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Acoustic effects on the free oscillations of the top membrane of a cylindrical container

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Objectives

Calculate the natural frequencies of the top membrane of a cylindrical container filled with a compressible and inviscid fluid

Introduction

Fluid-structure interaction problems
many technological applications
known for many years

Most works concerned underwater
applications,
nuclear reactor, naval engineering

Applications – Fluid-Membrane

* Microelectromechanical systems MEMS

Micropumps
Ultrasonic transducers
Pressure sensors

Performance this microsystems

Controlling resonant frequency

reducing power requirements (micropumps)

resonant frequency-fluid pressure (pressure sensors)

* Liquid containers

(nuclear reactor, aerospace vehicles)

Interesting to know the frequencies of the coupled system

Liquid frequencies can be close to other frequencies of the system resulting in a disturbance of the motion and even instability and failure

A shifting of these frequencies is a way of remedy the problem

This is done by covering the free liquid surface with a flexible structural member, such as a membrane or a thin elastic plate

* Pressure accumulators (hydraulic systems)

elastic cover separates fluid domains

prevent system instabilities

Other applications

* Sonar

fluid effects on vibration elastic cover of a cavity

* Biological prosthesis

Smart structures (membranes), low mass
Influence fluid low density important

Existing Methods (fluid-containers elastic membrane)

Variational formulation

Kinetic (plate + fluid) + Potential Energy

Aplication

incompressible
inviscid

Authors (problem studied)

Bauer

Cylindrical rectangular containers

Tariverdilo

Cylindrical container

Deformation modes

Vaccum modes Bessel, Harmonic - functions

Present Method

Wave equation - Velocity potential - Separation of variables
Pressure field - Bernouilli's linearized equation
Integration dynamic equation - frequencies

Aplication

Compressible, inviscid fluid
all modes

Problem formulation

The deformation equation

$$T\Delta\eta(r,\theta,t)-\rho_m t_h \frac{\partial^2\eta}{\partial t^2}=p(r,\theta,H,t)$$

Fluid-membrane boundary condition $z=H$

$$\frac{\partial p}{\partial z}=-\rho_f\cdot\frac{\partial^2\eta}{\partial t^2}$$

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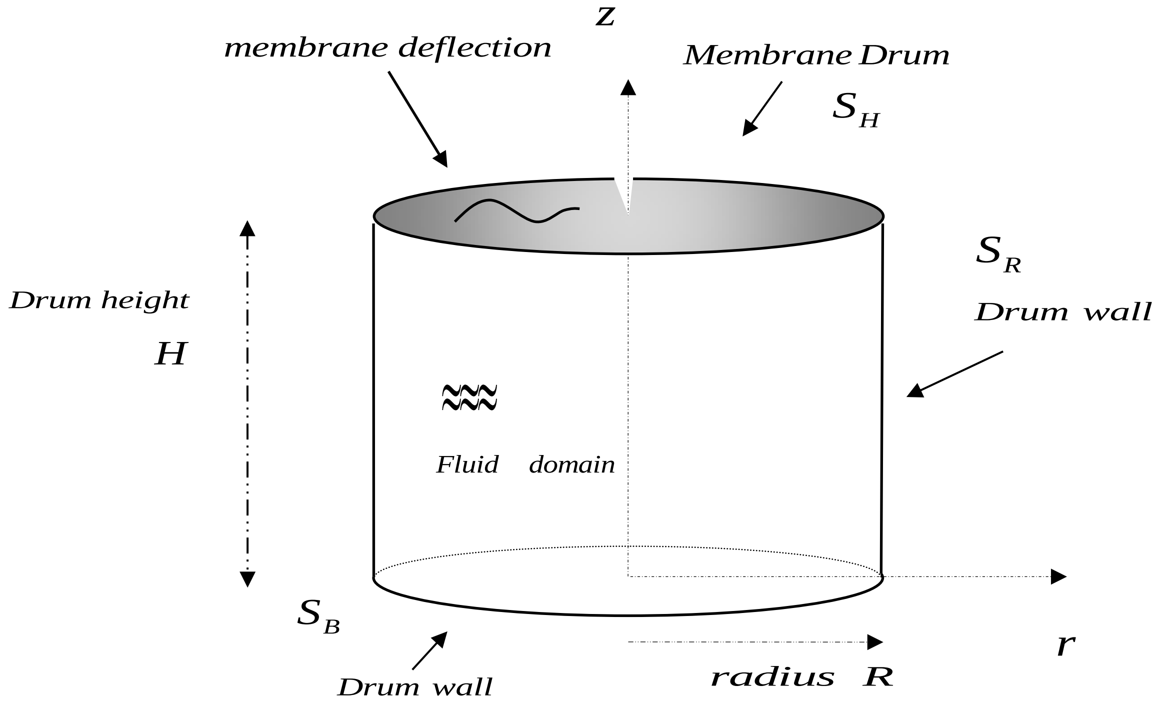
Fig. 1. Scheme of the cylindrical container with the top membrane showing the different geometrical parameters



Harmonic motion, frequency ω

(separation variables)

$$\eta(r,\theta,t) = \tilde{\eta}(r,\theta)\cdot e^{i\omega t} \qquad \varphi(r,\theta,z,t) = \tilde{\varphi}(r,\theta,z)\cdot e^{i\omega t}$$

Membrane deformation

$$\eta(r,\theta,t) = \sum_{m,n} \eta_m^n(r,\theta)\cdot \bar{d}_m^n \cdot e^{i\omega_m^n t}$$

Vacuum modes

$$\eta_m^n(r,\theta) = J_m\left(\beta_m^n\, r\right)\cdot\cos(m\theta)$$

Modal parameters m, n interger values

Number nodal circles n zero deformation

nodal diameters m

Deformation parameter

$$d_m^n(t) = \bar{d}_m^n \cdot e^{i\cdot\omega_m^n\cdot t}$$

Deformation weight coefficient $\bar{d}_m^n$

Deformation of the membrane

$$\eta(r,\theta,t)=\sum_{m,n}\eta_m^n(r,\theta)\cdot\bar{d}_m^n\cdot e^{i\omega_m^n t}$$

Bessel function m order $J_m\left(\beta_m^n r\right)$

Parameter β_m^n roots of equation $J_m\left(\beta_m^n R\right)=0$

Vaccum Frequency $\beta_m^n=\omega_{mv}^n\sqrt{\dfrac{\rho_m t_h}{T}}$

Fluid- Velocity Potential φ (compressible, inviscid fluid)

Wave equation (Helmhotz)

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = 0$$

Fluid velocity field

$$\vec{v}_f = \nabla\varphi$$

$$\Delta\tilde{\varphi} + k^2\tilde{\varphi} = 0$$

Boundary conditions

wave number $k = \frac{\omega}{c}$

Rigid walls

$$\frac{\partial\varphi}{\partial n} = 0$$

Membrane

$$\frac{\partial\varphi}{\partial z} = \frac{\partial\eta}{\partial t} \qquad z = H$$

