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Complete List of Authors:	Al-Hadithi, Basil; Universidad Politécnica de Madrid, Adánez, José Miguel; Intelligent Control Group, Universidad Politécnica de Madrid, Centre for Automation and Robotics UPM - CSIC., Jiménez, Agustín; Universidad Politécnica de Madrid madrid, Madrid, Spain, intelligent control group
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A Multi-Strategy Fuzzy Control based on Takagi-Sugeno model

Basil Mohammed Al-Hadithi[a,b], José Miguel Adánez[a,*], Agustín Jiménez[a]

[a] *Intelligent Control Group, Universidad Politécnica de Madrid, Centre for Automation and Robotics UPM - CSIC. C/ J. Gutierrez Abascal, 2. 28006-Madrid, Spain.*
[b] *Department of Electrical, Electronics, Control Engineering and Applied Physics, School of Industrial Design and Engineering, Universidad Politécnica de Madrid. C/ Ronda de Valencia, 3. 28012-Madrid, Spain.*

Abstract

In this work, a new method for the control of nonlinear systems is proposed, the Multi-Strategy Fuzzy Control (MSFC). This new nonlinear control method is a mixture of different control techniques by fuzzy interpolation using the fuzzy Takagi-Sugeno (T-S) model. This new method combines various control strategies to achieve smooth transient response and zero steady state error. The merged strategies in the proposed MSFC structure are: the optimal state feedback control to obtain an optimal transient response, the optimal control through an incremental state model to obtain zero steady state error and a constant input control to maintain the system behaviour within a predefined boundaries where the proposed control is applied.

Keywords: T-S fuzzy model, Multi-Strategy Fuzzy Control, Incremental state model, Optimal control

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1. Introduction

Fuzzy control method is widely used in control theory and control engineering systems due to its great theoretical value and successful applications in complex practical systems [1, 2].

*Corresponding author
Email address: jm.adanez@alumnos.upm.es (José Miguel Adánez)

Research on control of nonlinear systems over the years has produced many results [3, 4, 5]: control based on linearization, global feedback linearization, nonlinear H_∞ control, sliding mode control, variable structure control, state dependent Riccati equation control, etc.

Nonlinearities and uncertainties are always bothersome in controlling a real system, since a physical system is usually ill-known, is difficult to describe and has few measurements available, or is highly nonlinear. Different design techniques were developed for modeling and control of nonlinear systems. An important approach is to model the considered nonlinear systems as T-S fuzzy system [6].

In contrast with the linear feedback control theories, nonlinear control theories possess various merits which can make them a suitable choice for the control of nonlinear systems. The property of adaptiveness and robustness to the changes in the responses makes them a superior choice over linear feedback control schemes.

Fuzzy Logic Control (FLC) systems have recently shown growing popularity in nonlinear system control applications. A FLC system is essentially an effective way to decompose the task of nonlinear system control into a group of local linear controls based on a set of design-specific model rules. FLC also provides a mechanism to blend these local linear control problems all together to achieve overall control of the original nonlinear system. In this regard, FLC technique has its main advantage over other kinds of nonlinear control techniques. Latest research on FLC design is aimed to improve the optimality and robustness of the controller performance by combining the advantage of modern control theory with the T-S fuzzy model.

A novel optimal combined fuzzy Proportional-Integral-Derivative (PID) controller employing Dragonfly Algorithm (DA) was proposed for solving Automatic Generation Control (AGC) problem in interconnected power systems [7]. The DA algorithm was employed to optimize the controller parameters including the scaling factors of fuzzy logic and the PID gains.

A self-tuning fuzzy PID control strategy is proposed for improving the control performance of hydraulic crane due to pressure oscillation which has negative effect on hydraulic crane system, which requires high dynamic stability under a flexible operating condition [8]. The experimental results show that pressure amplitude reduced about 25% at low velocity and pressure oscillation of hydraulic cylinder is suppressed comparing with traditional PID. In addition, fuzzy PID control enables a smoother variation and a higher accuracy in changing processes of joint angle and crane tip position.

Adaptive FLC is efficient to control uncertain nonlinear system [9]. Since the adaptive fuzzy approaches can approximate the complex nonlinear functions through fuzzy rules, the application of the fuzzy logic to deal with the chattering problem is proven as an effective way; it has been used in many engineering applications [3].

A combining adaptive fuzzy-wavelet control algorithm is proposed in [10] for a class of continuous time unknown nonlinear systems. An application of wavelet networks to control problems of nonlinear systems is investigated. A wavelet network is constructed as an alternative to a Neural Network (NN) to approximate a nonlinear system. Based on this wavelet network and fuzzy approximation, suitable adaptive control laws and appropriate parameter update algorithms for nonlinear uncertain (or unknown) systems are developed to achieve tracking performance. The stability analysis for the proposed control algorithm is provided.

An improved approach through integration of improved Genetic Algorithm and Fuzzy Logic Control (GFLC) method is proposed in order to reduce the enlargement of coal floor deformation and the manual adjustment frequency of rocker arms. The enlargement of coal floor deformation is analyzed and a model is built. Then, the framework of proposed approach is built. Moreover, the constituents of GA such as tangent function roulette wheel selection, uniform crossover, and nonuniform mutation are employed to enhance the performance of GFLC [11].

Road tunnel ventilation system is of high non-linearity and uncertainty, and its exact mathematical model is acquired with very dificulty. In order to effectively control road tunnel ventilation system, a Combined Grey Prediction Fuzzy Control (CGPFC) law is proposed in [12]. The output of controller is formed by combining outputs of the CGPFC and the traditional FLC law. The grey predictor is used to predict the system outputs on line in rolling mode.

The influence of a faulted electrical power transmission line on a buried pipeline is investigated in [13]. A fuzzy logic system was used to simulate the problem. It was trained using data derived from finite element method calculations for different configuration cases of the above electromagnetic field problem. It is shown that the proposed method is very time efficient and accurate in calculating electromagnetic fields compared to the time straining finite element method. In order to create the rule base for the fuzzy logic system a special incremental learning scheme is used using genetic algorithms.

In [14], three fuzzy adaptive controllers are presented for a class of uncertain multivariable nonlinear systems with both sector nonlinearities and dead zones: two first controllers are state feedbacks and the last controller is an output feedback. The design of the first controller concerns systems with symmetric and positive definite control-gain matrix, while the second control design is extended to the case of non symmetric control-gain matrix. In the third controller, a high-gain observer is incorporated to estimate the unmeasurable states. An adaptive fuzzy logic system is used to approximate the uncertain functions. A Lyapunov approach is adopted prove the stability of those control systems.

Fuzzy adaptive output feedback optimal control problem for a class of strict-feedback nonlinear systems is investigated in [4]. With the help of fuzzy systems approximating the unknown nonlinear functions and cost function, the unmeasured states are estimated by designing fuzzy adaptive state observer. Combining state observer with backstepping design technique, a feedforward controller is designed. Finally, a fuzzy adaptive optimal controller with parameters adaptive laws is developed. The whole control scheme consists of a feedforward controller and a feedback optimal controller.

An optimal H_∞ tracking-based indirect adaptive FLC for a class of perturbed uncertain affine nonlinear systems without reaching phase is developed in [15]. First an Interval Type-2 (IT2) fuzzy system is used in an adaptive scheme to approximate the system using a nonlinear model and to determine the optimal value of the H_∞ gain control. Secondly, to eliminate the trade-off between H_∞ tracking performance and high gain at the control input, a modified output tracking error has been used.

A robust-optimal FLC for position and attitude stabilization and vibration suppression of a flexible spacecraft during antenna retargeting maneuver is presented in [16]. The FLC is based on T-S fuzzy model and uses the Parallel Distributed Compensator (PDC) technique to quadratically stabilize the closed-loop system. In addition, a fuzzy model-based observer is considered for estimating unmeasurable states. Using Lyapunov stability theory and Linear Matrix Inequalities (LMIs), the problem of designing an optimal-robust fuzzy controller/observer with actuator amplitude constraint is formulated as a convex optimization problem.

