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A MODEL FOR BURSTING OF NEAR WALL VORTICAL STRUCTURES IN BOUNDARY LAYERS

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SUMMARY

In the last decade, much effort has been devoted to the understanding of near wall turbulence, both through experimental and numerical observations. Different scenarios have been proposed, but a broad consensus on a basic model for the near wall events has still not been achieved. We present here a simple numerical model for some of the events occurring during the bursting phenomenon. In particular we study the behaviour of longitudinal vortices as they are brought near a wall by an x−dependent external shear, which is the near wall equivalent of a plane strain. We also study the effect of the interaction on the local wall stress, and we calculate the distributions of turbulent stresses to analyse the zones of maximum production in the transversal y-z plane.

INTRODUCTION

The direct simulation of the channel flow by Kim *et al.* (1987), using visualisation techniques similar to those in the experiments of Kim *et al.* (1971), showed the existence of streamwise vortical structures near the wall, very similar to those observed in experiments, and with similar spacings and lengths. Their computational box was 2300×1500 wall units in the streamwise and spanwise directions, and contained many structures, making it difficult to isolate the details of individual events. This is even harder in experimental studies. To achieve a better understanding of individual events near the wall, Jiménez and Moin (1991) introduced the use of numerical simulations in "minimal" channels. In that case only a few structures are present in the computational domain and it is easier to follow them in time and to study how they interact among themselves and with the wall. Because of the intrinsic three-dimensional character of the flow, it is difficult to obtain simple models for the near wall region, but both the full channel and the minimal channel simulations show that the structures are very elongated in the streamwise direction, and that the key event occurs when they come near the wall. Their interaction with the wall induces a layer of secondary wall vorticity of opposite sign, which then diffuses and interacts with the primary structure. During this interaction a very rapid growth of the wall shear stress is observed.

A similar mechanism was observed by Robinson (1991) analysing Spalart's (1988) turbulent boundary layer direct simulation. He came to the conclusion that, in the inner zone, single unpaired near-wall quasi-streamwise vortices generate persistent low-speed streaks, and that counter-rotating vortex pairs are rare. The fact that the whole structure is elongated suggests that a quasi two-dimensional model may be built, in the y-z transverse plane, in which the longitudinal variation is represented only through a variable forcing term in the equations. This permits the use of grids which are unaffordable in a three-dimensional simulations, and the observation of small details which would be unavailable otherwise. On the other hand, the longitudinal variations of the flow are only represented in an approximate manner. As a consequence we expect to find correspondence with experiments in the development of the small spatial scales, and for short time intervals, but not necessarily for large scales or for long times.

Ersoy and Walker (1986) were probably the first ones to investigate numerically two dimensional vortex motion near a wall in connection with the bursting phenomenon, although the basic model was proposed much earlier (Blackwelder and Eckelmann, 1979). They considered the viscous boundary layer induced by a pair of counter-rotating point vortices impinging on the wall at an arbitrary angle, and concluded that the wall layer always separates and generates a violent eruption as a consequence of the pressure field induced by the pair. They concluded moreover that, in asymmetric cases, the effect of vortex nearest to the wall dominates the motion. Their study was done in the context of non-interacting boundary layer theory and, as a consequence, the wall layer became singular in a finite time.

The present study tries to do a more realistic simulation by using the full two dimensional Navier Stokes equations and non singular vortices, and by using parameters derived from direct numerical simulations of the boundary layer. The aim is to study the formation of compact longitudinal cores from initially flat vorticity distributions, to follow the subsequent behaviour of those cores, and to quantify the average (streamwise) wall stress produced by them.

