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MILP and PSO approaches for solving a hydropower reservoirs intraday economic optimization problem

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Abstract

Short-term hydropower generation with several water reservoirs requires deciding, for each moment in time, the volume of water (outflow) that is released from every reservoir to be turbined and generate energy. Knowing the price of energy at every time period, the objective is to maximize the income earned from the generated energy. In this paper, we present (1) a Hydropower Reservoirs Operation Optimization problem with a higher level of detail than those found in the literature, encompassing temporal delays, water hammer effects, and increased temporal discretization, among others features, and (2) two distinct approaches for addressing this problem: MILP and PSO. These methods are compared across instances of varying nature to evaluate their performance. We make our code available on GitHub: https://github.com/baobabsoluciones/flowing-basin.

Keywords Daily optimization · Hydropower generation · Particle swarm optimization · Multireservoir · Mixed integer linear programming

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1 Introduction

The optimization of hydropower systems is crucial for maximizing economic benefits by utilizing the scarce resources they comprise. A typical hydropower system consists of a series of reservoirs that allow water to be stored and turbined when most convenient. The volume of released water (outflow) is limited by the capacity of the channel and the height of the reservoir, which is related to the volume of water it contains. Specifically, if the reservoir has minimum volume, no water can be released, and no energy can be generated. On the other hand, if the reservoir has an enormous volume, this optimization problem becomes irrelevant, as there is water to turbine indefinitely, and it ceases to be a scarce resource to manage.

Managing a hydropower system involves deciding the flow of water released from each dam at each time period. This optimization problem, known in the literature as Hydropower Reservoirs Operation Optimization (HROO) (Bernardes et al. 2022), can have varied objectives, such as maximizing the generated energy, minimizing the difference between demand and produced energy, or maximizing the income from generated energy, taking into account for the latter case that energy prices vary significantly in the short term. Ideally, if the objective is to maximize income, the system operator stores water when energy prices are low and releases it when energy prices become expensive. However, this task is complex due to several factors: on one hand, if the hydropower system has more than one reservoir, these reservoirs are not independent, as the water turbined in one power group enters the next reservoir. This means that the planning of the hydropower system must be done considering all reservoirs at once. On the other hand, it is important to consider that the price of energy in the electricity market can fluctuate significantly from one hour to the next, defining what are known as peak hours (expensive energy) and off-peak hours (cheap energy).

In this paper, we address a specific gap in the literature by introducing the *Hydropower Reservoirs Intraday Economic Optimization (HRIEO) problem* and providing two solution approaches. This problem focuses on daily management and economic benefits as the main objective, introducing complexities not present in the aggregated optimization over longer periods (days, months, or years) typically found in the literature. Fine time discretization, introduced by optimizing over shorter periods (hours or minutes), allows for the consideration of relevant aspects that improve the management of these systems, such as: (1) peak and off-peak hours, (2) the delay from water release to energy generation, and (3) various operational constraints such as the cost of starting power groups, restricted zones in turbine efficiency, channel gates operations limitations, and water hammer restrictions. Additionally, the European Regulation (2017/2195) (Commission 2017), which mandates price matching in the wholesale electricity market every 15 min, highlights the advantages of a finer time interval discretization. This new HRIEO problem captures better the impacts of such regulatory changes on the management of hydropower systems.

Our aim is to propose a model that considers all the aforementioned aspects, making it more reliable for the new HRIEO problem, and to solve it using both an exact and an approximate method in parallel. To achieve this, in Sect. 2, a literature review is conducted to justify these areas of research that remain unexplored. The third section provides a detailed description of the HRIEO problem proposed in this paper. In the fourth section, both the exact method, consisting of a Mixed-Integer Linear Programming (MILP) formulation solved using the branch and bound algorithm, and the approximate method, consisting of a Particle Swarm Optimization (PSO) algorithm are introduced. Finally, in Sect. 5, the results obtained from both approaches are presented for subsequent comparison and discussion.

2 Literature review

The HROO problem has been a subject of extensive study and analysis over the decades, evolving from its early exploration in the 1970s (Draper and Adamowski 1976) to a highly relevant research field in Operations Research. The inherent complexity in managing hydroelectric reservoirs has driven the development of various resolution approaches, all aiming to efficiently manage water resources and optimize energy production. Despite this wide variety of approaches, the modeling of the HROO problem exhibit significant similarities. General HROO models contain typical constraints such as maximum and minimum volume, and maximum allowable flow and generated power (Guedes et al. 2017). These models usually correlate the output flow in the reservoir with the generated power or energy, reflecting real-world operational dynamics.

However, key differences among various approaches stem from different temporal horizons and the discretization applied to them. HROO problems are generally categorized into short-term, mid-term, and long-term scheduling (Thaeer Hammid et al. 2020). Despite the term "short term", the models referenced operate over time horizons spanning days, even weeks (Zarghami 2018). These time horizons make it impossible, among other things, to effectively consider the price of the electricity market in these models, due to the inherently immediate nature of the latter. There are works that deal with a twenty-four-hour time horizon, discussing "daily production" (Xie et al. 2016; Yuan et al. 2008b; Wei and Hsu 2008; Li et al. 2022). Among all these, the largest and most common division of the time horizon found in the literature is into twenty-four one-hour intervals, with certain exceptions that work with longer intervals (Chen et al. 2013).

This maximum one-hour discretization in the literature fails to account for certain phenomena typical in many of the real multireservoir hydropower systems: **delays**. Although some literature addresses delays between cascaded reservoirs (Xie et al. 2016; Yuan et al. 2008a; Fu et al. 2011; Yuan and Yuan 2010), shorter time intervals could introduce delays in energy generation from when the water leaves the reservoir until it starts generating electricity, which are not currently modeled in the existance literature. These delays occur because the water must traverse a channel before arriving at the power groups. The channels of some hydropower systems are kilometres long and water may take more than an hour to traverse them. This delay should not be underestimated as it complicates the management of hourly energy price variations (Souza and Diniz 2012; Belsnes et al. 2016).

