COMPARISON OF WAVEPACKET MODELS FOR SUPERSONIC TWIN JETS AGAINST NEAR-FIELD PRESSURE MEASUREMENTS

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ABSTRACT

Plane-marching PSE wavepackets for a supersonic twin-jet operating at perfectly-expanded conditions are compared against near-field pressure measurements with the purpose of validating the modelling strategy. Two streamwise phased-microphone arrays distributed close to the external mixing-layer boundary of each jet are employed to obtain pressure measurements in the linear hydrodynamic fluctuation region. PM-PSE calculations are performed for the symmetric toroidal mode (mode SS0) based on a tailored twin-jet mean flow constructed via an analytical profile fitted from PIV measurements. Comparisons of the amplitude evolution in the streamwise direction for different frequencies reveal a very remarkable agreement between the SS0 PSE prediction and the symmetry-decomposed pressure signals, particularly when SPOD-filtered experimental measurements are considered. These results constitute a new validation of PM-PSE for modelling twin-jet wavepackets related to mixing noise.

Keywords: *jet noise, wavepackets, twin jets, supersonic*

1. INTRODUCTION

Twin-jet-powered aircraft and launch vehicles generate noise footprints with significant impact on both civil and military applications. The control of such systems at an aeroacoustic level requires a fundamental understanding of the noise-generation mechanisms in the turbulent flow. Under the knowledge gathered from the study of planar mixing layers and single round jets in the past decades [1], it is well established that the machinery of noise generation in turbulent shear flows is mainly powered by the large-scale coherent structures present in the flow [2–6], and that such structures can, in most cases, be predicted by linear stability theory in the form of what is commonly known as wavepackets [7]. On the modelling side, linear parabolized stability equations (PSE) have been found to deliver excellent wavepacket predictions for both subsonic and supersonic jets [8–10].

The existence wavepackets (and their significant role) in twin jets is expected by extension of the insights accumulated for single jets. In fact, late modelling efforts based on plane-marching PSE calculations [11, 12] have produced wavepacket models for twin jets that exhibit similar characteristics to those for single jets, but which no longer feature axisymmetric structures. However, experimental works aimed at their validation and characterization in twin-jet systems are recent and still very sparse, and mainly focused on the screech resonance phenomenon [13]. Following the need for experimental characterization of mixing-noise wavepackets in twin jets, in a recent work [14] we employ high-speed schlieren visualizations to educe coherent structures present in twin jets at perfectly-expanded conditions, and compare them against PM-PSE modes for validation purposes. While good agreement is found in terms of fluctuation structure, the comparison of PSE models with the schlieren images has inherent limitations due to the implicit line-of-sight integration: one of the symmetry planes of the system is inaccessible and the identification of the different oscillation modes (toroidal, flapping) is non-trivial. For this reason, the use of discrete-point experimental techniques

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such as microphone pressure measurements has also been recently considered as a complementary source of empirical data [15].

Using phased microphone arrays positioned just outside of the turbulent mixing layer, combined with cross-correlation techniques such as spectral proper orthogonal decomposition (SPOD), has proven effective to analyze coherent fluctuations in natural subsonic single jets [8, 9, 16–18]. By placing the microphones near the mixing-layer boundary, the measured pressure fluctuations mainly reflect linear irrotational components, with nonlinear perturbations linked to small-scale turbulent structures being, in turn, negligible. Applying SPOD to the phased measurements from multiple microphones enables the successful extraction of the wavepackets, which agree well with PSE predictions [9, 18]. In the case of supersonic jets, the use of the same idea to study mixing noise is virtually non-existent (mainly due to the ubiquity of screech), let alone its application to twin-jet configurations. Recently, we performed an experimental investigation [15] of near-field twin-jet pressure fluctuations employing an azimuthal array of microphones distributed following the mixing layer contours from RANS simulations. Although good agreement was obtained between the experimentally-determined amplitudes and the predicted wavepackets in a significant number of cases, the limitation of measuring one streamwise position at a time prevented the application of SPOD to filter the streamwise wavepacket structure.