PHYSICAL MODEL

We consider an infinite streamwise vortex subject to a variable shear that increases linearly with the streamwise coordinate, x. From continuity there is a downwash towards the wall which is independent of x. This mean flow is intended as a first approximation to a sweep event, in which a streamwise vortex is pushed towards the wall, and presumably initiates a new bursting cycle. The assumption is that the spatial and temporal scales of the pressure gradient that produces the sweep are large with respect to those of the evolution of the vortex. Under those conditions the mean flow can be assumed to be steady, and a good model is Hiemenz flow (Schlichting, 1987, p. 87), which has the advantage of being itself a steady solution of the Navier-Stokes equations. It can be expressed in terms of a dimensionless function $\phi(y)$ as $U_H(x,y) = ax\phi'(y)$, $V_H(y) = -a\phi(y)$. The function $\phi(y)$ behaves quadratically in y near the wall, so that the horizontal velocity drops linearly to zero in that region while the normal velocity vanishes quadratically, as in real profiles. Assuming the same x dependance for the perturbations as for the mean flow, we write the streamwise velocity perturbation as $u(x,y,z) = a\,x\,q(y,z)$

The equations for the transverse and longitudinal motion are weakly coupled through the vortex stretching term. The transverse velocity (v,w), is incompressible, and can be characterised by the streamwise vorticity component $\omega = \partial w/\partial y - \partial v/\partial z$, and by a stream function $\psi(y,z)$. From now on we will use dimensionless quantities normalised with Γ, the circulation of the streamwise vortex, and with r_0, the radius of the vortex, and a, related to the variable shear. The dimensionless equation for the longitudinal vorticity is

$$\frac{\partial \omega}{\partial t} + J(\omega,\psi) = Ru\left[(\phi'(y)+q)\,\omega + \phi(y)\,\frac{\partial \omega}{\partial y}\right] + Re\,\nabla^2\omega, \tag{1}$$

with $Re = \Gamma/\nu, Ru = ar_0^2/\Gamma$ and $J(\omega,\psi)$ collects the usual convective terms. The first term on the right hand side represents the effect of vortex stretching by the variable shear, while the second one represents advection of vorticity by the resulting normal velocity.

With the assumptions for the perturbation velocities, together with a corresponding one for the perturbation pressure, we can also write a transport equation for the perturbation streamwise velocity, q,

$$\frac{\partial q}{\partial t} + J(q,\psi) = -Ru\left[(q^2 + 2\,\phi' q) + \phi(y)\,\frac{\partial q}{\partial y}\right] + +\phi(y)''\,\frac{\partial \psi}{\partial z} + Re\,\nabla^2 q. \tag{2}$$

This quantity behaves approximately as a passive scalar evolving under the effect of viscous diffusion and of advection by the (q, v, w) field. Note that, due to the assumption of two dimensionality, these two equations are exact, except for a "quasi linear" approximation in (2) that $\partial p/\partial y = 0$. We integrate them to evaluate how the vortex evolves under its own induction, coupled to the stretching of the variable shear, and how the streamwise structures affect the longitudinal velocity distribution, including the mean wall stress averaged over the spanwise direction.

Note that the limit $Ru \to 0$, which is reached by letting $a \to 0$ and $x \to \infty$, is that of constant shear and no strain, but satisfies formally the equations. We will present later some simulations in this limit to separate the effects of self induction from those of stretching.

In our calculations, the equations are solved in a rectangular domain with periodic conditions in the spanwise direction. Free-slip conditions are imposed at the upper wall and a very efficient discretisation of the wall vorticity, described in (Orlandi, 1989), is used at the lower, non-slip wall. The main features of the numerical method are described in that paper. It is a scheme with second order time and space accuracy, and with global conservation of energy, enstrophy, and vorticity in the inviscid limit. The dimensionless units used in the numerical calculations can be related to the boundary layer "wall" units by fixing the vortex radius and the kinematic viscosity. The latter is defined as $\nu^+ = 1$, while the former, for reasons given below, will be taken as $r_0^+ = 10$. In the following all results will be expressed in those wall units.

