Recursive Active Filter with Metamaterial Unequal Wilkinson Power Dividers

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Abstract— **In this work, it is shown that it is possible to implement an unequal Wilkinson power divider with reduced dimensions by using metamaterial transmission lines based on complementary split ring resonators (CSRRs), in planar technology. In addition, this combiner has been used to implement a compact recursive active filter. Thanks to the use of the metamaterial combiner it is possible obtain a filter with small dimensions. Additionally, due to the asymmetry in the power divider it is possible obtain high gain and low noise at the same time that minimizes the number of lumped element.**

I. INTRODUCTION

In this last decade the resonant-type metamaterial transmission lines, either loaded with split ring resonators (SRRs) [1] or with complementary split ring resonators (CSRRs) [2], have been used in several planar passive devices due to the possibility to control their electrical characteristics (phase and characteristic impedance) and obtain smaller dimensions than with conventional transmission lines [3]. Some examples of these devices are filters [4], [5], branch-lines [6], rat-races [6] or power dividers [7], [8]. As an example, the layout of a typical metamaterial unit cell based on CSRRs is shown in Fig. 1(a). The equivalent circuit model of the unit cell is depicted in Fig. 1(b) where L and C_L are the inductance and the capacitance of the host line, C_S is the series capacitance of the gap, C_f is the fringing capacitance of the gap and the inductance and the capacitance of the CSRR are modelled by L_C and C_C respectively.

Due to the degrees of freedom (more than in conventional transmission lines) that these artificial transmission lines offer, it is possible to obtain values for the phase and the impedance not achievable with conventional lines [3].

Moreover, microwave active filters are an appropriate solution when a compact filtering structure with low losses and high selectivity is desired. Additionally, some issues as noise figure, electrical stability and power consumption must be addressed. There are different types of active filter technologies, which include devices based on negative resistances, active inductors, and transversal and recursive structures. This work deals with the design of a recursive filter, which are filtering topologies based on combining the signals coming from different paths, including feedback branches [9]. The most basic configuration is the first-order one, whose schematic representation is depicted in Fig. 2. For the practical implementation in microwave technology, it may require the use of an amplifier, transmission lines and power combiners.

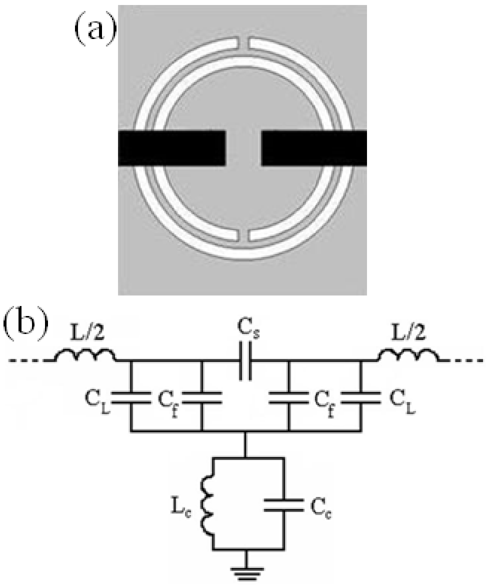

Fig. 1 Typical layout of a unit cell of a metamaterial transmission line based on CSRRs (a). Lumped element equivalent circuit model of the unit cell of a metamaterial transmission line based on CSRRs (b). In (a), the host line (depicted in black) is a microstrip transmission line with a gap and the CSRR is etched in the ground plane (depicted in gray).

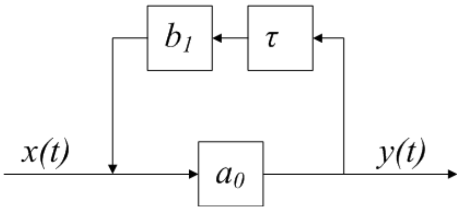

Fig. 2 Flow-graph of a first-order recursive active filter.

In this work, a compact recursive active filter with an unequal Wilkinson power divider using metamaterial transmission lines based on CSRRs has been designed and presented. This filter presents high gain and low noise and, simultaneously, small dimensions. Moreover, the use of an asymmetrical power divider in planar technology allows us to minimize the number of lumped elements necessary for the filter implementation.

II. UNEQUAL WILKINSON POWER DIVIDER

Power dividers and combiners are used in many kinds of RF/microwave devices. The Wilkinson topology shows a basic and simple concept to divide the power by a simple structure [10]. On the other hand, the unequal Wilkinson power divider has been used with strict restrictions in the design and fabrication because it requires a microstrip line with very high or low impedance. This makes the fabrication very complicated. This problem can be solved using different methods [11], [12], [13]. In this work we propose the use of metamaterial transmission lines based on CSRRs. With this kind of lines it is possible to obtain extreme values of impedance and, at the same time, reduced dimensions of the proposed circuits are achieved [3].

