

UNIVERSIDAD POLITÉCNICA DE MADRID
Escuela Técnica Superior de Ingeniería Aeronáutica y del Espacio



Modelling Friction-Saturated Limit-Cycle
Oscillations in Unstable Turbomachinery
Rotor Blades

DOCTORAL THESIS

Submitted for the degree of Doctor by:

Álvaro Escudero Saiz

Master of Science in Aeronautical Engineering

Madrid, 2025



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Title: Modelling Friction-Saturated Limit-Cycle Oscillations in Unstable Turbomachinery Rotor Blades

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Doctoral Programme: Aerospace Engineering

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This thesis has received financial support by the TASTE project under grant agreement No. RTC2019-007194-4 and the PANDORA project under grant agreement No. HORIZON-CL5-2022-D5-01.

*Even the most resistant matter yields,
not by force, but through the
quiet insistence of time and fire.*

Acknowledgements

First and foremost, I would like to express my deepest gratitude to my PhD supervisor, Professor Roque Corral. Thank you for giving me the opportunity to undertake this thesis and for guiding me throughout this journey. Your technical and business insights into turbomachinery have been invaluable, and I am certain your lessons will continue to resonate with me for years to come. There is a phrase I learned some years ago—*Duc in Altum*—which encourages one to sail into deep waters, to go beyond the known and the safe, and to trust that the results will come. I truly believe this reflects your teaching philosophy, and I am sincerely grateful for the way you have always encouraged me to take that step forward.

I would also like to extend my sincere thanks to the members of the thesis evaluation committee—Carlos Martel, Mehdi Vahdati, Loic Salles, Fanzhou Zhao, and Almudena Vega—for taking the time to read and evaluate my work. Your thoughtful comments, constructive feedback, and insightful suggestions have been instrumental in refining this thesis.

This work began at ITP Aero, and I thank the company for providing the opportunity and resources to conduct my research. There, I met colleagues who guided and helped me through the early stages of my thesis. I would like to acknowledge Michele Greco, Miguel Montiel, Ricardo Blázquez, Yago Blando, David Romera, and Marc Bolinches for their help and guidance.

I would also like to thank my colleagues at ETSIAE, with whom I have shared the final years of my research. It has been a pleasure to share lunch breaks, technical and non-technical discussions, join running trainings and share some beers along the way. A special mention goes to Michele Greco, with whom I have carried out all of my research; Javier González, Jesús Matesanz, Diego Caro, Alba Domínguez, Pedro Gatón, Ismael Ben-Yelun, and many others. Thanks also to Eduardo García for always being supportive with IT systems.

Two people, in particular, made my time at ETSIAE especially meaningful. Thank you to Salvador Rodríguez for your help during the most challenging part of the thesis, it has been a pleasure working alongside you. I am also deeply thankful to Venkata Chennuru; more than a colleague, I have found a true friend, and I will always be grateful for that.

A mis amigos y familiares Yago, Keyvan, Manu, Ortega, Bailo, Domingo, Pedro, Ramón, Felipe, Raúl, Carlos, Rubén, Cruz, Guille, Rubén, etc. y todos aquellos que quizá olvido mencionar; gracias por estar ahí en los momentos difíciles y, sobre todo, por ayudarme a disfrutar del camino con risas, planes y buenos momentos.

Quiero agradecer especialmente a mis padres y a mi hermano, por haber creído siempre en mí, por su apoyo incondicional y por animarme a seguir mi propio camino. La persona que soy hoy se ha forjado en los valores y enseñanzas que me habéis transmitido con tanto cariño.

Por último, gracias a Vega. Gracias por tu apoyo, amor, entendimiento y ánimo en los momentos más difíciles. Gracias por tu infinita paciencia, incluso cuando el trabajo se colaba en nuestros viajes y escapadas, y por convertir esos momentos en una divertida colección de fotos escribiendo artículos en los lugares más insospechados, siempre con tu cámara y tu sonrisa listas para capturar el momento. En definitiva, gracias por hacer que esta aventura tuviera aún más sentido y por ser el mejor ejemplo que habría podido tener. Lo mejor de este camino, que hemos recorrido y culminado juntos, ha sido compartirlo contigo.

Abstract

This thesis presents an analytical study on the friction-saturated limit-cycle oscillations (LCOs) in unstable turbomachinery rotor blades, focusing on the prediction of their vibration amplitude and its impact on blade lifespan.

In modern low-pressure turbine (LPT) design, reducing weight is a key objective. To achieve this, high-aspect-ratio blades are used, which lowers the reduced frequency of the blades, making them more prone to aeroelastic instability (flutter). When flutter occurs, the vibration amplitude increases until either the blade fails or a damping mechanism, such as dry friction in the blade attachment, dissipates energy and stabilizes the motion. If the resulting vibration amplitude and alternating stress levels remain low enough, the blade can withstand high-cycle fatigue (HCF) over its lifespan. Therefore, accurately predicting the saturated vibration amplitude is critical in blade design. While high-fidelity computational models exist to capture these vibrations in detail, they are complex and time-consuming. In contrast, simplified models that predict general trends remain scarce.

This thesis develops an analytical model to estimate vibration amplitude trends based on blade design and operating conditions. It builds on a previous dimensional analysis model, which had limitations in predicting the operational conditions where the maximum vibration occurs. The new model addresses these issues by incorporating additional physical effects, leading to improved accuracy. The model is introduced, validated using publicly available LPT test data, and then applied to predict the vibration amplitude of different turbomachinery blades.

The core principle of the model is an energy balance, where the aerodynamic work input per vibration cycle is counteracted by dry friction dissipation at the blade attachment. Since aerodynamic vibration amplitude and mechanical displacement in the blade root are different variables, a scaling relationship is established to link them. The model then predicts the vibration amplitude of friction-limited unstable blades as a function of rotational speed, aerodynamic work, and a dimensionless parameter grouping design and operational effects.

To validate the model, it is first applied to an experimental LPT rig blade, showing good agreement with test data and successfully capturing vibration trends. The model

is then used in an industrial application, where the vibration amplitudes of three different turbomachinery blades are evaluated to determine the effectiveness of friction-based stabilization. First, an engine LPT blade is analyzed, confirming its ability to saturate vibration levels. Next, a small civil engine (SCE) fan blade is examined, showing that while it can limit vibration growth, the alternating stress level is near the threshold for HCF endurance. Finally, a large civil engine (LCE) fan blade is studied, demonstrating that it cannot effectively limit vibration, leading to uncontrolled growth and eventual failure. These findings explain why friction-based stabilization is not commonly used in fan blades.

The key advantages of the presented model are: (a) its flexibility in rapidly predicting vibration amplitudes for different blade designs and operating conditions, and (b) its ability to provide insight into how design modifications affect blade behavior. As a result, this model is well-suited for early-stage engineering studies, offering a fast and practical way to anticipate the impact of design and operational changes on turbomachinery blade vibrations.

Keywords: Low-Pressure Turbine, Flutter, Friction, Vibration Amplitude, Unsteady Flows, Microslip, Linearised Navier-Stokes, Fan, Computational Fluid Dynamics, Dynamics and Structures.

Resumen

Esta tesis presenta un estudio analítico sobre las oscilaciones en ciclo límite saturadas por fricción en álabes de rotor inestables en turbomaquinaria, centrándose en la predicción de su amplitud de vibración y su impacto en la vida útil del álabe.

En el diseño moderno de turbinas de baja presión (LPT), la reducción de peso es un objetivo clave. Para lograrlo, se emplean álabes de alto alargamiento, lo que disminuye su frecuencia reducida y los hace más propensos a la inestabilidad aeroelástica (flameo). Cuando se produce flameo, la amplitud de vibración aumenta hasta que el álabe falla o un mecanismo de extracción de energía, como la fricción en el encastre del álabe, disipa trabajo y estabiliza el movimiento. Si la amplitud de vibración resultante y los niveles de tensión alternada son lo suficientemente bajos, el álabe puede soportar la fatiga de alto ciclo durante toda su vida útil. Por lo tanto, predecir la amplitud de vibración saturada es un aspecto fundamental en el diseño de álabes. Aunque existen modelos computacionales de alta fidelidad para capturar estos fenómenos con gran detalle, su complejidad y alto costo computacional limitan su uso. En contraste, los modelos simplificados que predicen tendencias generales son escasos.

Esta tesis desarrolla un modelo analítico para estimar tendencias en la amplitud de vibración en función del diseño y las condiciones de operación del álabe. Se basa en un modelo previo de análisis dimensional, que tenía limitaciones en la predicción de las condiciones operativas en las que se alcanza la máxima vibración. El nuevo modelo aborda estas deficiencias incorporando efectos físicos adicionales, lo que mejora su precisión. A lo largo de esta tesis se introduce el modelo, se valida con datos experimentales de una LPT y, posteriormente, se aplica para predecir la amplitud de vibración de diferentes álabes de turbomaquinaria.

El principio central del modelo es un balance energético, donde el trabajo aerodinámico aportado por ciclo de vibración se equilibra con la disipación por fricción en el encastre del álabe. Dado que la amplitud de vibración aerodinámica y el desplazamiento mecánico en la raíz del álabe son variables distintas, se establece una relación para vincularlas. El modelo predice la amplitud de vibración de álabes inestables limitados por fricción en función de la velocidad de rotación, el trabajo aerodinámico y un parámetro adimensional que agrupa efectos de diseño y operación.

Para validar el modelo, primero se aplica a un álabe experimental de una LPT en banco de pruebas, mostrando buena concordancia con los datos experimentales y capturando correctamente las tendencias de vibración. Después, se emplea en una aplicación industrial donde se evalúan las amplitudes de vibración de tres álabes de turbomaquinaria distintos para determinar la efectividad de la estabilización mediante fricción. En primer lugar, se analiza un álabe de LPT de motor, confirmando su capacidad para saturar los niveles de vibración. Posteriormente, se examina un álabe de fan de un motor civil pequeño, demostrando que, aunque puede limitar el crecimiento de la vibración, el nivel de tensión alternante está cerca del umbral de resistencia a fatiga. Finalmente, se estudia un álabe de fan de un motor civil grande, evidenciando que no puede limitar eficazmente la vibración, lo que lleva a un crecimiento descontrolado y eventual fallo estructural. Estos resultados explican por qué la estabilización basada en fricción no se emplea en álabes de fan.

Las principales ventajas del modelo presentado son: (a) su flexibilidad para predecir rápidamente las amplitudes de vibración en distintos diseños de álabes y condiciones operativas, y (b) su capacidad para proporcionar información sobre cómo las modificaciones en el diseño afectan el comportamiento del álabe. Como resultado, este modelo es una herramienta útil en estudios de ingeniería en etapas tempranas de diseño, ofreciendo una forma rápida y práctica de anticipar el impacto de cambios en el diseño y la operación sobre las vibraciones de álabes de turbomaquinaria.

Palabras clave: Turbina de baja presión, Flameo, Fricción, Amplitud de Vibración, Flujo no estacionario, Microdeslizamiento, Navier-Stokes linealizadas, Fan, Dinámica de Fluidos Computacional, Dinámica y Estructuras.

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Nomenclature

Abbreviations

ACAT	AneCom AeroTest
AF	Amplitude-Frequency
ARIAS	Advanced Research Into Aeromechanical Solutions
BEM	Beam Element Method
BTT	Blade Tip Timing
CFD	Computational Fluid Dynamics
CTA	Centro de Tecnologías Aeronáuticas
DFT	Discrete Fourier Transform
EO	Engine Order
FEM	Finite Element Method
FVM	Finite Volume Method
FSI	Fluid-Structure Interaction
FUTURE	Flutter-Free Turbomachinery Blades
GPU	Graphic Power Unit
HCF	High Cycles Fatigue
HPT	High-Pressure Turbine
IBPA	Inter-Blade Phase Angle
LCE	Large Civil Engine

LPT	Low-Pressure Turbine
MPI	Message Passage Interface
MUSCL	Monotonic Upstream-Centered Schemes for Conservation Laws
ND	Nodal diameter
NSV	Non Synchronous Vibration
NRBC	Non-Reflecting Boundary Conditions
OEM	Original Equipment Manufacturer
RANS	Reynolds-Averaged Navier Stokes
SCE	Small Civil Engine

Greek Symbols

α	Exponential constant of the penetration model
β	Southwell coefficient
δ	Vibration amplitude
κ	Scaling Factor
Λ	Aspect Ratio
λ	Microasperity contact parameter
Ω	Rotational speed
ψ	Fraction of volume removed in a hollow blade
ρ	Density
σ	Inter-Blade Phase Angle
ξ	Critical Damping Ratio
ζ	Fourier-mode Wavenumber

Roman Symbols

AF	Amplitude-Frequency
------	---------------------

I	Area Moment of Inertia
A_A	Apparent area of contact
C_A	Surface parameter of the asperities
c	Blade chord
C_p	Pressure Coefficient
E	Elastic Modulus
a_{FT}	Firtree width
f	Vibration frequency
G	Shear modulus
k	Reduced Frequency
$K_{n,i}$	Normal stiffness of an individual asperity
K_n	Total normal stiffness
L	Blade length
m	Blade mass
M	Mach Number
M_{is}	Isentropic Mach Number
$N_{A,C}$	Number of asperities in contact
s	Blade pitch
p	Pressure
R_A	Radius of the Asperity
R	Mean radius of the blade
$R_{C,A}$	Local contact radius of the Asperity
a	Speed of sound
D	Tip Diameter

W Work-per-cycle

Super-scripts

' Time perturbation

*

Limit of hysteresis cycle

^

Normalized values

~

Non-dimensional values

Sub-scripts

0

Total variables

a

Aerodynamic

b

Blade

e

Exit

m

Mechanical

1. Introduction

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1.1. Context

The air transport sector has experienced remarkable and sustained growth over recent decades, leading to significant environmental impacts such as CO₂ emissions, which are closely linked to the efficiency of aircraft engines. Currently, aviation contributes to 2.5% of global CO₂ emissions, a figure that has been rising with the growing global demand. Consequently, there is increasing pressure from governments and organizations to enhance the sustainability of the aviation industry.

A critical factor in reducing CO₂ emissions is engine efficiency and weight. Modern turbofan design trends focus on achieving higher bypass ratios, with the fan generating over 80% of the engine's power. This fan is typically driven by a multistage low-pressure turbine (LPT), which accounts for approximately 25-30% of the engine's total weight. While modern LPTs achieve efficiency values exceeding 90%, leaving limited room for further improvement, some studies suggest that a 15% reduction in LPT weight can yield up to a 1% reduction in fuel consumption [2]. Hence, there is significant pressure to design lighter LPT modules.

To achieve weight reduction, strategies have focused on using highly loaded blades to decrease blade count and minimizing the weight of blades and disks. This is typically accomplished by reducing the chord length of the blades, resulting in thinner disks and higher aspect ratio blades, which in turn leads to a lighter engine. However, high aspect ratio blades combined with increasingly thinner and more loaded blades, raise the

vulnerability to flow-induced vibrations leading to a high damage potential. Figure 1.1 illustrates the evolution of the aspect ratio, $\Lambda = L/c$ in the Rolls-Royce Trent series LPT blades, where L is the blade length and c is the blade chord [3].

In the early 1990s, turbine blades typically had an aspect ratio of around $\Lambda = 7$ and featured interlock configurations, such as the Trent 700 LPT rotor blade. These designs posed challenges in controlling and predicting vibration amplitudes, leading to the development of cantilevered welded-pair blades, as seen in the Trent 800. Despite their benefits, high aspect ratio cantilever blades tend to be aeroelastically unstable due to their low reduced frequencies and high loading. Initial efforts to mitigate vibration in unstable low-pressure turbine (LPT) blades employed tip dampers, but corrosion issues with these dampers created significant operational problems.