In [17] a robust FLC design approach for Dynamic Positioning (DP) system of ships using optimal H_∞ control techniques is proposed. The H_∞ control technique is used to exterminate the effects of environmental disturbances. Firstly, a T-S fuzzy model is applied to approximate the nonlinear DP system. Next, LMI is employed to find a positive definite matrix and controller gains. The stability of the controller is proven by using Lyapunov stability theorems.

A robust adaptive optimal tracking control design for missile systems with unknown parameters and external disturbance is proposed in [18], based on adaptive fuzzy technique. An adaptive FLC is equipped with an optimal robust to achieve the desired tracking performance for uncertain missile systems with external disturbance. The design procedure is divided into two steps. First, a fuzzy-based adaptive feedback linearization control scheme is designed to achieve the tracking of unknown (or uncertain) missile systems. Next, a combined optimal robust control scheme is employed to minimize the worst-case effect arising from adaptive fuzzy approximation error and external disturbance to improve the tracking performance of missile.

In [19], an adaptive nonlinear gain is introduced in the composite feedback controller to improve the system dynamic response. Using the Lyapunov analysis and the adaptive law for the nonlinear gain, it is shown that all the closed loop signals of the system are bounded. Finally, a double integrator Single-Input Single-Output (SISO) system and twin rotor Multi-Input Multi-Output (MIMO) system is used to demonstrate the application of the proposed scheme.

Recently, the optimal control has been paid considerable attention and important results have been achieved [20, 21]. The traditional optimal control of nonlinear systems usually requires to solve the Hamilton-Jacobi-Bellman (HJB) equation, however, obtaining a solution to the HJB equation is a challenging task since it does not have a closed-form analytical method. In [22, 23] it is proposed adaptive neural optimal control methods for continuous time SISO nonlinear systems. That results are extended to the nonlinear large-scale systems in [24], and it is developed a decentralized stabilization method using an online learning optimal control approach. It is proposed an online learning optimal control method for nonlinear systems with control constraints in [25], which solved the input saturating design problem. To solve the HJB equation, in [20] it is developed the approximator based Adaptive Dynamic Programming (ADP) optimal control scheme for discrete-time systems. However, the approaches in [20, 25] can only solve the control problem of the nonlinear systems satisfying the matching condition, but cannot be applied to those nonlinear systems in strict feedback form, especially to those nonlinear systems with unmeasured states.

Apart from the state feedback control, the observer-based output-feedback control, which can overcome the drawbacks of the state feedback control technique that requires the measurement of full state information, is one of the traditional main topics. Especially, several output-feedback optimal control methods for nonlinear systems have been proposed in [26] and [21]. In [26] it is proposed an observer-based online approximate optimal control scheme for nonlinear uncertain systems with immeasurable states, in which two NN have been used, an NN approximated unknown nonlinear function, a critic NN was designed to derive the optimal control input. In [21] it is proposed an adaptive optimal output feedback control approach for nonlinear systems with unavailable states, an NN state observer has been established, and this method can be applied to the system with higher degree of nonlinearity and without priori knowledge of system dynamic. However, the controlled systems are all required to satisfy the matching condition. To our best knowledge, to date, there are no results on adaptive backstepping output feedback fuzzy optimal control methods nonlinear systems without satisfying the matching condition.

Based on the Variable Structure Control (VSC) theory, an adaptive FLC system design method is proposed in [27] for uncertain T-S fuzzy models with norm-bounded uncertainties. As the local controller a VSC law is used with a switching feedback control term and an adaptation law to account for the norm-bounded uncertainties. In terms of LMIs, a sufficient condition is derived for the existence of linear sliding surfaces guaranteeing the asymptotic stability.

The problem in the original T-S identification method is that it cannot be applied when the triangular membership functions are overlapped by pairs [28]. Thus, these membership functions, which have been very popular in modelling and control applications, are restricted. In [28], a new approach is developed as a generalized version of T-S method to improve the T-S models estimation. This method is based on weighting of parameters and offers high accuracy without a substantially increasing in the computational cost. The generalized T-S method is extended to the multivariable case in [29].

In order to calculate the controller coefficients, an optimal selection of closed loop poles will lead to a trade-off between speed of dynamic response and control effort. The objective is to find the control action to transfer the system from any initial state to some final state in an infinite time. The solution is well known [30, 31, 32, 33] when the system is a linear one. However, in the case of nonlinear systems described by T-S fuzzy modelling [6], a general methodology is suggested in [5] to minimize the cost of each step instead of the global cost. The solution will be a suboptimal one but with a great advantage due to its easy calculation.

Several algorithms of FLC based on Linear Quadratic Regulator (LQR) are presented in [5]. The proposed controllers are based on the control action calculation at each time step according to the dynamic behaviour of the nonlinear system at each point of the state space. This control methodology offers a robust and well damped dynamic response.

Incremental state model is presented in [34] for the control of multivariable nonlinear delayed systems. Incremental state model solves the problem of computing the target state, choosing zero incremental state as an objective. Moreover, the control action in an incremental form is equivalent to introduce an integral action, thus obtaining zero steady state error. Then, an FLC based LQR is designed and an optimal observer for multivariable fuzzy systems is developed, since not all system states are measurable. Also, in [35] a multivariable optimal control based on incremental state model is proposed for the wind turbine control problem. Finally, incremental state model allows the disappearance of the affine terms in a natural way. A comparative study between incremental and traditional state models is performed, validating the proposed incremental state control.

To the best of the authors knowledge, there is no concrete approach which combines different control methodologies through the fuzzy T-S models. The most similar works are based on using a fuzzy system to modify the gains of a PID controller, nullifying some parameters of it in some rules to achieve, for example, a control that is purely proportional in the most remote areas, and introducing the integral and derivative components in the area closest to the operation point [36]. However, it is important to note that the same PID control strategy is used in all the rules, introducing only a parameter override to modify the strategies. On the other hand, what is proposed in this work is to use different control methodologies, with different algorithms and different state models.

Another similar approach is presented in [37]. This method is based on varying the different signals to be controlled according to the stages of the process. It is the same case found in the industrial control of wind turbines [38], in which, depending on their operation area, one or other control loops are closed, keeping some open loops through optimization tables based on aerodynamic models.

Moreover, in [39] the combination of several controllers is proposed, both Proportional-Integral (PI) but with different parameterization, depending on the frequencies at which the system operates. In this way, a separation of the frequencies is produced by means of both low pass and high pass filters. Then, the different controllers are applied to both signals, and finally they are combined again to obtain a single control signal.

In this work, a new MSFC which allows a mixture of different control methodologies through T-S model is proposed. The approach combines different control methodologies, with different control algorithms for different state models, more than combining diverse parameterization of the same control technique. As a concept, it allows great freedom in the selection of methodologies and the combination of various control structures (there are different ways of placing different methodologies). Depending on the system dynamics to be controlled, it is necessary to assess which of the control strategies are appropriate and if the structure fits the objectives set by the problem.

Note that, each control method has certain features. The main objective of the proposed algorithm is to apply the correct control strategy in the correct state.

The rest of this work is organized as follows. In section 2, the control strategies used in the proposed MSFC method are detailed. The new proposed MSFC is described in section 3. In section 4, an illustrative example of an inverted pendulum system is presented to show the effectiveness of the proposed approach.