The topology of the proposed unequal Wilkinson power divider is shown in Fig. 3. For this work we have designed the divider with a power ratio of 0.25 at 1.5 GHz. The method to obtain the impedance for the sections is reported in [14]. We have used this method due to the fact that it gives us an additional degree of freedom, the possibility to choose the impedance for one of the sections.

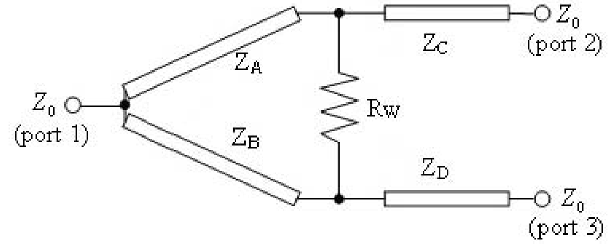

Fig. 3 Model for an unequal Wilkinson power divider.

The equations used to obtain the values of the characteristic impedance needed for the combiner with a ratio of 0.25 and a chosen impedance value Z_A are:

$$Z_B = \frac{Z_A}{Ratio} \tag{1}$$

$$Z_C = \frac{Z_A}{\sqrt{1+Ratio}} \tag{2}$$

$$Z_D = \frac{Z_C}{\sqrt{Ratio}} \tag{3}$$

$$R_W = \frac{Z_C^2}{Z_0} + \frac{Z_D^2}{Z_0} \tag{4}$$

For this case the value of the characteristic impedances, respect to the Fig. 3, are: $Z_0 = 50\ \Omega$, $Z_A = 13.83\ \Omega$, $Z_B = 55.06\ \Omega$, $Z_C = 12.36\ \Omega$, $Z_D = 24.67\ \Omega$ and $R_W = 15\ \Omega$. These values of characteristic impedance are obtained with the method reported in [14].

The layout obtained for the power divider can be seen in Fig. 4. The sections implemented by metamaterial cells have been designed to obtain a phase $\beta l = -90°$ and its corresponding characteristic impedance. For the design of each cell, the final layout of the power divider that minimizes the area and allows the connection of the resistance (R_W) has been considered. Although the topology of the cells can look complicated, due to the fact that the dimensions of each cell are smaller than the guided wave length (λ_g) at the operation frequency, the cells operate like a block with the corresponding phase and impedance (this is a characteristic of the metamaterials).

The complete power divider designed can be contained in a square with an area of ($\lambda_g / 4 \times \lambda_g / 4$), as can be seen in Fig. 4. The simulated frequency response for the metamaterial unequal Wilkinson power divider is shown in Fig. 5.

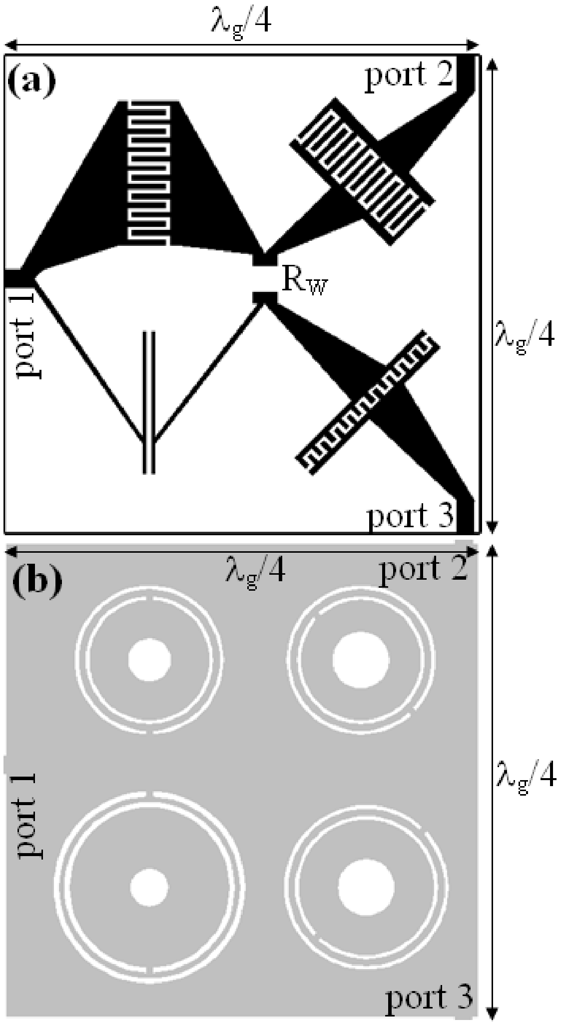

Fig. 4 Layout for an unequal Wilkinson power divider designed with metamaterial unit cells in all of their sections. Top (a)and bottom (ground plane) (b).