In response, subsequent engine designs reduced the blade aspect ratio to approximately $\Lambda = 4$, as in the Trent 500 and Trent 900. It is essential to note that rotor blade height is determined by engine architecture and tends to increase in newer engine generations to meet high efficiency requirements for driving larger fans. Consequently, the LPT blades depicted in Fig. 1.1 are ranked according to their aspect ratio. Since rotor blade height is constrained by engine architecture, aspect ratio reduction is achieved by increasing chord length.

To maintain consistent aerodynamic performance, the blade-to-pitch ratio, s/c , where s represents the blade pitch, must remain constant. As a result, increasing the blade chord results in a reduction in blade count. However, this also leads to an increase in rotor blade weight, and, as a consequence, in a heavier disk, since thicker disks are required to accommodate larger fir-trees and withstand the increased pull loads. Overall, reducing the aspect ratio by increasing the chord decreases the number of blades, while the total engine weight increases. Since this conflicted with weight reduction objectives, recent design trends have reverted to high-aspect-ratio blades, around $\Lambda = 8$, as exemplified by the Trent 1000 engine. Although these blades are aeroelastically unstable, their operation relies on controlling vibration amplitude through friction in the fir-tree attachment region, maintaining it below a threshold that allows the blade to sustain vibration indefinitely.

The most common aeroelastic problems encountered in turbomachinery are forced response, non-synchronous vibration and flutter (discussed in Section 1.3). This thesis focuses on the flutter problem, which is an aeroelastic instability of an isolated blade row. LPT blades in modern engines exhibit low reduced frequencies ($k \sim 0.1 - 0.2$) [4, 5], which are associated with low aerodynamic damping values. When aerodynamic damping turns negative, the blade becomes unstable and flutters, which may cause High Cycle Fatigue (HCF) if the vibration amplitude is sufficiently high. In the pursuit of reduced LPT weight, high-aspect-ratio blades, which may be aeroelastically unstable, are com-

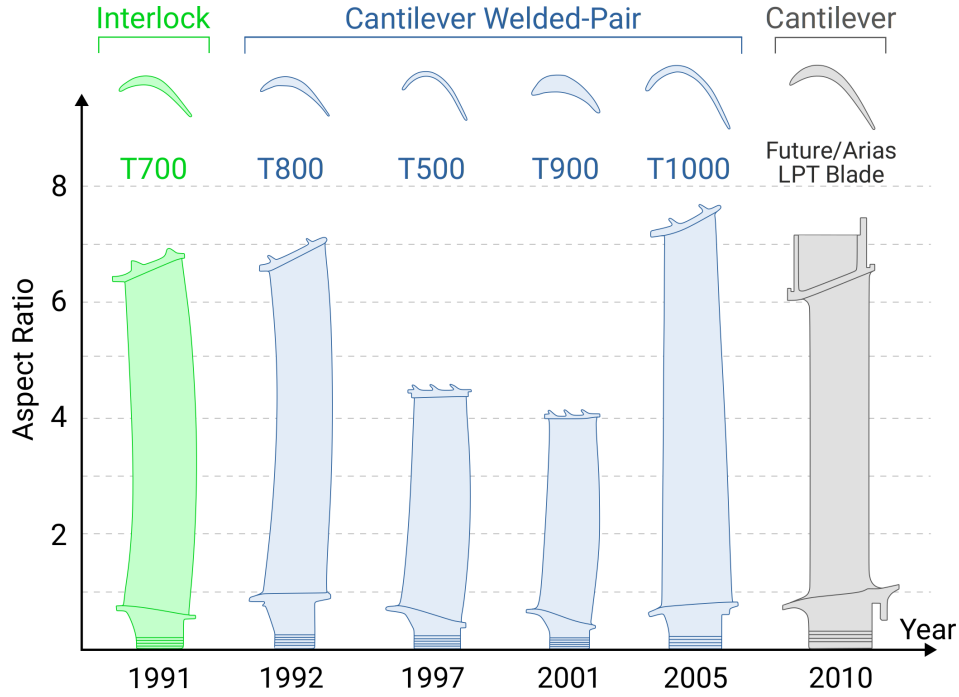


Figure 1.1.: EVOLUTION OF THE ASPECT RATIO AND 2D CHARACTERISTIC SECTION OF THE ROLLS-ROYCE LPTs DURING THE LAST DECADES. GREEN: INTERLOCK BLADE; BLUE: CANTILEVER WELDED IN PAIRS BLADES; GREY: FUTURE AND ARIAS PROJECTS BLADE AND ANALYZED THROUGHOUT THIS THESIS.

monly used in modern designs. As a result, it is generally accepted that these blades will experience vibration [6–8], making the vibration amplitude of fluttering blades a critical parameter for ensuring the safe operation of the engine.

While the aeroelastic interaction between the blade’s aerodynamics and structure is part of the turbomachinery flutter problem, other significant mechanical factors also influence the fluttering vibration amplitude. To reduce this amplitude and mitigate HCF issues, various mistuning and friction-based strategies have been previously employed [9–15]. Mistuning, on one hand, refers to small differences in mass and stiffness of the different bladed disk sectors that break the cycle symmetry. For flutter problem, the mistuning has a stabilising effect of the most unstable nodal diameters (ND), with the consequently reduction of the vibration amplitude [7, 16, 17]. On the other hand, mechanical dissipation arising from relative displacement between blades or between blade and disk, counteracts the aerodynamic work on the blade. In LPTs, this friction typically occurs in the microslip regime, known for its high dissipation capacity, which can effectively saturate the vibration amplitude.

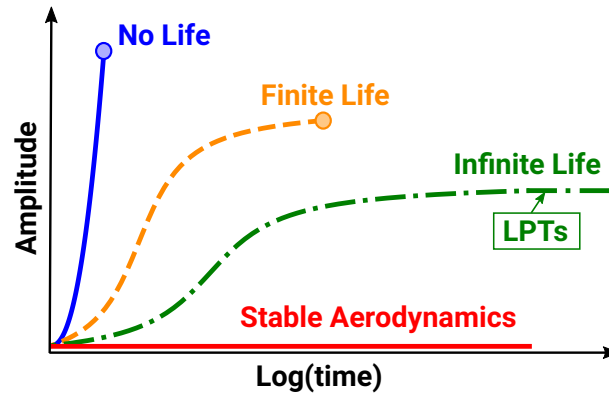


Figure 1.2.: EVOLUTION OF A FLUTTERING BLADE VIBRATION AMPLITUDE WITH TIME FOR DIFFERENT AERODYNAMICS AND DISSIPATION REGIMES.

1.2. Motivation

The vibration amplitude of fluttering blades increases until either structural failure occurs or until an energy dissipation mechanism is activated. In turbomachinery, this dissipation typically results from nonlinear dry friction present in contact joints between assembled parts, leading to a saturated vibration response [9]. Linear formulations can only predict system behaviour during the early stages when vibration amplitudes are small. During this phase, unstable modes grow exponentially, with their growth rate dictated by aerodynamics. However, this exponential growth cannot be sustained over the long term, and once the vibration amplitude becomes large enough, nonlinear effects come into play, leading to the saturation of the vibration amplitude at a finite level [1, 6].

Various scenarios are illustrated in Figure 1.2. If the blade is stable, the vibration amplitude remains zero, as all modes are stable and perturbations decay to zero. When unstable modes are present, different outcomes are possible. If the instability is severe and cannot be effectively mitigated by nonlinear mechanisms, the amplitude may become large enough to cause rotor blade failure. In an intermediate scenario, the amplitude saturates at a finite level. If this amplitude, and consequently the alternating stresses, are low enough, the blade may sustain vibration indefinitely, making flutter acceptable since the stresses correspond to infinite life on the Goodman diagram. Alternatively, if the vibration amplitude is high, the blade may only sustain the vibration for a finite time, resulting in a limited lifespan [3].

Depending on the rotor blade configuration, various contact joints, such as shrouds, dampers, and blade-disk attachments, play different roles in vibration control. Interlock blades, which make contact at the shroud, increase natural frequencies compared to cantilever configurations, thereby enhancing flutter stability [18]. However, this configuration has disadvantages, including difficulties in predicting the final amplitude and

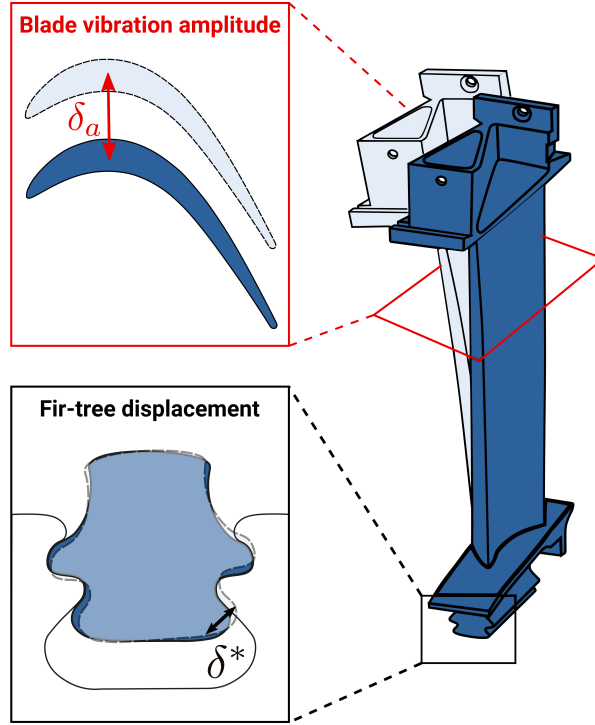


Figure 1.3.: SKETCH OF A VIBRATING LPT BLADE WHERE THE DISPLACEMENT IN THE FIRTREE REGION BALANCES THE AERODYNAMIC WORK INJECTED IN THE SYSTEM

potential issues related to contact. Dampers, typically located on the blade platform, are metallic devices placed in grooves under the platforms of adjacent blades, held in place by centrifugal forces during operation. When the blades vibrate, the relative displacement between the platforms and the damper creates friction at the contact interface, dissipating energy and damping the system [19, 20]. In cantilever rotor blades, dissipation is restricted to the blade attachment to the disk, where dry friction occurs in the fir-tree, as these designs avoid contact between neighbouring shrouds or platforms. Figure 1.3 illustrates how an unstable LPT blade, vibrating in its first flap mode, receives energy from the flow and how it is dissipated through friction at the fir-tree attachment.

Several authors have studied limit-cycle oscillation (LCO) solutions arising from the equilibrium between unsteady aerodynamics and nonlinear friction with varying levels of accuracy. The interaction between negative aerodynamic damping and friction damping has been explored using a single degree-of-freedom blade model with a Coulomb damper [21, 22]. More recently, analyses have been conducted using different microslip models and high-fidelity methods with realistic large-scale finite element models [14, 15, 23, 24]. Advances in computational resources have allowed the prediction of blade vibration amplitude due to coupled aerodynamic and friction effects using Computational Fluid Dynamics (CFD) and Finite Element Method (FEM) simulations [12].

While high-fidelity contact simulations are crucial for understanding and quantifying LCOs, low-fidelity models are valuable for describing global trends. Corral and Gallardo [6] studied the dynamic evolution of an unstable cantilever LPT rotor blade with multiple unstable modes and characterizing friction with the Sellgren and Olofsson microslip model [25]. A similar model was employed in [26] to analyze the mistuned flutter response, with results successfully compared against experimental data.

Corral and Gallardo [1] predicted the vibration amplitude of a family of unstable cantilever LPT rotor blades using a simplified friction model within the limit of small hysteresis loops in the fir-tree attachment, representative of a deep microslip regime. However, this approach becomes inaccurate as rotor blade vibration increases, which is often the case in high-aspect-ratio cantilever LPT rotor blades in aeronautical turbines, which typically operate in a microslip regime with larger hysteresis cycles. In such cases, the frictional behavior can transition between sticking (microslip) and slipping (macroslip) when dry friction is insufficient to counteract the aerodynamic work, leading to instability and unbounded vibration amplitude growth.

The vibration amplitude predictions obtained using the Corral and Gallardo model [1] are compared with experimental measurements in Fig. 1.4. The experimental data comes from a test campaign conducted as part of the European Union research project *Flutter-Free Turbomachinery Blades* (FUTURE) [8], an aeromechanical study aimed at evaluating different design strategies for flutter control in LPT rotor blades. The FUTURE LPT blade, illustrated in Fig. 1.1 is a cantilever LPT rotor blade with an aspect ratio of $\Lambda \approx 7$, representative of modern LPT rotor blade designs. This blade was specifically designed to induce flutter and measure its vibration amplitude. The model's predictions deviate from the experimental results primarily due to inaccuracies in estimating the rotational speed at which maximum vibration occurs. The model predicts this peak $\Omega = 185\%$, whereas the experimental measurements show the highest vibration amplitude near the design conditions, at $\Omega = 103\%$.

While the model proposed in [1] captures certain physical phenomena, such as the qualitative behaviour of the fir-tree and the aerodynamic work scaling, its accuracy is limited by key assumptions. Specifically, it assumes that the relative displacement between the fir-tree and the disk is negligible due to small vibration amplitudes, and it does not account for the variation of aerodynamic work with rotational speed. As a result, the aerodynamic stabilization at high rotational speeds due to reduced loading is not included, nor is the modification of the hysteresis cycle size at large vibration amplitudes. In this thesis, the impact of these assumptions is analyzed, and a revised version of the model that addresses these limitations is proposed. The improved model demonstrates significantly better agreement with experimental measurements in predicting blade vibration

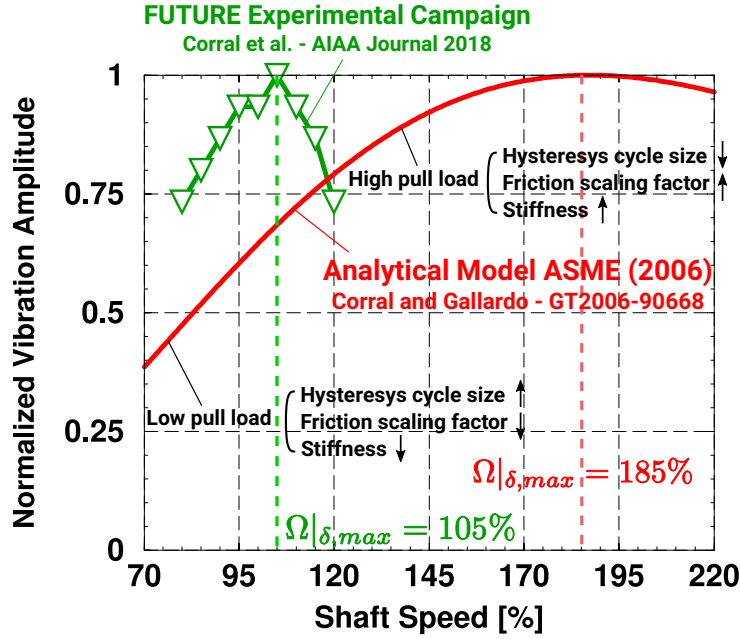


Figure 1.4.: NORMALIZED VIBRATION AMPLITUDE MEASUREMENTS OF THE FUTURE EXPERIMENTAL CAMPAIGN AND NORMALIZED VIBRATION AMPLITUDE PREDICTED BY THE MODEL PROPOSED BY CORRAL AND GALLARDO [1].

amplitudes.

