response of the controlled system using the proposed method in front of zero reference and without disturbances. The variance is $var(y) = 0.1250$.

To compare this method with traditional GPC, we calculate, using the approximation given in (24), the polynomial

$$C_G(q^{-1}) = 1 - 1.9q^{-1} + 0.98q^{-2}$$

We take a prediction horizon 10 with weighting prediction matrices $Q = 10I$ and the input $R = 1$. The controlled system response in front of step and disturbance is very similar to the one already shown in figure 3. But the big difference is the variance of the controlled system using the GPC method with zero reference signal and no disturbances (Fig. 6), where the variance is $var(y) = 0.3986$, which is significantly higher than the one achieved with the proposed method.

7. Conclusion

In this work, a new incremental state formulation has been proposed for the modelling of multivariable delayed systems with colored noise. This formulation allows us to pro-

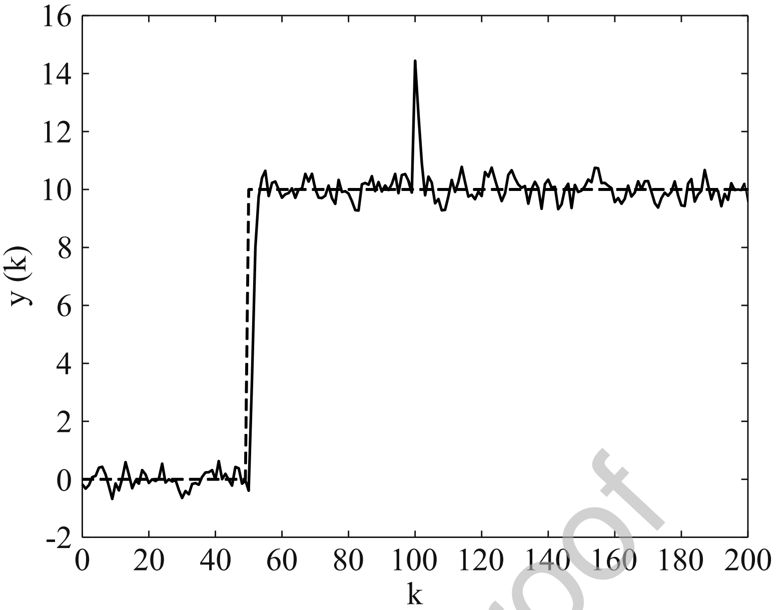

Figure 3. Transient response of the controlled system by the proposed method, with reference change and in the presence of a step disturbance.

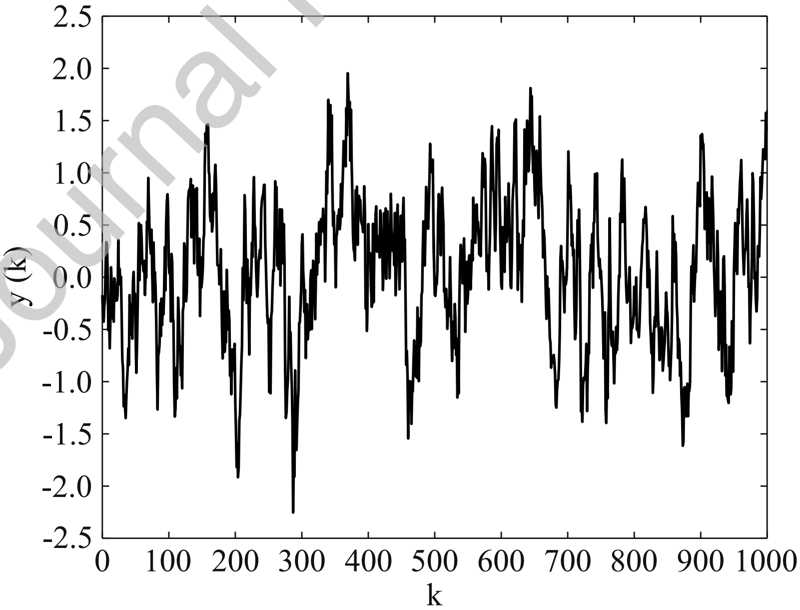

Figure 4. Output noise in the system without control.