All the calculations are performed within a domain of size equal $z^+ = 100$ in the spanwise direction and $y^+ = 60$ normal to the wall. This corresponds to the average full spacing between consecutive sublayer "streaks" in the spanwise direction but excludes the logarithmic region in the vertical one. All the simulations are performed in a 256×256 grid.

INITIAL CONDITIONS

Robinson (1991), analysing Spalart's (1988) turbulent boundary layer data base from direct simulations, computed distributions of diameters, strengths and wall distances for the quasi-streamwise vortices. He found that the radii have a distribution with a maximum around $r^+ = 5-20$, the wall distance, around $y^+ = 10-50$, and the Reynolds number, $Re = \Gamma/\nu = 60-250$. The value $r_0^+ = 10$ chosen in our computations is intended to represent an average of these vortices.

Some preliminary computations using those values showed that the vorticity was rapidly dissipated. Since the goal of the study was to investigate the vortex dynamics, we decided to increase the Reynolds number to allow for longer lifetimes, and all the simulations presented here are done using $\Gamma^+ = Re = 500$. The discrepancy is most probably due to the three dimensional effects that are neglected in our model. In a strictly two dimensional approximation, like ours, the vorticity rotation terms $\omega_y\partial U/\partial y$ and $\omega_z\partial U/\partial z$ cancel identically, and there is no way to regenerate the streamwise vortices from the transverse vorticity in the mean shear. As a consequence any streamwise vortex is eventually dissipated. On the contrary, it was shown in (Jiménez and Moin, 1991) that those reorientation terms play a significant role in the evolution of the streamwise vorticity in the real boundary layer.

The initial streamwise velocity profile, U_H, has two effects. First, through the scale factor a, it generates the external strain and the normal velocity field that brings the streamwise vortices closer to the wall. Second it is itself convected by the transverse velocity field and generates the shear layers and the wall stress that, although essentially passive in our approximation, are such important features of real flows. Both effects can be studied approximately by using an initially linear profile near the wall, although the consistency of the equations forces us to use a more realistic model such as the Hiemenz flow. Still, all that is required is that $\partial U_H/\partial y \to 0$ as $y \to \infty$, and the detailed profile is quite irrelevant. In particular, the vertical scale of the Hiemenz profile is arbitrarily, and was adjusted in our simulation so that $\phi' = 0.9$ at $y = 4.5$ ($y^+ = 45$), giving an essentially linear profile over most of the region of interest, but levelling to $\phi' = 1$ at the upper edge of the computational domain.

The strain parameter, a, was estimated from direct simulation data. In those, the wall shear has a spotty distribution, in the range $\partial U^+/\partial y^+ = 0-3$, with a streamwise length scale of the order of $x^+ = 50-100$ (Jiménez and Moin, 1991). The corresponding values for the strain are, $a^+ = (\partial^2 U^+/\partial y^+ \partial x^+)/(d\phi'/dy^+)_w \approx 0.5-2.0$. From these data the dimensionless strain in Eqs.(4) and (5) can be approximately evaluated as $Ru = 0.1-0.5$. Because we are interested in investigating the effect of the strain, we considered two cases: the first one with constant shear, $Ru = 0$, and the second one with a strong strain, $Ru = 0.243$.

The choice of the initial vorticity distribution and location for the longitudinal vortices influences in a large measure the evolution of the flow. Rather than introducing circular vortices *ab initio* we decided to study whether they might form naturally by self induction, and we used vortices whose initial shapes were very elongated in the spanwise direction,