In addition to the occurrence of delays, the typical scheduling aggregation of hours (Zhang et al. 2013), days (Fang and Popole 2020), or weeks (Yin et al. 2022) found in the existing literature excludes common phenomena that are invisible in this aggregation. However, these phenomena must be addressed when managing such a system over shorter periods spanning minutes. These **additional short-term phenomena** include:

- Frequent operations of the channel gate that regulates the outflow of the reservoir. These operations, which involve heavy mechanisms, result in energy costs and wear. Estimating these costs accurately is challenging and, consequently, the frequency of these operations should be controlled.
- Excessively frequent gate operations may also disrupt the steady flow of water exiting the reservoir, making it difficult to estimate the flow rate for electricity generation. Moreover, this irregular flow behavior can cause disturbances leading to fatigue and other undesirable effects in the system's channels. This is another reason to keep the frequency of gate operations as low as possible.
- The possibility of having abrupt closures of the channel gate. These can lead to sudden increases in pressure, potentially causing water hammer phenomena (Miao et al. 2021). This poses a risk of channel rupture and should be avoided as much as possible.
- The consideration of the power groups in the hydroelectric plant. This introduces phenomena into the model such as the cost of starting power groups and restricted zones in turbine efficiency, which are scarcely addressed in the literature (Borghetti et al. 2008). These restricted zones are parts of the electric power curve that should be avoided. They should be avoided because they are adjacent to the start-up zones of power groups. This fact adds uncertainty to the produced power curve, complicating its prediction and not clearly defining how many power groups need to operate in these zones. Moreover, these areas should also be avoided from an efficiency standpoint, as this poor prediction can lead to the same electric power output for different values of turbinated flow.

Lastly, concerning the objective function, there is a wide variety of objective functions to consider in the literature. Some very common ones include maximizing energy production (Zarghami 2018; Yoo 2009) or minimize operating cost (Hota et al. 2009; Rabêlo et al. 2012), as well as more specific ones like minimizing deviation from demand (Zhang et al. 2013) or minimizing deviation from the maximum production level (Moeini and Babaei 2017). However, few works in the literature aim to maximize **economic income** (Belsnes et al. 2016), despite being, in the real world, one of the main objectives of the managers of this type of systems. This objective, besides being uncommon, introduces the particular electricity market into the model. This market is characterized by the occurrence of peak and off-peak hours and a price that varies by the hour, or even by the quarter-hour, according to the new European Regulation (2017/2195) (Commission 2017), which mandates price matching in the wholesale electricity market every 15 min. This fact necessitates that any model aiming for economic profit must incorporate a discretization that captures the behavior of this market.

To the best of the authors' knowledge, this work represents the first in which the HROO problem is presented with a temporal discretization that allows the introduction of all the previously mentioned phenomena, presenting a new problem in the literature: **the Hydropower Reservoirs Intraday Economic Optimization (HRIEO) problem**.

To solve this new problem, we pay attention to the different techniques that have been used to solve the HROO problem, observing a wide variety of alternatives. These include mathematical programming, dynamic programming, metaheuristic algorithms, and reinforcement learning:

- **Mathematical programming**, including Linear Programming (Feng et al. 2017a), Mixed-Integer Linear Programming (Rodriguez et al. 2018), and Non-Linear Programming (Zambon et al. 2012). Linear Programming is effective in achieving globally optimal solutions but often simplifies nonlinear aspects in reservoir management, leading to less precise outcomes and poor performance of the reservoir system (Hossain and El-Shafie 2013). To address these limitations, Mixed-Integer Linear Programming is used, which can handle the non-convex and non-linear traits primarily introduced by the hydroelectric power production function. However, Non-Linear Programming directly incorporates complex nonlinear characteristics through polynomial functions, although it may fall into local optima.
- **Dynamic Programming**: in the field of hydroelectric energy scheduling, Dynamic Programming has been successfully applied (Zhao et al. 2012). Nevertheless, its application is challenged by the notorious "curse of dimensionality", which becomes particularly problematic in extensive reservoir systems, so common in real systems (Ming et al. 2015).
- **Metaheuristic algorithms**: in the domain of hydroelectric resource optimization (HROO), algorithms such as Genetic Algorithms (GA) (Feng et al. 2017b), Particle Swarm Optimization (PSO) (Mandal et al. 2008), and its variants like MOPSO (Niu et al. 2018) and HQPSO (Niu et al. 2020), have been extensively utilized. These metaheuristic approaches are adept at handling the nonlinear and non-convex characteristics inherent in HROO challenges. However, the introduction of some elements of a random nature and the requirement for tuning of parameters inherent in these algorithms often result in unstable outcomes (Kumar and Yadav 2022), thus limiting their practical applicability in real-world scenarios.
- **Reinforcement Learning**: given its effectiveness in solving combinatorial problems such as scheduling, it is not surprising that Reinforcement Learning (RL) and Deep Reinforcement Learning (DRL) have been widely applied to the HROO problem (Xu et al. 2021; Matheussen et al. 2019). However, these approaches to the HROO problem encounter typical challenges associated with RL and DRL: convergence difficulties, obstacles in defining rewards and states, huge action spaces, among others.

Given the complex nature of the HROO problem and the various techniques previously applied, we chose to use **Mixed-Integer Linear Programming (MILP)** and

Particle Swarm Optimization (PSO) for our approach. Since the HRIEO problem is a novel problem in the literature, we found it beneficial to attempt solving it using both an exact and an approximate method in parallel to compare their performance on the same problem, a comparison not found in the literature for similar problems.

First, for the exact approach, MILP formulation provides a structured approach to approximate the non-convex and nonlinear characteristics of the hydroelectric power production function through linearization techniques, making it possible to model all the features of the HRIEO problem.

Second, for the heuristic approach, PSO was selected for several reasons. PSO has been effectively employed in similar problems, demonstrating its suitability for HROO challenges. Its straightforward encoding of solutions and the conversion of a particle into a flow proposal made it particularly appealing. Additionally, the method of modifying particle positions suggested that PSO would perform well in our context, a notion confirmed by our results. PSO required relatively low development efforts while promising good outcomes, making it a practical and efficient choice. Our exploration of this method yielded successful results.

In conclusion, given the unique characteristics of the HRIEO problem and the historical context of the techniques used for HROO problems, our approach to presenting and solving this new problem is justified by two key contributions. First, the HRIEO problem introduces short-term phenomena complexities such as delays in energy generation, conditions to avoid during the operation of the reservoir system, or the intricate management of economic objectives in a highly dynamic market, which are not adequately addressed by existing models. These unique features necessitate a novel problem definition to better reflect real-world operational challenges in hydroelectric reservoir management. Second, our choice of MILP and PSO is informed by their proven effectiveness in similar contexts. MILP's ability to manage non-convex and nonlinear traits ensures precise modeling of the HRIEO problem, while PSO's flexibility and efficiency offer a robust heuristic solution. By leveraging both techniques, we aim to provide a comprehensive comparison and robust solution framework, addressing the limitations of previous studies and advancing the field of hydroelectric reservoir optimization.