The aim of this work is to provide a new quantitative validation of PM-PSE wavepacket models for supersonic twin jets against near-field pressure measurements obtained using a linear microphone array distributed in the streamwise direction, thus enabling the application of SPOD to educe the coherent information contained in the pressure signals. These comparisons complement the aforementioned quantitative analyses based on schlieren visualizations and on an azimuthal microphone array. The manuscript is organized as follows. Section 2 describes the twin-jet experimental apparatus and the adopted processing strategy, section 3 summarizes the numerical methodology employed for the calculation of PM-PSE wavepacket models, section 4 presents the results for the measured pressure signals and the comparison against the computed PM-PSE modes, and section 5 provides concluding remarks.

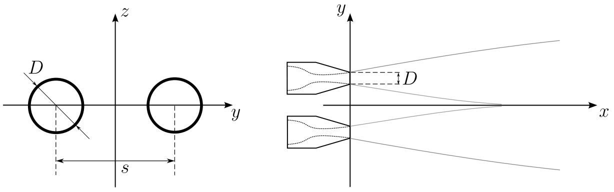

Figure 1. Sketch of the studied twin-jet configuration.

Figure 2. T200 experimental setup: (left) twin-jet system with linear microphone arrays installed; (right) twin-jet system with the 3D-printed part used for microphone positioning.

2. EXPERIMENTAL SETUP

The studied twin-jet configuration consists of two round convergent-divergent nozzles separated by a distance $s = 1.76D$ operating at perfectly-expanded conditions ($M_j = 1.54$), see Fig. 1. The jets are not isothermal and are subject to a total temperature equal to the ambient temperature ($T_0 = T_\infty = 300$ K).

The employed experimental setup is illustrated in Fig. 2, which is built on the T200 wind tunnel facility at Institut Pprime (Poitiers, France). Near-field pressure measurements are carried out using two symmetric streamwise arrays of microphones distributed near the outer mixing-layer boundaries of the twin-jet system. The microphones are located at the symmetry plane containing both jets (xy plane at $z = 0$), with 8 microphones mounted on each array. The left picture on Fig. 2 shows the microphone arrays mounted on the experimental model. The respective streamwise positions of the microphones on each array are $x/D = [0.5, 1, 1.5, 2, 3, 4, 5, 6]$. Their relative position with respect to the twin-jet mixing layers can be observed in Fig. 3.

A 3D-printed part designed using streamwise velocity contours of RANS calculations is employed for adjusting the position of each microphone (see the right picture in Fig. 2). The geometry of this part approximately follows the evolution of the mixing-layer boundary of the twin jet for the conditions under study, such that microphones located right at its surface would be placed in the desired near-field region. More details on the strategy employed for microphone positioning can be found in [15].

2.1 Processing of microphone signals

The symmetries of the twin-jet geometry can be exploited to decompose the microphone signals into symmetric and antisymmetric fluctuations. For the current experimental setup, the symmetry of the configuration across the xz plane allows the calculation of the following symmetry-decomposed pressure signals:

$$p_{0\mathrm{S}} = \frac{p_t + p_b}{2}, \quad p_{0\mathrm{A}} = \frac{p_t - p_b}{2}, \tag{1}$$

where p_t refers to those pressure signals measured by microphones located near the top mixing layer ($y > 0$) and p_b to those measured near the bottom mixing layer ($y < 0$).

The power spectral density (PSD) of each signal is calculated following Welch's method. First, the pressure fluctuation is obtained by removing the mean from the measured signal:

$$p'(t) = p(t) - \frac{1}{N}\sum_{i=1}^{N} p_i. \tag{2}$$

Then, the power spectral density is computed as:

$$P_{xx}(f) = \frac{1}{\Delta f N_b G_w}\sum_{j=1}^{N_b} |\hat{p}_j(f)|^2, \tag{3}$$

where $\hat{p}_j(f)$ denotes the windowed Fourier-transform of the pressure fluctuation signal for block j, expressed in terms of the fast Fourier transform (FFT) as:

