

THE TURBULENT CASCADE IN PHYSICAL SPACE

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Cascades are required whenever a conserved quantity has to be transferred across a range of scales but, beyond that generic idea, every particular instance of multiscale transport requires a physical implementation that does not have to be the same in all cases. In fact, it is probably always true in high-dimensional systems that cascades include a variety of mechanisms that transfer the conserved quantity in different directions, in such a way that a one-directional transfer across scales is only true as a statistical average. The main problem is the traditional idea of scale, which, because of the Fourier uncertainty principle, only takes a definite meaning when averaged over a region of space larger than the scale in question. Based on the general idea that the Navier–Stokes equations are local PDEs in physical, but not in scale space (e.g. Fourier), we have tried for some time in our group to ascertain whether some component of the various transfers in turbulence can be identified locally in physical space, and to clarify its mechanisms. We primarily do this by isolating individual intense structures, and tracking them in space and time (and occasionally across scale).

The best-known example of a turbulent cascade is the transfer of kinetic energy in equilibrium isotropic turbulence. If energy is fed by some large-scale forcing, it has to be transferred to the Kolmogorov scale to be dissipated by viscosity, but the manner in which this happens has been the subject of endless discussions. We will first present evidence that it is local in scale, in the sense that, when the energy is separated into scale bands by filtering and the forcing is unsteady, the unsteadiness moves towards bands of smaller scale as a ‘wave’ [1]. Moreover, the transfer rate is self-similar, as posited by Kolmogorov [4]. The delay required by the energy to cross an octave of scales centred at ℓ is proportional to the local eddy turnover, $\ell^{2/3} \epsilon^{-1/3}$, where ϵ is the ensemble-averaged energy dissipation rate.

Another conserved quantity whose transfer is important is the wall-normal flow of streamwise momentum in turbulent shear flows. Traditionally, momentum is considered to be carried by the tangential Reynolds stress, and structures (Qs) of particularly intense uv are treated as important momentum carriers [7]. The organisation of these intense Qs is different from that of the energy structures in isotropic turbulence. Only ‘large’ structures couple strongly enough to the mean shear to carry net momentum on average, but the shear changes with the distance from the wall. The result is that momentum-carrying Qs form an inertial self-similar hierarchy of many different scales, but stratified in physical space by their distance from the wall. Most relevant Qs are attached to the wall, in the sense that their size is proportional to their distance from it. The transfer of momentum across this hierarchy is a problem that has to be understood as much as the Kolmogorov energy cascade, because equilibrium has to be maintained across scales, and the result sets the scaling parameters for the whole flow; for example, it defines the friction velocity as the uniform velocity scale in wall-bounded turbulence (and therefore the drag coefficient).

Three-dimensional Qs are studied for channels in [5], and tracked individually in time in [6]. They are found to grow and decay while they merge and split in complicated temporal graphs that have to be parsed to understand their evolution. In agreement with their inertial character, their lifetime is proportional to their size, but, somewhat surprisingly given their wall-attached character, they are not found to be particularly connected with the wall. Roughly half of the Qs are born near the wall and move away from it, while the other half do the opposite. In fact, Qs were studied in [3] for homogeneous shear turbulence, with no walls, and shown to be a property of the shear, not of the neighbourhood of the wall. The relevant condition is that they should be larger than the local Corrsin scale, but most Qs satisfying that property in wall-bounded flows are too large to fit in the flow without hitting the wall, and are therefore attached. From the point of view of the present meeting, the interesting behaviour is the merging and splitting. Because Qs are defined by thresholding an intense property, they are born and die as small structures, which at first grow larger and later shrink. Part of this evolution is a consequence of their intensification and weakening, but between 50% and 70% of the volume change is due to mergers and splits. If we define the mergers as an inverse cascade to larger volumes, and the splits as a direct cascade, the direct cascade predominates, but not by a large margin (rough 1.3 on average). These interactions are fairly deterministic, and local in physical space. Eddies (Qs) predominantly break or merge from specific locations related to their geometry, and the ratio between both processes depends on the relative sizes of the fragments. Small fragments of large eddies have almost the same probability of merging as of splitting, but a large attached Q is roughly twice more likely to break in half than to be created from two equal fragments.

The energy cascade in isotropic turbulence is also local in physical space [2, see elsewhere in this meeting]. The band-filtered fields used in [1] can be segmented into individual structures of intense energy, as in the case of the momentum transfer in channels, and these structures can be followed in time. The evolution in this five-dimensional space (three spatial dimensions, scale, and time) is too complex to track each structure individually, but it can be handled statistically. A measure of how related two scales are is the volume of the intersection of their intense structures, which has to be

compared to the null hypothesis of randomly located point sets with the same overall volume fractions. For example, it is found that structures in energy bands separated by a factor of 2 are correlated, but those separated by 4 or more, are not. This could simply be a sign of spectral leakage in the filter, but the structures are tracked in space-time, as in channels, so that each of them has a lifetime (which, not surprisingly, is proportional to $\ell^{2/3}$). It is then found that structures of size ℓ are more strongly correlated with those of size 2ℓ at the beginning of their life than at the end. The opposite is true for their correlation with smaller structures of size $\ell/2$, in strong agreement with a process in which energy passes from large to small structures at the same physical location.

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