Solution of the Potential

Separation of variables

Being α_m^n the roots of

And the parameter $\lambda_m^n = \sqrt{\left(\alpha_m^n\right)^2 - k^2}$

Applying Boundary Condition over Membrane, after integration

$$\frac{\partial \varphi}{\partial z} = \frac{\partial \eta}{\partial t}$$

Expanding in series, after integration of squared integrands

Orthogonality properties

Momentum's linearized equation

$$\frac{\partial \varphi}{\partial t} + \frac{p}{\rho_f} = 0$$

Pressure over membrane

$$\ddot{d}_m^n(t) = \frac{d^2(d_m^n(t))}{dt^2} = -\omega_m^{n\,2} \cdot \bar{d}_m^n \cdot e^{i \cdot \omega_m^n \cdot t}$$

Harmonic motion - Pressure jump over membrane

$$p(r,\theta,t) = \sum_{m,n} P_m^n(r,\theta)\ \bar{d}_m^n e^{i\omega_m^n t}$$

Pressure mode, $P_m^n(r,\theta)$

$$P_m^n(r,\theta) = \frac{\rho_f \coth\left(\lambda_m^n H\right)}{\lambda_m^n} \sqrt{\frac{\Lambda_{\beta m}^n}{\Lambda_{\alpha m}^n}} \varphi_m^n\left(r,\theta\right) \omega_m^{n\,2}$$

Integration of squared dynamic equation

(orthogonality condition)

Frequency equation

$$A_m^n \cdot \omega_m^{n\,4} + B_m^n \cdot \omega_m^{n\,2} + C_m^n = 0$$

Coefficients

$$A_m^n = \left[\frac{\rho_f \coth\left(\lambda_m^n H\right)}{\lambda_m^n} \right]^2 - \left(\rho_m t_h\right)^2$$

$$B_m^n = 2\rho_m t_h T \beta_m^{n\,2}$$

$$C_m^n = -T^2 \beta_m^{n\,4}$$

Iteration procedure , $\lambda_m^n(k)$ wave number k

$$\lambda_m^n = \sqrt{\left(\alpha_m^n\right)^2 - k^2}$$

Fluid mass parameter

Multiplying the membrane equation expanded in series by the deformation mode $\eta_u^v(r,\theta)$, and after integration over the membrane circular surface

With ortogonallity properties, and making the sum in u , v indices, the following formula for natural frequency is obtained for each value of indices m , n

$$\omega_m^n = \sqrt{\frac{K_m^n}{M_m^n + M_{Fm}^n}}$$

$$K_m^n = \pi \cdot \Lambda_{\beta m}^n \cdot T \cdot \left(\beta_m^n R\right)^2 \qquad M_m^n = \pi \cdot \Lambda_{\beta m}^n \cdot \rho_m \cdot t_h \cdot R^2$$

$$M_{Fm}^n = \pi \frac{\rho_f \coth\left(\lambda_m^n H\right)}{\lambda_m^n} \sqrt{\frac{\Lambda_{\beta m}^n}{\Lambda_{\alpha m}^n}} R^2 \cdot \sum_i \frac{\beta_m^i R}{\left(\beta_m^i R\right)^2 - \left(\alpha_m^n R\right)^2} J_m\left(\alpha_m^n R\right)\left(J_{m-1}\left(\beta_m^i R\right) - J_{m+1}\left(\beta_m^i R\right)\right)$$

And the Fluid mass parameter $$C_{MF_m^n} = \frac{M_{Fm}^n}{\rho_f \pi R^2 H}$$

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Results

Table 1 Natural frequencies (Hz) for the membrane of the cylindrical drum (in vacuum and water) for the present method and the work of Tariverdilo et al. [13], of radius R=60 mm , drum height H=60 mm and membrane thickness $t_h = 0.5mm$

m,n	f_v	f_l (Tarverdilo)	f_l (present method)	Relative discrepancy (%)
0 , 0	54.9	--	15.4	
0 , 1	46.5	--	46.5	
0 , 2	85.3	--	85.3	
0 , 3	123.3	--	123.3	
1 , 0	87.5	19.2	17.0	11.4
1 , 1	160.1	52.9	52.4	0.95
1 , 2	232	94.6	93.2	1.48
1 , 3	304	159	138.9	12.6
2 , 0	117	32.7	29.6	2.77
2 , 1	192	70.8	69.6	1.69
2 , 2	265	115.7	113.5	1.90
2 , 3	337	185.9	161.5	1.31
3 , 0	145	46.7	43	7.92
3 , 1	222	88.9	87	2.13
3 , 2	297	136.8	134	2.05
3 , 3	370	212.8	184	13.5
4 , 0	173	61.2	57	6.86
4 , 1	252	107.3	105	2.14
4 , 2	328	157.9	155	1.84
4 , 3	402	239.5	207	13.5

Membrane -Cylindrical Container - Properties

Container Height $H = 2\,m$

Fliud density $\rho_f = 1,225\,\dfrac{kg}{m^3}$

Radius $R = 1\,m$

Membrane Thickness $t_h = 0,02\,mm$

Frequency parameter $\Omega = \sqrt{\dfrac{\rho_m t_h}{T}} R \cdot \omega$

Membrane Material $\rho_m = 2700\,\dfrac{kg}{m^3}$

Membrane Tension $T = 100\,\dfrac{N}{m}$

Fig. 2 Variation of the frequency parameter with the radius of the membrane for different modes, n=0,1,2 nodal circles and zero nodal diameter m= 0