3 Problem statement

A multi-reservoir hydropower system may be divided into several subsystems, each containing a reservoir, a dam, a channel, and a power group, as depicted in Fig. 1. The dam allows storing water in the reservoir, which is released when the channel

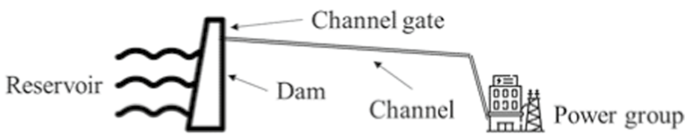

Fig. 1 Elements of each hydropower subsystem

gate is opened. The opening degree of the gate determines the volume of water per second (flow) that enters the channel. After some time, this water will reach the power group, generate energy, and arrive at the next reservoir, if any.

The Hydropower Reservoirs Intraday Economic Optimization (HRIEO) problem aims to determine the optimal outflow from each reservoir at each time interval throughout a full day. The objective is to maximize the income generated for the day while respecting the operational constraints specific to HRIEO.

For simplicity in the model without losing generality, we make the following assumptions:

- The water turbined at one hydropower subsystem is immediately available downstream at the next subsystem's dam without any delay.
- The turbined flow in the power group of each subsystem is calculated as a moving average of the past outflows from the subsystem's dam. The numbers of periods of this delay were defined for each subsystem based on the physical characteristics of its channel.
- The generated electric power is calculated based on the turbined flow at each moment. The relationship between these two variables were defined by a piecewise linear function. This function elucidates how the utilization of the various power groups impacts hydropower generation.
- The height (or volume) of water in each reservoir determines the maximum flow allowed through its channel based on Bernoulli's principle (Sánchez-Sánchez et al. 2013), which governs the phenomenon by which the maximum velocity at which water can exit the channel varies with the reservoir's height (or volume). This relationship was established using a piecewise linear function.

Solving the HRIEO problem requires having a forecast for two input information regarding the day being solved:

- The price of energy for every hour, or for every quarter-hour (Commission 2017), of the day.
- The amount of water that will enter each reservoir from the river or rain at each point in time. This flow of water mainly feeds the reservoir of the first subsystem in the cascade. The main source of water of the second, third, and subsequent reservoirs will generally not be the river or rain, but the water turbined in the previous subsystem.

Time is discretized in intervals or periods of fixed length. In each of these periods, the value of the outflow is decided for every reservoir, and the outflow is assumed to be constant throughout each interval.

The income obtained in each period is calculated by multiplying the price of energy at the corresponding hour and the generated power in the period. The power generated in the period is proportional to the water passing through the power group, which is called 'turbined flow.'

At the end of every period, the volume of each reservoir is updated with a simple volume balance, considering the flow that enters the reservoir and the flow that exits it. The flow entering the reservoir comes from the river/rain or the preceding subsystem, while the flow exiting the reservoir is the problem's decision variable.

As stated above, a solution to the problem contains the outflow decided for every reservoir and for every period. The objective function used to evaluate this solution is the income from the generated power, subtracting operational costs.

Finally, in our HRIEO problem, the short-term phenomena discussed in the previous section are considered. These affect the calculation of the objective function and impose additional constraints on the outflows of the solution:

- **Channel delays.** To consider channel delays, the water passing through the power group is not equal to the water released during the same period. Instead, the turbined flow is calculated using the flows that exited the reservoir in the past. The number of periods comprised by this delay depends on the the length of the channel.
- **Power groups.** The generated power depends on the turbined flow according to a non-linear function characteristic of each power group. Moreover, different amounts of turbined flow require varying numbers of active turbines, and activating an additional turbine to process a higher turbined flow incurs a 'startup cost'. Finally, some values of turbined flow require an uncertain number of turbines and have limited power efficiency, so they incur a 'limit zone' penalty.
- **Frequent gate operations.** To prevent excessively frequent operations of the channel gate, we forbid sudden changes in the direction of change of the outflow from each reservoir. If the outflow has been increased, it cannot start decreasing unless it remains constant for a number of intervals, and vice versa.
- **Water hammer.** To prevent abrupt closures of the gate and, therefore, water hammer, the operations of the channel gate will be limited from one decision to the next. This limit will be set as a fraction of the maximum outflow.

4 MILP formulation

The exact method we selected, as previously explained and justified in earlier sections, consists of a MILP formulation solved using the branch and bound algorithm provided by the Gurobi commercial solver.

In the following tables, the sets (Table 1), parameters (Table 2), and variables (Table 3) required for the MILP formulation of the HRIEO problem are presented.

With the aforementioned Sets, Parameters, and Constraints already presented, we can construct the MILP formulation, as detailed in Table 4.

The objective function is formulated in (1), aiming to maximize the economic profit, considering penalties for limit zones and power group startups.

Constraints (2) and (3) refer to the volume balance calculated for the reservoir in the first time interval and the subsequent time intervals, respectively. Constraint (4) defines the inflow into the first reservoir as the sum of the inflow into the first reservoir and the unregulated inflow into it, while constraint (5) defines the inflow into reservoir i, different from the first, as the sum of the turbine discharge in the predecessor reservoir $i-1$ and the unregulated inflow into reservoir i. Constraint (6)

Table 1 Sets for the MILP formulation

Name	Description	Index
I	Set of reservoirs	i
T	Set of time slots to consider	t
L_i	Set of relevant lags (delays) for reservoir i	l
PCW_i^{PQ}	Set of different linear segments in the power - turbined flow function for reservoir i	a
BKP_i^{PQ}	Set of breakpoints for each linear segment in the power - turbined flow function for reservoir i	b
PCW_i^{VQ}	Set of different linear segments in the volume - maximum flow function for reservoir i	c
BKP_i^{VQ}	Set of breakpoints for each linear segment in the volume - maximum flow function for reservoir i	d
PG_i	Set of different power groups for reservoir i	p

calculates the turbine discharge in the power station of reservoir i as the arithmetic mean of the outflow from the reservoir in the relevant previous lags.

