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The Turbulence Cascade in Physical Space

Javier Jiménez, José I. Cardesa and Adrián Lozano-Durán

Abstract Some recent developments on the physical mechanism of turbulence cascades are summarised. It is first shown that the energy cascade in statistically steady isotropic turbulence is local in scale, at least on average, and that temporal variations of the large-scale forcing are transferred to smaller scales as a 'wave' consistent with the classical Kolmogorov model. It is further shown that, when energy-containing structure are individually tracked in band-pass filtered velocity fields, they also behave classically. The correlation of their physical position with larger (or smaller) structures is highest towards the beginning (or end) of their lifetimes. The analysis is then extended to the structures of momentum flux in the logarithmic layer of turbulent channels. Small structures grow and shrink smoothly along their lifetimes, but larger ones change size mostly by splits and mergers involving structures of similar size. For the largest structures, splits predominate, although not overwhelmingly.

1 Introduction

Cascades are required whenever a conserved quantity has to be transferred across a range of scales but, beyond that generic idea, every particular instance of multiscale transport requires a physical implementation that does not have to be the same in all cases. In fact, it is probably always true in high-dimensional systems that cascades include a variety of mechanisms that transfer the conserved quantity in different directions, in such a way that a one-directional transfer across scales is only true as a statistical average. The main problem lies in the traditional idea of scale, which, because of the Fourier uncertainty principle, only takes a definite meaning when averaged over a region of space larger than the scale in question. Based on the general idea that the Navier–Stokes equations are local PDEs in physical, but not in scale space (e.g. Fourier), our group have tried for some time to ascertain whether

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some component of the various transfers in turbulence can be identified locally in physical space, and to clarify their mechanisms. We primarily do this by isolating individual intense structures, and tracking them in space, time and, occasionally, also across scales.

2 The Energy Cascade

The best-known example of a turbulent cascade is the transfer of kinetic energy in equilibrium isotropic turbulence [4]. If energy is fed by some large-scale forcing, it is transferred to the Kolmogorov scale, η, to be dissipated by viscosity, but the manner in which this happens has been the subject of endless discussions. We will first present evidence that the transfer is local in scale, at least on average. For example, Fig. 1a shows that, when the flow is separated into scale bands by filtering, and the forcing is unsteady, the unsteadiness of the sub-grid energy transfer, $\Sigma = -\langle \tau_{ij} \overline{S_{ij}} \rangle$, where τ_{ij} is the subgrid Reynolds stress tensor at the chosen filter width, and $\overline{S}_{ij}$ is the filtered rate of strain, moves towards bands of smaller scale as a 'wave' [1]. The mean $\langle \cdot \rangle$ refers to the time-dependent instantaneous average over the whole computational box. Moreover, the evolution of the transfer rate is self-similar, as posited by Kolmogorov, in the sense that the delay required by the energy to cross an octave of scales centred at r is proportional to the local eddy turnover, $r^{2/3}/\varepsilon^{1/3}$, where ε is the ensemble-averaged energy dissipation rate (Fig. 1b).

The energy cascade in isotropic turbulence is also local in physical space [2]. The band-filtered fields used in [1] can be segmented into individual structures of intense energy, and these structures can be followed in time. In a first approach, the evolution in this five-dimensional space (three spatial dimensions, scale, and time)