$$\Omega = \sqrt{\frac{\rho_m t_h}{T}} R \cdot \omega$$

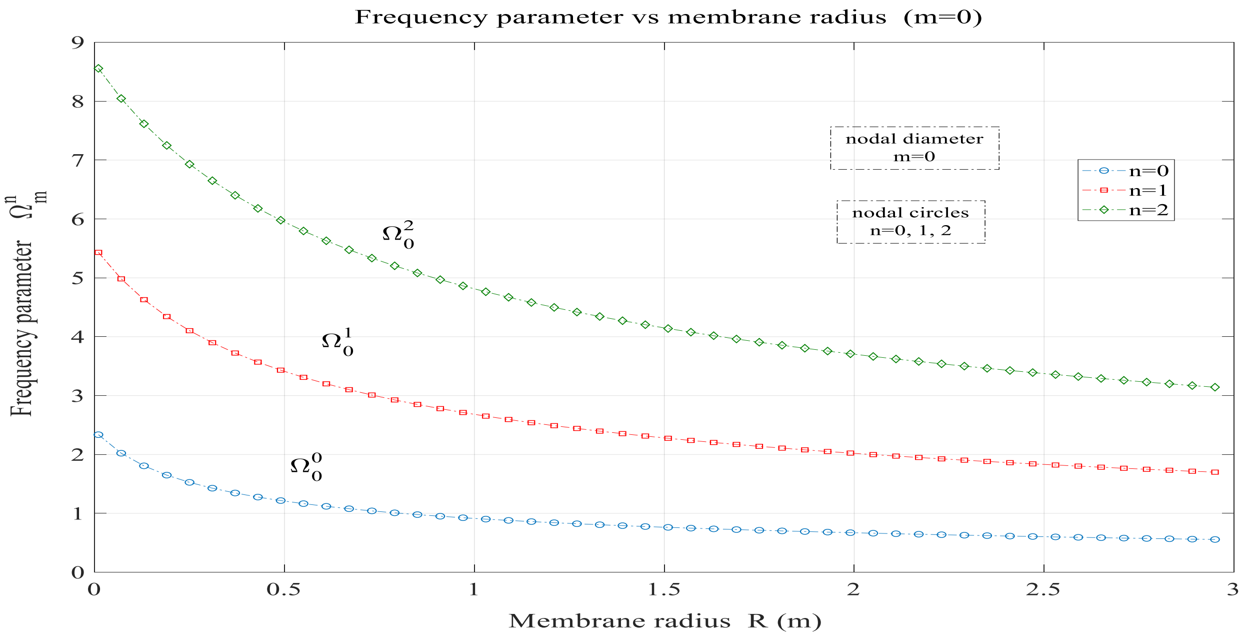

Fig. 3 Variation of the frequency parameter with the radius of the membrane for different modes, n=0,1,2 nodal circles and one nodal diameter m=1

$$\Omega = \sqrt{\frac{\rho_m t_h}{T}} R \cdot \omega$$

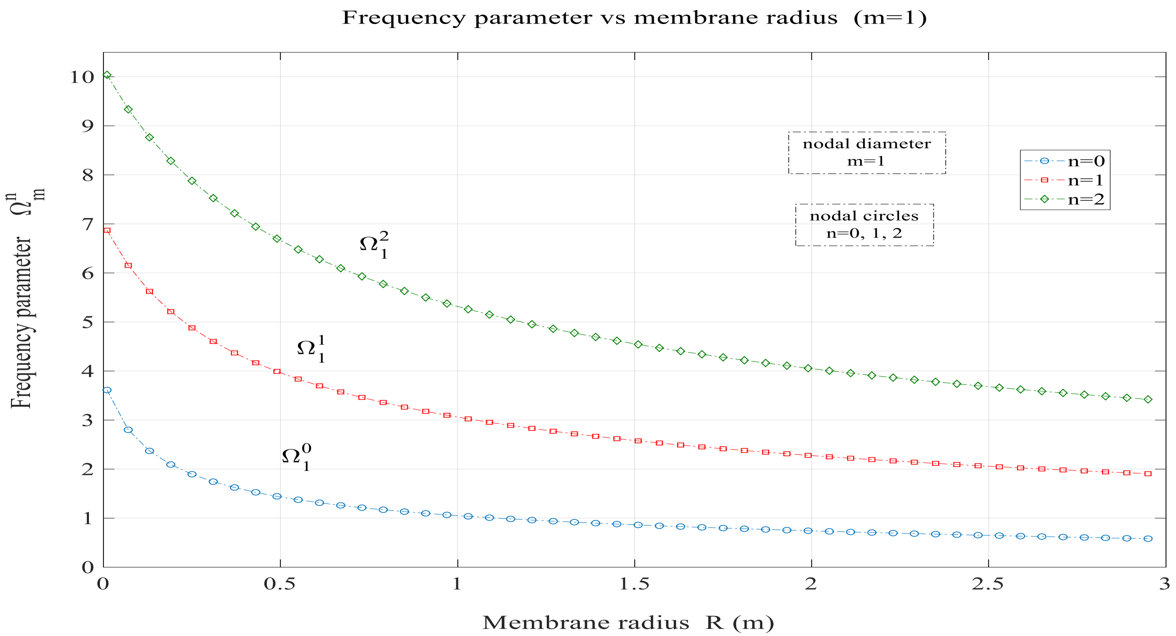

Fig. 4 Variation of the frequency parameter with the drum height H for different modes, n=0,1,2 nodal circles and zero nodal diameter m=0

$$\Omega = \sqrt{\frac{\rho_m t_h}{T}} R \cdot \omega$$

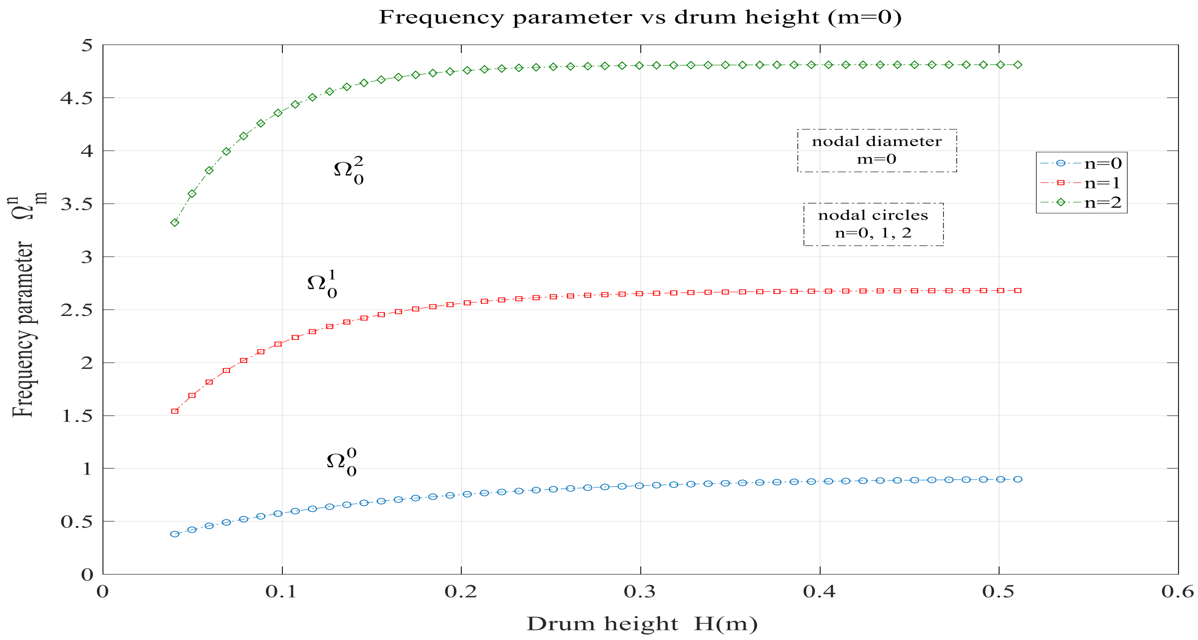

Fig. 5 Variation of the frequency parameter with the drum height H for different modes, n=0,1,2 nodal circles and one nodal diameter m=1