Within the problem, there are two intricately complex aspects that necessitate the use of piecewise linear functions, giving rise to what is commonly referred to as the Piecewise Linear Function problem in our model: the curve that links generated power to the turbined flow, and the curve that associates the maximum flow allowed by the channel with the reservoir volume at any given moment. On one hand, the first of the aforementioned curves is modeled using the constraints outlined from (7) to (11). On the other hand, the second of these curves is represented by the constraints from (12) to (16). To model both piecewise linear functions, convex combinations were employed among the coordinates of the different breakpoints for each of these curves.

In constraint (17), the variation in channel flow is calculated as the difference between the channel flow at time t and the previous time $t-1$, a variation necessary to define the behavior previously described to constraint the channel gate operations in constraints from (18) to (22), where the system will be required to keep the gate stationary for a specified period before changing the direction of the outgoing flow through the gate. Flow variation is also used in constraints (23) and (24) to prevent water hammer by avoiding large changes in outflow from one decision to the next.

Constraints (25) and (26) require maintaining the reservoir volume within its upper and lower bounds.

Constraints (27) and (28) limit the outflow from the reservoir that can flow through the channel. The first constraint is based on the channel section, and the second constraint is based on the reservoir volume, as described in this document.

In constraint (29), the total benefit due to the total power generated in each reservoir i is calculated as the sum, for each reservoir, of all the profits generated in each time interval, taking into account that power is measured in MW, and money is in €/MWh.

Finally, the constraints associated with penalties for limit zones and power group startups are collected. In constraint (30), for each reservoir, all time intervals in which

Table 2 Parameters for the MILP formulation

Name and Index(es)	Description
D	Duration of each time slot [s]
$D1$	Slot used for volume target comparison
QNR_{it}	Unregulated inflow into reservoir i in time slot t [m^3/s]
$Q0_t$	Inflow into the first reservoir in time slot t [m^3/s]
$QMAX_i$	Maximum flow that the channel of reservoir i is capable of transporting [m^3/s]
$QTBP_{ib}$	Turbined flow at the breakpoints b of the piecewise linear power-turbined flow function for reservoir i [m^3/s]
$POWBP_{ib}$	Power at the breakpoints b of the piecewise linear power-turbined flow function for reservoir i [MWh]
$QMAXBP_{id}$	Maximum flow at the breakpoints d of the piecewise linear volume-maximum flow function for reservoir i [m^3/s]
$VOLBP_{id}$	Volume at the breakpoints d of the piecewise linear volume-maximum flow function for reservoir i [m^3]
$V0_i$	Initial volume of reservoir i [m^3]
$VMAX_i$	Maximum volume of reservoir i [m^3]
$VMIN_i$	Minimum volume of reservoir i [m^3]
K	Minimum number of slots in which the flow rate through the channel must remain constant before changing the direction of the variation of this same outflow flow rate through the channel (for the gate operations constraint)
R	Percentage of the maximum allowed flow $QMAX_i$ that can change the outflow from one decision to the next (for the water hammer constraint)
P_t	Electricity price in time slot t [€/MWh]
P^{ZL}	Penalty for time slots in the limit zone [€/slots]
P^{SU}	Penalty for power group start-up [€/start-ups]
ZL_i^{PQ}	Segments a considered limit zones in the piecewise linear power-turbined flow function for reservoir i
PC_{ip}^{PG}	Segments a of the piecewise linear power-turbined flow function belonging to each power group p of reservoir i
PC_{ip}^{SUP}	Segments a of the power-turbined flow function for reservoir i that belong to a higher power group than p

segments considered as limit zones have been reached are calculated. Constraints (31) and (32) are used to calculate all power group startups that have occurred. Then, with constraint (33), all these power group startups are summed for each reservoir.

Table 3 Variables for the MILP formulation

Name and Index(es)	Description
vol_{it}	Volume of reservoir i in time slot t [m^3]
q_{it}^{IN}	Inflow into reservoir i in time slot t [m^3/s]
q_{it}^{OUT}	Outflow through the channel of reservoir i in time slot t [m^3/s]
q_{it}^{TB}	Turbined flow in the power plant of reservoir i in time slot t [m^3/s]
q_{it}^{MAX}	Maximum flow allowed by the channel of reservoir i in time slot t [m^3/s]
q_{it}^{CH}	Change in channel flow for reservoir i between time slots $t-1$ and t [m^3/s]
pow_{it}	Electric power generated in the power plant of reservoir i in time slot t [MW]
ben_i^{POW}	Total benefit due to the total electric energy generated in the power plant of reservoir i [€]
x_{it}^+	= 1 if there is a positive change in the outflow flow rate in reservoir i in time slot t = 0 otherwise
x_{it}^-	= 1 if there is a negative change in the outflow flow rate in reservoir i in time slot t = 0 otherwise
w_{ita}^{PQ}	= 1 if the turbined flow value in the PQ curve is in segment a for reservoir i in time slot t = 0 otherwise
z_{itb}^{PQ}	Proportion of each breakpoint b in the calculation of power and turbined flow in the piecewise PQ function in time slot t for reservoir i
w_{itc}^{VQ}	= 1 if the maximum flow value in the VQ curve is in segment c for reservoir i in time slot t = 0 otherwise
z_{itd}^{VQ}	Proportion of each breakpoint d in the calculation of volume and maximum flow in the piecewise VQ function in time slot t for reservoir i
$zltot_i$	Total number of time slots in limit zones for reservoir i
$pwch_{itp}$	= 1 if a higher power group than group p is started in reservoir i during time slot t = 0 otherwise
$pwchtot_i$	Total number of power group startups in reservoir i [m^3]

5 Particle swarm optimization

5.1 Introduction

Particle Swarm Optimization or PSO (Kennedy and Eberhart 1995) requires encoding any solution as an array of numbers. The algorithm starts with many