2. Control methods applied in the proposed MSFC

In this work, a new nonlinear control method is developed, using the fuzzy T-S model, based on the combination of different control strategies according to the system operating point. This MSFC is able to combine the advantages of these control methods. The control methods used are: the control by optimal state feedback to obtain an optimal transient response [28], the control by incremental state models to achieve zero steady state error [34, 35] and a constant input control to guarantee the convergence of the system states. Moreover, a state observer should be added since the states are not directly measurable. The aforementioned control methods used in the proposed MSFC are briefly recalled:

2.1. T-S identification method using T-S fuzzy model

For the nonlinear system modelling and identification, a generalized version of T-S fuzzy model is used [28]. This method estimates the nonlinear system parameters minimizing a quadratic performance index, using a parameters weighting method. The method models nonlinear functions as a set of difference equations by the following IF-THEN rules for an n^{th} order system:

$$\begin{aligned}
&\mathrm{S}^{(i_1\dots i_m)}\text{: If } z_1(k) \text{ is } M_1^{i_1} \text{ and } \dots \text{ and } z_m(k) \text{ is } M_m^{i_m} \text{ then:}\\
&y(k+1) = a_0^{(i_1\dots i_m)} + a_1^{(i_1\dots i_m)} y(k) + \dots + a_n^{(i_1\dots i_m)} y(k-n+1) + \\
&\qquad + b_1^{(i_1\dots i_m)} u(k) + \dots + b_n^{(i_1\dots i_m)} u(k-n+1)\\
&\qquad \vdots
\end{aligned} \tag{1}$$

Where $\{z_1(k), z_2(k), \dots z_m(k)\}$ are measurable nonlinear variables, which hereinafter will be called fuzzy variables. Moreover, $M_j^{i_j}$ are the fuzzy sets associated with the fuzzy membership functions $\mu_j^{i_j}(z_j(k))$. In this fuzzy notation, j is the fuzzy variable index and i_j is the fuzzy rule index associated with the fuzzy variable.

From the described fuzzy discrete system, all the fuzzy rules can be transformed into state model as shown in [34].

$$\begin{aligned}
&\mathrm{S}^{(i_1\dots i_m)}\text{: If } z_1(k) \text{ is } M_1^{i_1} \text{ and } z_2(k) \text{ is } M_2^{i_2} \text{ and } \dots \text{ and } z_m(k) \text{ is } M_m^{i_m} \text{ then:}\\
&x(k+1) = a_x^{(i_1\dots i_m)} + A^{(i_1\dots i_m)} x(k) + B^{(i_1\dots i_m)} u(k)\\
&\quad y(k) = a_y^{(i_1\dots i_m)}(k) + C^{(i_1\dots i_m)} x(k)\\
&\qquad \vdots
\end{aligned} \tag{2}$$

2.2. Optimal control based on discrete state model

The applied optimal control is based on discrete LQR method. In this method, since the target state is not null, the goal is to minimize the cost index J, which require the tuning of state Q and input R weighting matrices and the knowledge of state x_r and input references u_r.

$$J = \sum_{k=0}^{\infty} [\, (x(k) - x_r)^T Q (x(k) - x_r) + (u(k) - u_r)^T R (u(k) - u_r) \,] \quad (3)$$

However, from the point of view of real applications, the aim is to achieve a desired output value y_r. In [35], a systematic method is proposed whose objective is to obtain the state and input reference for a desired output reference y_r, as follows:

$$\begin{aligned} &\mathrm{S}^{(i_1 \ldots i_m)}\text{: If } z_1(k) \text{ is } M_1^{i_1} \text{ and } z_2(k) \text{ is } M_2^{i_2} \text{ and } \ldots \text{ and } z_m(k) \text{ is } M_m^{i_m} \text{ then:} \\ &\begin{bmatrix} x_{r\,(i_1 \ldots i_m)} \\ u_{r\,(i_1 \ldots i_m)} \end{bmatrix} = \begin{bmatrix} (A^{(i_1 \ldots i_m)} - I) & B^{(i_1 \ldots i_m)} \\ C^{(i_1 \ldots i_m)} & 0 \end{bmatrix}^{-1} \begin{bmatrix} -a_x^{(i_1 \ldots i_m)} \\ y_r - a_y^{(i_1 \ldots i_m)} \end{bmatrix} \\ &\vdots \end{aligned} \quad (4)$$

and the control action in each fuzzy rule becomes:

$$\begin{aligned} &\mathrm{S}^{(i_1 \ldots i_m)}\text{: If } z_1(k) \text{ is } M_1^{i_1} \text{ and } z_2(k) \text{ is } M_2^{i_2} \text{ and } \ldots \text{ and } z_m(k) \text{ is } M_m^{i_m} \text{ then:} \\ &\qquad u(k)_{(i_1 \ldots i_m)} = K^{(i_1 \ldots i_m)} \left(x_{r\,(i_1 \ldots i_m)} - x(k) \right) + u_{r\,(i_1 \ldots i_m)} \\ &\qquad \vdots \end{aligned} \quad (5)$$

2.3. Optimal control based on incremental state model

Based on the nonlinear discrete system modelled as (2), the incremental state model [34, 35] can be obtained in each rule as follows :

$$
\begin{aligned}
&\mathrm{S}^{(i_1\ldots i_m)}\text{: If } z_1(k) \text{ is } M_1^{i_1} \text{ and } z_2(k) \text{ is } M_2^{i_2} \text{ and } \ldots \text{ and } z_m(k) \text{ is } M_m^{i_m} \text{ then:}\\
&\begin{bmatrix} y(k+1) \\ \delta x(k+1) \end{bmatrix} = \begin{bmatrix} I & C^{(i_1\ldots i_m)}A^{(i_1\ldots i_m)} \\ 0 & A^{(i_1\ldots i_m)} \end{bmatrix} \begin{bmatrix} y(k) \\ \delta x(k) \end{bmatrix} + \\
&\qquad \begin{bmatrix} C^{(i_1\ldots i_m)}B^{(i_1\ldots i_m)} \\ B^{(i_1\ldots i_m)} \end{bmatrix} \delta u(k)\\
&y(k) = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} y(k) \\ \delta x(k) \end{bmatrix}\\
&\vdots
\end{aligned}
\tag{6}
$$

where incremental state and incremental input are defined as:

$$
\begin{aligned}
\delta x(k) &= x(k) - x(k-1) \\
\delta u(k) &= u(k) - u(k-1)
\end{aligned}
\tag{7}
$$

And thus, an expanded state model is obtained

$$
\begin{aligned}
&\mathrm{S}^{(i_1\ldots i_m)}\text{: If } z_1(k) \text{ is } M_1^{i_1} \text{ and } z_2(k) \text{ is } M_2^{i_2} \text{ and } \ldots \text{ and } z_m(k) \text{ is } M_m^{i_m} \text{ then:}\\
&x_a(k+1) = A_a^{(i_1\ldots i_m)}(k)x_a(k) + B_a^{(i_1\ldots i_m)}(k)\delta u(k)\\
&y(k) = C_a^{(i_1\ldots i_m)}x_a(k)\\
&\vdots
\end{aligned}
\tag{8}
$$

Incremental state feedback controllers can be designed by any method. In [34, 35], a discrete LQR method is applied. The objective is using the incremental control action $\delta u(k)$ which minimizes the following cost index:

$$
J = \sum_{k=0}^{\infty} \left(x_r - x_a(k)\right)^T Q \left(x_r - x_a(k)\right) + \delta u^T(k) R \delta u(k) \tag{9}
$$

The reference state x_r is approached as follows:

$$x_{ar} = \begin{bmatrix} y_r \\ \delta x_r \end{bmatrix} = \begin{bmatrix} y_r \\ 0 \end{bmatrix} \tag{10}$$

Finally, the control action in each fuzzy rule becomes:

$$\begin{aligned} &\mathrm{S}^{(i_1 \ldots i_m)}\text{: If } z_1(k) \text{ is } M_1^{i_1} \text{ and } z_2(k) \text{ is } M_2^{i_2} \text{ and } \ldots \text{ and } z_m(k) \text{ is } M_m^{i_m} \text{ then:} \\ &\quad \delta u(k)_{(i_1 \ldots i_m)} = K^{(i_1 \ldots i_m)}((x_r - x_a(k)) \\ &\quad u(k)_{(i_1 \ldots i_m)} = u(k-1) + \delta u(k)_{(i_1 \ldots i_m)} \\ &\quad \vdots \end{aligned} \tag{11}$$

2.4. T-S state observer

In general, the state $x(k)$ is not accessible. So an observer will be required and therefore, the optimal state observer formulation developed in [34, 35] is used. The system model is described as T-S fuzzy state model (Eq. 2).