$$\Omega = \sqrt{\frac{\rho_m t_h}{T}} R \cdot \omega$$

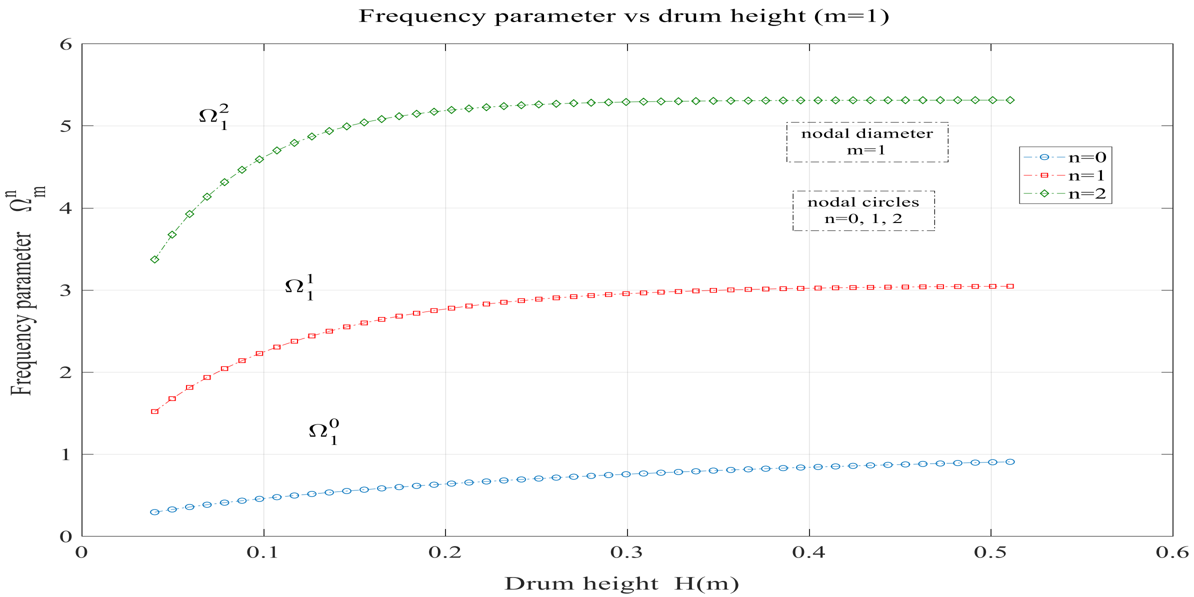

Fig. 6 Variation of the frequency parameter with the membrane thickness for different modes, n=0,1,2 nodal circles and zero nodal diameter m=0

$$\Omega = \sqrt{\frac{\rho_m t_h}{T}} R \cdot \omega$$

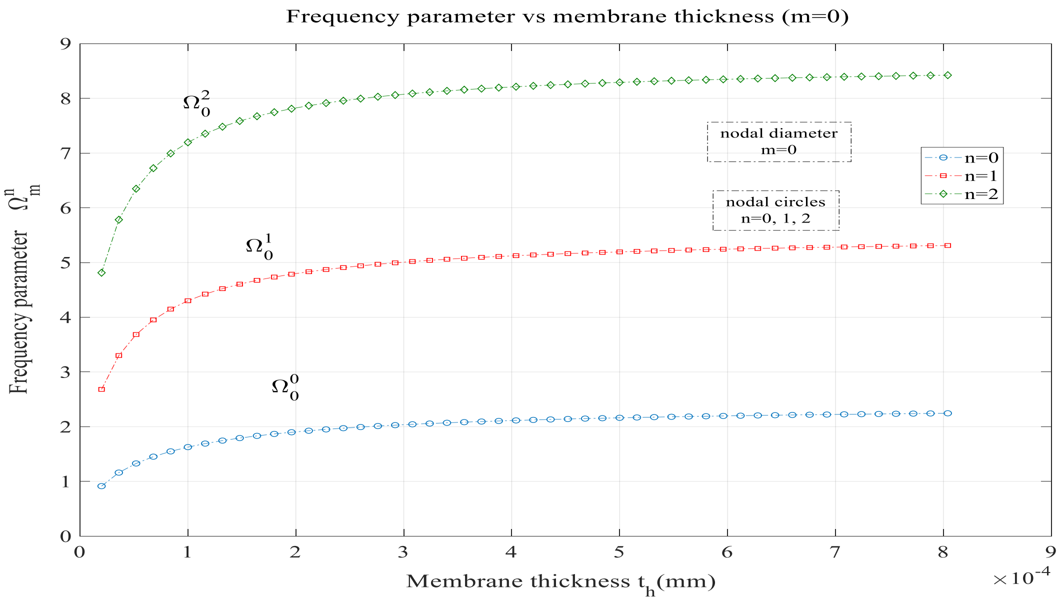

Fig. 7 Variation of the frequency parameter Ω with the membrane thickness for different modes, n=0,1,2 nodal circles and one nodal diameter m=1

$$\Omega = \sqrt{\frac{\rho_m t_h}{T}} R \cdot \omega$$

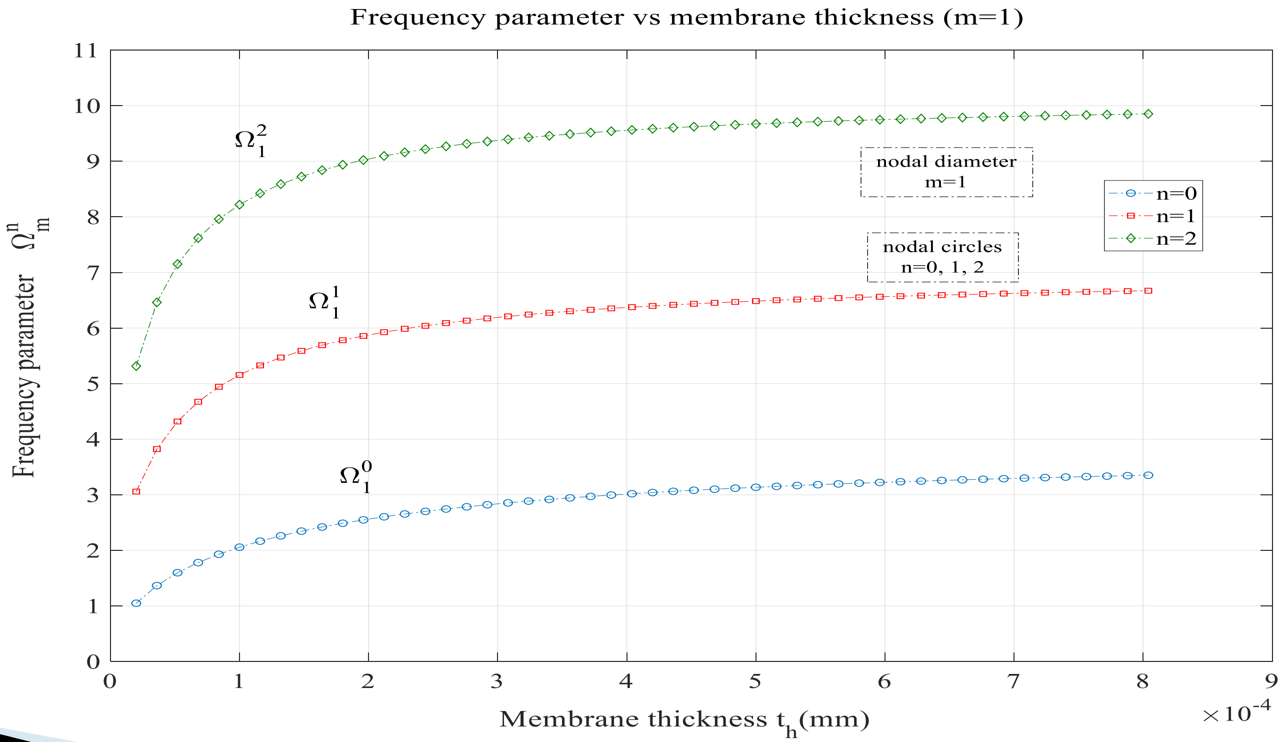

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Fig. 8 Variation of the frequency parameter with the fluid density for different modes, n=0,1,2 nodal circles and zero nodal diameter m=0