Table 4 MILP formulation

$\max\ z = \sum_i [ben_i^{POW} - P^{ZL} zltot_i - P^{SU} pwchtot_i]$		(1)
$vol_{i1} \leq V0_i + D(q_{i1}^{IN} - q_{i1}^{OUT})$	$\forall i \in I$	(2)
$vol_{it} \leq vol_{it-1} + D(q_{it}^{IN} - q_{it}^{OUT})$	$\forall i \in I, \forall t \in T, t \neq 1$	(3)
$q_{1t}^{IN} = Q0_t + QNR_{1t}$	$\forall t \in T$	(4)
$q_{it}^{IN} = q_{i-1t}^{TB} + QNR_{it}$	$\forall i \in I, i \neq 1, \forall t \in T$	(5)
$q_{it}^{TB} = \sum_l \frac{q_{it-l}^{OUT}}{\lvert L_i \rvert}$	$\forall i \in I, \forall t \in T$	(6)
$pow_{it} = \sum_b z_{itb}^{PQ} POWBP_{ib}$	$\forall i \in I, \forall t \in T$	(7)
$q_{it}^{TB} = \sum_b z_{itb}^{PQ} QTBP_{ib}$	$\forall i \in I, \forall t \in T$	(8)
$z_{itb}^{PQ} \leq \sum_{a=b-1}^{b} w_{ita}^{PQ}$	$\forall b \in BKP_i^{PQ}, \forall i \in I, \forall t \in T$	(9)
$\sum_b z_{itb}^{PQ} = 1$	$\forall i \in I, \forall t \in T$	(10)
$\sum_a w_{ita}^{PQ} = 1$	$\forall i \in I, \forall t \in T$	(11)
$q_{it}^{MAX} = \sum_d z_{itd}^{VQ} QMAXBP_{id}$	$\forall i \in I, \forall t \in T$	(12)
$vol_{it} = \sum_d z_{itd}^{VQ} VOLBP_{id}$	$\forall i \in I, \forall t \in T$	(13)
$z_{itd}^{VQ} \leq \sum_{c=d-1}^{d} w_{itc}^{VQ}$	$\forall d \in BKP_i^{VQ}, \forall i \in I, \forall t \in T$	(14)
$\sum_d z_{itd}^{VQ} = 1$	$\forall i \in I, \forall t \in T$	(15)
$\sum_c w_{itc}^{VQ} = 1$	$\forall i \in I, \forall t \in T$	(16)
$q_{it}^{CH} = q_{it}^{OUT} - q_{it-1}^{OUT}$	$\forall i \in I, \forall t \in T$	(17)
$q_{it}^{CH} \leq x_{it}^{+} QMAX_i$	$\forall i \in I, \forall t \in T$	(18)
$q_{it}^{CH} \geq -x_{it}^{-} QMAX_i$	$\forall i \in I, \forall t \in T$	(19)
$x_{it}^{+} + x_{it}^{-} \leq 1$	$\forall i \in I, \forall t \in T$	(20)
$x_{it}^{+} + x_{it-k}^{-} \leq 1$	$\forall i \in I, \forall t \in T, k = \{1, 2, \ldots, K\}$	(21)
$x_{it}^{-} + x_{it-k}^{+} \leq 1$	$\forall i \in I, \forall t \in T, k = \{1, 2, \ldots, K\}$	(22)
$q_{it}^{CH} \leq QMAX_i R$	$\forall i \in I, \forall t \in T$	(23)
$q_{it}^{CH} \geq -QMAX_i R$	$\forall i \in I, \forall t \in T$	(24)
$vol_{it} \leq VMAX_i$	$\forall i \in I, \forall t \in T$	(25)
$vol_{it} \geq VMIN_i$	$\forall i \in I, \forall t \in T$	(26)
$q_{it}^{OUT} \leq QMAX_i$	$\forall i \in I, \forall t \in T$	(27)
$q_{it}^{OUT} \leq q_{it}^{MAX}$	$\forall i \in I, \forall t \in T$	(28)
$ben_i^{POW} = \sum_t pow_{it} P_t \frac{D}{3600}$	$\forall i \in I$	(29)
$zltot_i = \sum_t \sum_{a \in ZL_i^{PQ}} w_{ita}^{PQ}$	$\forall i \in I$	(30)
$\sum_{a \in PC_{ip}^{PG}} w_{it-1a} + \sum_{a \in PC_{ip}^{SUP}} w_{ita} - 1 \leq pwch_{itp}$	$\forall i \in I, \forall t \in T, \forall p \in PG_i$	(31)
$\sum_{a \in PC_{ip}^{PG}} w_{it-1a} + \sum_{a \in PC_{ip}^{SUP}} w_{ita} \geq 2pwch_{itp}$	$\forall i \in I, \forall t \in T, \forall p \in PG_i$	(32)
$pwchtot_i = \sum_t \sum_p pwch_{itp}$	$\forall i \in I$	(33)

random solutions, each of which is considered the “position” x^p of a particle p. Each particle also has a random initial “velocity”, v^p.

After every iteration k of the algorithm, the velocity and position of each particle are updated according to the Eq. 1, with which the particle is accelerated towards the best positions.

$$v_{k+1}^p = wv_{k+1}^p + c_1 r_1 (b_k^p - x_k^p) + c_2 r_2 (B_k - x_k^p) \text{ and } x_{k+1}^p = x_k^p + v_{k+1}^p \qquad (1)$$

In these equations, w, c_1, c_2 are parameters of the algorithm (usually set between 0 and 3), r_1, r_2 are random numbers between 0 and 1, b_i^p is the best position found by the particle p so far, and B_i is the best position found by the whole swarm so far.

The parameters w, c_1 and c_2 are named *inertia weight*, *cognitive coefficient*, and *social coefficient*, respectively. Their ideal values for the HRIEO problem have been found through parameter tuning.

To determine b_k^p and B_k after every iteration k, the solution represented by the position of each particle must be evaluated. To be specific, the income obtained with the outflows represented by each solution must be given to a evaluation function to calculate the income.

The following sections describe the evaluation function used to compute the income of every solution, the approaches used to transform solutions into outflows, and several adjustments made to the algorithm to improve its results.

5.2 Evaluation function

The evaluation function estimates the objective function value of any given solution. It is used in the PSO algorithm to evaluate the generated solutions and improve them.

The evaluation function has the following input and output variables, alongside some constant parameters:

- **Input variables:** the decided outflows (given by the PSO) and the inflow and price of energy throughout the day.
- **Output variable:** the total income obtained with the given solution.
- **Constant parameters:** the constants of the hydropower system, including minimum and maximum volumes, flows, and lags.

To compare the MILP model with the PSO, the evaluation function was built using the MILP’s equations (see Table 4). As such, the income estimated by the evaluation function for a given solution is the same as the MILP’s objective function value for that solution. In this way, both methods use the same assumptions and simplifications to solve the problem, so the performance of the underlying algorithms can be compared.

5.3 Solution encoding

The position of each particle needs to represent a solution to the problem, encoding the outflow decided for each dam and for every time interval. Thus, if the basin has $|I|$ dams and we want to do the planning for $|T|$ time intervals, the position x_k^p will be an array of size $|T| \times |I|$.