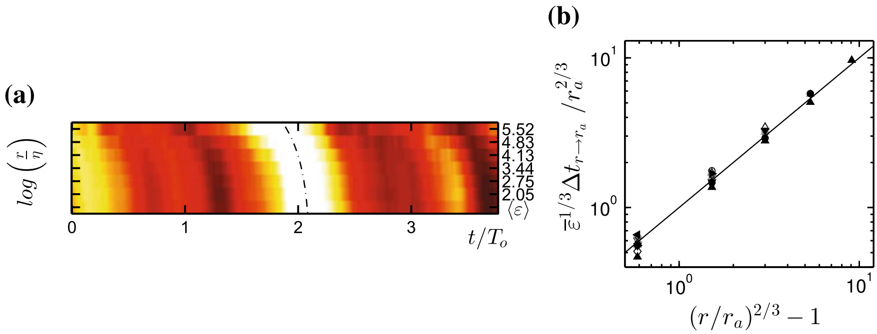

Fig. 1 **a** Time-scale diagram of $\Sigma(r)$, with the filter width r decreasing from top to bottom and the instantaneous mean dissipation $\langle\varepsilon\rangle$ added as the bottom band. Isotropic turbulence at microscale Reynolds number $Re_\lambda = 384$. The dashed-dotted line corresponds to $\varepsilon^{1/3}\Delta t = (250\eta)^{2/3} - r^{2/3}$. **b** Dimensionless average delay between the energy transfer at two filter widths, $r > r_a$. Several homogeneous flows. The slope of the solid line is linear. Reproduced with permission from [1]

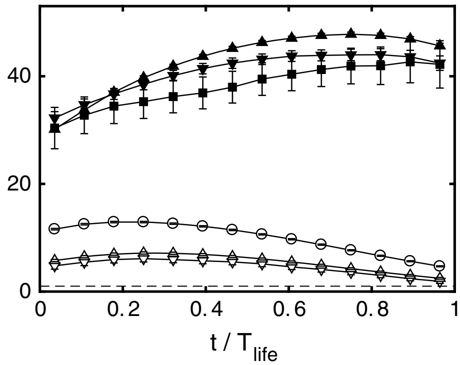

Fig. 2 Intersection ratio of individual structures of size r_A with the set of all structures of size r_B, separated from r_A by a factor of two. Normalised to unity for random sets, and plotted as function of the time within the life of each structure. ∘, $r_A/\eta = 30$; △, 60; ▽, 120; □, 240. Solid symbols are for $r_B = r_A/2$; open ones are $r_B = 2r_A$. Error bars are two standard deviations. Adapted with permission from [2]

is too complex to have allowed tracking each structure individually up to now, but it can be handled statistically. A measure of how related are the location of two flow scales is the volume of the intersection of their intense structures, which has to be compared to the null hypothesis of randomly located point sets with the same overall volume fractions. It is found that structures in energy bands separated by a factor of 2 are more correlated than random, but that those separated by 4 or more are not [2]. This could simply be a sign of spectral leakage in the filter, but the structures are also tracked in time, so that each of them has a lifetime (which, not surprisingly, is proportional to $r^{2/3}$), and the evolution of its correlation can be measured. Figure 2 shows that structures of size r are more strongly correlated with those of size $2r$ at the beginning of their life than at the end. The opposite is true for their correlation with smaller structures of size $r/2$, in strong agreement with a process in which energy passes from larger to smaller structures at the same physical location.

3 Momentum Transfer in Shear Flows

Another conserved quantity whose transfer is important is the flux along the wall-normal (y) direction of streamwise momentum in turbulent shear flows. Traditionally, momentum is considered to be carried by the tangential Reynolds stress, uv, defined as the product of the streamwise and wall-normal velocity fluctuations. The structures (Qs) of particularly intense uv are often treated as important momentum carriers [8], and their organisation is different from that of the energy structures in isotropic turbulence. Only 'large' structures couple strongly enough to the shear of the mean velocity, $S = \partial_y U$, to carry net mean momentum. But the shear in the

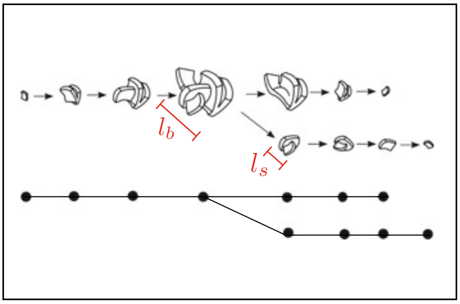

Fig. 3 Sketch of a simple evolution graph for Qs. Time increases from left to right, and the graph includes a split. Structure size is characterised by the diagonal ℓ of its circumscribed parallelepiped

inertial (logarithmic) layer of a wall-bounded flow changes as the inverse of the distance from the wall, and the result is that the inertial momentum-carrying Qs form a self-similar hierarchy of different scales in which most relevant Qs are attached to the wall, in the sense that their size is proportional to their distance from it. The transfer of momentum across this hierarchy is a problem that has to be understood as much as the Kolmogorov energy cascade, because equilibrium has to be maintained across scales, and the result determines the scaling parameters for the overall flow; for example, it defines the friction velocity as the uniform velocity scale in wall-bounded turbulence (and therefore the drag coefficient).