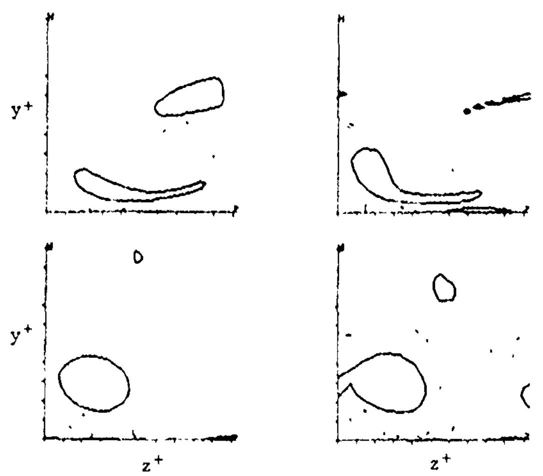

Fig. 1 – Time sequence of ω_x maps in the transverse section of a minimal channel, moving with the approximate convection velocity of the vorticity structures at the wall. Flow is minimal channel from Ref.[3]. Time from left to right and top to bottom.

described by the expression

$$\omega = \omega_M \exp\left(-\frac{y-y_0}{\sigma_y}\right)^2 \sin\left(\pi\,\frac{x-x_0}{\sigma_x}\right), \quad x \in (0, \sigma_x) \quad (3)$$

where ω_M was adjusted so that $\Gamma = 1$, and with $\sigma_y^+ = 0.75$, $\sigma_z^+ = 10 - 40$, and $y_0^+ = 7.5$. A typical initial condition is shown in Fig. 3.a. These vortex sheets are found often in the near wall region of real boundary layers (see Fig. 1), and it is not immediately obvious whether they result from the deformation of initially circular vortices, or whether they are precursors to them. Our goal in choosing them as initial conditions was to see whether they would evolve spontaneously into circular cores, and how fast. In the case of the simulations presented here, a vortex sheet with $\Gamma^+ = Re = 500$ and $\sigma_z^+ = 20$ rolls up into a compact core in $t^+ \approx 1$, which is shorter than other relevant time scales in the flow.

RESULTS AND DISCUSSION

We were also interested in investigating whether our quasi-two-dimensional model would predict the type of events depicted in Fig. 1, whose most characteristic feature is the presence of intense compact vortices of one sign and weaker, more diffuse, vortices of the opposite sign which seem strained by them into thin sheets. Approximately symmetric vortex pairs are conspicuously absent. We performed several calculations with different initial configurations to study their evolution.

With counterrotating vortices of similar initial circulations and shapes, either with or without external strain, both vortices roll up in approximately half an eddy turnover time. The final configuration is that of Fig. 2, with a vortex pair receding away from the wall. Configurations similar to those of Fig. 1 cannot be achieved. The layers of vorticity of opposite sign which are formed at the wall play an important role in the roll-up process. They are initially very intense, but they later diffuse away from the wall and, subjected to the shear of the main vortex, are dissipated quickly. When an external strain is applied, the vortices are pushed against the wall and become more compact under the effect of the stretching, but the wall interaction mechanism does not change appreciably.

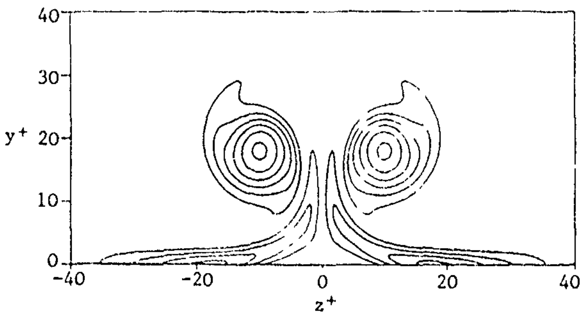

Fig. 2 – Contour plots of vorticity at $t^+ = 4.8$ for a symmetric initial condition. ($\omega_{max} = .52$, $\Delta\omega$ increment $= .10$, $Re = 500$, $Ru = 0$).