The T-S observer is formulated in each fuzzy rule as follows:

$$\begin{aligned} &\mathrm{S}^{(i_1 \ldots i_m)}\text{: If } z_1(k) \text{ is } M_1^{i_1} \text{ and } z_2(k) \text{ is } M_2^{i_2} \text{ and } \ldots \text{ and } z_m(k) \text{ is } M_m^{i_m} \text{ then:} \\ &x_e(k+1) = a_x^{(i_1 \ldots i_m)} + A^{(i_1 \ldots i_m)} x_e(k) + B^{(i_1 \ldots i_m)} u(k) + \\ &\qquad H^{(i_1 \ldots i_m)} \left(y(k) - a_y^{(i_1 \ldots i_m)} - C^{(i_1 \ldots i_m)} x_e(k) \right) \\ &\qquad \vdots \end{aligned} \tag{12}$$

From observer state estimation, the incremental state can be estimated as follows [35]:

$$x_{ae}(k) = \begin{bmatrix} y(k) \\ x_e(k) - x_e(k-1) \end{bmatrix} \tag{13}$$

The optimal state observer [34, 35] solves the problem of calculating a matrix H which minimizes the observer cost index $J_{obs.}$:

$$J_{obs.}(H) = \alpha^T (A - HC)(A - HC)^T \alpha \tag{14}$$

CI	CI	CI	CI	CI
CI	LQR	LQR	LQR	CI
CI	LQR	INC	LQR	CI
CI	LQR	LQR	LQR	CI
CI	CI	CI	CI	CI

Figure 1: MSFC proposed structure of control methodologies. LQR control based on discrete state model (LQR). Incremental state feedback controller based on LQR (INC). Constant input control (CI).

$$\forall \alpha \in \mathfrak{R}^n$$

and it is verified that for any value of α, it fulfills that:

$$H = AC^T(CC^T)^{-1} \tag{15}$$

3. Multi-Strategy Fuzzy Control

In this section, the proposed fuzzy T-S controller based on the combination of above mentioned control methods according to the system operating point, is described.

The structure to merge the different control strategies may vary depending on the dynamic behaviour of the system under study and the different objectives to be achieved. In this work, the following configuration is proposed for MSFC method (see figure 1):

- Incremental state feedback controller based on LQR: It is used in the central fuzzy rule (INC label in figure 1) to deal with the nominal operating point of the system. In this way, it is expected to use the incremental state control to obtain zero steady state error.

- Optimal LQR control based on discrete state model: It is used in the intermediate rules (LQR labels in figure 1), around the nominal operating point. This optimal control strategy is used to bring the system states optimally from any point of the state space to the fuzzy central rule where control by incremental model acts.

- Constant input control: It is used in the exterior rules (CI labels in figure 1). This control action uses a predefined maximum control signal allowed by the actuator. In case that there is an excessive disturbance which forces the states to get in the exterior zone, this control method must produce an action to return the system states to the intermediate rules, where the other control actions act.

Therefore, each of the different control strategies, detailed above in section 2, will lead to the following control signals:

- Incremental state feedback controller based on LQR. In the central rule (INC label in figure 1):

$$\begin{aligned} &\mathrm{S}^{(i_1 \dots i_m)}\text{: If } z_1(k) \text{ is } M_1^{i_1} \text{ and } \dots \text{ and } z_m(k) \text{ is } M_m^{i_m} \text{ then:} \\ &\delta u(k)_{(i_1 \dots i_m)} = K_{INC}^{(i_1 \dots i_m)} \left(\begin{bmatrix} y_r \\ 0 \end{bmatrix} - \begin{bmatrix} y(k) \\ x_e(k) - x_e(k-1) \end{bmatrix} \right) \\ &u(k)_{(i_1 \dots i_m)} = u(k-1) + \delta u(k)_{(i_1 \dots i_m)} \end{aligned} \quad (16)$$

- Optimal LQR control based on discrete state model. In the intermediate rules (LQR labels in figure 1):

$$\begin{aligned} &\mathrm{S}^{(i_1 \dots i_m)}\text{: If } z_1(k) \text{ is } M_1^{i_1} \text{ and } \dots \text{ and } z_m(k) \text{ is } M_m^{i_m} \text{ then:} \\ &\begin{bmatrix} x_{r(i_1 \dots i_m)} \\ u_{r(i_1 \dots i_m)} \end{bmatrix} = \begin{bmatrix} (A^{(i_1 \dots i_m)} - I) & B^{(i_1 \dots i_m)} \\ C & 0 \end{bmatrix}^{-1} \begin{bmatrix} -a_x^{(i_1 \dots i_m)} \\ y_r - a_y^{(i_1 \dots i_m)} \end{bmatrix} \\ &u(k)_{(i_1 \dots i_m)} = K_{LQR}^{(i_1 \dots i_m)} \left(x_{r(i_1 \dots i_m)} - x_e(k) \right) + u_{r(i_1 \dots i_m)} \\ &\vdots \end{aligned} \quad (17)$$

- Constant input control. In the exterior rules (CI labels in figure 1):

$$\begin{aligned} &\mathrm{S}^{(i_1 \ldots i_m)}\text{: If } z_1(k) \text{ is } M_1^{i_1} \text{ and } \ldots \text{ and } z_m(k) \text{ is } M_m^{i_m} \text{ then:} \\ &u(k)_{(i_1 \ldots i_m)} = cst_{CI}^{(i_1 \ldots i_m)} \qquad (18) \\ &\vdots \end{aligned}$$

where $cst_{CI}^{(i_1 \ldots i_m)}$ are constants associated to constant input control strategy in the fuzzy rules.

In the MSFC, the different control actions calculated in all the fuzzy rules are defuzzified to obtain a single control signal

$$u(k) = \frac{\sum_{i_1=1}^{r_1} \cdots \sum_{i_m=1}^{r_m} w(z(k))_{(i_1 \ldots i_m)} u(k)_{(i_1 \ldots i_m)}}{\sum_{i_1=1}^{r_1} \cdots \sum_{i_m=1}^{r_m} w(z(k))_{(i_1 \ldots i_m)}} \qquad (19)$$

where $w_{(i_1 \ldots i_m)}$ is the weight which corresponds to the rule $S^{(i_1 \ldots i_m)}$:

$$w(z(k))_{(i_1 \ldots i_m)} = \prod_{j=1}^{m} \mu_j^{i_j}(z_j(k)) \qquad (20)$$

Since the states are not directly accessible, a fuzzy state observer is used. This state observer can be unique for all control strategies, using discrete state estimation in each rule, as follows:

$$\begin{aligned} &\mathrm{S}^{(i_1 \ldots i_m)}\text{: If } z_1(k) \text{ is } M_1^{i_1} \text{ and } \ldots \text{ and } z_m(k) \text{ is } M_m^{i_m} \text{ then:} \\ &x_e(k+1)_{(i_1 \ldots i_m)} = a_x^{(i_1 \ldots i_m)} + A^{(i_1 \ldots i_m)} x_e(k) + B^{(i_1 \ldots i_m)} u(k) + \\ &\qquad H^{(i_1 \ldots i_m)} \left(y(k) - \left(a_y^{(i_1 \ldots i_m)} + C x_e(k) \right) \right) \qquad (21) \\ &\vdots \end{aligned}$$

and the estimated state calculated in all the fuzzy rules can be defuzzified to obtain a single signal

$$x_e(k+1) = \frac{\sum_{i_1=1}^{r_1} \cdots \sum_{i_m=1}^{r_m} w(z(k))_{(i_1 \ldots i_m)} x_e(k+1)_{(i_1 \ldots i_m)}}{\sum_{i_1=1}^{r_1} \cdots \sum_{i_m=1}^{r_m} w(z(k))_{(i_1 \ldots i_m)}} \qquad (22)$$

From observer state estimation, the incremental state can be estimated as shown in (13):

$$x_{ae}(k) = \begin{bmatrix} y(k) \\ x_e(k) - x_e(k-1) \end{bmatrix}$$

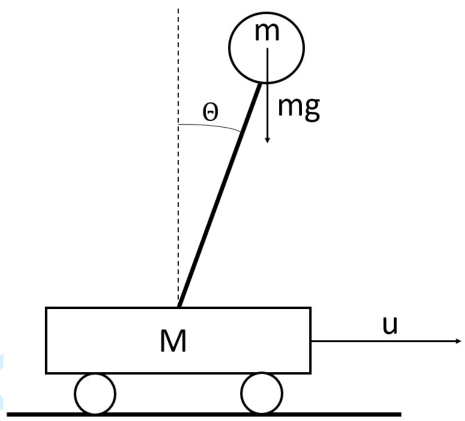


Figure 2: Inverted pendulum system.