$$\hat{p}_j(f) = \frac{1}{N_{\mathrm{FFT}}}\mathrm{FFT}[w(t)p'_j(t)], \tag{4}$$

where $w(t)$ is a window function and N_{FFT} denotes the block size. The quantity Δf is the frequency bin size, N_b denotes the total number of blocks, and G_w is the energy correction associated to the applied window function (a Hamming window in this case), defined by

$$G_w = \frac{1}{N_{\mathrm{FFT}}}\sum_{i=1}^{N_{\mathrm{FFT}}} w_i^2. \tag{5}$$

The pressure fluctuation signals are split into 487 blocks of 8192 samples each featuring a 50% overlap. The PSD results are converted to dB using the auditory threshold pressure ($p_{\mathrm{ref}} = 20$ µPa) as a reference, i.e. P_{xx} [dB SPL] $= 10\log(P_{xx}/p_{\mathrm{ref}}^2)$.

Since all microphone signals are recorded at the same time and the twin-jet dynamics are assumed to be statistically stationary, the spatio-temporal coherence present in the data can be extracted via spectral proper orthogonal decomposition (SPOD). In this case, the SPOD eigenvalue problem takes the form [19]:

$$\mathbf{S}_k\mathbf{\Psi}_k = \mathbf{\Psi}_k\mathbf{\Lambda}_k, \tag{6}$$

where $\mathbf{S}_k$ is the 8×8 cross-spectral density tensor for the kth frequency, defined as:

$$\mathbf{S}_k = \frac{1}{\Delta f N_b G_w}\hat{\mathbf{Q}}_k\hat{\mathbf{Q}}_k^H, \tag{7}$$

with $\hat{\mathbf{Q}}_k$ being the matrix of frequency-domain realizations. $\mathbf{\Psi}_k$ is the matrix containing the eigenvectors for the kth frequency and $\mathbf{\Lambda}_k$ denotes the diagonal matrix containing the associated eigenvalues λ_i, which represent the PSD associated with the ith SPOD mode. The superscript H denotes the complex-conjugate transpose.

The same block size, overlap and window function used for the calculation of the PSD are employed for the SPOD. Since for the current analysis the number of blocks is much larger than the number of spatial points, the formulation of the SPOD problem given by equation (6) requires much less computational effort than the more typical snapshot method formulation [20].

3. NUMERICAL METHODOLOGY

3.1 Mean flow

In this work, the twin-jet mean flow is constructed from the linear superposition of two single-jet mean flows, usually known in previous investigations [11, 12] as a tailored

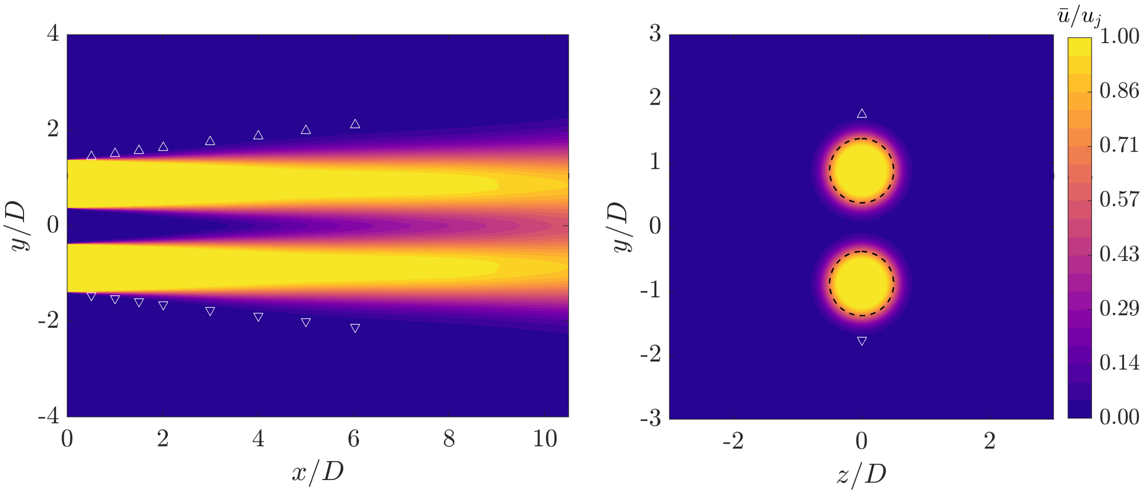

Figure 3. Contours of the tailored twin-jet mean flow employed for the PM-PSE calculations: (left) symmetry plane at $z = 0$; (right) cross-stream plane at $x/D = 3$. The white symbols illustrate the measurement position of the microphones with respect to the mean flow solution.