When the two counterrotating vortex sheets have the same initial circulation magnitude, but their shapes and distances from the wall are different, the evolution is asymmetric for a while. An example is given in Fig. 3a–c, in which one of the vortices is relatively compact and close to the wall, while the other one is more elongated and located

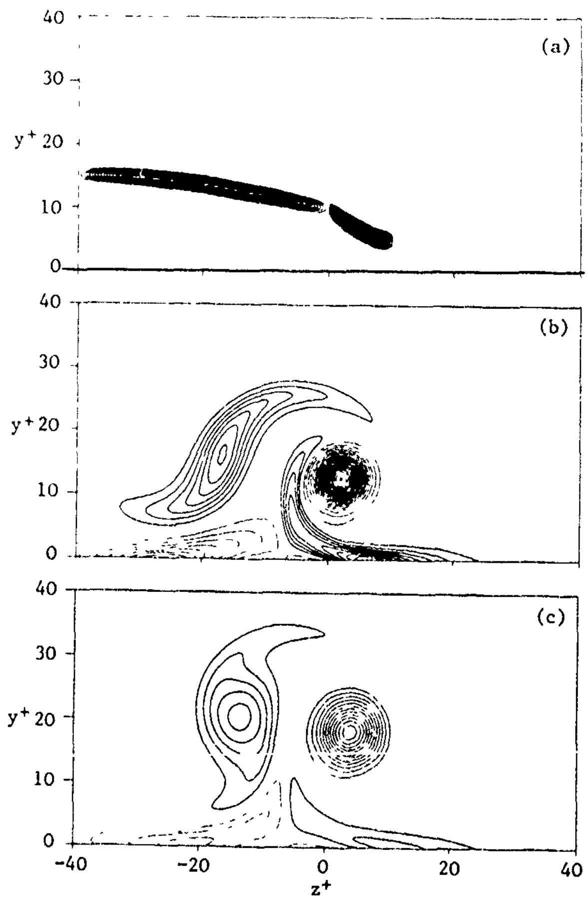

Fig. 3 – Contour plots of vorticity with asymmetric initial conditions. ($\Delta\omega$ increment $= .1$, $Re = 500$, $Ru = 0$). a) $t^+ = 0$; b) $t^+ = 2.4$, $\omega_{max} = .90$; c) $t^+ = 4.8$, $\omega_{max} = .55$.

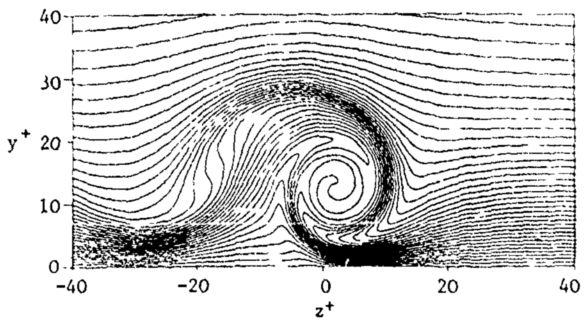

Fig 4 – Contour plots of streamwise velocity corresponding to Fig. 3.b. (Δq increment = .025)

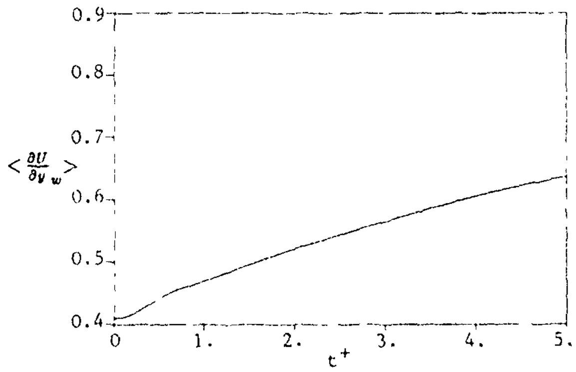

Fig. 5 – $<\partial U/\partial y_w>$ time history for flow in Figures 3–4.