4. Illustrative example of inverted pendulum

Consider the problem of stabilizing an inverted pendulum (see Fig. 2). The control of this system is a well-known one, since this system is unstable and highly nonlinear. The objective is to maintain the inverted pendulum upright despite disturbance effects due to external forces. The inverted pendulum can be represented as follows:

$$\ddot{\theta} = \frac{g \sin\theta - \cos\theta \left(\frac{u + ml\dot{\theta}^2 \sin\theta}{M+m} \right)}{l \left(\frac{4}{3} - \frac{m \cos^2\theta}{M+m} \right)} \tag{23}$$

where θ denotes the angular position deviated from the equilibrium position (vertical axis) of the pendulum and $\dot{\theta}$ is the angular velocity, the gravity acceleration is $g = 9.81\ m/s^2$, the mass of the cart is $M = 1\ kg$, the mass of the pole is $m = 1\ kg$ and the distance from the center of the mass of the pole to the cart is $l = 1\ m$.

The main objective is to move the pendulum to its equilibrium position $\theta = \dot{\theta} = 0$. In order to control the system, a discrete controller is proposed with a sampling time of $0.01s$ and a sample-hold device is used for the control action.

Five triangular rules are used for the angular position θ and five triangular rules for the angular velocity $\dot{\theta}$ as shown in figure 3. Then the fuzzy model is

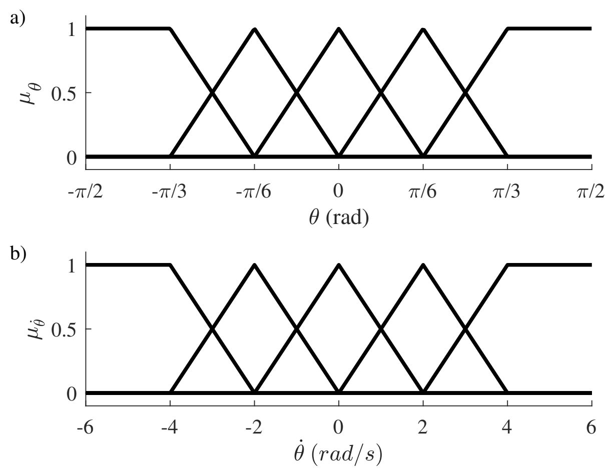


Figure 3: Fuzzy rules for MSFC of the inverted pendulum system. a) fuzzy rules for the angular position. b) Fuzzy rules for the angular velocity.

identified through T-S generalized identification method and it is constructed with a set of 25 rules:

$$S^{(1,1)}\text{: If } \theta(k) \text{ is } M_\theta^1 \text{ and } \dot\theta(k) \text{ is } M_{\dot\theta}^1 \text{ then:}$$

$$\theta(k) = -0.0141 + 1.9579\theta(k-1) - 0.9674\theta(k-2) -$$
$$0.00004u(k-1) - 0.00001u(k-2)$$
$$\vdots$$

The MSFC structure is defined as shown in figure 4, which indicates the methodology selected for each fuzzy rule. In this way, the MSFC structure combines the advantages of each control method through the fuzzy interpolation defined by the fuzzy T-S system.

Thus, in the figures 5, 6 and 7 the weights of the rules for each control method are shown.

Then, discrete model for the fuzzy rules $S^{(2,2)}$, $S^{(2,3)}$, $S^{(2,4)}$, $S^{(3,2)}$, $S^{(3,4)}$,

θ (rad) \ $\dot{\theta}$ (rad/s)	-4	-2	0	2	4
$-\pi/3$	CI	CI	CI	CI	CI
$-\pi/6$	CI	LQR	LQR	LQR	CI
0	CI	LQR	INC	LQR	CI
$\pi/6$	CI	LQR	LQR	LQR	CI
$\pi/3$	CI	CI	CI	CI	CI

Figure 4: MSFC structure for inverted pendulum. LQR denotes the control based on discrete state model. INC denotes incremental state feedback controller based on LQR. CI denotes constant input control.

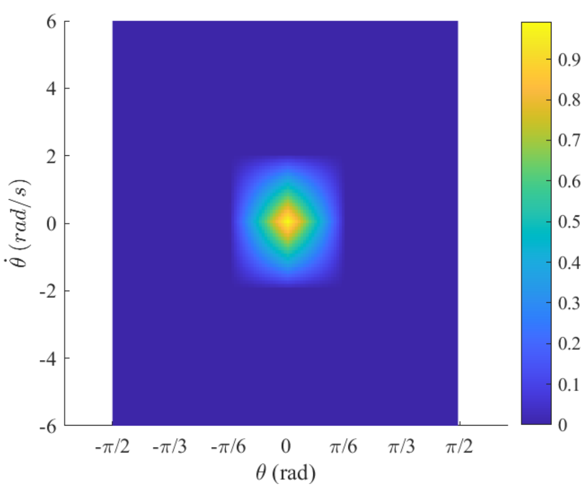


Figure 5: Weight of the rule associated with the incremental state feedback controller based on LQR.

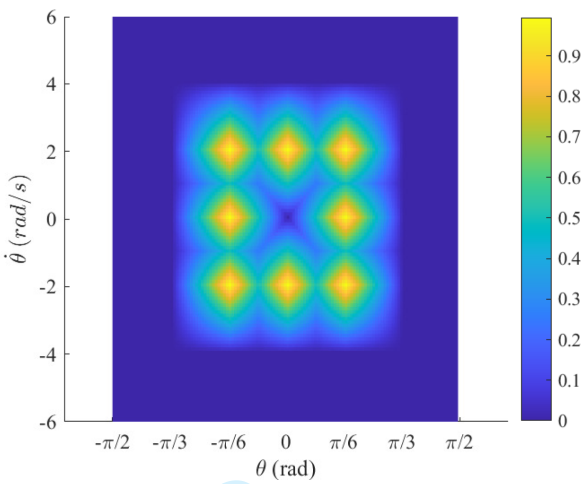


Figure 6: Weight of the rules associated with the LQR control based on discrete state model.

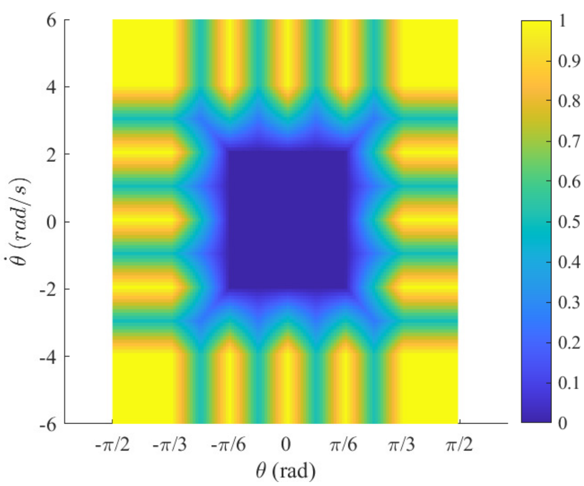


Figure 7: Weight of the rules associated with constant input control.