Let $x_k^p(i,t)$ be the value of the array x_k^p that refers to the interval t and the dam i. This value can directly represent the flow exiting the reservoir i at time t, q_{it}^{OUT} (see Eq. 2). With this solution encoding (which we called "**flows solution encoding**"), the value of $x_k^p(i,t)$ ranges from 0 to the maximum physical flow of the channel i, $QMAX_i$.

$$q_{it}^{OUT} = x_k^p(i,t) \tag{2}$$

Alternatively, $x_k^p(i,t)$ may represent the increase of flow with respect to the previous time interval, as indicated in Eq. 3. With this representation (which we called "**variations solution encoding**"), the values of the position range from -1 to 1.

$$q_{it}^{OUT} = \max\left\{ \min\left\{ q_{it-1}^{OUT} + x_k^p(i,t) \cdot QMAX_i, QMAX_i \right\}, 0 \right\} \tag{3}$$

5.4 Boundary handling

Many problems have constrained solution spaces, where solution values have upper and lower bounds. This is the case of the HRIEO problem, where particles have the bounds $(0, QMAX_i)$ when using the flows solution encoding or $(-1, 1)$ when using the variations solution encoding.

Using a boundary handling strategy is necessary to repair particles that leave the feasible solution space. Choosing the right strategy greatly impacts the quality of the final solutions. This study experimented with the following boundary handling strategies (Helwig 2010):

- **Nearest**: Move the particle to the closest boundary.
- **Random**: Place the particle randomly within the boundaries.
- **Shrink**: Reduce the particle's velocity so it lands on the boundary.
- **Reflective**: Reflect the particle's position back inside the boundary.
- **Intermediate**: Adjust the particle's position to the midpoint between its location and the boundary, only for the axes that exceed the limits.
- **Periodic**: Reposition the particle within the boundaries using the modulo function, creating a repeating solution space.

The optimal boundary handler for the HRIEO problem was determined through parameter tuning.

5.5 Topology

In the PSO algorithm, each particle accelerates towards the best position discovered by the entire swarm, known as a **global topology** or **star topology**.

Alternatively, particles may connect to only a subset of particles, called a **local topology**, which can perform better on some problems. This work experimented with these local topologies (Bratton and Kennedy 2007):

- **Ring topology**: Each particle is linked to its k closest neighbors (Bratton and Kennedy 2007).
- **Pyramid topology**: Particles are interconnected through n-dimensional simplices, where n is the dimension of the solution space (Lane et al. 2008).
- **Random topology**: Each particle is connected to k randomly selected particles (Ni and Deng 2013).

In Ring and Random topologies, k is the number of neighbors. The Ring topology also includes the *Minkowski p-norm* parameter for measuring distance: $p = 1$ for Manhattan distance and $p = 2$ for Euclidean distance.

The best topology for the HRIEO problem was found through parameter tuning.

5.6 Initialization with random biased optimization

The basic PSO algorithm generates random solutions for the first population of particles. In this section, we explain an alternative way to initialize the population of particles which improves the PSO's performance. This alternative initialization method uses Random Biased Optimization, RBO, and a heuristic method specifically designed for the HROO problem.

The heuristic method works by sorting the periods in decreasing order of energy price. Then, following this order, the highest possible outflows are assigned to each period. This means that the period with highest price is assigned the maximum flow of the channels, and the period with the second highest price gets assigned the maximum possible flow (depending on the volume available after the first flow assignment). Since the volume available in the reservoir also depends on the turbined flow of the previous dam, all the outflows of the first dam are assigned before applying the heuristic to the second dam.

The algorithm followed when applying the heuristic to one particular dam is detailed in Algorithm 1. It takes the dam's inflows as input, which equal the flows from the river or the turbined flows from the previous dam.

Note the algorithm shown here is simplified; additional steps must be added to account for limit zones, the water hammer constraint, the maximum outflow allowed depending on the volume, the time delays between outflow and energy generation, and the presence of subsequent dams.

Algorithm 1 Heuristic method applied to dam i (simplified)

Input: set of dams I; set of periods T; inflows $q_{it}^{IN}, \forall t \in T$; energy price $P_t, \forall t \in T$; initial volume $V0_i$; minimum volume $VMIN_i$; maximum volume $VMAX_i$; maximum outflow $QMAX_i$; period length D
Output: outflows $q_{it}^{OUT}, \forall t \in T$
Initialize outflows, $q_{it}^{OUT} \leftarrow 0, \forall t \in T$
Initialize volumes, $vol_{it} \leftarrow \min(vol_{it-1} + D(q_{it}^{IN} - q_{it}^{OUT}), VMAX_i), \forall t \in T$
Initialize set of periods, $S \leftarrow T$
while $|S| > 0$ **do**
  Get period in S with the highest energy price, $\hat{t} \leftarrow \arg\max_{t \in S} P_t$
  Remove period $\hat{t}$ from S, $S \leftarrow S - \{\hat{t}\}$
  Find the first period $\bar{t} > \hat{t}$ such that $vol_{i\bar{t}} = VMAX_i$
  Calculate the volume that can safely be removed in $\hat{t}$ without making future periods reach minimum volume, $safevol \leftarrow \min\{vol_{it} : t = \hat{t}, \ldots, \bar{t}\} - VMIN_i$
  Assign the equivalent outflow in $\hat{t}$, $q_{i\hat{t}}^{OUT} \leftarrow \min(safevol/D, QMAX_i)$
  Recalculate volumes, $vol_{it} \leftarrow \min(vol_{it-1} + D(q_{it}^{IN} - q_{it}^{OUT}), VMAX_i), \forall t \in T$
end while

To generate multiple alternative (pseudo-random) initial solutions with this heuristic, a Random Biased Optimization method (RBO) is applied to the heuristic.