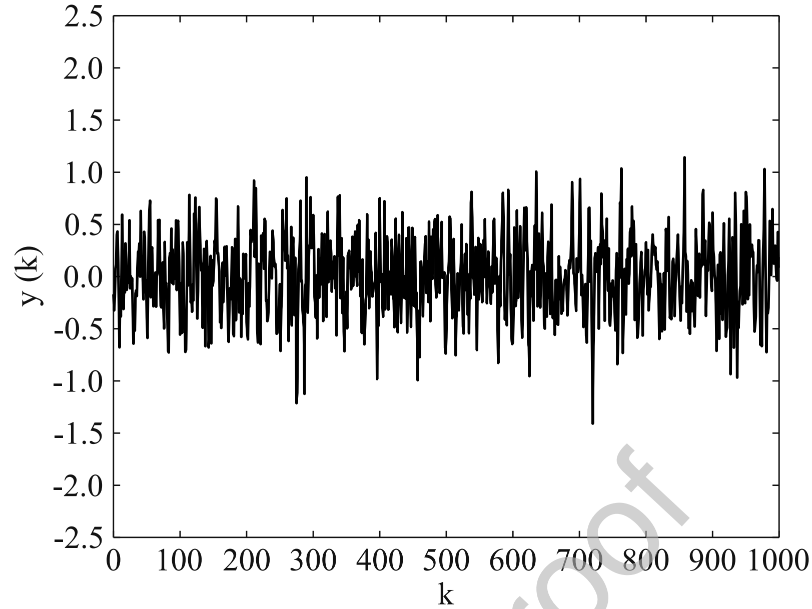

Figure 5. Output noise in the system controlled by the proposed method.

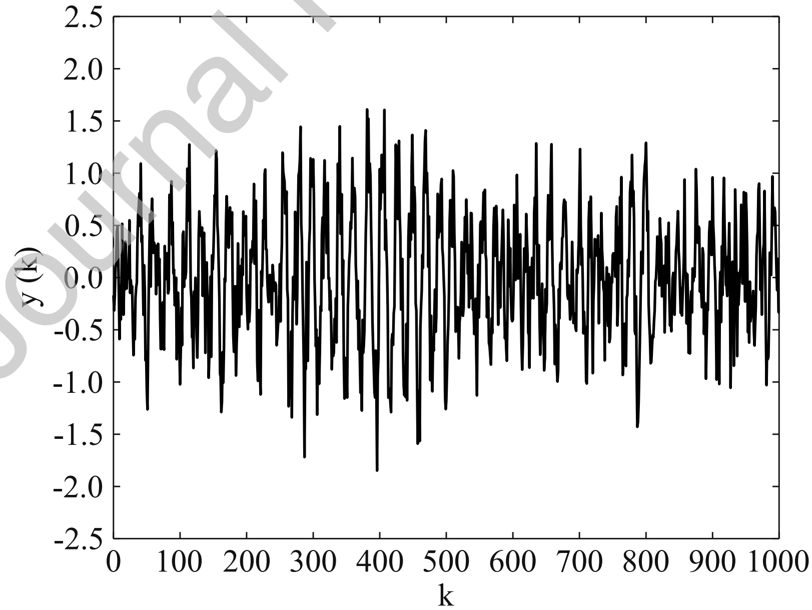

Figure 6. Output noise in the system controlled by GPC method.

pose a control method that is optimal against noise both in the state estimation, using Kalman filter, and in the state feedback control, using the LQR algorithm. Finally, since we consider incremental state variables instead of the states themselves, we solve two problems of the non-incremental state control. The first one is the calculation of the reference state, since a zero incremental state can be taken as an objective, while the second one is the presence of steady state errors, since the resultant controller is equivalent to introduce integral actions and then the controlled system has zero steady state error, even in presence of disturbances.

Declaration of Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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