twin-jet model. The tailored twin-jet mean streamwise velocity component $\bar{u}$ is obtained as the sum (linear combination) of the streamwise velocity fields of two single jets with centers respectively located at $y = \pm s/2$. Here, the velocity components in the y and z directions are neglected. The mean temperature field of the twin jet is estimated from the constructed streamwise velocity field through the following Crocco-Busemann relation:

$$\frac{\bar{T}}{T_j} = \left[1 + \frac{\gamma - 1}{2} M_j^2 \left(1 - \frac{\bar{u}}{u_j}\right)\right] \frac{\bar{u}}{u_j} + \left(1 - \frac{\bar{u}}{u_j}\right) \frac{T_\infty}{T_j}, \tag{8}$$

where T_j and u_j refer to the isentropic jet-exit conditions fixed by M_j and T_0, while the mean density field is obtained from the perfect gas equation of state under the assumption of a constant static pressure field $\bar{p} = p_\infty = 10^5$ Pa.

To obtain the single-jet mean flow, an analytical profile based on the Gaussian function introduced by [21, 22] is fitted to PIV measurements of the xy twin-jet symmetry plane. This profile has been used in recent studies of single jets [9] and twin jets [11, 12] with satisfactory results. It is defined as follows:

$$\bar{u}(x, r) = \begin{cases} \bar{u}_c(x), & r < R(x), \\ \bar{u}_c(x) \exp\left[-\dfrac{(r - R(x))^2}{\delta(x)^2}\right], & r \geq R(x), \end{cases} \tag{9}$$

where r denotes the radial coordinate. The parameters $\bar{u}_c$, R and δ respectively represent the mean streamwise velocity of the jet along the nozzle axis, the radial extent of the potential core and the thickness of the mixing layer, which are functions of the streamwise coordinate. These parameters are fitted from the PIV data using a cubic spline, similarly to [9, 12]. More details on the employed PIV measurements can be found in [23]. Although the PIV data corresponds to a twin-jet mean flow and therefore contains both the outer and inner shear layers of each jet, only the external mixing layer of one of the two jets is used for the estimation of $\delta(x)$.

Fig. 3 depicts contours of the tailored twin-jet mean flow obtained following the aforementioned procedure. The triangular symbols represent the position of the different microphones involved in the experimental pressure measurements, showing that they are placed right outside of the external mixing layer of each jet.

3.2 Wavepacket modelling via plane-marching PSE

Twin-jet wavepackets under perfectly-expanded conditions are modelled by means of linear plane-marching parabolized stability equations (PM-PSE). This extension of the classical PSE formulation accounts for mean flows featuring a strong inhomogeneity in two spatial directions, namely, the cross-stream planes in this case. Details on the employed PM-PSE formulation can be found in [11]. In brief, a perturbation ansatz of the following form is considered:

$$\hat{\mathbf{q}}_\omega(x,y,z) = A_\omega \tilde{\mathbf{q}}_\omega(x,y,z) \exp\left(\mathrm{i} \int_{x_0}^{x} \alpha_\omega(\xi) \mathrm{d}\xi \right), \tag{10}$$

where $\hat{\mathbf{q}}_\omega$ denotes the fluctuation at the angular frequency $\omega = 2\pi f$, which is split into a three-dimensional shape function with a slow evolution in the streamwise direction, given by $\tilde{\mathbf{q}}_\omega$, and a wave function featuring a rapid variation along x, given by the exponential term. A_ω represents the initial amplitude and $\alpha_\omega = \alpha_r + \mathrm{i}\alpha_i$ corresponds to the streamwise wavenumber, for which a slow variation with x is also assumed.