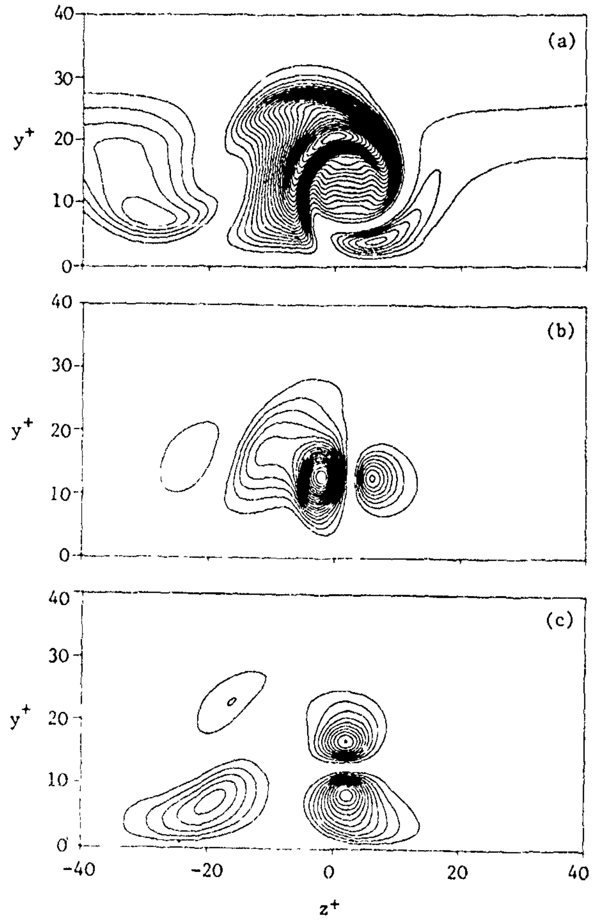

Fig. 6 – Normal stresses distributions for Fig. 3.b. a) q'^2, b) v'^2, c) w'^2. (Δ increment = .00125)

at a greater distance. The small core on the right, having a higher vorticity, assumes a circular shape in a very short time and prevents for a while the roll-up of the more diffuse sheet. The results at $t^+ = 2.4$ (Fig. 3.b) is quite similar to some of the structures in Fig. 1. The main difference is the behaviour of the secondary vortex sheet at the wall, which looks weaker in the three dimensional case. This sheet tends to protrude in between the two members of the vortex pair, shielding the diffuse vortex and allowing it to roll up even in the presence of the strong core. In fact, in the two dimensional case, in which that secondary vorticity is strong, the two cores have essentially rolled up at $t^+ = 4.8$ (Fig. 3.c) and the final result is quite symmetric, similar to that in figure 2. Therefore, a first conclusion of our simulation is that the single structures observed Robinson (1991) have to be caused by very non-symmetric events, e.g. legs of hairpin vortices lying at different distance from the wall, and that the vortex closest to the wall is always dominant.

It is generally accepted that these longitudinal structures are responsible for the formation of the low velocity streaks in the wall region. Fig. 4 shows that this process is caused by the convection of the streamwise velocity field, $U = U_H + u$, by the longitudinal vorticity. In the region of updraft, the initial shear is convected away from the wall, generating a low velocity streak and a detached layer of intense spanwise shear. These are regions of negative u' and positive v' (II quadrant) which generate negative $u'v'$ and production of turbulent energy.

In real three dimensional flows these shear layers probably become unstable and lead to the formation of new longitudinal vortices and of new turbulent cycles, but these effects are absent in our simplified two dimensional model. To the sides of these low velocity regions the downwash pushes the background shear towards the wall and generates a high velocity streak. It is not *a priori* clear whether the two effects would cancel at the wall and whether the global effect would be a increase or a decrease in the wall stress averaged over the span. Figure 5 shows that the former is the case and that the averaged wall stress increases by 50% in $t^+ = 5$, which is a time scale comparable to that estimated for the growth of a burst in (Jiménez and Moin, 1991). Since the eddy turnover time can be estimated as $t_e^+ = (2\pi r_0^+)^2/\Gamma^+ \approx 8$, the increase in wall stress is very rapid, and occur in time which are short compared to an eddy rotation, although longer that those associated to the formation of the compact cores.