$$\Omega = \sqrt{\frac{\rho_m t_h}{T}} R \cdot \omega$$

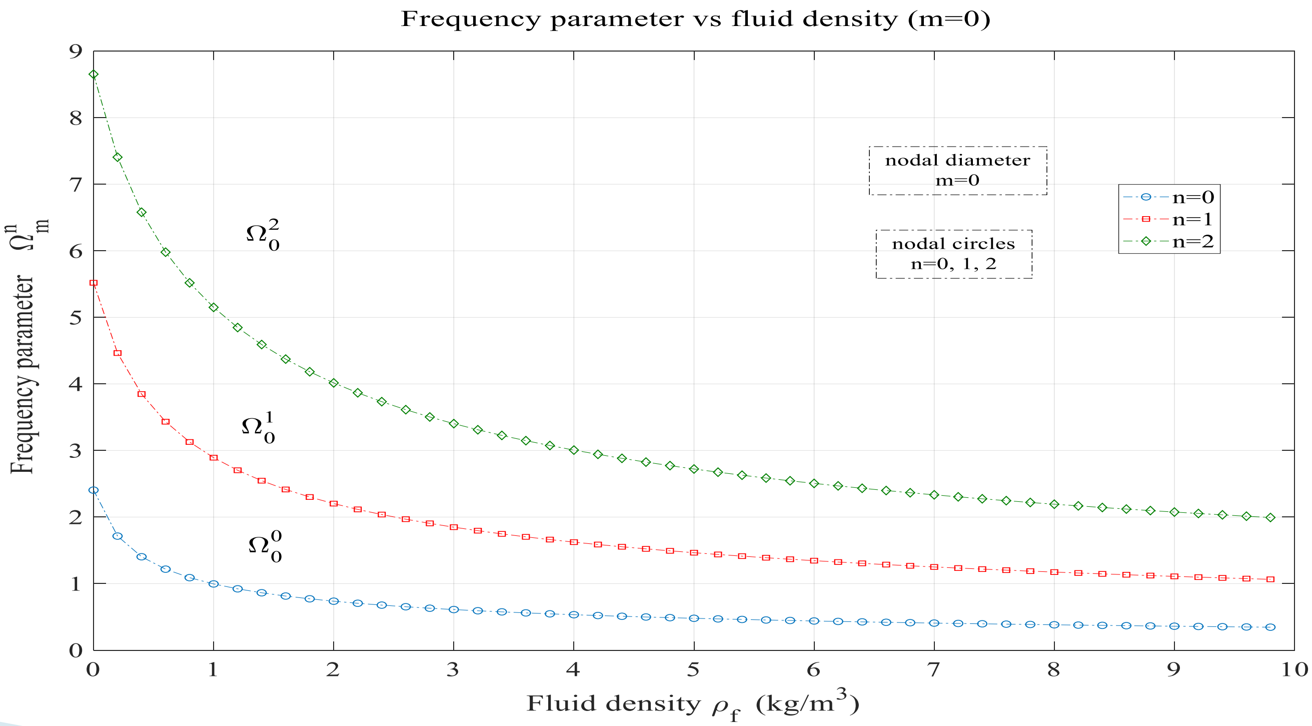

Fig. 9 Variation of the frequency parameter with the fluid density for different modes, n=0,1,2 nodal circles and one nodal diameter m=1

$$\Omega = \sqrt{\frac{\rho_m t_h}{T}} R \cdot \omega$$

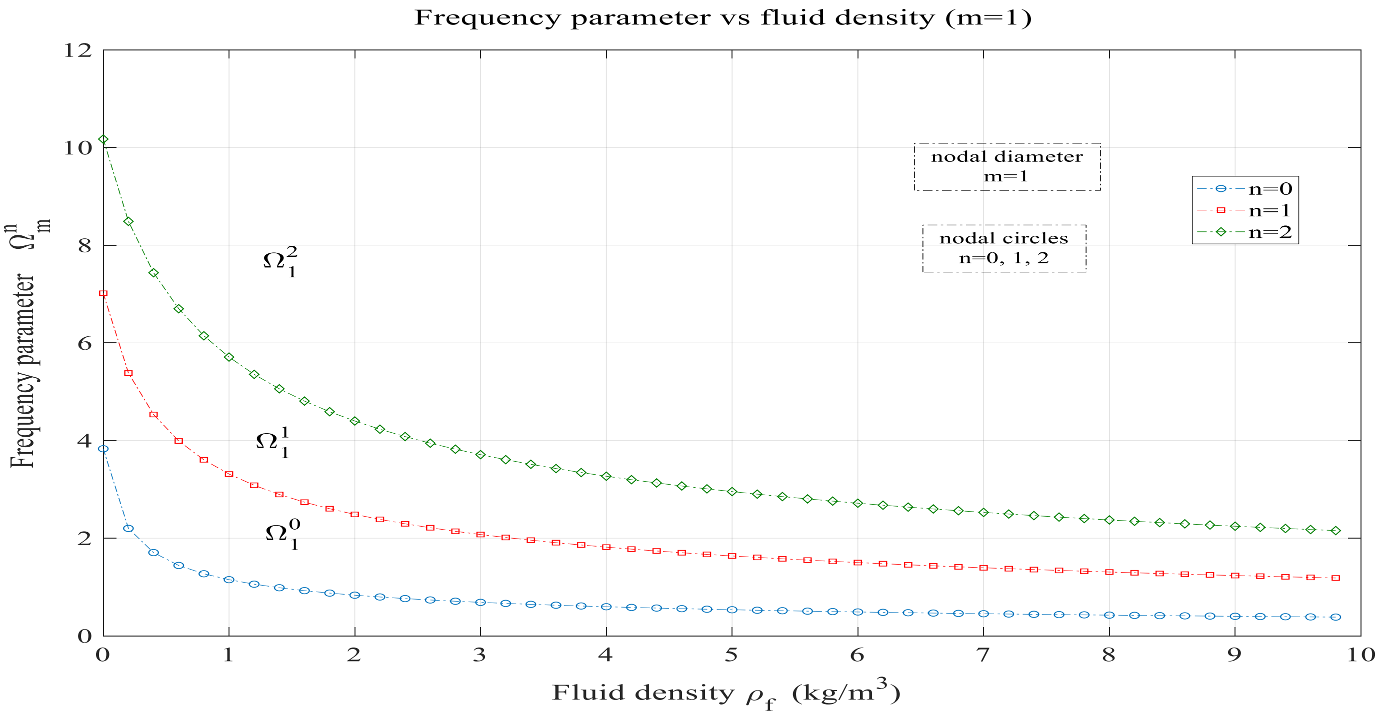

Fig. 10 Variation of the frequency parameter with the membrane material density for different modes, n=0,1,2 nodal circles and zero nodal diameter m=0