The model presented in this thesis enhances the understanding of how unsteady aerodynamic forces and friction effects in the fir-tree influence the vibration amplitude of fluttering cantilevered blades. During the design phase, several decisions regarding the blade's geometrical and structural characteristics significantly affect its vibration amplitude. Furthermore, since the LPT operates under diverse conditions that can influence blade vibrations, accurately predicting vibration amplitudes is essential for assessing component lifespan. This model serves as a valuable tool for industrial applications, offering insights into how design choices and operating conditions affect flutter-induced vibrations in LPT blades. It enables engineers to refine blade designs with aeromechanical considerations in mind, thereby controlling blade lifespan.

1.3. Theoretical Background

1.3.1. Turbomachinery Aeroelasticity

Aeroelasticity is the field of study that focuses on the interaction between an elastic structure and the surrounding fluid, driven by the interplay of aerodynamic, inertial, and elastic forces. Collar's triangle [27] illustrates the relationships between these forces, categorizing different types of aeroelastic phenomena. When only elastic and inertial

forces are involved, the problem is reduced to classical vibration analysis. If aerodynamic forces interact primarily with inertial forces, it is classified as a stability and control issue. On the other hand, when aerodynamic forces act in combination with elastic forces, it becomes a problem of static aeroelasticity.

Dynamic aeroelasticity arises when aerodynamic, elastic, and inertial forces interact simultaneously. This process involves a feedback loop: aerodynamic forces deform an elastic structure, with the magnitude of the deformation depending on the structure's elastic and inertial properties. The deformation, in turn, alters the aerodynamic forces acting on the structure, restarting the loop. In turbomachinery, the main dynamic aeroelastic problems are forced response, non-synchronous vibrations (NSV), and flutter. These phenomena can affect blades, vanes, seals, and other components exposed to unsteady aerodynamic loads.

Forced response is a synchronous phenomenon that occurs when an external excitation aligns with one of a component's natural vibration frequencies. In turbomachinery, the excitation frequency is determined by the combination of the Engine Order (EO) and the rotational speed, expressed as $f = \text{EO } \Omega$. Unsteady aerodynamic forces cause forced response and can be categorized into two types: high engine order (HEO) and low engine order (LEO) excitations. HEO excitations result from the wake or potential field interactions between rotating and stationary components, such as rotor-stator interactions, and typically excite high-frequency vibration modes. In contrast, LEO excitations arise from flow non-uniformities, including gusts, staggered blades, crosswind, or combustion instabilities, and excite low-frequency vibration modes.

Non-synchronous vibration (NSV) shares similarities with forced response in that unsteady aerodynamic loads also drive it. However, the key difference is the excitation frequency. Forced response is termed "synchronous" because its frequency is dependent on the engine's rotational speed. In contrast, NSV arises from aerodynamic instabilities that generate excitations with frequencies independent of rotational speed, which then interact with the natural frequencies of the components.

Examples of NSV include acoustic resonances caused by boundary layer separation or periodic vortex shedding [28, 29], and rotating stall [30]. Figure 1.5 illustrates a rotating stall in the front stage of a high-pressure compressor, where the pressure distribution in the casing is shown. The stall cell propagates at approximately 70% of the engine's rotational speed in the absolute frame of reference, $\Omega_{\text{RS}} \approx 0.7 \Omega$, corresponding to a counter-rotating speed of 30% in the relative frame of reference, as shown in the figure.

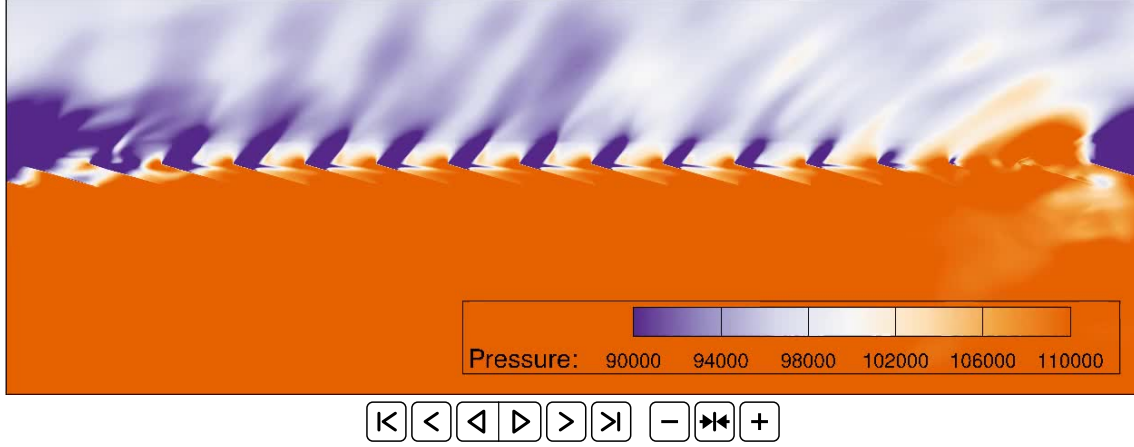


Figure 1.5.: EVOLUTION OF THE PRESSURE IN THE CASING OF A TRANSONIC COMPRESSOR OVER A REVOLUTION OF THE ROTATING STALL.

1.3.2. Flutter

Flutter is an aeroelastic instability that occurs when the airflow surrounding an elastic structure injects energy into the system, leading to self-excited vibrations without any external forcing [31, 32]. Classical flutter occurs when only a single unstable mode is present, and the aerodynamic forces remain linear, with no significant nonlinear effects such as shock waves or flow separation. In this case, the disturbed unsteady flow can be regarded as a minor deviation from the primary flow, allowing it to be analyzed with linear techniques. The key assumptions are that the flow stays attached to the airfoil surface and that the instability arises purely from the phase shift between the structural motion and the resulting aerodynamic forces. Additionally, it is assumed that only a single mode is involved as the other modes are well-separated by frequency. This type of flutter is typically observed in subsonic, low-pressure turbines and compressors.

Flutter can be effectively understood by examining whether the airflow transfers or draws energy from the airfoil. The standard approach to analyze this phenomenon involves calculating the work-per-cycle done by the fluid on the airfoil, which may be represented as

$$W_{\text{cycle}} = \int_0^T \int_{\Sigma_{\text{blade}}} p \mathbf{V}_{\text{blade}} \cdot d\mathbf{A} dt \quad (1.1)$$

where the viscous terms are neglected. If neither the pressure on the blade surface, p , nor the velocity of the blade, $\mathbf{V}_{\text{blade}}$, depend on the blade position then the work-per-cycle reduces to

$$W_{\text{cycle}} = \int_0^T \mathbf{F}(t) \cdot \mathbf{V}_{\text{blade}}(t) dt. \quad (1.2)$$

Under the assumption of harmonic motion, the position of the blade is given by $\mathbf{x}_{\text{blade}} = \mathbf{x}_0 \sin \omega t$, thus the blade velocity is $\mathbf{V}_{\text{blade}} = \omega \mathbf{V}_0 \cos \omega t$. The blade force is defined as

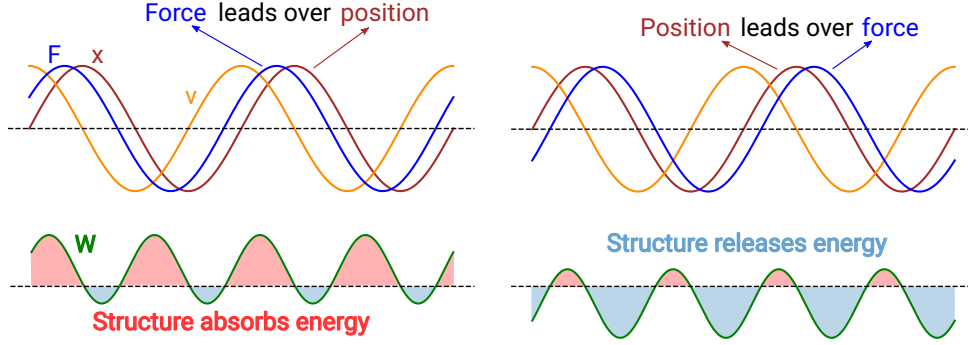


Figure 1.6.: POSITION, VELOCITY, FORCE AND WORK PER CYCLE FOR TWO DIFFERENT PHASING BETWEEN THE FORCE AND THE DISPLACEMENT. LEFT: FORCE LEADS, STRUCTURE ABSORBS ENERGY. RIGHT: FORCE LAGS, STRUCTURE RELEASES ENERGY.

$\mathbf{F} = \mathbf{F}_0 \sin(\omega t + \psi)$, where ψ represents the phase shift relative to the position. Thus, the work done per cycle can be expressed as

$$W_{\text{cycle}} = \omega \mathbf{x}_0 \mathbf{F}_0 \int_0^{2\pi/\omega} \cos \omega t \sin(\omega t + \psi) dt = \pi \mathbf{x}_0 \mathbf{F}_0 \sin \psi \quad (1.3)$$

The phase shift angle, ψ , is a critical factor in determining the system's stability. When $\psi > 0$, energy is transferred from the flow to the structure, causing the system to become unstable. Conversely, when $\psi < 0$, energy is transferred from the structure to the flow, resulting in a stable system. Thus, flutter is determined solely by the phase shift between blade displacement and the aerodynamic force. Figure 1.6 illustrates the phase shift between blade motion and aerodynamic forces under both stable and unstable conditions, as well as the aerodynamic work either injected into or absorbed by the system. If a force acts on the blade and, as a consequence, the blade is displaced, positive work is performed and the structure absorbs energy. When forces and displacements are perfectly in phase $\psi = 0$ or in anti-phase $\psi = \pi$, the system is conservative, with no network dissipated per cycle.

In turbomachinery, flutter is driven by the rotor blade's construction, the cyclic symmetry of the system, and aerodynamic coupling effects between neighbouring blades. The primary mechanism behind turbomachinery flutter is blade-to-blade aerodynamic coupling, which is the one closely related to the subject of this thesis. There are several other flutter mechanisms in turbomachinery, including mode coalescence, acoustic flutter, stall flutter, and whirl flutter. Mode coalescence typically occurs in blades with a low mass ratio and closely spaced natural frequencies, where aerodynamic forces can significantly modify the system's mode shapes and frequencies. This phenomenon has been observed in modern composite fan blades used in high-bypass ratio engines [33] and in turbine sector vanes [34]. Acoustic flutter arises from the reflection of acoustic waves within the engine

intake, which then feed into blade vibrations [35, 36]. Stall flutter is driven by the interaction between a rotating stall cell and the rotor blade's induced vibration [37]. Whirl flutter occurs when rotor aerodynamic forces couple with the flexibility of components such as wings, pylons, or shafts, typically during flight in propeller-driven aircraft [38].

Blade-to-blade coupling occurs due to aerodynamic interactions among neighboring blades in motion. When a blade vibrates, it generates aerodynamic forces that influence adjacent vibrating blades, and the stability of the system depends on the phase difference between the aerodynamic forces generated and the displacement of the blades, as shown in figure 1.6. This interaction, known as aerodynamic coupling, can lead to aeroelastic instability in a blade arrangement, even when a comparable isolated blade would not exhibit flutter. To illustrate the effect of neighbouring blades, the relative motion and the aerodynamic influence between them must be addressed.

The oscillatory motion of a tuned blade row during flutter can be characterized as a traveling wave mode, meaning that all blades vibrate with the same mode, amplitude, and frequency, but exhibit a phase lag, referred to as the Inter-Blade Phase Angle (IBPA), between adjacent blades. IBPA can be expressed as a function of the nodal diameter, n , which denotes the number of diameters with zero displacement across the airfoils, and the number of blades in the row, N as

$$\sigma = \frac{2\pi n}{N}. \quad (1.4)$$

Figure 1.7 illustrates the IBPA and nodal diameter concept, which can only take discrete values to satisfy full-cycle periodicity constraints. In this figure, the row is composed of 24 blades. The top section of the figure shows $ND = 1$, corresponding to one diameter where the displacement of the blades is zero. Here, the phase difference between adjacent blades, or IBPA, is $\sigma = 15^\circ$ requiring all 24 blades to capture each distinct vibration phase. Consequently, this setup would involve simulating the entire wheel. The middle section represents $ND = 3$, where the IBPA is $\sigma = 45^\circ$ and a minimum flow unit of eight passages is enough to represent this travelling-wave mode accurately.

In the bottom section, $ND = 6$ is shown, corresponding to an IBPA of $\sigma = 90^\circ$, with four passages required to represent this vibration. At this nodal diameter, blades 2 and 4 reach maximum deflection and vibrate in anti-phase, while blades 1 and 3 pass through their zero deflection position with opposing velocities. The highest nodal diameter $ND = 12$ (not shown in the figure), has the shortest wavelength of two passages, corresponding to $\sigma = 180^\circ$. For a rotor blade, if vibration propagates in the same direction as the rotor, it is classified as a forward traveling-wave mode with a positive nodal diameter number. Conversely, when the vibration mode rotates opposite to the rotor's direction, it is referred to as a backward traveling-wave mode with a negative nodal diameter number.

In a blade cascade, the flutter-induced aerodynamic response results from the combined

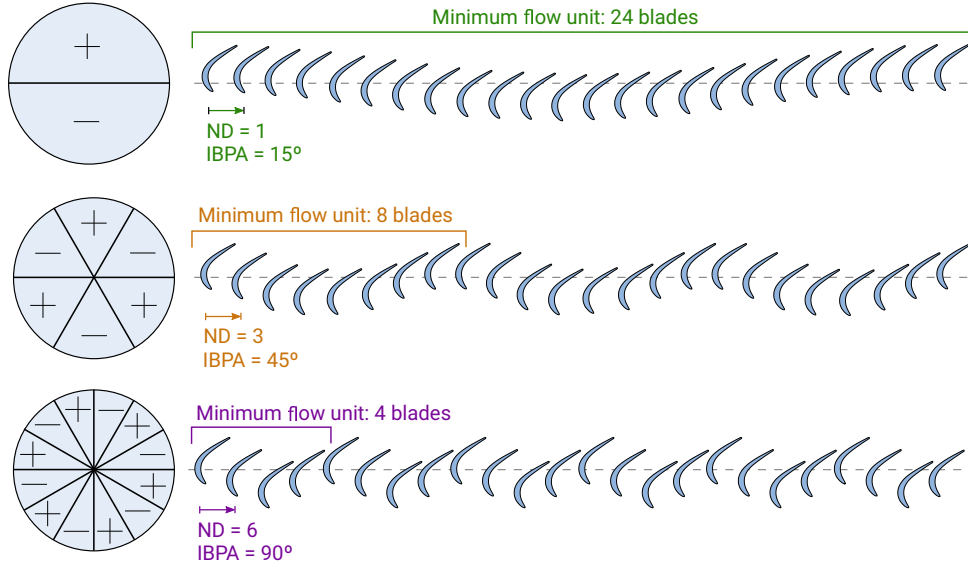


Figure 1.7.: NODAL DIAMETERS AND THE CORRESPONDING POSITION OF THE BLADES, INTER-BLADE PHASE ANGLE AND MINIMUM FLOW UNIT TO REPRESENT THEM.

influence of several blades. For small perturbations, these influences add linearly, as shown in [39]. Thus, the total unsteady response on a blade is the cumulative effect of the response from itself and neighbouring blades, lagged by a multiple of the IBPA. The traveling wave mode response can be represented as:

$$\hat{c}_p^{m,\sigma} = \sum_{n=-N/2}^{n=N/2} \hat{c}_p^{n,m} e^{-i\sigma n} \quad (1.5)$$

where $\hat{c}_p^{m,\sigma}$ is the complex pressure coefficient, acting on blade m and $\hat{c}_p^{n,m}$, which is typically referred to as local aerodynamic influence coefficient, is the pressure coefficient generated by the vibrating blade n on blade m . Equation 1.5 highlights the central role of IBPA in shaping the traveling wave mode response, as it introduces a phase shift to each influence coefficient in the superposition process.