Fig. 6 shows the spatial distribution, in the y-z plane, of the velocity perturbations at the particular moment in time which corresponds to Fig. 3.b. These perturbations would contribute to the turbulent intensities u', v' w' in the three dimensional boundary layer, but can only be related directly to them after a randomising and averaging step that was not attempted here. Still, the location of the different contributions is interesting. The fact that the u' perturbation that defines the low velocity streak is wider in the spanwise direction, and stronger, away from the wall than close to it, is well known from experiments. The association, in the cen-

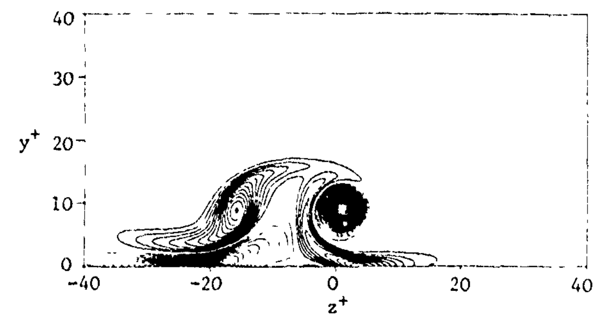

Fig. 7 – Contour plots of vorticity with strain at $t^+ =$ 2.4, ($\omega_{max} = 1.30$, $\Delta\omega$ increment = .1, $Re =$ 500, $Ru = 0.243$).

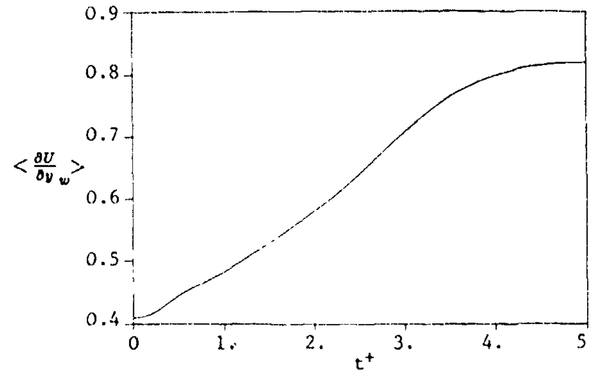

Fig. 9 – $< \partial U/\partial y_w >$ time history for flow in Figures 7-8.

tre of the streak, of $v' > 0$ and $u' < 0$, defines an "ejection" event, and suggests once again that ejections are not transient events in the boundary layer but long lasting states that pass by the stationary probes used in experimental observations (Moin, 1987, Jiménez and Moin, 1991)

From this first simulation, without external strain, we see that our simplified model reproduces some of the features of the bursting event: it is very rapid, it is governed by the presence of longitudinal vortices located near the wall, and during the bursting event there is a large growth of the wall stress.

To analyse how the external strain modifies the flow structure, a simulation was run with $Ru = 0.243$ and with the same initial conditions as those in the previous calculation. In Fig. 7 we show the vorticity distribution at a time comparable to that of Fig. 3.b. The main difference is that now the whole structure is closer to the wall, being convected by the normal velocity created by the strain. This is true both for the original vortex cores and for the secondary vorticity layers at the wall Also, the general intensity of the vorticities has increased, due to the stretching term. As a consequence, all the interactions are stronger and, in particular, the effect of the wall vortex sheet becomes more important than before. The resulting shielding effect prevents the straining of the left vortex by the right one, and the pair becomes more symmetric, even for the severely unsymmetrical initial conditions used here. The end result is that a symmetric pair is always formed, which flies away from the wall even in the presence of downwash created by the strain. The resulting longitudinal velocity distribution is shown in Fig. 8, and also shows the formation of a low velocity streak surrounded by high velocity regions.