Three-dimensional Qs are studied for turbulent channels in [5], and tracked individually in time in [7]. They are found to grow and decay while they merge and split in complicated temporal graphs that have to be parsed to understand their evolution (Fig. 3). In agreement with their inertial character, their lifetime is proportional to their size, but, somewhat surprisingly given that they are mostly attached to the wall, they are not found to be particularly connected with it. Roughly half of the Qs are born near the wall and move away from it, while the other half do the opposite. In fact, Qs were studied in [3] for homogeneous shear turbulence, which shares with wall-bounded turbulence the role of the shear as a source of momentum transfer and of turbulent energy, but which has no walls. Its Qs differ very little from those in wall-bounded flows, showing that they are a consequence of the shear, rather than of the neighbourhood of the wall. The relevant condition for Qs to participate in momentum transfer is that they should be larger than the local Corrsin scale, $L_c = (S^3/\varepsilon)^{1/2}$. But $L_c \sim y$ in the logarithmic layer of wall-bounded flows, and most Qs larger than L_c are also too large to fit in the flow without hitting the wall. They are therefore attached.

From the point of view of the present paper, the most interesting question is the relevance of merging and splitting in the growth and decay of the momentum-carrying structures. Because Qs are defined by thresholding an intense property (uv), they are born and die as small structures, which at first grow in volume and later shrink. Part of this evolution is a smooth variation with time as a consequence of their intensification and weakening. Figure 4a shows that this accounts for most of the change in volume for Qs smaller than about 100 wall units (defined by the kinematic viscosity and the friction velocity, and denoted by a '+' superscript). Above that threshold, all Qs

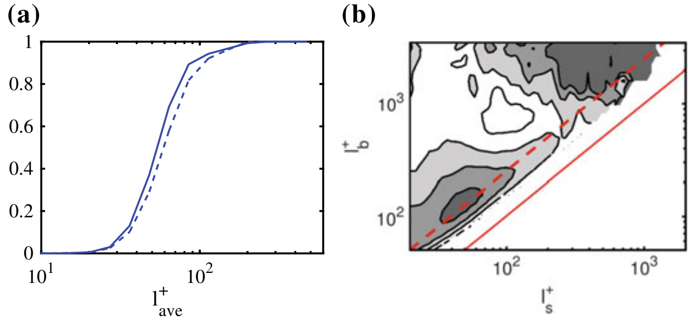

Fig. 4 **a** Fraction of the number of Qs that split (solid line) or merge (dashed) at least once in their lives, as a function of their mean diagonal size averaged over their lifetime. **b** Volume ratio between the direct (splits) and inverse (merge) cascade events, as a function of the size of the smallest and largest fragments in the interaction. The dashed line is $\ell_s = 0.4\ell_b$, and the solid one in $\ell_s = \ell_b$. Contours are, from light to dark, $1.1(0.2)1.7$. Both figures refer to Qs above the viscous wall layer of a turbulent channel with friction Reynolds number $Re_\tau = 4200$ [6]

merge or split at least once in their life, and it can be shown that between 50 and 70% of the volume change of the largest Qs is due to these discontinuous events. If we interpret the mergers as an inverse cascade to larger volumes, and the splits as a direct cascade, the direct cascade predominates, although not by a large margin (rough 1.3 on average). Figure 4b shows that, disregarding the smallest Qs that cascade seldom, the direct cascade is a property of large structures that break into (or merge from) fragments of comparable size. Small fragments of large eddies have almost the same probability of merging as of splitting, but a large attached Q is roughly twice more likely to break in half than to be created from two comparable fragments.

In summary, the above examples show that cascades in turbulence can be associated to definite interactions that take place locally in physical space between entities of comparable size. In general, both direct and inverse events occur, leading respectively to the generation of smaller and of larger structures, but, in all our examples, the direct cascade predominates.

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