The unsteady pressure field generated by a bladed-row vibrating at a stable IBPA of $\sigma = 90^\circ$ is shown in figure 1.8, where two flow units, with four blades each, are represented. The travelling-wave moves in the same direction than the rotor does, so it is referred to as a forward travelling wave. Figure 1.9 provides a closer view of two adjacent blades vibrating at the same IBPA, with blade 2 leading in displacement.

Flutter due to blade-to-blade coupling is particularly common in low-pressure turbines for several key reasons. First, their high aerodynamic loading increases the energy injected into the system, as the unsteady aerodynamic forces exhibit greater fluctuations. Second, the low reduced frequency typical of LPT blades makes them more susceptible to flutter,

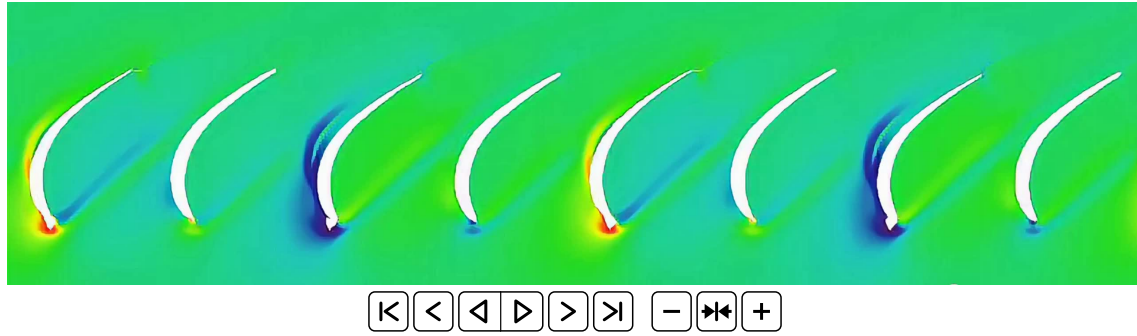


Figure 1.8.: UNSTEADY PRESSURE ON DIFFERENT BLADES FOR $\sigma = 90^\circ$ (FORWARD NODAL DIAMETER).

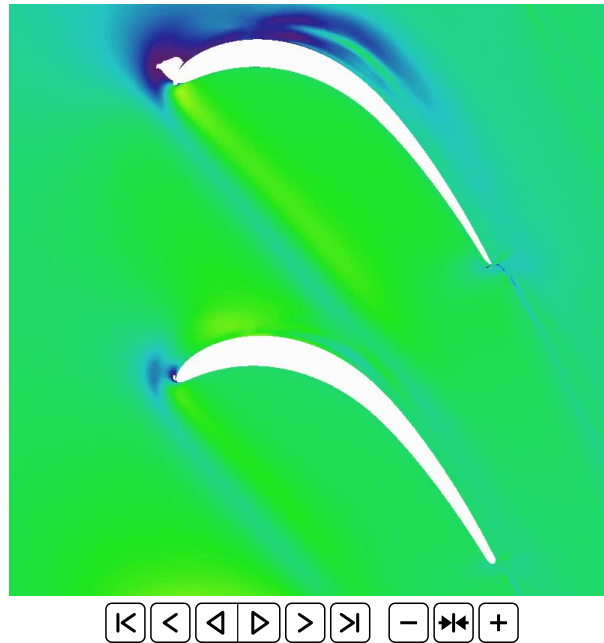


Figure 1.9.: DETAIL OF THE UNSTEADY PRESSURE ON TWO BLADES OVER A VIBRATION CYCLE FOR $\sigma = 90^\circ$ (FORWARD NODAL DIAMETER).

as the average aerodynamic work-per-cycle increases linearly with reduced frequency. In contrast to fans, which typically have a lower mass ratio (~ 20), LPT blades have a higher mass ratio (around ~ 100), meaning that aerodynamic effects have a relatively smaller impact on the structural properties of the blades. As a result, LPT blades tend to flutter in a single mode shape, close to their natural frequencies and mode shapes in a vacuum.

1.3.3. Friction models

Friction arises at the physical interface between two surfaces in contact when there is relative motion, opposing the movement of one surface against the other. This force depends on the nature of the surfaces in contact as well as the normal force applied

between them. Key contributors to friction include elastic and plastic deformations (or the ploughing effect), molecular adhesion, and surface roughness [40, 41]. At contact points, mechanical energy is converted into thermal energy due to friction, which contributes to damping in vibrating structures. Many systems with vibrating components, such as gas turbine engines, consist of multiple interconnected parts that are joined under various conditions (e.g., welded, bolted, loosely connected). Friction at these interfaces dissipates energy and thereby generating damping.

Frictional systems are highly nonlinear, and the performance depends on several parameters. Generally, modeling friction in nonlinear systems requires a few key parameters, such as material properties, load frequency and contact conditions. The latter mainly refers to the relative displacement between the bodies in contact, and it has a significant influence on the modelization of the friction. If there is no relative displacement, it is referred to as a rigid contact, resulting in negligible dissipation of energy. On the other hand, large displacements between the two bodies lead to a macroslip contact, characterized by an even distribution of pressures and a simultaneous transition of the entire contact surface from contact to slip. Between these two limits, a regime known as micro slip arises, where only a fraction of the contact surface is sliding, assuming an uneven distribution of pressures in the contact and an uneven and progressive transition of the contact surface into contact or slip.

The Coulomb friction model, one of the most widely used, considers parameters like material characteristics, contact area, normal load, and repose time. More advanced models, such as the Stribeck model, extend the Coulomb approach by introducing a velocity-dependent function (the Stribeck curve) to represent friction better. The Coulomb and Stribeck models are most effective for conditions where large displacement, or macroslip, occurs at the interface. These models are commonly applied due to their mathematical simplicity, particularly when normal loads are relatively small.

However, in applications where the friction contact pressure is high, microslip models become essential for accurately representing the friction interface. The main hypothesis when estimating mechanical work is that the blade attachment is macroscopically fixed, rendering macro-slip friction models (such as the classical Coulomb model) unsuitable, especially when applied to describe the overall behavior of the contact. Various microslip models were derived to address this challenge without delving deeper into the local behaviour of the different parts of the contact surface. These models establish a connection between the tensional state in the contact and the macroscopic displacement, employing distinct laws.

Mindlin [42] was among the first to develop an analytical model to represent micro-slip at the contact interface between two spheres. Afterwards, Hagman [43] proposed a model

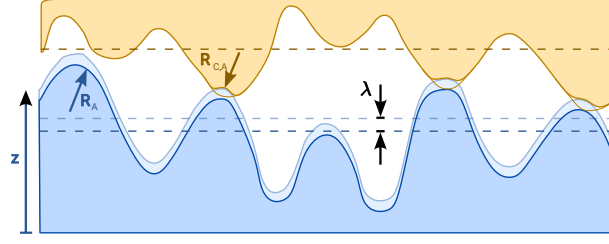


Figure 1.10.: SKETCH OF TWO ROUGH SURFACES APPROACHING.

that assumes a uniform distribution of spherical asperities on the contact surfaces, all with the same radius and height. Based on Hagman's model, Sellgren and Ollofson [25, 44] developed a model for cases of harmonic loading, as those typically encountered in turbomachinery.

Hagman model

A micro-slip model for computing the frictional load in the contact is presented in [43]. In this model, the shape of the asperities is assumed to be spherical, and they have the same radius, R_A . The contact between surfaces is treated as elastic, and individual asperity behavior is governed by Hertz's contact theory. The number of asperities in contact, $N_{A,C}$, increases linearly with the relative approach, z , of the surfaces as $N_{A,C} = C_A z$, where C_A is a surface parameter linking the asperity count to the surface approach. A representation of this contact mechanism is provided in 1.10. The normal stiffness, $K_{n,i}$, for an individual asperity can be expressed as

$$K_{n,i} = 2E'R_{C,A} \quad (1.6)$$

where $R_{C,A} = \sqrt{R_A(\lambda - z)}$ represents the contact radius, and λ is the penetration length of the surfaces. The total stiffness, K_n , is determined as

$$K_n = K_{n,i}N_{A,C} = K_{n,i}A_A C_A z. \quad (1.7)$$

The normal load is derived from the total stiffness and depends on the penetration length λ as

$$N = \int_0^\lambda K_n dz = \frac{8}{15} E' C_A A_A R_A^{1/2} \lambda^{5/2}. \quad (1.8)$$

The normal load on an individual asperity is expressed as

$$N_i = \frac{4}{3} E' R_A^{1/2} (\lambda - z)^{3/2} \quad (1.9)$$

Following Mindlin's work [42], where the tangential displacement of elastic spheres in contact is studied, the tangential force for an individual asperity, F_i , is related to its displacement, δ_i , as

$$F_i = \mu N_i \left[1 - \left(1 - \frac{16 R_{C,A} G' \delta_i}{3 \mu N_i} \right)^{3/2} \right] \quad (1.10)$$

which establishes a limit in the tangential force for the elastic regime, i.e. $F_i = \mu N_i$.

The total frictional load F consists of two components: F_{Spring} from the asperities still in the elastic regime, and F_{Slip} , from asperities that have reached their slip limit:

$$F = F_{\text{Spring}} + F_{\text{Slip}} \quad (1.11)$$

The elastic contribution, F_{Spring} , is

$$F_{\text{Spring}} = \int_0^{z_{Li}} F_i C_A A_A dz = \frac{8}{15} \mu C_A A_A E' R_A^{1/2} \lambda^{5/2} \left[1 - \left(1 - \frac{4 G' \delta}{\mu E' \lambda} \right)^{5/2} - \left(\frac{4 G' \delta}{\mu E' \lambda} \right)^{5/2} \right] \quad (1.12)$$

and the slip contribution, F_{Slip} , is

$$F_{\text{Slip}} = \int_{z_{Li}}^{\lambda} \mu N_i C_A A_A dz = \frac{8}{15} \mu C_A A_A E' R_A^{1/2} \lambda^{5/2} \left(\frac{4 G' \delta}{\mu E' \lambda} \right)^{5/2}. \quad (1.13)$$

Therefore, the total frictional load is

$$F = \frac{8}{15} \mu C_A A_A E' R_A^{1/2} \lambda^{5/2} \left[1 - \left(1 - \frac{4 G' \delta}{\mu E' \lambda} \right)^{5/2} \right] = \mu N \left[1 - \left(1 - \frac{4 G' \delta}{\mu E' \lambda} \right)^{5/2} \right] \quad (1.14)$$

This model defines the micro-slip regime limit by the maximum tangential displacement

$$\delta_c = \frac{\mu E' \lambda}{4 G'}. \quad (1.15)$$

Introducing dimensionless variables for tangential displacement, i.e. $\tilde{\delta} = \delta / \delta_c$ and friction force, $\tilde{F} = F / \mu N$, equation 1.14 becomes

$$\tilde{F} = \frac{F}{\mu N} = 1 - \left(1 - \tilde{\delta} \right)^{5/2} \quad (1.16)$$

When the displacement is $\delta = \delta_c$, i.e. $\tilde{\delta} = 1$, the shear stiffness reduces to zero, and all asperities transition to sliding, marking the microslip zone's end. Beyond this, steady-state sliding corresponds to the macroslip regime.

Sellgren and Olofsson model

Building on Hagman's model, Sellgren and Olofsson [25,44] proposed a modified approach specifically tailored for vibratory scenarios such as those in fluttering turbomachinery blades. Their model incorporates a harmonic friction force, making it well-suited to describing the hysteresis behaviour observed during cyclic loading and unloading in such applications.

Assume the frictional load reaches a peak value F^* before reducing. Using Mindlin's work on tangential displacements in elastic sphere contacts [42,45], the friction force during unloading for an individual asperity, $F_{U,i}$, can be described as:

$$F_{U,i} = 2\mu N_i \left[1 - \left(1 - \frac{16R_{C.A}G'\delta_{U,i}}{6\mu N_i} \right)^{3/2} \right]. \quad (1.17)$$

The unloading deflection limit is twice the loading limit [46]. Thus, the maximum elastic asperity height during unloading, $z_{U,Li}$, is

$$z_{U,Li} = \lambda - \frac{2G'\delta}{\mu E'} \quad (1.18)$$

Mindlin et al. [42,45] demonstrated that during unloading, the slip direction reverses while maintaining its magnitude. Consequently, the tangential load's slip component during unloading is twice that of loading. The friction force for the entire contact during unloading, F_U , is then expressed as

$$F_U = \frac{16}{15}\mu C_A A_A E' R_A^{1/2} \lambda^{5/2} \left[1 - \left(1 - \frac{2G'|\delta^* - \delta|}{\mu E' \lambda} \right)^{5/2} \right] \quad (1.19)$$

which must be transformed into the original coordinate system shown in figure 1.11, accounting for unloading beginning at F^* . Thus, the friction force during the unloading branch of the hysteresis loop, $F_{H,U}$, is

$$F_{H,U} = F^* - 2\mu N \left[1 - \left(1 - \frac{2G'|\delta^* - \delta|}{\mu E' \lambda} \right)^{5/2} \right]. \quad (1.20)$$

In dimensionless terms, this becomes

$$\tilde{F}_{H,U} = \tilde{F}^* - 2 \left[1 - \left(1 - \frac{|\tilde{\delta}^* - \tilde{\delta}|}{2} \right)^{5/2} \right]. \quad (1.21)$$

A sketch of the hysteresis loop in figure 1.11 highlights three phases: Preloading (P), Un-

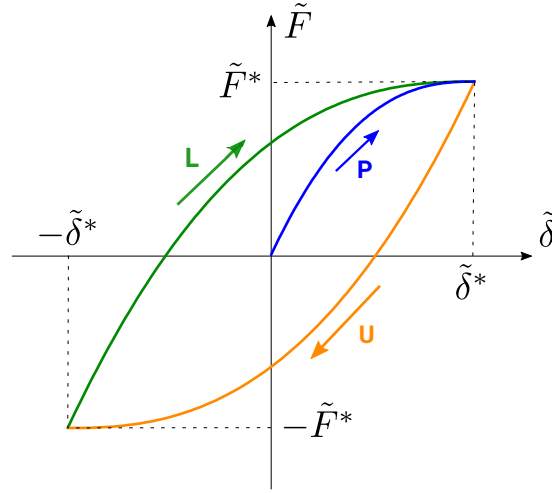


Figure 1.11.: HYSTERESIS LOOP SKETCH. P: PRELOADING, U: UNLOADING, L: LOADING.

loading (U), and Loading (L). When the frictional load oscillates symmetrically between F^* and $-F^*$ the states at $F = -F^*$ and $F = F^*$ are identical except for a sign reversal. As the load increases from $-F^*$ to F^* , events mirror those during the decrease from F^* to $-F^*$, with reversed signs.