$S^{(4,2)}$, $S^{(4,3)}$ and $S^{(4,4)}$ become:

$S^{(2,2)}$: If $\theta(k)$ is M_θ^2 and $\dot{\theta}(k)$ is $M_{\dot{\theta}}^2$ then:

$$x(k+1) = \begin{bmatrix} -0.0032 \\ 0.0016 \end{bmatrix} + \begin{bmatrix} 1.9961 & 1 \\ -1.0041 & 0 \end{bmatrix} x(k) + \begin{bmatrix} -0.00001 \\ -0.00002 \end{bmatrix} u(k)$$
$$y(k) = -0.0016 + \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$
$$\vdots$$

The following weighting matrices are used for LQR state feedback matrices calculation:

$$Q = \begin{bmatrix} 1000 & 0 \\ 0 & 1000 \end{bmatrix}$$
$$R = 1$$

The state controller matrices, calculated in each fuzzy rule associated with the optimal control method based on discrete state model using the discrete LQR algorithm are:

$S^{(2,2)}$: If $\theta(k)$ is M_θ^2 and $\dot{\theta}(k)$ is $M_{\dot{\theta}}^2$ then:

$$K^{(2,2)} = \begin{bmatrix} -286.5963 & -287.6285 \end{bmatrix}$$
$$\vdots$$

Moreover, the incremental model for the fuzzy rule $S^{(3,3)}$ is:

$S^{(3,3)}$: If $\theta(k)$ is M_θ^3 and $\dot{\theta}(k)$ is $M_{\dot{\theta}}^3$ then:

$$\begin{bmatrix} y(k+1) \\ \delta x(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 2.0016 & 1 \\ 0 & 2.0016 & 1 \\ 0 & -0.9996 & 0 \end{bmatrix} \begin{bmatrix} y(k) \\ \delta x(k) \end{bmatrix} + \begin{bmatrix} -0.00005 \\ -0.00005 \\ -0.00005 \end{bmatrix} \delta u(k)$$
$$y(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} y(k) \\ \delta x(k) \end{bmatrix}$$

The following weighting matrices are used for the LQR controller by incre-

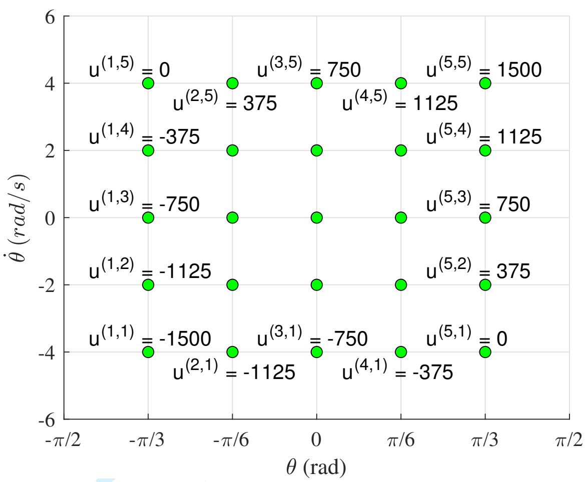


Figure 8: Control signals associated with rules that apply constant input (CI) control strategy.

mental state model:

$$Q = \begin{bmatrix} 1000000 & 0 & 0 \\ 0 & 1000000 & 0 \\ 0 & 0 & 1000000 \end{bmatrix}$$

$$R = 1$$

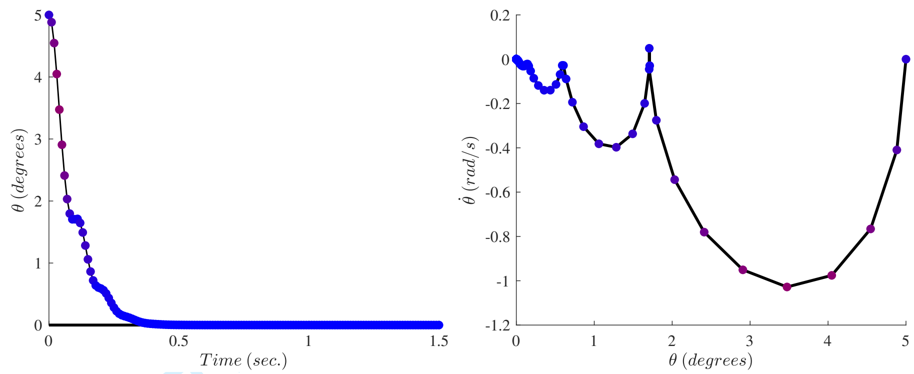
The state controller matrix, calculated in the fuzzy rule associated with the optimal control method based on the incremental state model using the discrete LQR algorithm is:

$S^{(3,3)}$: If $\theta(k)$ is M_{θ}^{3} and $\dot{\theta}(k)$ is $M_{\dot{\theta}}^{3}$ then:

$$K^{(3,3)} = \begin{bmatrix} -616.3911 & -10564.4711 & -7643.4405 \end{bmatrix}$$

Finally, in the fuzzy rules $S^{(1,1)}$, $S^{(1,2)}$, $S^{(1,3)}$, $S^{(1,4)}$, $S^{(1.5)}$, $S^{(2,1)}$, $S^{(2,5)}$, $S^{(3,1)}$, $S^{(3,5)}$, $S^{(4,1)}$, $S^{(4,5)}$, $S^{(5,1)}$, $S^{(5,2)}$, $S^{(5,3)}$, $S^{(5,4)}$ and $S^{(5,5)}$, the control signals associated with the rules that apply constant input control are as shown in figure 8:

On the other hand, the state observer matrices are calculated for all fuzzy



(a) Transient response. (b) Phase-plane diagram.

Figure 9: Response of the inverted pendulum system controlled by the proposed MSFC when the initial angular position is set to $\theta(0) = 5°$.

rules using the optimal state observer formulation as:

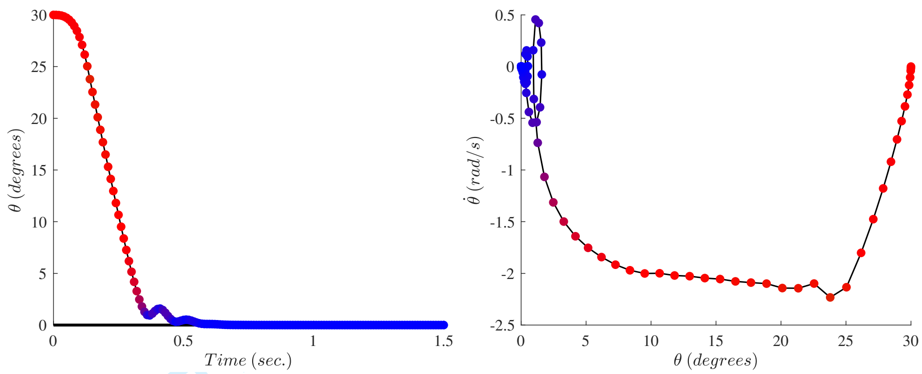
$$S^{(1,1)}\text{: If } \theta(k) \text{ is } M_\theta^1 \text{ and } \dot{\theta}(k) \text{ is } M_{\dot{\theta}}^1 \text{ then:}$$

$$H^{(1,1)} = \begin{bmatrix} 1.9579 \\ -0.9674 \end{bmatrix}$$

$$\vdots$$

Figure 9a shows the transient responses of output angular position θ when the inverted pendulum is set to a initial position of $\theta(0) = 5°$. In order to show the different methodologies behaviour of the MSFC response, several colors have been used to indicate which control strategy is working at each moment: Blue colour is used for the control by incremental state model, red colour for the LQR control by discrete state model and green one for constant input control. Mixed colours represent the merging of the different strategies through the fuzzy interpolation. Figure 9b shows the phase-plane diagram of the controlled system.