$$\Omega = \sqrt{\frac{\rho_m t_h}{T}} R \cdot \omega$$

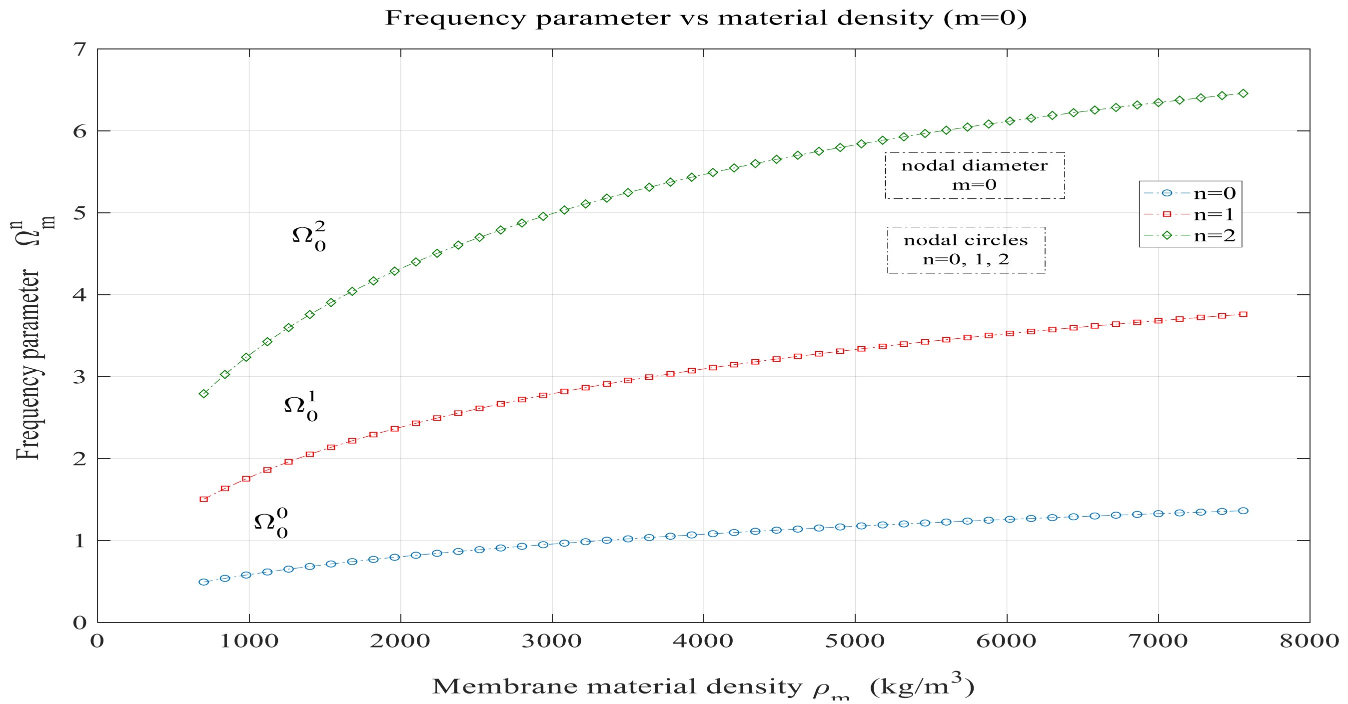

Fig. 11 Variation of the frequency parameter with the membrane material density for different modes, n=0,1,2 nodal circles and one nodal diameter m=1

$$\Omega = \sqrt{\frac{\rho_m t_h}{T}} R \cdot \omega$$

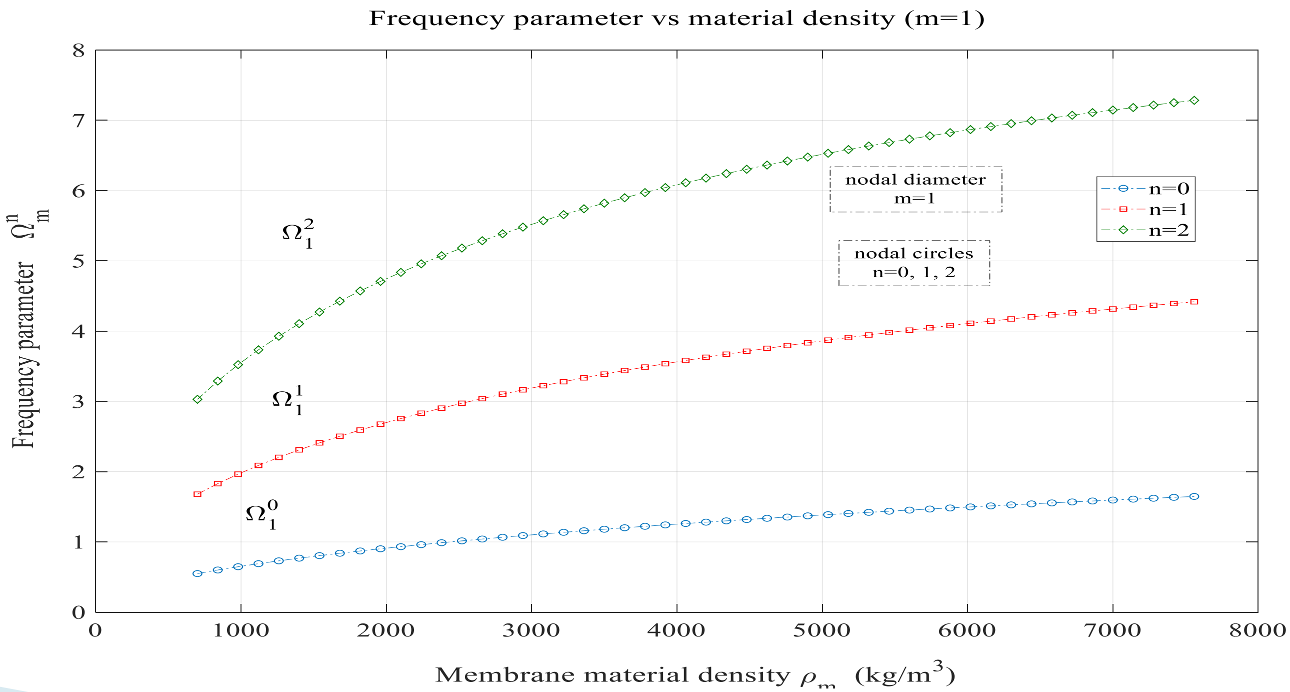

Fig. 12 Variation of the frequency parameter with the fluid sound speed for different modes, n=0,1,2 nodal circles and zero nodal diameter m=0