The frictional load of the loading branch of the hysteresis cycle, $F_{H,L}$, in the original coordinate system is

$$F_{H,L} = -F^* + 2\mu N \left[1 - \left(1 - \frac{2G'|- \delta^* + \delta|}{\mu E' \lambda} \right)^{5/2} \right] \quad (1.22)$$

which can be expressed in dimensionless form as

$$\tilde{F}_{H,L} = -\tilde{F}^* + 2 \left[1 - \left(1 - \frac{|-\tilde{\delta}^* + \tilde{\delta}|}{2} \right)^{5/2} \right]. \quad (1.23)$$

Subsequent oscillations will produce a repetition of the first oscillation.

1.4. Objectives and Outline

This thesis aims to advance the understanding of aeroelastic phenomena in turbomachinery, with a particular focus on the vibration amplitudes of unstable LPT rotor blades controlled by friction. The study builds upon the predictive model proposed in [1], which serves as a foundation for investigating the interaction between flutter-induced blade vibrations and friction dissipation in the fir-tree region. However, the original model is

grounded in simplified assumptions that limit its applicability. To address these limitations, the current research extends the formulation of the analytical model to incorporate previously unaddressed physical mechanisms relevant to the flutter behaviour of friction-saturated LPT blades. The extended model is then validated using experimental data obtained from a wind tunnel test campaign.

The initial model is based on two primary assumptions that limit its applicability. First, it assumes minimal displacements in the fir-tree attachment region, an assumption contradicted by recent experimental observations [8, 47], which reveal significantly higher vibration amplitudes in fluttering LPT blades. Second, the model assumes that aerodynamic work-per-cycle remains constant across rotational speeds, an oversimplification that fails to capture off-design operational conditions.

The experimental campaign underpinning this research was conducted within the framework of the ARIAS EU Project (*Advanced Research Into Aeromechanical Solutions*), which investigates aeromechanical challenges in aircraft engines. This thesis focuses on the project's turbine work package, as summarised in [48]. As part of this project, a realistic LPT blade was tested in a vacuum test rig within a high-speed wind tunnel at the *Centro de Tecnologías Aeronáuticas* (CTA) in Zamudio, Spain. The experimental data collected during these tests have been critical for validating the extended analytical model developed in this thesis

The primary objective of this research is to extend and validate a predictive model to evaluate the influence of operating conditions and design parameters on the vibration amplitudes of turbomachinery blades. The methodology is detailed in subsequent chapters, with validation and discussion of key physical aspects integrated into the analysis. The proposed model is applied to a representative LPT case, and its applicability is further extended to fan blades in the final chapter. To achieve the stated objectives, the thesis is organized as follows:

Chapter 1 motivates this research and a comprehensive theoretical background on turbomachinery aeroelasticity and friction modeling, laying the foundation for subsequent chapters, is given.

Chapter 2 details the development and implementation of the analytical model. This includes the formulation of the aerodynamic and mechanical models, their coupling to close the model, and practical considerations for industrial applications.

Chapter 3 examines the results generated by the model. The chapter introduces the CFD solver used to characterise the LPT rotor blade and calculate the aerodynamic work-per-cycle. It explores the effects of Mach number and rotational speed on the blade dynamics, including a sensitivity analysis.

Chapter 4 focuses on validating the model. This chapter presents the experimental campaign, analyzes the experimental results, and discusses the model’s accuracy under varying Mach numbers and rotational speeds. The results demonstrate the model’s robustness across a wide range of operating conditions.

Chapter 5 extends the model’s applicability to aeronautical fan blades. Modifications are introduced to adapt the model to fan blades, and the feasibility of using friction to stabilize both fan and LPT blades is discussed for various engine sizes by scaling blade dimensions.

Chapter 6 summarizes the main technical results and revisits the key discussions of this study.

Chapter 7 concludes the thesis with a summary of key findings and offers recommendations for future research.

1.5. Original Contributions

Several original contributions have been achieved within this thesis. To summarise:

- Extension of the original model including a mechanical dissipation formulation valid in all the microslip regime and not only in the limit of small microslip displacements.
- Extension of the original model, including the effect of the rotational speed and mistuning on the aerodynamic work-per-cycle, which drives the model behaviour specially at off-design conditions.
- Experimental validation of the analytical model. The model’s predictions have been compared with the experimental results from the ARIAS EU Project.
- Extension of the model to fan blades and sensitivity analysis of the engine size on the possibility to saturate by friction unstable turbomachinery blades.

These contributions have been published or are still in the publication process in several journals:

- Escudero, A., Corral, R., and Blando, Y., 2024. “Mach number influence on low-pressure turbine flutter”. *Journal of the Global Power and Propulsion Society*, **8**, pp. 62–72. doi:10.33737/jgpps/182890
- Escudero, A., Rodríguez-Blanco, S., and Corral, R., 2024. “Validation of a methodology to assess the flutter limit-cycle oscillation amplitude of low-pressure turbine bladed disks-part i: Mach number effects”. *Journal of Engineering for Gas Turbines and Power*, **147**(6), 11, p. 061001. doi:10.1115/1.4066587

- Escudero, A., Rodríguez-Blanco, S., and Corral, R., 2024. “Validation of a methodology to assess the flutter limit cycle oscillation amplitude of low-pressure turbine bladed disks-part ii: Rotational speed effects”. *Journal of Engineering for Gas Turbines and Power*, **147**(6), 11, p. 061002. doi:10.1115/1.4066584

as well as in several international conferences:

- Escudero, A., Corral, R., and Blando, Y., 2023. “Mach number influence on low-pressure turbine flutter”. In Proceedings from the Hong Kong23 Turbomachinery Technical Conference, Global Power and Propulsion Society
- Escudero, A., Rodríguez-Blanco, S., and Corral, R., 2024. “Validation of a methodology to assess the flutter limit cycle oscillation amplitude of low-pressure turbine bladed-disks - part i: Mach number effects”. *Turbo Expo: Power for Land, Sea, and Air*, 06
- Escudero, A., Rodríguez-Blanco, S., and Corral, R., 2024. “Validation of a methodology to assess the flutter limit cycle oscillation amplitude of low-pressure turbine bladed-disks - part ii: Rotational speed effects”. *Turbo Expo: Power for Land, Sea, and Air*, 06

2. Analytical Model for Friction Saturated LPT blades

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2.1. Analysis methodology

Unstable LPT rotor blades reach a limit cycle oscillation after thousands or tens of thousands oscillations due to nonlinear friction effects. The amplitude of these vibratory states determines the structure's life to High Cycle Fatigue (HCF) [1,8,55]. In the final equilibrium state, the aerodynamic work-per-cycle of the airfoil, W_a , and the dry friction in the blade attachment, W_m , must balance, thus

$$W_a = W_m. \quad (2.1)$$

The unsteady aerodynamics due to the vibration of the airfoil is linear [6,55] and hence the aerodynamic work scales with the square of the vibration amplitude of the airfoil, δ_a , i.e, $W_a \propto \delta_a^2$. In contrast, the mechanical damping relationship with vibration amplitude is nonlinear, and can be expressed as $W_m = f(\delta)$. This relation depends on the relative displacement between the rotor blade and the disk in the fir-tree contact, δ . This relationship is illustrated in figure 1.3, showing that the fir-tree displacement, though much smaller than the macroscopic vibration amplitude of the blade, is directly tied to it.

For very small vibration amplitudes of the blade ($\delta < \delta_{\text{offset}}$), there is no relative displacement between the rotor blade and the disk, resulting in negligible dissipation at the

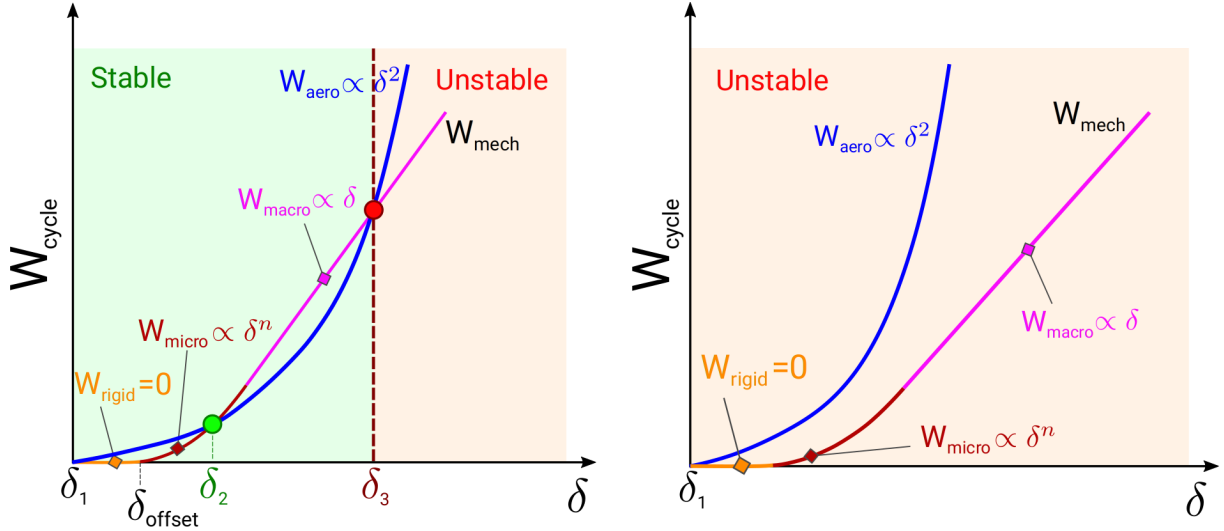


Figure 2.1.: SCENARIOS OF THE FLUTTER-SATURATED PROBLEM. LEFT: STABLE CONDITION WITH EQUILIBRIUM BETWEEN THE AERODYNAMIC WORK AND THE NON-LINEAR FRICTION DISSIPATION. RIGHT: UNSTABLE CONDITION WITHOUT ENOUGH DISSIPATION TO SATURATE THE VIBRATION AMPLITUDE.

attachment and in the whole rotor blade, as the material's dissipation is much smaller than the dry friction and the aerodynamic damping. On the other hand, large vibration amplitudes lead to macro displacements of the rotor blade in the attachment, where the mechanical damping relationship with the vibration amplitude becomes linear, $W_m \propto \delta_a$. Between these two limits, a regime known as microslip arises, where only a fraction of the contact surface is sliding. In this regime, the work done by the nonlinear dissipation in the contact can be described as $W_m \propto \delta_a^n$ with $n > 2$.

Depending on the relative scaling of aerodynamic and mechanical work, two distinct scenarios can arise, as depicted in Fig. 2.1. When there exists a range of fir-tree displacements where friction dissipation exceeds aerodynamic work, three solutions emerge, as illustrated in Fig. 2.1a. The first solution, $\delta_1 = 0$, is unstable and any small perturbation from $\delta = 0$ moves the system towards the solution 2.

This second solution represents a stable cyclic limit where the vibration amplitude reaches δ_2 . For displacements $\delta < \delta_2$, aerodynamic work dominates over friction dissipation, causing the amplitude to increase. Conversely, for $\delta > \delta_2$, the friction dissipation is higher than the aerodynamic work, decreasing the vibration amplitude. Hence, solution 2 acts as an attractor, drawing the system back to this stable cyclic limit after perturbations unless the amplitude reaches δ_3 , which is an absolute stability limit. Beyond this point ($\delta > \delta_3$), friction dissipation becomes insufficient to balance the aerodynamic work, and the system enters an unstable regime where vibration amplitudes cannot be controlled.

Alternatively, as shown in Fig. 2.1b, if the aerodynamic self-excitation is too strong or the

frictional dissipation too weak, the system fails to stabilize entirely, leaving instability as the only outcome.

2.2. Aerodynamic work model

The aerodynamic damping of LPT airfoils can be accurately computed under most operating conditions using linearized aerodynamic models [31]. In this thesis, the aerodynamic damping is calculated with the *MusT* – *L* solver, a linearized Navier-Stokes solver that has been extensively validated [56]. The primary hypothesis is that the flow can be decomposed into two components: a steady mean flow, computed with a nonlinear RANS solver [57, 58], and an unsteady harmonic perturbation, resolved using a linearized frequency-domain Navier-Stokes solver. This approach assumes negligible influence of aerodynamics on the blade’s vibration frequency and mode shape, allowing these properties to be pre-computed and used as inputs to the aerodynamic solver. While recent methods have been developed to couple nonlinear friction at contact interfaces with unsteady aerodynamics in fluttering fans [12] and LPTs [15], the model used here considers the aerodynamic damping separately from friction effects. Mode displacements are imposed on the blade surface, and moving mesh source terms are incorporated into the linearized solver.

At low reduced frequencies, typical of LPT flutter, the aerodynamic work [59–61] can be expressed as

$$W_a = \frac{1}{2} \rho_a U_e^2 S c^{-1} \delta_a^2 \tilde{W}_a(\text{IBPA}, k, C_L) \quad (2.2)$$

where ρ_a is the air density at the rotor exit, U_e is the exit velocity, S is the rotor surface, c is the blade chord, and δ_a represents the vibration amplitude of the rotor. The subscript a emphasizes the macroscopic airfoil displacement, which should not be confused with the displacement in the rotor fir-tree δ .

The air density and the exit velocity are the two parameters affected by the operation conditions. Numerical and experimental studies [8] have shown that the aerodynamic work scales directly with density, as does the vibration amplitude, aligning with Eq. 2.2. Furthermore, measurements at different density levels of the vibration amplitude and the unsteady pressure obtained during the experimental campaign levels scale proportionally to the density. The dimensionless aerodynamic work-per-cycle $\tilde{W}_a$ depends on the Inter-Blade Phase Angle, IBPA, the reduced frequency $k = \omega c / U_e$ and the lift coefficient, C_L , which reflects airfoil loading. The last two parameters vary with the shaft speed Ω as the natural frequency of the blade, load distribution and incidence angle change with the shaft speed. The influence of Reynolds and Mach numbers on $\tilde{W}_a$ is comparatively minor.

At low reduced frequencies, the minimum aerodynamic damping, defined as $W_a^{min}(\Omega) = \min_{IBPA} \tilde{W}_a(IBPA, \Omega)$, exhibits a linear relationship with k [60, 61]. Flutter severity increases with the loading but in a more indirect way than the C_L since the modeshape is involved [62].

The Mach number primarily influences aerodynamic damping by affecting the dynamic pressure ($U_e \propto M_e$), though its impact in dimensionless terms is relatively small. Nonetheless, the Mach number modifies the pressure coefficient distribution and lift coefficient $C_L(M_e)$, leading to slight variations in the dimensionless aerodynamic work [63]. Using Eq. 2.2 and assuming isentropic flow, the relationship between minimum aerodynamic damping and the exit Mach number can be expressed as:

$$\frac{W_a^{min}}{\frac{1}{2}\gamma P_0 S C^{-1} \delta_a^2} = \mathcal{F}(M_e^2) \tilde{W}_a^{min}[k(M_e, \Omega), C_L(M_e, \Omega)] \quad (2.3)$$

where the Mach dependent scaling factor $\mathcal{F}(M_e^2)$ is given by:

$$\mathcal{F}(M_e^2) = \frac{M_e^2}{\left[1 + \frac{1}{2}(\gamma - 1)M_e^2\right]^{\gamma/(\gamma-1)}}. \quad (2.4)$$

This factor is derived by expressing the dynamic pressure in terms of the specific heat ratio γ , the total pressure P_0 and the exit Mach number M_e . It is important to highlight that, from a technical perspective, the sole genuine dependence on the Mach number (i.e., compressibility) occurs through the loading (C_L) and ρ_a .