The substitution of (10) into the linearized Navier-Stokes equations, together with an order of magnitude analysis that neglects terms of order $1/Re^2$, yields a parabolized initial value problem that can be solved by marching in the streamwise direction.

4. RESULTS

4.1 Measured pressure spectra

The obtained PSD of the symmetry-decomposed pressure signals is displayed in Fig. 4, which shows maps of P_{xx} as a function of x and St for $p_{0\mathrm{S}}$ and $p_{0\mathrm{A}}$. These maps illustrate the range of frequencies at which mixing noise is dominant in the studied twin jet, namely $St \approx [0.1, 1]$. This is the range of interest for comparison against the wavepackets predicted by PM-PSE. As expected in the near-field region where hydrodynamic fluctuations remain important, the streamwise range of linear amplitude growth becomes shorter as the frequency is increased.

Both the symmetric and antisymmetric pressure signals reveal similar spectral signatures, indicating that both kinds of fluctuations are present and are equally important in the twin-jet system.

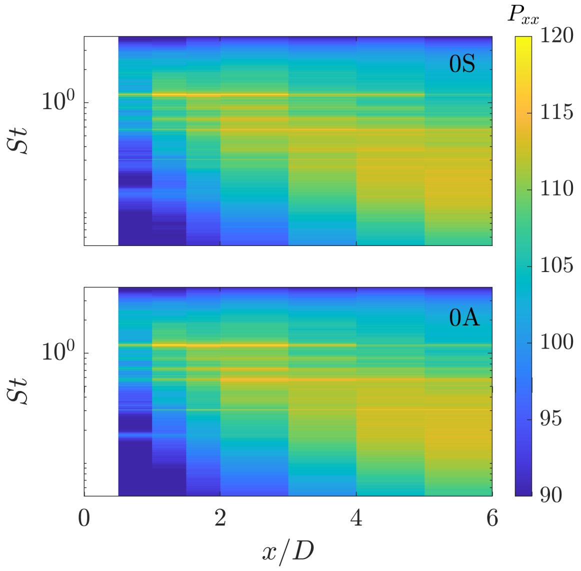

Figure 4. PSD (in [dB SPL/Hz]) of the symmetry-decomposed pressure signals ($p_{0\mathrm{S}}$ and $p_{0\mathrm{A}}$) as a function of x and St.

4.2 Comparison of modelled wavepackets against measured signals

The results presented herein focus on the comparison between the symmetric pressure signal $p_{0\mathrm{S}}$ and the PM-PSE mode SS0, which corresponds to a wavepacket that is symmetric with respect to both twin-jet symmetry planes and represents toroidal Kelvin-Helmholtz instabilities which are analogous to those of $m = 0$ modes in axisymmetric single jets. To illustrate its three-dimensional structure, Fig. 5 displays contours of the PM-PSE mode SS0 for $St = 0.7$ in two different cutplanes: the twin-jet symmetry plane at $z = 0$ and the cross-stream plane at $x/D = 3$. The ring-like structures are clearly visible in the cross-stream plane, which are not axisymmetric owing to the close spacing between the two jets. The fluctuation contours in the symmetry plane highlight the Mach-wave radiation associated with the wavepacket that is typical of mixing noise in supersonic jets. The relative position of the microphone probes with respect to the predicted wavepacket structure are indicated by the triangular symbols.

The initial amplitude of wavepackets calculated via linear stability theory is defined up to an arbitrary constant. For the purpose of comparison against experimen-