The simulation has been obtained from full wave electromagnetic simulation by means of the *AGILENT ADS/MOMENTUM* commercial software. The substrate is *ARLON 25N* with thickness $h = 0.5$ mm, dielectric constant $\varepsilon_r = 3.38$ and loss tangent $\delta = 0.0025$.

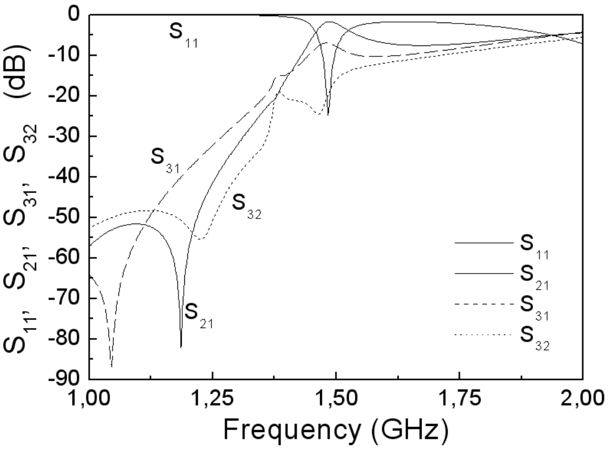

Fig. 5 Simulated response for the metamaterial unequal Wilkinson power divider represented in Fig. 4 with a rate of 0.25 at 1.5 GHz.

III. RECURSIVE FILTER DESIGN

The circuit scheme of a first-order active filter is shown in Fig. 6. In this case, two unequal Wilkinson power dividers are used in order to combine the signals at the input and output, as well as to provide isolation between the feedback line and the input in order to avoid instabilities in the circuit. These combiners are assumed to be asymmetrical, with a certain power balance given by the transmission parameters α_i and β_j. In the direct branch of the filter an amplifier is placed with a gain value A. Some fraction of the output power is combined with the input by means of a feedback transmission line with transmission parameter γ. All the blocks of the scheme are assumed to be well matched to 50 Ω. Thus, the transfer function of the structure can be obtained as

$$H_f = \frac{A\alpha_1\alpha_2}{1-\beta_1\beta_2\gamma A}. \tag{5}$$

This transfer function H_f is dependent with frequency, and it presents a band-pass response, with the pass bands located at the frequencies at which the overall phase response of the filter loop is multiple of 2π,

i.e.: $\angle A + \angle\gamma + \angle\beta_1 + \angle\beta_2 = k2\pi$, (6)

being k any integer number. Thus, given the two power combiners and the corresponding amplifier, the designer can adjust the phase of the feedback line in order to obtain the filtering response at the desired frequency.

The noise figure of the filter presented in Fig. 7 can be obtained as a function of the noise figure of the amplifier as [15]

$$F_f = 1 + \frac{1}{\alpha_1^2}(F_a - 1) + \left(\frac{\beta_2 - \beta_1 A\gamma}{\alpha_1\alpha_2 A}\right)^2, \tag{7}$$

where F_a is the noise figure of the gain block. Since the last term of the equation is always positive, and the term α_1 is always lower than one, it can be checked that the noise figure of the filter presents a lower bound given by the noise figure of the isolated amplifier. That is, the noise performance of the filter will unavoidably be worse than the noise figure of the isolated amplifier.

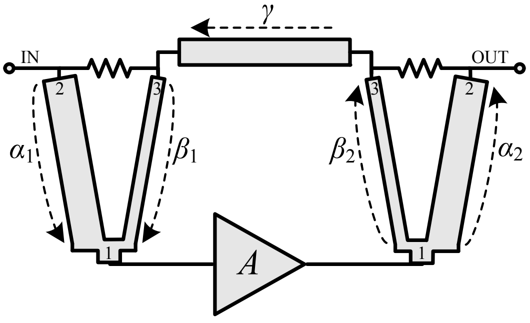

Fig. 6 Circuit schematic of a first-order recursive active filter.