2.3. Mechanical work model

An accurate estimate of the mechanical damping is paramount in determining rotor blade vibration amplitude and the measured unsteady pressure. Material damping is negligible compared to the effects of dry friction and can be safely ignored. Dry friction in the rotor blade attachment acts as a nonlinear mechanism that limits the vibration amplitude of unstable rotor blades, leading to saturation [6]. In this work, a heuristic approach, previously used to predict the vibration amplitude of realistic rotor blades [1, 55], is applied to estimate the final saturated vibration amplitude of an aerodynamically unstable rotor and the associated unsteady pressure.

The primary assumption for estimating mechanical work is that the rotor blade attachment behaves macroscopically as a fixed contact, rendering traditional macro-slip friction models (e.g., the Coulomb model) inadequate for describing the overall contact behavior. Instead, micro-slip friction models are more suitable, as they capture the relationship between the contact's tensional state and macroscopic displacement without requiring

detailed analysis of local contact behavior. These models use distinct governing laws to establish this relationship.

The microslip model proposed by Sellgren and Ollofson [25, 44], introduced earlier in section 1.3.3, is used in this study. Figure 1.11 illustrates the hysteresis loop, showing the relationship between the dimensionless tangential force at the contact interface, $\tilde{F}$, and the dimensionless fir-tree displacement, $\tilde{\delta}$, during loading and unloading cycles. This relationship is derived from Eqs. 1.21 and 1.23 and is expressed as:

$$\tilde{F} = \mp \tilde{F}^* \pm 2 \left[1 - \left(1 - \frac{|\tilde{\delta} - \tilde{\delta}^*|}{2} \right)^{5/2} \right] \quad (2.5)$$

where the friction force is non-dimensionalized as $\tilde{F} = F/\mu N$, where μ is the friction coefficient and N the normal load. The dimensionless displacement in the attachment is $\tilde{\delta} = \delta/\delta_c$, and the characteristic length of the micro-slip is

$$\delta_c = \frac{\mu E' \lambda}{4G'} \quad (2.6)$$

where E' and G' are the composite elasticity and shear modulus respectively, and λ is the contact penetration length, which can be expressed as a function of the normal stress as

$$\lambda = \lambda_0 \left[\frac{\sigma_n}{E'} \right]^\alpha. \quad (2.7)$$

In this expression, λ_0 and α are constants depending on the material surface [64, 65] and the elastic regime.

$\tilde{F}^*$ and $\tilde{\delta}^*$ are the friction and displacement values at the turning points of the cycle, and each of the signs of the expression correspond to the loading and unloading states. It may be shown that, if the hysteresis loop is symmetric, i.e.: $\tilde{F}^* = |-\tilde{F}^*|$ and $\tilde{\delta}^* = |-\tilde{\delta}^*|$, the mechanical work-per-cycle becomes

$$W_m = \mu N \delta_c \tilde{W}_m \quad (2.8)$$

where $\tilde{W}_m$ is the dimensionless mechanical work-per-cycle, which is derived assuming harmonic friction displacement in the attachment as

$$\tilde{W}_m = \int_{cycle} \tilde{F} d\tilde{\delta} = 4(2 - \tilde{F}^*)\tilde{\delta}^* + \frac{16}{7} \left[(1 - \tilde{\delta}^*)^{7/2} - 1 \right] \quad (2.9)$$

where the dimensionless tangential force is given by

$$\tilde{F}^* = \frac{F^*}{\mu N} = 1 - (1 - \tilde{\delta}^*)^{5/2}. \quad (2.10)$$

The derivation of this expression is provided in Appendix A. If $\tilde{\delta}^* \ll 1$, or in other words, if the tangential force $F \ll \mu N$ then it can be shown that

$$\tilde{W}_m \simeq \frac{15}{6} \tilde{\delta}^{*3}. \quad (2.11)$$

The derivation details of this approximation are presented in Appendix B. Corral and Gallardo [1] used Eqs. 2.2 and 2.11, which can be expressed as $W_a = W'_a \delta_a^2$ and $W_m = W'_m \delta_a^3$, respectively, to derive the vibration amplitude of a whole family of rotor blades. If the constants W'_a and W'_m were known then the vibration amplitude could be simply computed as

$$\delta_a = W'_a / W'_m \quad (2.12)$$

However, the only reason to assume that $\tilde{\delta}^* \ll 1$ is simplicity since Eq. 2.12 allows the elimination of several unknown factors if it is employed to performed a read accross among a familiy of rotor blades. The analysis of the simplified and the complete nonlinear mechanical models reveal that the simplification of the model does not predict accurately the equilibrium of the system as the rotor blade vibration increases and $\tilde{\delta}^* \sim 1$. Therefore the complete nonlinear Eq. 2.9 instead its approximation for small amplitudes (Eq. 2.11) is employed consistently throughout this paper.

Figure 2.2 compares the mechanical work-per-cycle as a function of the vibration amplitude on the fir-tree contact, obtained by the small amplitude approximation, $W_{\text{Mech}}^{\text{Approx}}$, and the complete expression, $W_{\text{Mech}}^{\text{Exaxt}}$. Both expressions scale as $\tilde{\delta}^{*3}$ for small values of $\tilde{\delta}^*$, but differ significantly for $\tilde{\delta}^* \sim 1$. The aerodynamic work, W_{Aero} , scales with the square of the vibration amplitude, $\tilde{\delta}^{*2}$. It is remarkable that the equilibrium solution, $\tilde{\delta}_e^*$, is noticeable different depending on which expression of the mechanical work is used for large vibration amplitudes.

The assessment of the impact of operational conditions on the mechanical work-per-cycle involves a detailed examination of the terms in Eq. 2.8. The normal force, N , is originated from the centrifugal force exerted by the rotating bladed-disk. Consequently, it can be expressed as a function of the shaft speed by

$$N = m_b \omega_{n,0}^2 \tilde{\Omega}^2 R \quad (2.13)$$

where m_b represents the rotor blade mass, R is the mean blade radius, $\omega_{n,0}$ is the vibration natural frequency of the blade when the rotational speed is zero and the shaft speed, Ω , has been non-dimensionalized as

$$\tilde{\Omega} = \Omega / \omega_{n,0}. \quad (2.14)$$

On the other hand, as shown in Eq. 2.7, the microslip's characteristic length, δ_c , depends

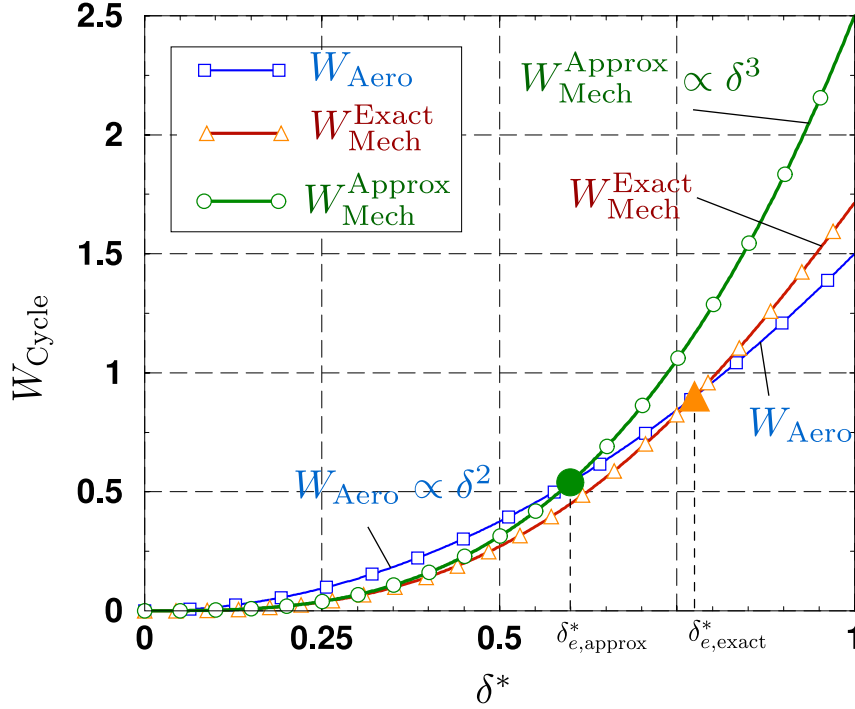


Figure 2.2.: MECHANICAL WORK-PER-CYCLE AS A FUNCTION OF THE VIBRATION AMPLITUDE IN THE CONTACT FOR TWO DIFFERENT APPROXIMATIONS.

in turn on the contact penetration length, λ , which is related with $\tilde{\Omega}$ as

$$\lambda = \lambda_0 \left(\frac{m_b \omega_{n,0}^2 \tilde{\Omega}^2 R}{E' A_c} \right)^\alpha \quad (2.15)$$

where A_c is the area of the contact surface.

Hence, the mechanical work-per-cycle can be expressed as a function of the shaft speed using Eqs. 2.13 and 2.15 as

$$W_m = \mu m_b \omega_{n,0}^2 R \frac{\mu E'}{4G'} \lambda_0 \left(\frac{m_b \omega_{n,0}^2 R}{E' A_c} \right)^\alpha \tilde{\Omega}^{2(1+\alpha)} \tilde{W}_m \quad (2.16)$$

2.4. Relation between the two scales

The vibration amplitudes associated with the aerodynamic damping, δ_a , (see Eq. 2.2), and the mechanical work, δ^* , (see Eq. 2.9) differ not only in their physical interpretation, but also in magnitude, often by several orders. While δ_a represents the macroscopic displacement of the rotor blade, δ corresponds to the much smaller relative displacement between the rotor blade and the disk at the fir-tree attachment, such that $\delta \ll \delta_a$. To bridge these two scales and provide closure to the problem, an approximate relationship between these two scales was introduced in [1,55] in terms of dimensional relations, leaving

a constant undetermined. In this work, the closure of the problem is thoroughly presented by providing two approaches—Beam Element Method (BEM) and FEM—to compute this constant. Additionally, the methodology used to compute it with FEM is explained in detail.

Due to the significant disparity in magnitude between the displacements at the rotor blade tip and the fir-tree attachment, it is advantageous to express the mechanical work in terms of the dimensionless alternate tangential force at the contact, $\tilde{F}^*$, which is more directly connected to the macroscopic blade displacement. It is worth noting that standard finite element (FE) dynamic models typically used in industry for bladed-disk frequency analysis assume that the rotor blade is rigidly clamped to the disk. More sophisticated contact boundary conditions can be implemented, enabling the evaluation of stress distributions within the contact even when the relative displacement is assumed to be zero [14, 15]. Using Eq. 2.10 to express the dimensionless fir-tree displacement, $\tilde{\delta}^*$, as a function of the dimensionless tangential force, $\tilde{F}^*$, Eq. 2.9 can be reformulated as

$$\tilde{W}_m = 4(2 - \tilde{F}^*) \left[1 - \left(1 - \tilde{F}^* \right)^{2/5} \right] + \frac{16}{7} \left[(1 - \tilde{F}^*)^{7/5} - 1 \right]. \quad (2.17)$$

The above expression, conversely than in [1], is a nonlinear relationship between $\tilde{W}_m$ and $\tilde{F}^*$. The next step is to relate the dimensionless alternate tangential force or stress at the attachment to the blade vibration. To achieve this, and considering that $\delta \ll \delta_a$, the rotor blade is conceptualized as a beam clamped in the attachment. The friction's tangential force is related to the beam attachment's tangential stress. By enforcing momentum equilibrium at the attachment, the following equation is derived

$$M = EI y_{zz} \simeq m_b L \omega_n^2 \delta_a Y_{\xi\xi}(0) \quad (2.18)$$

where M is the external momentum, L the height of the blade including the shank, and ω_N the natural frequency of the blade. $Y_{\xi\xi}(0)$ is a constant related to the beam's boundary conditions at the attachment. If the high-frequency internal dynamics of the attachment is disregarded [1] then this momentum must be counterbalanced by the vertical alternate reaction in the fir-tree.

$$R_j = \frac{M}{a_{FT} n_{lob}} \quad (2.19)$$

where R_j is the reaction in the j -th lobe, which is deemed equal for all the lobes, a_{FT} is the width of the fir-tree, and n_{lob} is the number of lobes in the attachment. The above expression assumes that the mode-shape corresponds to a bending mode. The normal alternate reaction is related to the tangential force or stress as

$$\tau_j = R_j \sin \theta / A_c \quad (2.20)$$

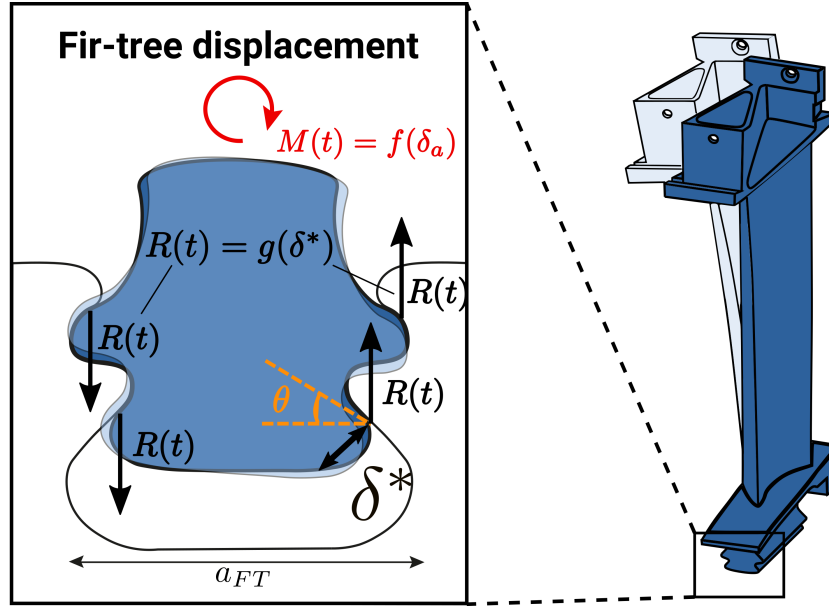


Figure 2.3.: SCHEME OF THE BALANCE OF FORCES IN THE FIR-TREE FOR A FLAP MODE.

where θ is the inclination angle of the contact surface, A_c . Figure 2.3 provides a detailed view the fir-tree geometry. Using equations 2.18, 2.19 and 2.20, the alternate stress on the contact surface is expressed as

$$\tau = B \frac{m_a L \omega_n^2 \sin \theta}{A_c a_{FT}} \delta_a \quad (2.21)$$

where B is a constant of proportionality that must be obtained from experimental data or using a FEM. Equation 2.21 is a linear relationship between the alternate stress on the contact surface, τ , and the blade vibration amplitude, δ_a , that depends on the geometry and the stiffness of the rotor blade. In this work, the constant B is determined through a FEM simulation, where fixed boundary conditions at the fir-tree contact nodes are applied to prevent any displacement. A static load is applied to represent the centrifugal force acting on the blade, which contributes to its overall stiffening.

The blade's natural frequencies, mode shapes, and forces within the structure are calculated. The reaction force, R_j , is computed as the radial component of the force acting at the constrained contact nodes, as illustrated in Fig. 2.4. Once R_j is determined, the normal alternating reaction is evaluated using Eq. 2.20. The alternating reaction is linked to the vibration amplitude—defined as the maximum displacement of the mode shape—through Eq. 2.21, which incorporates the constant B . By rearranging this relationship, the value of B is derived.