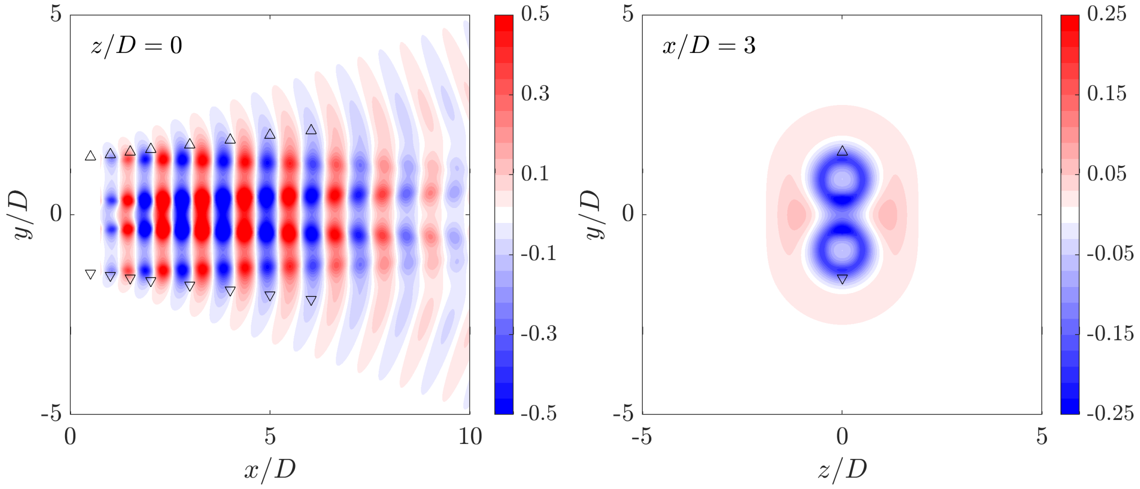

Figure 5. Contours of the real part of the pressure fluctuation for PM-PSE mode SS0 at $St = 0.7$: (left) symmetry plane at $z/D = 0$; (right) cross-stream plane at $x/D = 3$. The contours are normalized with respect to the maximum value of the real part of $\hat{p}$. The black symbols denote the measurement positions of the microphones.

tal measurements, the value of A_ω for a given PM-PSE mode at a given frequency is calibrated by means of a least squares fit to the amplitude of the measured pressure signals, resulting in the following estimation:

$$A_{\omega,p} = \frac{\sum_{j=1}^{N_{\text{mic}}} |\hat{p}_\omega(x_j, y_j, z_j)| \sqrt{P^j_{xx,\omega}}}{\sum_{j=1}^{N_{\text{mic}}} |\hat{p}_\omega(x_j, y_j, z_j)|^2}, \tag{11}$$

where $\hat{p}_\omega(x_j, y_j, z_j)$ denotes the PM-PSE pressure fluctuation for frequency ω evaluated at the measurement position of microphone j, while $P^j_{xx,\omega}$ represents the PSD of the pressure signal measured by microphone j for frequency ω. This is also the approach followed by previous works on single jets (see e.g. [9]).

The comparison between the measured symmetric pressure signal and PM-PSE mode SS0 is presented in Fig. 6, which shows the evolution of the amplitude as a function of the streamwise coordinate for different frequencies spanning the previously identified range of interest. Two different sets of experimental data are included for each case, namely, the original symmetric pressure signal, marked with star symbols, and the first SPOD mode obtained from the SPOD-filtered symmetric pressure signal, marked with square symbols. Correspondingly, two different curves are represented for mode SS0: that with the initial amplitude calibrated with the original measurements (solid lines, labelled A_p), and that with the amplitude calibrated using the first SPOD mode extracted from the measurements (dashed line, labelled A_Ψ). Overall, a very good agreement is obtained between the PM-PSE wavepacket predictions and the experimental measurements for most of the studied frequencies. Except for the lowest frequency ($St = 0.1$), the match between the PM-PSE model and the measured amplitudes is significantly improved when the SPOD-filtered measurements are considered, especially near and downstream of the saturation point for higher frequencies.

These results suggest that, even though no azimuthal mode decomposition is applicable in the twin-jet case, and therefore the measured signals are expected to contain contributions from multiple modes, the SS0 wavepacket is one of the most dominant contributions to the dynamics of the system, since by itself it can represent the measured amplitude evolution with remarkable accuracy for most frequencies. This comparison constitutes a new quantitative validation of the ability of PM-PSE to model mixing-noise wavepackets in twin jets, which complements our previous quantitative comparisons based on the coherent structures educed from high-speed experimental measurements [14].