$$\Omega = \sqrt{\frac{\rho_m t_h}{T}} R \cdot \omega$$

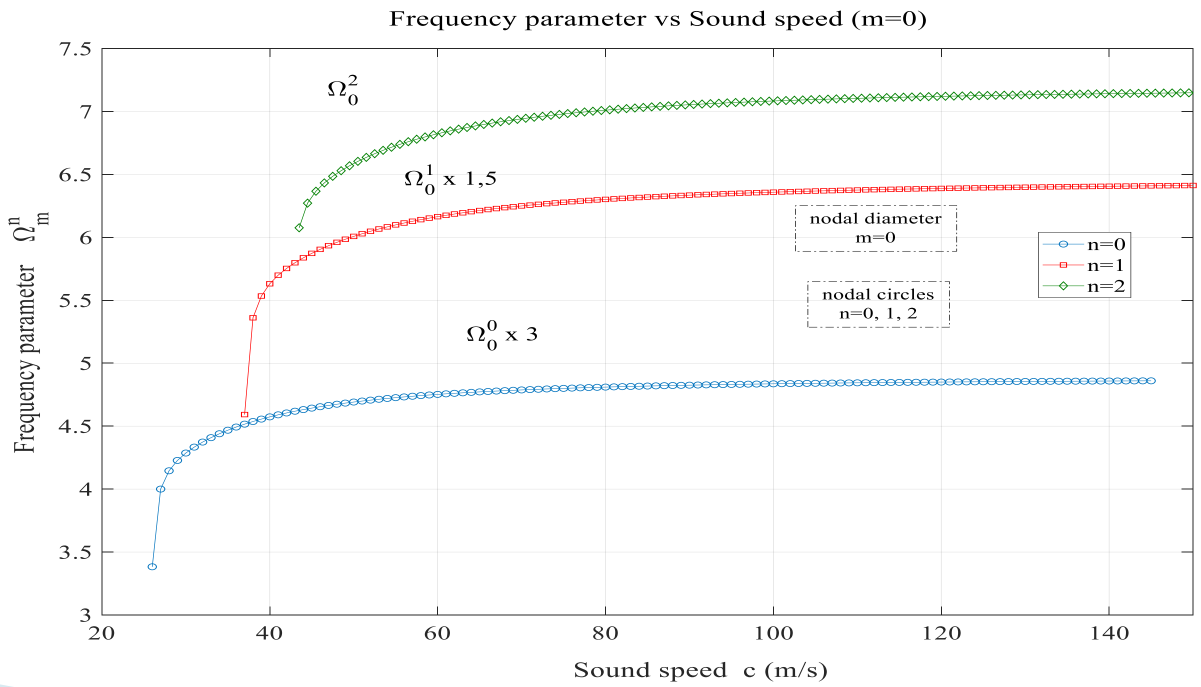

Fig. 13 Variation of the frequency parameter with the fluid sound speed for different modes, n=0,1,2 nodal circles and one nodal diameter m=1

$$\Omega = \sqrt{\frac{\rho_m t_h}{T}} R \cdot \omega$$

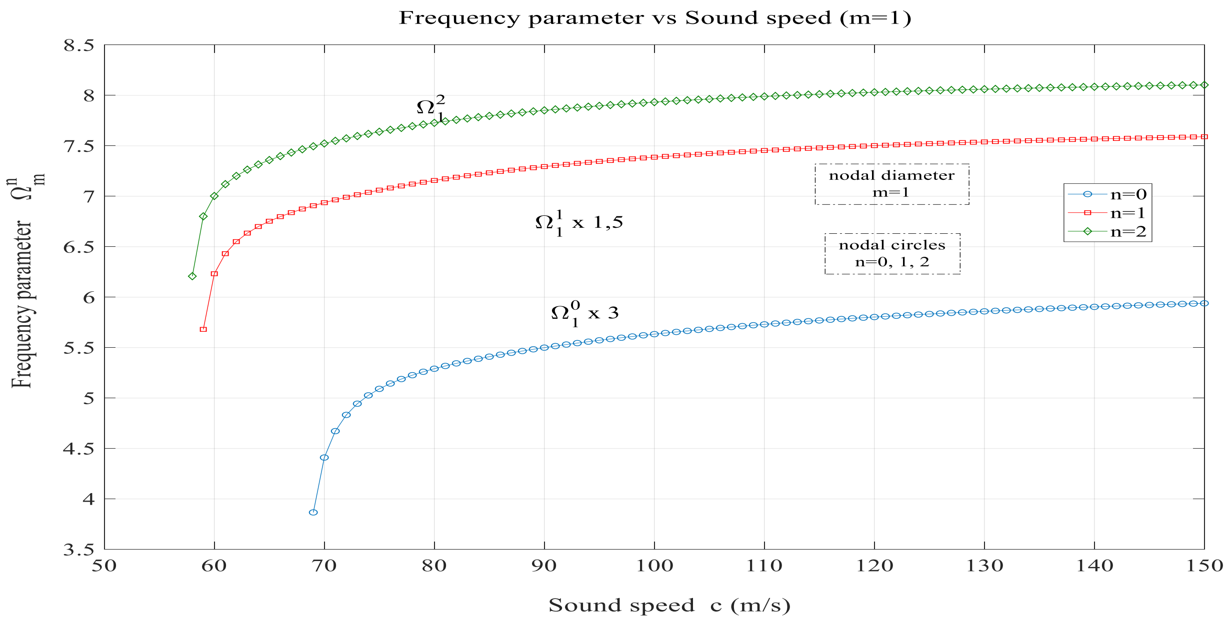

Fig. 14 Variation of the fluid mass parameter with the excitation frequency for the mode m = 0 and n = 0