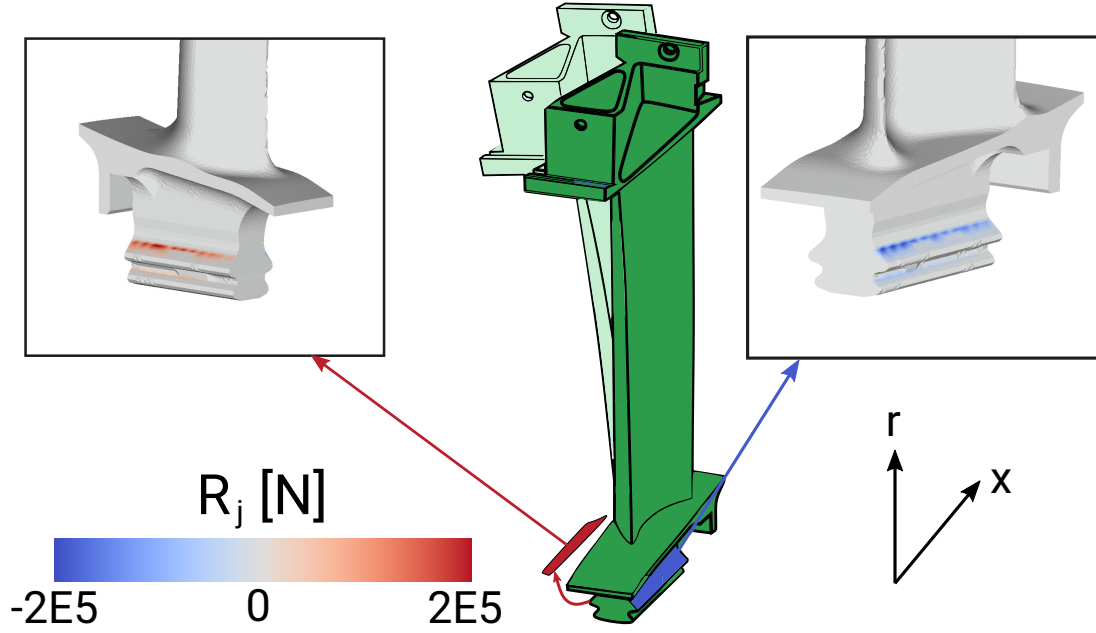


Figure 2.4.: REACTION FORCE IN THE RADIAL DIRECTION FOR THE 1ST FLAP MODE.

The alternate force can be expressed in terms of the alternate stress as $F = \tau A_c$, and it can be non-dimensionalized as (see Eq. 2.10)

$$\tilde{F}^* = B \frac{m_a L \omega_n^2 \sin \theta}{\mu N a_{FT}} \delta_a, \quad (2.22)$$

which is a linear relationship between the small scales of the contact displacements and the large scales of the rotor blade motion. Furthermore, the dimensionless alternate tangential force in the attachment also depends on the shaft speed, factored through the changes in the normal force and the natural frequency of vibration $\omega_n^2 = \omega_{n0}^2 (1 + \beta \tilde{\Omega}^2)$, where $\tilde{\Omega} = \Omega / \omega_{n0}$ and β is a constant dependent on the details of the centrifugal stiffening of the rotor blade that can be easily obtained from a FE model. Consequently, Eq. 2.22 becomes

$$\tilde{F}^* = \frac{1 + \beta \tilde{\Omega}^2}{\tilde{\Omega}^2} \tilde{\delta}_a \quad (2.23)$$

with the vibration amplitude non-dimensionalized as $\tilde{\delta}_a = \delta_a / \delta_{ac}$, where

$$\delta_{ac} = \mu a_{FT} R / (BL \sin \theta) \quad (2.24)$$

is the characteristic blade vibration amplitude. Since there is a linear relationship between the dimensionless tangential force and the dimensionless vibration amplitude, Eq. 2.23 can be expressed as a $\tilde{F}^* = f(\beta, \tilde{\Omega}) \tilde{\delta}_a$, where $f(\beta, \tilde{\Omega})$ includes the dependencies with β and $\tilde{\Omega}$.

Injecting Eq. 2.23 into Eq. 2.17, the dimensionless mechanical work is linked to the

vibration amplitude and the shaft speed. Its functional dependence can be expressed as

$$\tilde{W}_m = \tilde{W}_m \left[f(\beta, \tilde{\Omega}) \tilde{\delta}_a \right]. \quad (2.25)$$

This simplified approach offers advantages in several aspects by omitting the attachment details. Whether a beam or a finite element model is employed to calculate the natural frequencies of the rotor blade, the model can be constructed under the assumption that the rotor blade is clamped at the attachment. This assumption enables the displacement at the contact to be replaced as the main unknown of the model for the tangential alternating stress in the contact.

In the limit of small vibration amplitude in the fir-tree ($\tilde{\delta}^* \ll 1$), when Eq. 2.12 is used to relate to geometrically similar rotor blades or the same rotor at different operating conditions, many of the unknown or difficult-to-estimate constants in the model cancel out. The low sensitivity to unknown data is one of the keys to the model's success, which is presented in subsection 3.4. For a particular rotor blade, the linear relation between the stress state in the contact and the macroscopic displacement can also be derived from an FE model with increased accuracy but at the expense of less agility and flexibility.

2.5. Vibration amplitude

The calculation of blade vibration amplitude is obtained by equating the aerodynamic work (Eq. 2.2) and the mechanical work (Eq. 2.16) and solving the nonlinear equation

$$\Re^* \tilde{\Omega}^{-2(1+\alpha)} \tilde{W}_a^{\min}(\tilde{\Omega}) \tilde{\delta}_a^2 = \tilde{W}_m \left[f(\beta, \tilde{\Omega}) \tilde{\delta}_a \right] \quad (2.26)$$

where $\Re^*$ is a dimensionless constant that controls the level of non-linearity of the model and is expressed as

$$\Re^* = \frac{\frac{1}{2} \gamma P_0 F(M_e^2) a_{FT}^2 \mu}{m_b L \omega_{n0}^2 B^2 \sin^2 \theta} \frac{4G'}{E'} \frac{R}{\lambda_0} \left(\frac{E' A_c}{m_b \omega_{n0}^2 R} \right)^\alpha. \quad (2.27)$$

The parameter $\Re^*$ represents the influence of the physical parameters of the model on the relation of the blade's capacity to absorb aerodynamic work over its capacity to dissipate work through dry-friction in the fir-tree. This parameter depends on the operating condition, material, blade size and geometry.

In the microslip regime, characterized by the limit $\Re^* \ll 1$ limit, the mechanical work can be expressed as $\tilde{W}_m \propto \tilde{\delta}^{*3}$ while the dimensionless fir-tree displacement is linearly related to the dimensionless vibration amplitude, i.e. $\tilde{\delta}^* \propto \tilde{\delta}_a$. Hence, the mechanical work can

be approximated with a cubic function of $\tilde{\delta}_a$, i.e. $\tilde{W}_m \approx 4/25 \left[f(\beta, \tilde{\Omega}) \tilde{\delta}_a \right]^3$, effectively simplifying Eq. 2.26 for straightforward calculation of the dimensionless vibration amplitude as

$$\tilde{\delta}_a = \frac{25}{4} \mathfrak{R}^* \frac{\tilde{\Omega}^{2(2-\alpha)}}{(1 + \beta \tilde{\Omega}^2)^3} \tilde{W}_a^{\min}(\tilde{\Omega}), \quad (2.28)$$

showing a linear effect of $\mathfrak{R}^*$ on $\tilde{\delta}_a$.

Equation 2.26 relates the different physical parameters, grouped in the constant $\mathfrak{R}^*$, the minimum dimensionless aerodynamic damping $\tilde{W}_a^{\min}$, the dimensionless rotational speed $\tilde{\Omega}$, the Southwell coefficient β [66] and the dimensionless vibration amplitude $\tilde{\delta}_a$. Hence, the model depends on the operating point, the geometry and the material's properties. While most of these properties can be obtained from CFD and FEM analyses, some, particularly those related to frictional contact, are difficult to estimate. The parameter λ_0 , for instance, represents the intersection of microasperities between the contacting solids [25, 44]. This challenge of estimating λ_0 is addressed by calibrating the model with experimental data.

3. Vibration amplitude results

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The analytical model presented in Section 2 is applied to predict the vibration amplitude of an LPT rig blade, which has been experimentally analyzed and tested within the scope of the ARIAS project [24, 47, 50, 51, 67]. The test aimed to reproduce the aeromechanic behaviour of a real-engine LPT blade. Hence, some modifications on the blade have been included to reproduce all the nondimensional parameters of the engine, including those related to the aeroelastic behavior.

This chapter begins by characterizing the studied blade, detailing the CFD solver used for its aerodynamic and aeroelastic characterization. The influence of the Mach number on the blade's performance is then discussed, followed by an analysis of the rotational speed and its effects on both the aerodynamic and contact mechanical behavior. Finally, the chapter concludes with a sensitivity analysis of the model parameters that are particularly difficult to estimate, providing insight into their impact on the model's predictions.

3.1. Characterization of the LPT blade

3.1.1. CFD Solver

The aerodynamic and aeroelastic analysis of the LPT blade presented in this thesis has been conducted using Mu^2s^2T [56, 57, 68], a robust and extensively validated suite of solvers developed by ITP Aero. Mu^2s^2T is a second-order finite volume solver that runs over hybrid grids using an edged-base data structure [57, 69]. The solver is parallelized via the

Message Passing Interface (MPI) library and optimized for execution on Graphics Processing Units (GPUs), offering a cost-efficient solution for industrial Reynolds-averaged Navier-Stokes (RANS) simulations [58].

The analysis of LPT blade behavior necessitates performing numerous simulations, driven by the wide range of nodal diameters at which flutter may occur and the various operating conditions that must be evaluated for stability. Consequently, the computational workload involves an extensive case matrix with significant resource demands. To mitigate these costs, frequency-domain solvers based on linearization can be employed. For perspective, a single 3D unsteady linear simulation of a low-pressure turbine rotor blade comprising approximately 2.5M grid points, achieves convergence within about one hour using a single GPU (*Nvidia RTX 3080*). This chapter provides an overview of the fundamental principles underlying the linearized Navier-Stokes solver applied in this work.

3.1.1.1. Linearised Navier-Stokes Solver

Frequency-domain linearized Navier-Stokes solvers were introduced for turbomachinery studies in the 1990s [70, 71] as an efficient method to model unsteady harmonic flows, particularly in aeroelastic and aeroacoustic applications. These solvers demonstrated computational speeds one to two orders of magnitude greater than direct time-domain simulations of the Navier-Stokes equations [70].

The 3D RANS equations, expressed in integral form and assuming no heat sources [72], for a deformable control volume $\Omega(t)$ in the relative frame of reference, are given by:

$$\int_{\Omega(t)} \frac{\partial \mathbf{U}}{\partial t} d\Omega + \int_{\Sigma(t)} [\mathbf{F}^I(\mathbf{U}) - \mathbf{F}^v(\mathbf{U}, \nabla \mathbf{U}) - \mathbf{U} \mathbf{v}_c^T] \cdot d\mathbf{A} = \int_{\Omega(t)} \mathbf{S} d\Omega \quad (3.1)$$

where $\Sigma(t)$ denotes the control volume surface, $\mathbf{U}$ represents the vector of conservative variables, $\mathbf{F}^I$ and $\mathbf{F}^v$ are the matrices of the inviscid and viscous fluxes, $\mathbf{S}$ are the source terms including the inertial forces, $\mathbf{A}$ is the outward-pointing surface vector and $\mathbf{v}_c$ is the velocity of the control volume surface. The vector of conservative variables is:

$$\mathbf{U} = (\rho \quad \rho u \quad \rho v \quad \rho w \quad \rho E)^T \quad (3.2)$$

the inviscid flux tensor components are:

$$\mathbf{F}^I = (\mathbf{f}^I \quad \mathbf{g}^I \quad \mathbf{h}^I) = \begin{bmatrix} \rho u & \rho v & \rho w \\ \rho u^2 + p & \rho uv & \rho uw \\ \rho uv & \rho v^2 + p & \rho vw \\ \rho uw & \rho vw & \rho w^2 + p \\ u(\rho E + p) & v(\rho E + p) & w(\rho E + p) \end{bmatrix} \quad (3.3)$$

while the viscous flux tensor is defined as:

$$\mathbf{F}^v = (\mathbf{f}^v \quad \mathbf{g}^v \quad \mathbf{h}^v) = \begin{bmatrix} 0 & 0 & 0 \\ \tau_{xx}^v & \tau_{xy}^v & \tau_{xz}^v \\ \tau_{xy}^v & \tau_{yy}^v & \tau_{yz}^v \\ \tau_{xz}^v & \tau_{yz}^v & \tau_{zz}^v \\ u(\tau_{xx}^v + \tau_{xy}^v + \tau_{xz}^v) & v(\tau_{xy}^v + \tau_{yy}^v + \tau_{yz}^v) & w(\tau_{xz}^v + \tau_{yz}^v + \tau_{zz}^v) \end{bmatrix} \quad (3.4)$$

where the viscous stress tensor τ_{ij}^v for a Newtonian fluid is defined as:

$$\tau_{ij}^v = \mu \left(\frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right) - \left(\mu_v - \frac{2}{3}\mu \right) \frac{\partial v_i}{\partial x_i} \delta_{ij} \quad (3.5)$$

with μ_v being the volume viscosity coefficient which is typically set to zero in aerodynamic applications. The viscous fluxes account for both laminar and turbulent viscosity contributions, $\mu = \mu^{\text{lam}} + \mu^{\text{tur}}$, with laminar viscosity determined using Sutherland's law [73], while turbulent viscosity is calculated via turbulence models. The Mu^2s^2T suite includes several turbulence models; however, in the linear solver, the eddy viscosity remains constant and is precomputed using the Wilcox $k - \omega$ model [74] within the non-linear solver [57]. Finally, the inertial forces $\mathbf{f}_m = -\rho\Omega_R \times (\Omega_R \times \mathbf{x}) - 2\rho\Omega_R \times \mathbf{v}$, contribute to the forcing term:

$$\mathbf{S} = [0 \quad \rho\mathbf{f}_m \quad \rho\mathbf{f}_m \cdot \mathbf{v}]^T = \mathbf{S}(\mathbf{U}, \mathbf{x}) \quad (3.6)$$

indicating with Ω_R the vector of the rotational speed.