Fig. 9 shows the growth of the mean wall shear, which is initially faster than in the unstrained case, but levels off

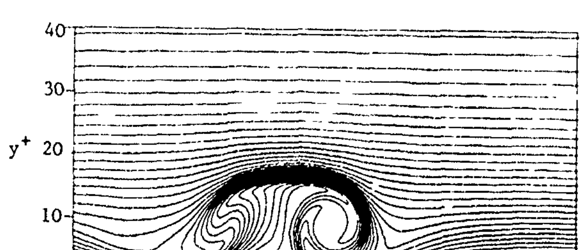

Fig. 8 – Contour plots of streamwise velocity corresponding to Fig. 7. (Δq increment = .025)

in a shorter time. This levelling corresponds to the drifting of the symmetric pair away from the wall under its own induction. These characteristics are common to most of the simulations that were carried out including a strain. Asymmetric evolutions could only be achieved in those cases under the most extreme initial conditions. Because the whole boundary layer is contracted vertically towards the wall by the strain, the general level of the longitudinal vorticity gradients is higher, but the structure of the streak is the same as in the unstrained case.

Robinson (1991) found both single vortices and counter-rotating pairs in the wall region, but he notes that the probability of the former is larger than that of the latter. The same was observed in the minimal channel by (Jiménez and Moin, 1991). The present simulations suggest that events with a single vortex are associated to weak streamwise strains while events with vortex pairs correspond to intense strains. At longer times our quasi-two-dimensional simulations always tend to produce symmetric configurations, but it was pointed in the introduction that the model can not be trusted in that range. More interesting is the possible relation of our strained model to the boundary layer in a positive pressure gradient. The results suggest that symmetric pairs should be more common in that case, but we could not find any independent experimental evidence for or against this prediction.

CONCLUSIONS

We have seen that the present simplified quasi-two-dimensional model for the near wall region of the turbulent boundary layer reproduces several of the aspects of the bursting phenomenon, as well as of the structure of the low velocity streaks. In particular it has been shown that flat sheets of streamwise vorticity near the wall roll into compact cores, and that this process, which is a prerequisite for the appearance of strong longitudinal vortices, is fast and relatively independent of the presence of a longitudinal strain. Moreover, the roll-up mechanism is essentially two dimensional (in the y-z plane) and unrelated to the formation of hairpins by longitudinal tilting of the spanwise vorticity. While the latter process has been shown repeatedly to be important in transition of wall flows into turbulence, this new mechanism provides an alternative way for the regeneration of turbulent structures once fully developed turbulent flow has been established. This new mechanism also makes less surprising the relative rarity of symmetrical vortex pairs that has been reported in several recent investigations of the wall region.

On the other hand, our simulations show that even very asymmetric vortex pairs would eventually roll-up into a more or less symmetric configuration, which then lifts away from the wall under its own induction, and that this tendency is greater in the presence of a positive strain. This suggests that symmetric hairpins should be more common in the outer region of the boundary layer than near the wall, and that the same should be true in layers subject to a favourable pressure gradient. While there is some experimental support for the first trend, we know of no experimental or numerical evidence relating to the second.

The effect of the longitudinal vortices on the average streamwise shear also reproduces well the general structure of the longitudinal streaks found in the wall region. While it has been clear for some time that low speed streaks are associated with the advection of slow fluid away from the wall by the vertical velocities induced by the longitudinal vortices, we believe that this is the first time that it has been shown quantitatively that the net effect of this transport is to increase the *averaged* wall stress. In other words, that the transport of high speed fluid towards the wall is more important than that of low speed fluid away from it. The generation of a high average wall shear is, of course, one of the most characteristic properties of the turbulent boundary layer, and one of its most technologically interesting ones.

On the other hand, most of the three dimensional effects are absent from our model, and the main consequence is that our turbulence will not self sustain. The Reynolds number that we had to use for our vortices to survive for a reasonable time were at least twice larger than those observed in direct numerical simulations. We believe that, even if the present model does give interesting indications of the behaviour of bursting structure for short times, their long time behaviour, and their genesis, depends on the correct understanding of the three dimensional effects.

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