The aim of using asymmetrical power combiners, as the one presented in the previous section, is justified by several reasons. Firstly, the noise performance of the filter highly depends on the power balance given by such combiners [15]. Thus, increasing the transmission factor of the coefficients in the direct branch, especially α_1, can significantly improve the noise performance of the filter. This effect can be seen in Fig. 7, where the noise performance of an active filter as a function of the noise figure of the gain block is shown for two different combiners: the asymmetric Wilkinson of the previous section and a conventional 3 dB hybrid. On the other hand, the total gain of the filter loop should be limited to be under 0 dB, in order to avoid stability problems [16]. A typical solution to solve this problem is to place an attenuator in series with the gain block [17]. As it can be seen in Eq. (5), the problem of limiting the gain factor A is that the transfer function of the filter will decrease as well. The option used in this case is to limit the power transmission in the loop by means of reducing the transmission coefficients β_j of the combiners. In this way, a passive attenuator is not required and higher filter gain values are reachable. However, despite the mentioned advantages when using unequal-split combiners, the main limitation of such devices is their size. Thus, the use of a compact Wilkinson based on metamaterial cells can be very useful to reduce the overall size of the filter. Furthermore, the same technology can be used to implement the feedback transmission line with the same purpose.

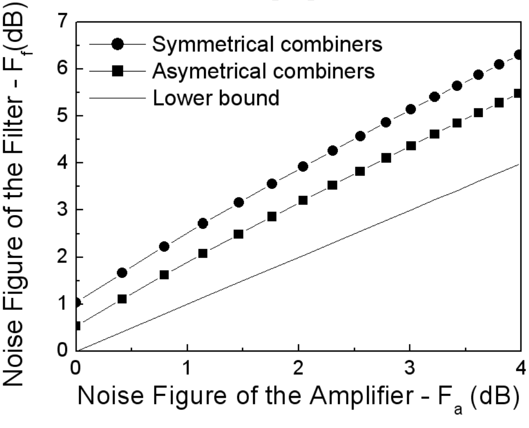

Fig. 7 Noise performance of the active filter as a function of the noise figure of the amplifier: for a symmetrical combiner (α_1=β_1=3 dB), and for the implemented asymmetrical combiner (α_1=-2 dB and β_1=-8 dB), both with the same gain in the pass-band of the filter and with ideal feedback line.

Fig. 8 shows the simulated results obtained with a first-order filter based on the topology shown in Fig. 6, and that makes use of the asymmetrical Wilkinson presented in the previous section (Fig. 4). The gain block has been implemented by using a commercial monolithic amplifier, model *ERA-5+* of *MINICIRCUITS*. It presents a gain of 12.3 dB, and a noise figure of 3 dB, at 1.5 GHz. The feedback line has been implemented by using a metamaterial cell (Fig. 1(a)) as the ones used in the design of the combiners. Its phase has been adjusted to obtain the pass band centred at 1.5 GHz, the same frequency at which the combiners were designed. As it can be seen in Fig 8, the gain of the filter is around 20 dB, which is more than 7 dB higher than the isolated amplifier. Attending to Fig. 7, the expected noise figure of the filter is 4.3 dB, which is around 0.9 dB better than a hypothetical design based on symmetrical combiners.

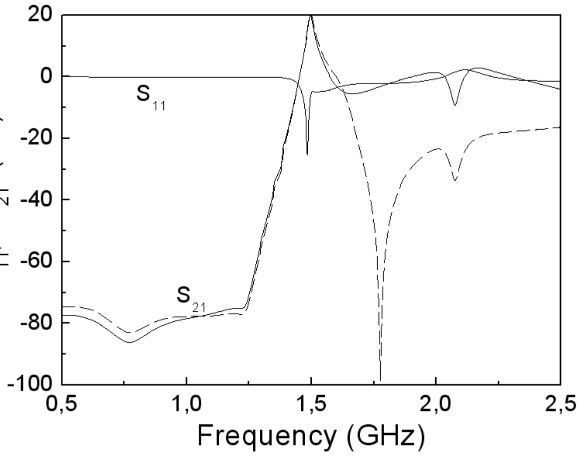

Fig. 8 Simulated reflection (S_{11}) and transmission coefficient (S_{21}) response for the recursive active filter (solid lines). Simulated transmission coefficient for the same filter with CSRRs in both ports (input and output) to obtain the transmission zero (dashed line) at high frequencies.

In order to obtain a transmission zero above the central frequency of the filter, we have added two CSRRs, one at the input port and another at the output port, which introduce a notch to enhance the selectivity of the filter, as can be seen in Fig. 8 (dashed line).

IV. CONCLUSIONS

In conclusion, in this work it has been designed a metamaterial unequal Wilkinson power divider with a ratio of 0.25 at 1.5 GHz. This asymmetrical combiner has been used to implement a recursive active filter with high gain and low noise. Due to the use of metamaterials, the dimensions of the filter are smaller than a filter with the same characteristics implemented by conventional transmission lines.

ACKNOWLEDGMENT

The authors would like to acknowledge the financial support of the Universidad Carlos III de Madrid under Programa Alianza 4 Universidades and TEC2009-14525-C02-01

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