Figure 10a shows the transient response of output angular position θ when the inverted pendulum is set to an initial position of $\theta(0) = 30°$. Figure 10b shows the phase-plane diagram of the inverted pendulum.



(a) Transient response. (b) Phase-plane diagram.

Figure 10: Response of the inverted pendulum system controlled by the proposed MSFC when the initial angular position is set to $\theta(0) = 30°$.

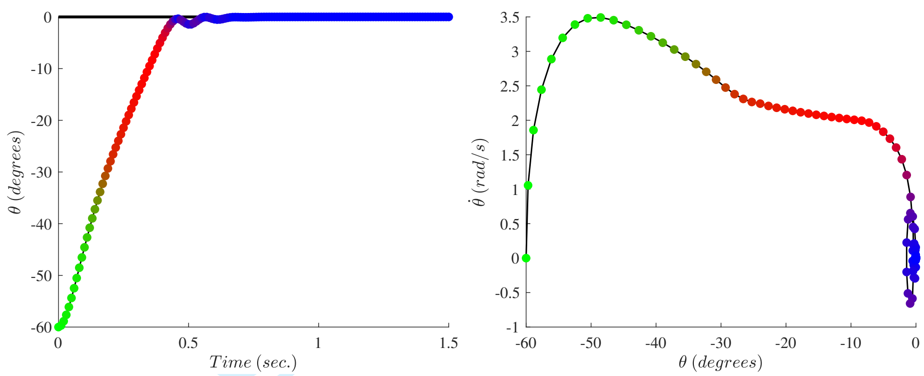
Figure 11a shows the transient responses of output angular position θ when the inverted pendulum is set to a initial position of $\theta(0) = -60°$. Figure 11b shows the phase-plane diagram of the inverted pendulum.

Figure 12a shows the transient responses of output angular position θ when the inverted pendulum is set to a initial position of $\theta(0) = 85°$. Figure 12b shows the phase-plane diagram of the inverted pendulum.

As it can be shown in the above figures, the MSFC can stabilize the inverted pendulum system in a wide range of initial angular positions. Depending on the operating point, different control strategies can be activated and moreover, in the intermediate points, several strategies are working simultaneously through the fuzzy interpolation.

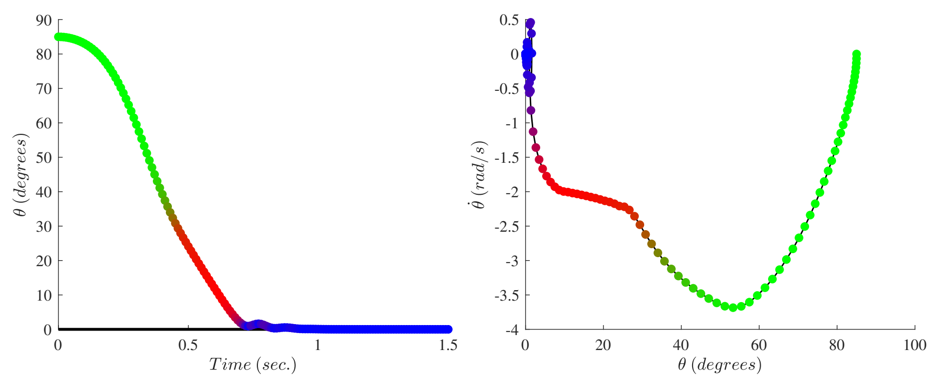
The constant input control is chosen to avoid that the inverted pendulum system falls completely out of the operating range. The LQR optimal control applied over discrete state model is chosen to obtain a fast and optimal transient response in the main operation zone. On the other hand, the incremental state feedback controller based on LQR is chosen to obtain a zero steady state error.

In order to show the different behaviours of the system controlled by different control methodologies, the following figures show a comparison of the responses



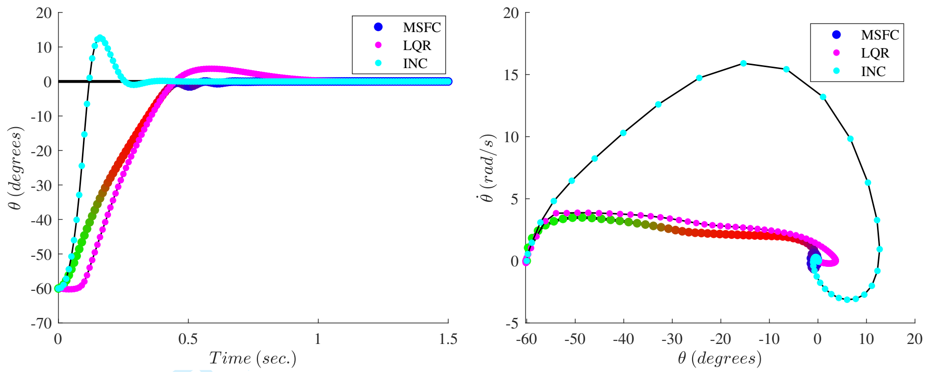
(a) Transient response. (b) Phase-plane diagram.

Figure 11: Response of the inverted pendulum system controlled by the proposed MSFC when the initial angular position is set to $\theta(0) = -60°$.



(a) Transient response. (b) Phase-plane diagram.

Figure 12: Response of the inverted pendulum system controlled by the proposed MSFC when the initial angular position is set to $\theta(0) = 85°$.



(a) Transient response. (b) Phase-plane diagram.

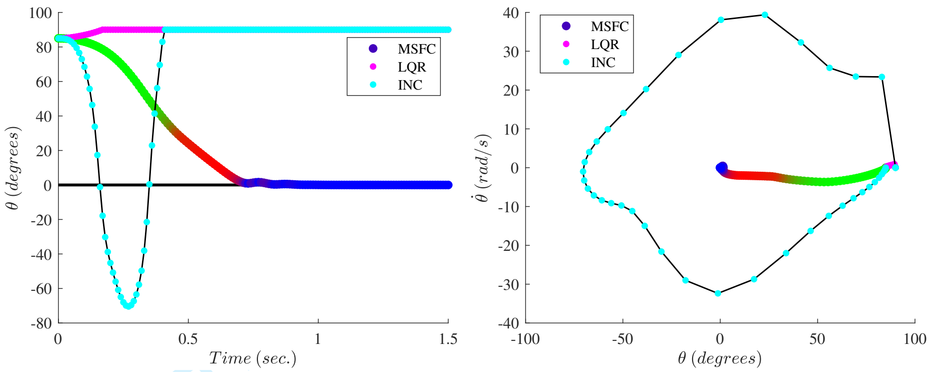
Figure 13: Response of the inverted pendulum system controlled by the proposed MSFC, LQR fuzzy control based on discrete state model and LQR fuzzy control based on incremental state model; when the initial angular position is set to $\theta(0) = -60°$.

of the system controlled by proposed MSFC method (merged green, red and blue colours), LQR fuzzy control based on discrete state model [32] (magenta colour) and LQR fuzzy control based on incremental state model [34] (cyan colour). Figure 13a shows the transient responses of output angular position θ when the inverted pendulum is set to a initial position of $\theta(0) = -60°$. Figure 13b shows the phase-plane diagram of the inverted pendulum.

Figure 14a shows the transient responses of output angular position θ when the inverted pendulum is set to a initial position of $\theta(0) = 85°$. Figure 14b shows the phase-plane diagram of the inverted pendulum.

Note that, with the MSFC method, the system can be stabilized with an initial angular position of $\theta(0) = 85°$, while the methods based on fuzzy optimal control using discrete state model (LQR) and fuzzy optimal control using incremental state model (INC) cannot stabilize the system. Thus the pendulum pole falls to $\theta = 90°$ (horizontal position), with the same initial angular position $\theta(0) = 85°$.

The following figures show the responses of the system controlled by proposed MSFC method when the system is affected by an abrupt change in the



(a) Transient response. (b) Phase-plane diagram.