The RBO method introduces randomness into the heuristic in two ways. Firstly, it employs random biased period selection, where the heuristic may choose any period instead of always selecting the one with the highest energy price. However, periods with higher prices have a higher probability of being chosen. The probabilities are determined by Eq. 4, where $|S|$ represents the number of remaining periods, p_k denotes the probability of selecting the period with the k-th highest energy price, and $0 < r < 1$ is a parameter indicating the *common ratio* of the probability sequence (Juan et al. 2009).

$$p_k = r \cdot p_{k-1}, \quad p_1 = \frac{r-1}{r^{|S|}-1} \tag{4}$$

Secondly, the RBO method incorporates random biased outflow assignment, where the heuristic does not always assign the maximum outflow based on the available volume. Instead, it can assign any outflow value, with higher values being more likely than lower ones. This is achieved by multiplying the maximum outflow by a random number generated according to Eq. 5, where u is a random number between 0 and 1, and $n > 1$ is a parameter indicating the *bias* towards higher outflow values.

$$p = u^{1/n} \tag{5}$$

These modifications introduce a level of randomness into the algorithm, enabling the computation of multiple alternative initial solutions for the PSO algorithm.

Table 5 Characteristics of each subsystem

Variable	Dam 1	Dam 2
Minimum volume	$VMIN_1 = 34{,}045\ m^3$	$VMIN_2 = 17{,}117\ m^3$
Maximum volume	$VMAX_1 = 70{,}882\ m^3$	$VMAX_2 = 58{,}343\ m^3$
Maximum outflow	$QMAX_1 = 14.15\ m^3\ s^1$	$QMAX_2 = 11.27\ m^3\ s^1$
Lags	$L_1 = \{1\}$	$L_2 = \{3, 4, 5\}$
Maximum power	4.6 MW	8.48 MW
Turbines installed	2	3

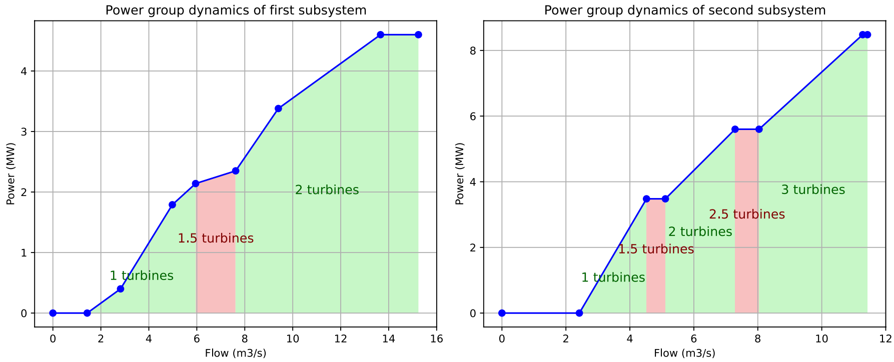

Fig. 2 The characteristic power-turbine flow curve of each power group. The function maps turbine flow to generated power and required number of turbines. When the number of turbines is a decimal number, the power group is in a limit zone

6 Computational results

6.1 Available data

To conduct our experiments, we used real data from a hydropower system consisting of two reservoirs referred to as "dam1" and "dam2." The specific properties of these reservoirs can be found in Table 5.

Additionally, the power groups within the system are illustrated in Fig. 2. This figure depicts the characteristic non-linear function of each power group.

6.2 Scenarios and instances

The implemented methods were compared in various *scenarios*. Each scenario involves different constraints related to dam operations and costs specific to the HRIEO problem. The following scenarios were considered:

- **Gate operations constraint:** This scenario requires satisfying the constraint that, once the outflow is increased, it must remain constant for $K = 2$ periods (half an

hour) before it can be decreased (and vice versa). In addition, the startup cost and limit zone penalty are both 50€.
- **Water hammer constraint:** Under this scenario, changes in outflow from one period to the next cannot exceed 20% of the channel's maximum outflow ($R = 0.2$). Startup costs and limit zone penalties are also penalized with 50€.
- **No operational costs:** This scenario represents the simplest case where the gate operations and water hammer constraints are ignored. Startup costs and limit zone penalties are negligible, and the income directly corresponds to the revenue from generated energy.

The MILP always satisfies the gate operations and water hammer constraints when relevant, since they are part of its formulation. However, the PSO may not always satisfy these constraints. To minimize the violations, the following adjustments were made for the PSO algorithm:

- In the gate operations constraint scenario, the outflows of each solution of the PSO were 'flattened' using Eq. 6, where $q_{i,t}^{CH} = q_{i,t}^{OUT} - q_{i,t-1}^{OUT}$. This forces the decided outflows to satisfy the constraint, but the real outflows, limited by volume, may not satisfy it.

$$q_{i,t}^{OUT}{}' = \begin{cases} q_{i,t}^{OUT} & \text{if } q_{i,t}^{CH} \cdot q_{i,t-1}^{CH} \cdot \ldots \cdot q_{i,t-K}^{CH} \geq 0 \\ q_{i,t-1}^{OUT} & \text{otherwise} \end{cases} \tag{6}$$

- In the water hammer constraint scenario, the variations solution encoding was used and the particles were bounded between $-R$ and $+R$ instead of the range -1 to $+1$.

In addition to these three alternative operational scenarios, the study considered two different hydropower system sizes:

- **A small station with two dams**.
- **A big station with six dams:** data for the additional subsystems was generated by duplicating data from the original two. Specifically, the third and fourth subsystems were copies of the second subsystem, while the fifth and sixth were copies of the first.

Each *instance* corresponds to one day of the operation of the system, for which the inflows, initial volumes, and energy prices are known.

The methods were compared using their performance on eleven specific problem instances, carefully selected to be representative. These instances include the driest day of the dataset, the rainiest day, and nine intermediate cases.

6.3 Parameter tuning

The parameters of the PSO, including the choice for the boundary handler and topology, were tuned for each scenario using the Optuna Pyhton library (Akiba et al. 2019).

Tuning involved computing the average income of the PSO with 100 alternative parameters, and then returning the parameters with the best results. To ensure fairness in the comparison between the PSO and MILP, the average income was not computed using the eleven instances of the experimentation. Instead, two other instances were used.

Tuning was done separately for the PSO with random initialization and the PSO with RBO initialization. Then, the best-performing approach on the scenario was selected.

The best parameters found for each scenario by following this process are presented in Table 6.

6.4 Configuration

To solve each of the instances, the MILP model was implemented in Python and solved using the Gurobi commercial solver. The PSO was also implemented in Python with the PySwarms library (Miranda 2018).

The MILP model was solved to a final gap of 1%. The PSO was stopped early if it did not improve the income of the solution for more than 0.5% in 2.5 min.

When applying these methods in a real hydropower system, they would be used to decide the outflows of the next period only. Then, they would be executed again with the updated information, deciding the outflows of the subsequent period, and so on. Consequently, and since each period is 15 min long, the solvers are not given more than 15 min to solve the problem.