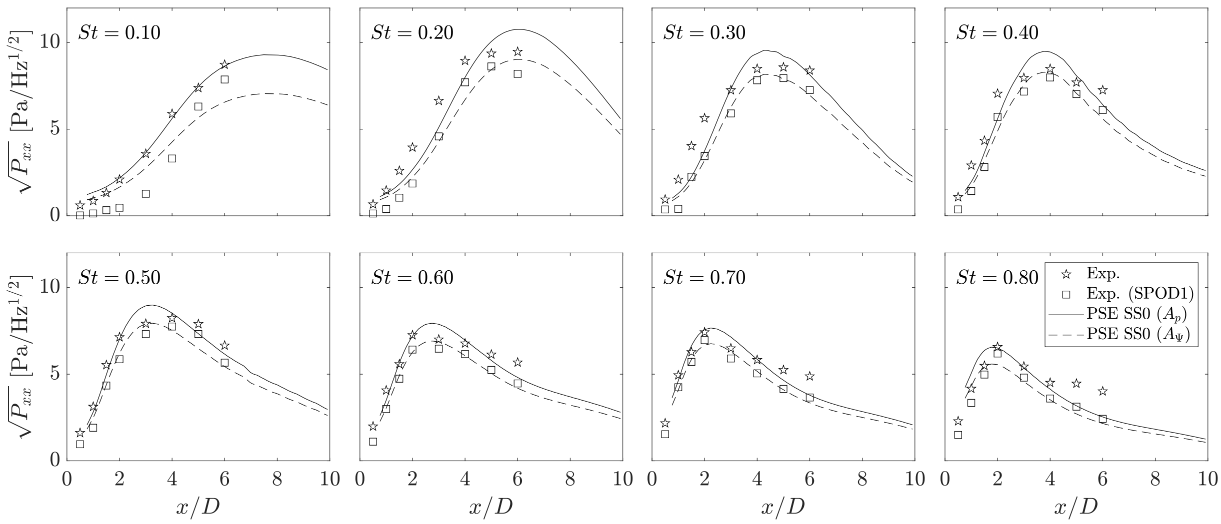

Figure 6. Amplitude comparison between the measured symmetric pressure signal (p_{0S}) and the PM-PSE wavepacket for mode SS0 as a function of the streamwise coordinate for different frequencies. The star symbols denote the experimental measurements; the squared symbols represent the first SPOD mode extracted from the experimental signals; the solid lines indicate the PM-PSE amplitude evolution calibrated directly with the experimental measurements (A_p); the dashed lines correspond to the PM-PSE amplitude evolution calibrated with the first SPOD mode of the experimental results (A_Ψ).

5. CONCLUSIONS

Wavepacket models for mixing noise in twin jets are still premature and require comparison against experimental measurements to assess their validity and accuracy. In this work, wavepackets predicted by plane-marching PSE for a supersonic twin jet operating at perfectly-expanded conditions have been compared against experimental pressure measurements in the near field.

Two streamwise arrays with 8 microphones each are employed to obtain phased measurements in the symmetry plane containing the two jets, with the microphones positioned right outside of the external mixing layer of each jet to target the linear region of the hydrodynamic fluctuation field. The spectral characteristics of the obtained measurements exhibit the desired features expected in the linear hydrodynamic regime and reveal the range of frequencies at which mixing noise is dominant ($St \approx [0.1, 1]$), which in turn guide the wavepacket modelling.

PM-PSE calculations are performed based on a tailored twin-jet mean flow obtained by fitting an analytical axisymmetric jet profile to PIV measurements of the twin jet system. The analysis focuses on the symmetric toroidal mode SS0, for which the predicted wavepackets feature three-dimensional Kelvin-Helmholtz instabilities which deviate from the familiar axisymmetric structures known from single jets.

The comparison between the amplitude of the symmetry-decomposed pressure measurements and the PM-PSE mode SS0 yields a very good agreement for most frequencies in the studied range, particularly when the SPOD-filtered pressure measurements are considered. The obtained results provide a new quantitative assessment of the capability of plane-marching PSE to model twin-jet wavepackets associated with mixing noise. Future work should involve the linear combination of multiple PM-PSE modes in the comparison, as well as the validation of antisymmetric fluctuations.

6. ACKNOWLEDGMENTS

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