Figure 14: Response of the inverted pendulum system controlled by the proposed MSFC, LQR fuzzy control based on discrete state model and LQR fuzzy control based on incremental state model; when the initial angular position is set to $\theta(0) = 85°$.

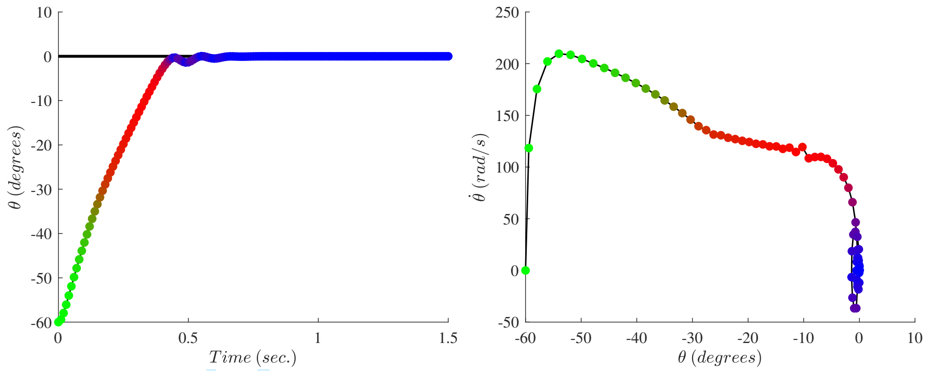
pendulum mass. The mass of the pole changes from $m = 1\ kg$ to $m = 1.25\ kg$ in time $t = 0.25\ s$. Figure 15a shows the transient responses of output angular position θ when the inverted pendulum is set to a initial position of $\theta(0) = -60°$. Figure 15b shows the phase-plane diagram of the inverted pendulum.

Figure 16a shows the transient responses of output angular position θ when the inverted pendulum is set to a initial position of $\theta(0) = 85°$. Figure 16b shows the phase-plane diagram of the inverted pendulum.

The proposed MSFC method can stabilize the system even with a mass increment in the mass of pole.

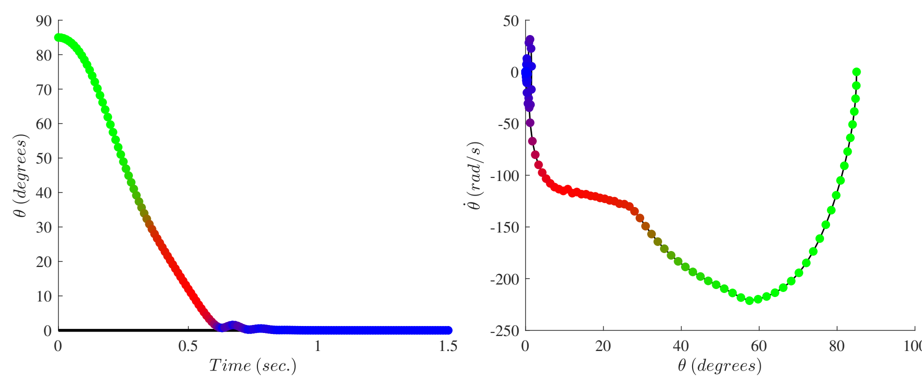
Figures 17 and 18 show the responses of the system controlled by proposed MSFC method when the system is affected by a noise in the angular position sensor. There is defined a zero mean normal noise $N(0, \sigma)$ with $\sigma = 0.1°$. Figure 17a shows the transient responses of output angular position θ when the inverted pendulum is set to a initial position of $\theta(0) = -60°$. Figure 17b shows the phase-plane diagram of the inverted pendulum.

Figure 18a shows the transient responses of output angular position θ when the inverted pendulum is set to a initial position of $\theta(0) = 85°$. Figure 18b



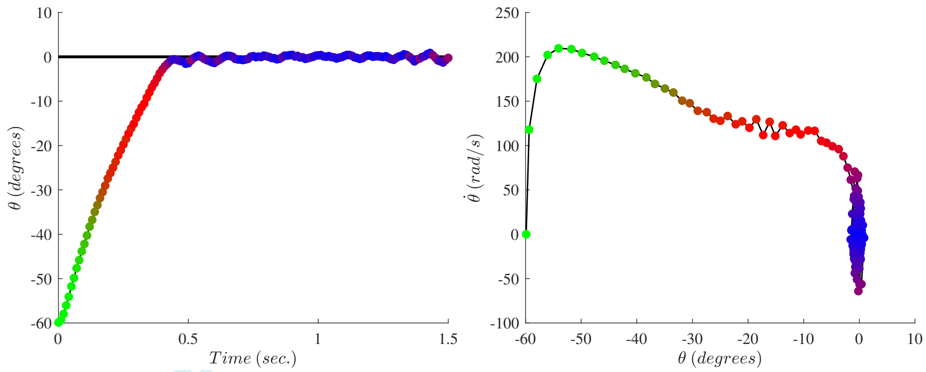
(a) Transient response. (b) Phase-plane diagram.

Figure 15: Response of the inverted pendulum system controlled by the proposed MSFC when the system is affected by an abrupt change in the pendulum mass and the initial angular position is set to $\theta(0) = -60°$.



(a) Transient response. (b) Phase-plane diagram.

Figure 16: Response of the inverted pendulum system controlled by the proposed MSFC when the system is affected by an abrupt change in the pendulum mass and the initial angular position is set to $\theta(0) = 85°$.



(a) Transient response. (b) Phase-plane diagram.

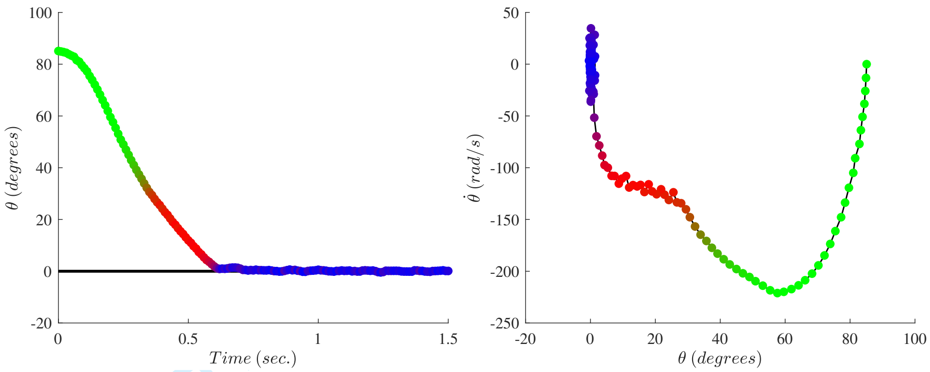
Figure 17: Response of the inverted pendulum system controlled by the proposed MSFC when the system is affected by a noise in the angular position sensor and the initial angular position is set to $\theta(0) = -60°$.

shows the phase-plane diagram of the inverted pendulum.

As it can be seen the proposed MSFC method can stabilize the system even a noise in the angular position sensor.

5. Conclusions

In this work, a new method for the T-S fuzzy control is proposed for the control of nonlinear models. A new MSFC is developed which combines different control techniques by a fuzzy interpolation using the fuzzy T-S model. The proposed MSFC can extract the advantages of each strategy to obtain a fast and smooth transient response with zero steady state error. The applied control methodologies in the MSFC in this work are the following: the optimal state feedback control to obtain an optimal transient response, the optimal control through an incremental state model to obtain zero steady state error and a constant input control to maintain the system behaviour within a predefined boundaries where the proposed control is applied. In order to show the controller behaviour, an inverted pendulum is chosen as an illustrative example. The controller presents a fast and smooth transient response and zero steady state



(a) Transient response. (b) Phase-plane diagram.

Figure 18: Response of the inverted pendulum system controlled by the proposed MSFC when the system is affected by a noise in the angular position sensor and the initial angular position is set to $\theta(0) = 85°$.

error. The main feature of the proposed MSFC is its versatility, since this new methodology allow the fuzzy combination of so many control techniques.

Acknowledgements

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