Table 6 Best parameters of the PSO for each scenario

Parameter	2 dams, gate operations constraint	2 dams, water hammer constraint	2 dams, no costs	6 dams, gate operations constraint	6 dams, water hammer constraint	6 dams, no costs
Initialization	RBO	Random	RBO	RBO	Random	Random
Number of particles	850	670	760	140	740	680
Inertia weight	0.05	0.08	0.30	0.03	0.07	0.64
Cognitive coefficient	3.84	3.75	3.78	2.30	4.09	1.94
Social coefficient	3.65	0.519	2.08	3.17	4.60	1.13
Bounds handling	Shrink	Shrink	Shrink	Shrink	Shrink	Reflective
Topology	Ring	Ring	Star	Random	Random	Ring
Number of neighbors	605	390	–	45	25	605
Minkowski p-norm	2	2	–	–	–	2
Solution encoding	Variations	Variations	Flows	Variations	Variations	Variations
Initial RBO solutions	49%	–	31%	96%	–	–
RBO common ratio	0.44	–	0.70	0.95	–	–
RBO bias	–	–	4.64	–	–	–

6.5 Benchmark method

The MILP and PSO approaches were compared against a trivial method, called "**Greedy**", that consists of leaving the gates completely open at every period. In other words, the outflow of every dam is always assigned its maximum value. This approach reflects the former strategy of the hydropower station that motivated this study.

6.6 Results and discussion

The comparison of MILP, PSO and Greedy solutions for the described instances across all scenarios is indicated in Table 7. Each value in the table represents the average over the eleven representative instances, with PSO values also averaged across five replications.

The average income for the Greedy method is displayed, and the performance of other methods is indicated by their percentage improvement over Greedy's average income. From this information, we see the MILP is the best method when the station has two dams, but the PSO gives better solutions when the station has six.

This fact is further supported by the average final gap values of the MILP. In cases where there are two dams, these values are small, indicating that the MILP successfully obtains solutions that are close to optimal. However, when there are six dams, the final gap values are considerably higher.

In addition, the table shows the average convergence time of each method in seconds. We see the PSO converges faster than the MILP method in most scenarios.

Lastly, the table gives the percentage of periods with violations of either the gate operations or water hammer constraints. The MILP always satisfies

Table 7 Comparison of the PSO and MILP methods across various scenarios, benchmarked against the trivial Greedy approach

Method	2 dams, gate operations constraint	2 dams, water hammer constraint	2 dams, no costs	6 dams, gate operations constraint	6 dams, water hammer constraint	6 dams, no costs
Greedy (average income)	10517€	10538€	11867€	30618€	30620€	35486€
MILP (average final gap)	3.3%	3.22%	1.09%	43.3%	25.95%	8.45%
MILP (% over Greedy)	+20.9%	+20.7%	+9.7%	−6.7%	+1.3%	+1.5%
PSO (% over Greedy)	+16.1%	+14.1%	+7.5%	+13.3%	+15.5%	+3.5%
MILP (convergence time, s)	592	541	281	775	725	596
PSO (convergence time, s)	305	391	325	408	473	327
MILP (constraint violation)	0%	0%		0%	0%	
PSO (constraint violation)	3.7%	0.8%		2.7%	0.7%	
Greedy (constraint violation)	15%	2.5%		5.6%	1.2%	

the constraints since they are included in its formulation when relevant. The described adaptions for the PSO do not eliminate the constraint violations but make them very infrequent.

6.7 Solution analysis

Figure 3 presents the solution of the MILP for a two-reservoir system under the gate operations constraint. The solution consists of the outflow of every reservoir for every period of the day, shown in green. The figure also shows the price curve of the day (in red) and the predicted volumes (in blue). Values of flow, volume and price are normalized to be between 0 and 1.

This solution highlights the intricacies of solving the HRIEO problem:

- First dam, periods 70–75: The outflow is kept at zero. This increases the reservoir's volume, preparing it to generate more energy during the second price peak at period 85.
- Second dam, periods 50–80: The outflow is kept at 40% despite having enough volume to have a higher value. However, having only 40% of outflow ensures the volume stays at a high level, enabling a substantial outflow during the price peak at period 85.
- First dam, periods 8–30: There is a high outflow even though energy prices are low. This benefits the second reservoir, which uses the released water to generate energy during the first price peak at period 37. This strategy is preferred because the second subsystem has a larger power capacity.

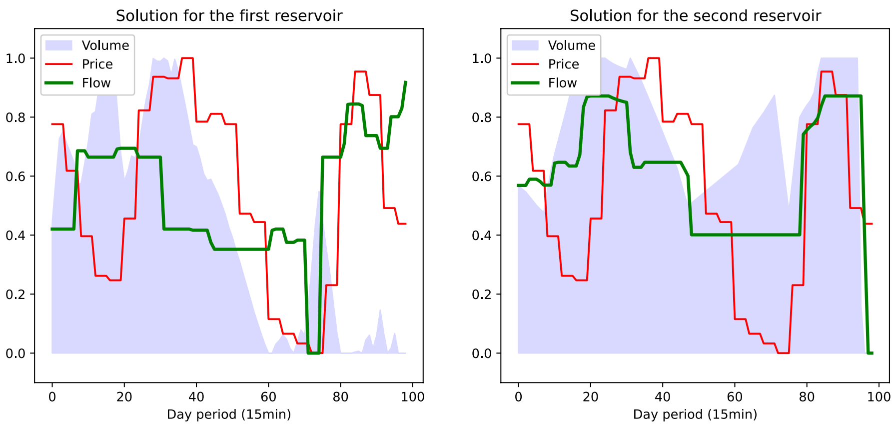

Fig. 3 Solution found by the MILP for 2 dams, gate operations constraint

7 Conclusions

A new HRIEO problem was defined considering the time delays of the channels, the power group startup costs, the gate operations or water hammer constraint, and the objective of maximizing income. Using these assumptions, two formulations were implemented and their performance was compared: the MILP and PSO.

Our results show that, when the hydropower station has two dams, the MILP's solutions are near-optimal and they outperform the PSO solutions. However, when the station has six dams, the PSO scales better and significantly outperforms the MILP. The relative performance of the methods does not depend on the constraint or operational costs considered, but the PSO does not always satisfy the relevant constraints.

Further research can be done to study the uncertainty of the input data (particularly the inflows) as time intervals further in the future are considered.

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