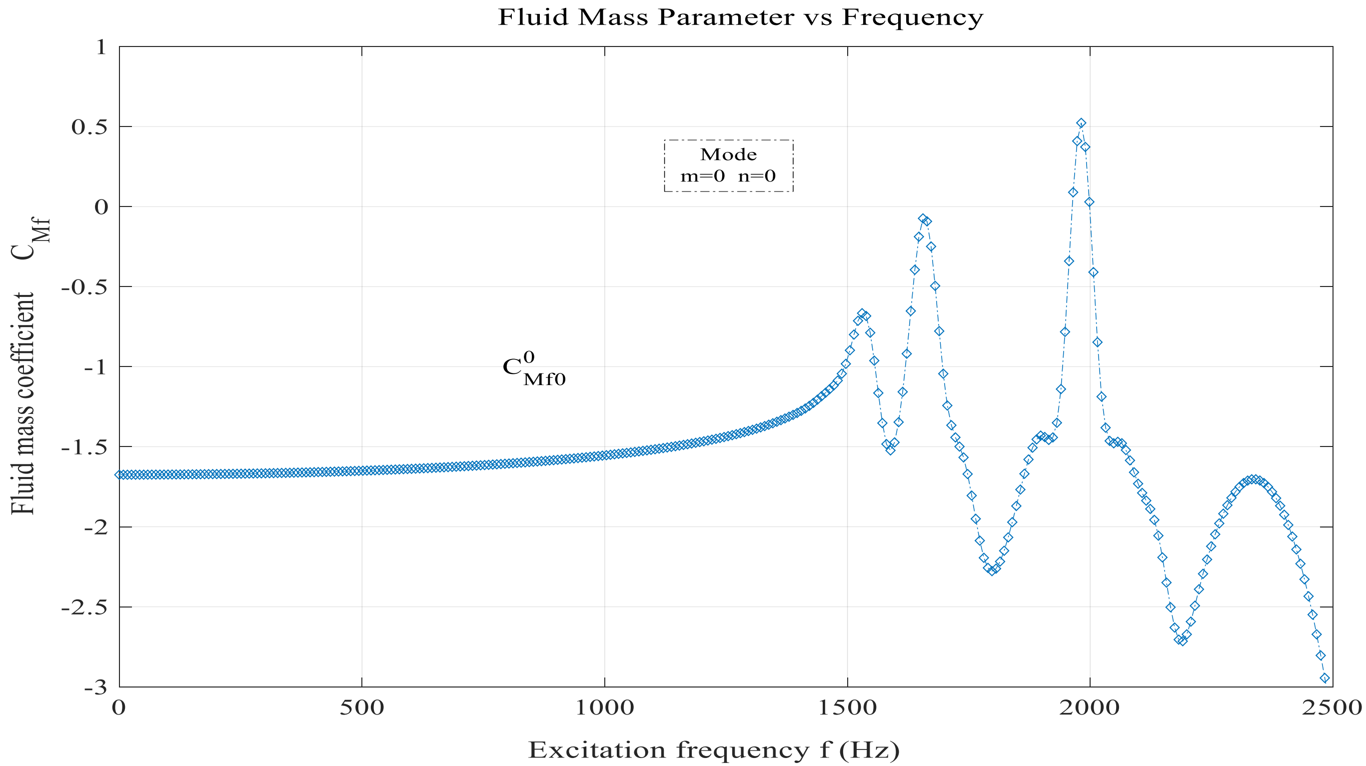

Fig. 13 Variation of the fluid mass parameter with the excitation frequency for the mode m = 1 and n = 0

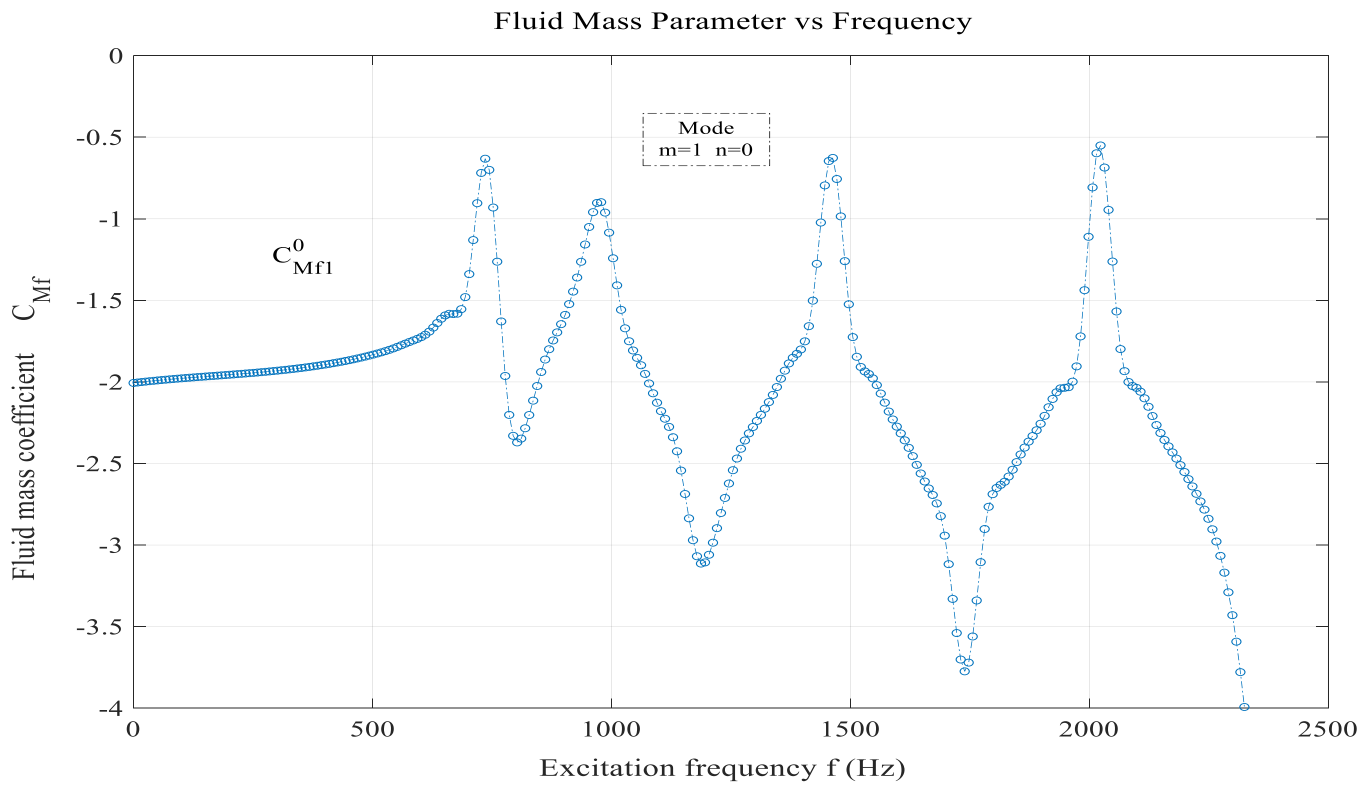

Table 2. Natural frequencies (Hz) for the aluminum membrane of a cylindrical container filled with fluid (air density) and in the vacuum case for different modes, with radius $R = 1\,m$, container height $H = 2\,m$ and membrane thickness $t_h = 0{,}02\,mm$

m , n	f_v	f_a	Frequency reduction (%)
0 , 0	16.47	6.26	62.0
0 , 1	37.81	18.37	51.4
0 , 2	59.27	32.96	44.4
0 , 3	80.76	49.10	39.2
1 , 0	26.24	7.18	72.6
1 , 1	48.05	20.94	56.4
1 , 2	69.68	36.40	47.8
1 , 3	91.26	53.19	41.7

Conclusions

Method calculate frequency top membrane of cylindrical container filled compressible inviscid fluid

Wave equation - Fluid velocity Potential - Separation variables

Momentuum's Linearized equation - Pressure field

Dynamic equation integration – membrane coupled frequencies

Frequency parameter Variation: –Radius, Drum Height, Sound speed, Membrane Thickness, Material density, and Fluid density

Fluid mass parameter frequency Spectrum

Validation other methods