When the vibration amplitude of airfoils is sufficiently small, the unsteady flow behavior can be accurately modeled by linearizing the governing equations. This simplification captures the small-disturbance dynamics of the system. Assuming harmonic blade motion with a temporal angular frequency ω and a constant inter-blade phase angle σ , the solution, $\mathbf{U}(\mathbf{x}, t)$, is decomposed into a steady base flow, $\mathbf{U}_0(\mathbf{x})$, and a small perturbation $\mathbf{u}(\mathbf{x}, t) \ll \mathbf{U}_0$, which in turn can be expressed as a Fourier series in time. By retaining only the first harmonic of the perturbations, any flow variable can be expressed as:

$$\mathbf{U}(\mathbf{x}, t) = \mathbf{U}_0(\mathbf{x}) + \text{Re}(\hat{\mathbf{u}}_\omega(\mathbf{x})e^{-I\omega t}). \quad (3.7)$$

In turbomachinery applications, accuracy of the numerical scheme can be enhanced by using a harmonically deforming computational grid that conforms to the vibrating blade mode shapes [71]. Flow unsteadiness results from the periodic motion of solid boundaries, such as the LPT airfoil in this case. The grid, conforming to these vibrating surfaces, undergoes harmonic deformation, which can be described as:

$$\mathbf{x}(t) = \mathbf{x}_0 + \text{Re}(\hat{\mathbf{x}}(\mathbf{x})e^{-I\omega t}). \quad (3.8)$$

Consequently, the control volume Ω of each cell, its boundary area $\mathbf{A}$ and the velocity of its surface $\mathbf{W}_c$ are also expressed as the sum of a mean value and a small harmonic perturbation:

$$\begin{aligned} \Omega(\mathbf{x}, t) &= \Omega_0(\mathbf{x}) + \text{Re}(\hat{\Omega}(\mathbf{x})e^{-I\omega t}) \\ \mathbf{A}(\mathbf{x}, t) &= \mathbf{A}_0(\mathbf{x}) + \text{Re}(\hat{\mathbf{a}}(\mathbf{x})e^{-I\omega t}) \\ \mathbf{W}_c(\mathbf{x}, t) &= \mathbf{W}_0(\mathbf{x}) + \text{Re}(\hat{\mathbf{w}}(\mathbf{x})e^{-I\omega t}) \end{aligned} \quad (3.9)$$

where the perturbation terms are significantly smaller than their mean counterparts, i.e., $\hat{\Omega}(\mathbf{x}, t) \ll \hat{\Omega}_0(\mathbf{x})$ and $\hat{\mathbf{a}}(\mathbf{x}) \ll \mathbf{A}_0(\mathbf{x}, t)$. Additionally, the mean mesh velocity is zero, $\mathbf{W}_0(\mathbf{x}) = 0$. The steady-state flow is first computed by solving the stationary problem:

$$\int_{\Sigma} [\mathbf{F}^I(\mathbf{U}_0) - \mathbf{F}^v(\mathbf{U}_0, \nabla \mathbf{U}_0)] \cdot d\mathbf{A} = \int_{\Omega} \mathbf{S}_0 d\Omega \quad (3.10)$$

Once the steady solution is established, the unsteady linear solution is derived. By substituting Eqs. 3.7, 3.8 and 3.9 into Eq. 3.1, discarding the second-order terms, and applying Eq. 3.10, the linearized Navier-Stokes equations in conservative form are obtained:

$$-I\omega \int_{\Omega} \hat{\mathbf{u}} d\Omega_0 + \int_{\Sigma} \left[\left(\frac{\partial \mathbf{F}^{I+v}}{\partial \mathbf{U}} \right)_0 \hat{\mathbf{u}} \right] \cdot d\mathbf{A}_0 - \int_{\Omega} \left(\frac{\partial \mathbf{S}}{\partial \mathbf{U}} \right)_0 \hat{\mathbf{u}} d\Omega_0 = \hat{\mathbf{X}}_{\omega} \quad (3.11)$$

where $(\partial \mathbf{F}^{I+v}/\partial \mathbf{U})_0$ and $(\partial \mathbf{S}/\partial \mathbf{U})_0$ represent the Jacobian matrices of fluxes and source terms with respect to conservative variables, evaluated at the mean flow state. Note that the former is actually a vector of matrices since $(\partial \mathbf{F}^{I+v}/\partial \mathbf{U})_0 = (\partial \mathbf{f}/\partial \mathbf{U})_0 \mathbf{i} + (\partial \mathbf{g}/\partial \mathbf{U})_0 \mathbf{j} + (\partial \mathbf{h}/\partial \mathbf{U})_0 \mathbf{k}$ and therefore $(\partial \mathbf{F}^{I+v}/\partial \mathbf{U})_0 \hat{\mathbf{u}}$ is a matrix. Concerning the viscous fluxes, the matrix contains spatial derivative operators. For the linearized solver, the laminar and turbulent viscosity coefficients $\mu = \mu^{\text{lam}} + \mu^{\text{tur}}$ are held constant over time, simplifying to $\mu = \mu_0(\mathbf{x})$. This eliminates the need for turbulence modeling, unlike the non-linear solver.

The source terms and fluxes induced by time-dependent control volume variations are

included in $\hat{\mathbf{X}}_\omega$, expressed as [71]:

$$\hat{\mathbf{X}}_\omega = +I\omega \int_{\Omega} \mathbf{U}_0 d\hat{\Omega}_\omega - \int_{\Sigma} \mathbf{F}_0 \cdot d\hat{\mathbf{A}}_\omega + \int_{\Omega} \mathbf{S}_0 d\hat{\Omega}_\omega - I\omega \int_{\Sigma} \mathbf{U}_0 \hat{\mathbf{x}}_\omega \cdot d\mathbf{A}_0 + \int_{\Omega} \left(\frac{\partial \mathbf{S}}{\partial \mathbf{x}} \right)_0 \hat{\mathbf{x}}_\omega \cdot d\mathbf{A}_0 \quad (3.12)$$

where the first, second and third terms arise from the harmonic change of the volume and the surface. The fourth term is caused by the harmonic motion of the control volume, and finally, the last one is due to the change of the inertial forces with the volume's position.

3.1.1.2. Spatial discretization

The computational domain is discretized using hybrid unstructured grids with an arbitrary number of faces, providing the flexibility needed to handle the complex geometries typical of turbomachinery. The linearized equations 3.11 are solved using the Finite Volume Method (FVM) on a dual mesh [75], as illustrated in Fig. 3.1. In edge-based solvers, finite volumes are constructed by connecting the centroids of cells to the midpoints of edges formed by neighboring nodes. The solution vector, $\hat{\mathbf{u}}_\omega$, is stored at the mesh vertices. Fluxes across control volume faces are calculated as the average of the fluxes at the edge's two nodes. Finally, a pseudo-time derivative is included in order to solve Eq. 3.11. Similarly as in the steady-state solvers, the governing equations are marched in the pseudo-time, t^* [76]. Hence, the semi-discrete formulation of the Linearised Navier-Stokes Equations for cell i is expressed as:

$$\Omega_i \left(\frac{\partial \hat{\mathbf{u}}_\omega^i}{\partial t^*} - I\omega \hat{\mathbf{u}}_\omega^i - \left(\frac{\partial \mathbf{S}}{\partial \mathbf{U}} \right)_0 \hat{\mathbf{u}}_\omega^i \right) + \sum_{j=1}^{n_{\text{edg}}} \left[\hat{\mathbf{F}}_{ij} - \hat{\mathbf{D}}_{ij} \right] \cdot \mathbf{n}_{ij} \sigma_{ij} = 0. \quad (3.13)$$

where n_{edg} is the number of edges of the cell, σ_{ij} is the area associated with the edge that joins the node i with its neighbour j , with a normal vector $\mathbf{n}_{ij}$. The fluxes at the edge are indicated with $\hat{\mathbf{F}}_{ij}$, and the artificial viscosity term, $\hat{\mathbf{D}}_{ij}$. This artificial viscosity, essential for stabilizing the numerical scheme, is formulated in matrix form to appropriately scale both convective and acoustic wave contributions.

To enhance the accuracy of the discretization, the Monotonic Upstream-Centered Schemes for Conservation Laws (MUSCL) scheme [77] is employed. In practical applications, this scheme achieves an accuracy level between second and third order, with the latter being limited to one-dimensional problems on uniform grids. The numerical dissipation terms are determined using a parabolic reconstruction of the flow variables at the cell edges, denoted as $\hat{\mathbf{u}}_\omega^{i+}$ and $\hat{\mathbf{u}}_\omega^{j-}$. The fluxes are computed using the following expression:

$$\hat{\mathbf{F}}_{ij}^I - \hat{\mathbf{D}}_{ij} = \frac{1}{2} \left\{ (\hat{\mathbf{F}}^I(\hat{\mathbf{u}}_\omega^{i+}) + \hat{\mathbf{F}}^I(\hat{\mathbf{u}}_\omega^{j-})) - |\mathbf{A}_{ij}(\mathbf{U}_0^{i+}, \mathbf{U}_0^{j-})|(\hat{\mathbf{u}}_\omega^{i+} - \hat{\mathbf{u}}_\omega^{j-}) \right\} \quad (3.14)$$

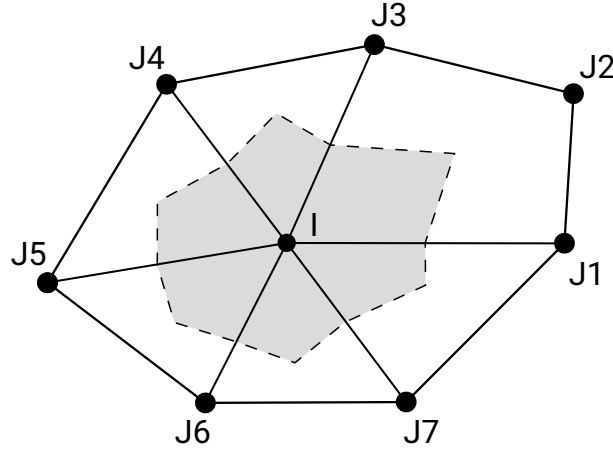


Figure 3.1.: SKETCH OF A HIBRID CELL. DASHED LINES REPRESENT THE DUAL MESH.

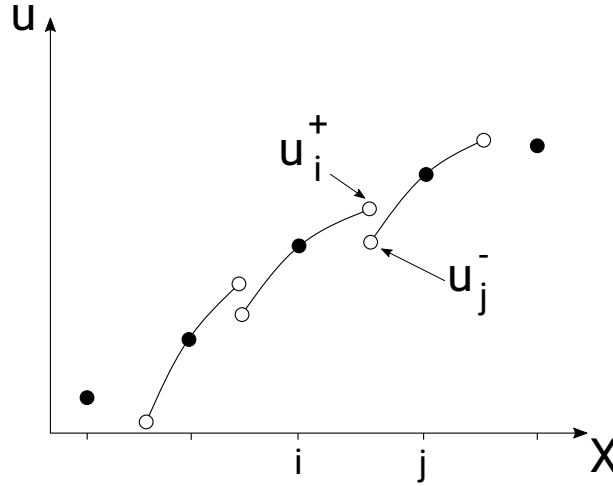


Figure 3.2.: SCHEMATIC REPRESENTATION OF THE RECONSTRUCTION OF THE LINEAR VARIABLES AT THE CELL BOUNDARY.

where $|\mathbf{A}_{ij}|$ represents the Roe Matrix [78]. The Harten entropy fix condition [79] is employed to prevent numerical singularities that can occur near supersonic flow conditions, otherwise arising when employing Roe's numerical flux, i.e. Eq. 3.14. Figure 3.2 illustrates the parabolic reconstruction of the flow variables at the cell edges, $\hat{\mathbf{u}}_j^{i+}$ and $\hat{\mathbf{u}}_\omega^{j-}$ which can be written as [77]:

$$\hat{\mathbf{u}}_\omega^{i+} = \hat{\mathbf{u}}_\omega^i + \frac{S_i}{4}[(1 - \kappa S_i)\Delta_i^- + (1 + \kappa S_i)(\hat{\mathbf{u}}_\omega^j - \hat{\mathbf{u}}_\omega^i)] \quad (3.15)$$

$$\hat{\mathbf{u}}_\omega^{j-} = \hat{\mathbf{u}}_\omega^j - \frac{S_j}{4}[(1 - \kappa S_j)\Delta_j^+ + (1 + \kappa S_j)(\hat{\mathbf{u}}_\omega^j - \hat{\mathbf{u}}_\omega^i)] \quad (3.16)$$

where S is a numerical flux limiter described next, and κ a free parameter such that if $\kappa = 1/3$ the spatial discretisation is third-order accurate for a one-dimensional uniform mesh. In Eqs. 3.15 and 3.16 the backward and forward difference operators, Δ_i^- and Δ_j^+

respectively are defined as:

$$\Delta_i^- = 2\nabla \hat{\mathbf{u}}_\omega^i \cdot (\mathbf{x}_j - \mathbf{x}_i) - (\hat{\mathbf{u}}_\omega^j - \hat{\mathbf{u}}_\omega^i) \quad (3.17)$$

$$\Delta_j^+ = 2\nabla \hat{\mathbf{u}}_\omega^j \cdot (\mathbf{x}_j - \mathbf{x}_i) - (\hat{\mathbf{u}}_\omega^j - \hat{\mathbf{u}}_\omega^i). \quad (3.18)$$

which, on a one-dimensional uniform grid, simplify to $\Delta_i^- = \hat{\mathbf{u}}_\omega^i - \hat{\mathbf{u}}_\omega^{i-1}$ and $\Delta_j^+ = \hat{\mathbf{u}}_\omega^{j+1} - \hat{\mathbf{u}}_\omega^j$. To limit spurious oscillations near shock waves or steep mean flow gradients, the numerical scheme is designed to maintain monotonicity. The Van Albada limiter [80] is used to modify the reconstruction, ensuring the scheme reverts to first-order near flow discontinuities:

$$S_i = \max \left(0, \frac{2\Delta_i^-(\hat{\mathbf{u}}_\omega^j - \hat{\mathbf{u}}_\omega^i) + \delta}{(\Delta_i^-)^2 + (\hat{\mathbf{u}}_\omega^j - \hat{\mathbf{u}}_\omega^i)^2 + \delta} \right) \quad (3.19)$$

$$S_j = \max \left(0, \frac{2\Delta_j^+(\hat{\mathbf{u}}_\omega^j - \hat{\mathbf{u}}_\omega^i) + \delta}{(\Delta_j^+)^2 + (\hat{\mathbf{u}}_\omega^j - \hat{\mathbf{u}}_\omega^i)^2 + \delta} \right) \quad (3.20)$$

where δ is a small value added to recover the baseline formulation $S_i = S_j = 1$ in smooth regions of the flow. Conversely, near shock-waves $S_i = S_j \simeq 0$ resulting in a stable first-order upwinding scheme. For a uniform grid with element size Δx , the scheme in smooth regions simplifies to:

$$\begin{aligned} \frac{\partial \hat{\mathbf{u}}_\omega^j}{\partial t^*} - I\omega \hat{\mathbf{u}}_\omega^j + \mathbf{A}_0 \left[\left(\frac{3-\kappa}{2} \right) \left(\frac{\partial \hat{\mathbf{u}}_\omega}{\partial x} \right)_j^{\Delta x} - \left(\frac{1-\kappa}{2} \right) \left(\frac{\partial \hat{\mathbf{u}}_\omega}{\partial x} \right)_j^{2\Delta x} \right] = \\ = \left(\frac{1-\kappa}{2} \right) \Delta x |\mathbf{A}_0| \left[\left(\frac{\partial^2 \hat{\mathbf{u}}_\omega}{\partial x^2} \right)_j^{\Delta x} - \left(\frac{\partial^2 \hat{\mathbf{u}}_\omega}{\partial x^2} \right)_j^{2\Delta x} \right] = - \left(\frac{1-\kappa}{8} \right) \Delta x^3 |\mathbf{A}_0| \left(\frac{\partial^4 \hat{\mathbf{u}}_\omega}{\partial x^4} \right)_j^{\Delta x} \end{aligned} \quad (3.21)$$

where $\mathbf{A}_0 = (\partial \mathbf{f} / \partial \mathbf{U})_0$ and $(\partial^n \hat{\mathbf{u}}_\omega / \partial x^n)_j^{m\Delta x}$ is the n -th second-order centred finite difference with spacing $m\Delta x$ in the node j . Note that for $\kappa = 1$ a standard second-order centred scheme is recovered whereas for $\kappa = 1/3$ the first derivative becomes fourth-order, and hence the whole scheme is third-order accurate in space.

The viscous fluxes are calculated based on the gradients of the mean and linearized flow variables:

$$\mathbf{F}_{ij}^v = \mathbf{F}^v(\nabla \hat{\mathbf{u}}_\omega^{ij}, \hat{\mathbf{u}}_\omega^{ij}, \mathbf{U}_0^{ij}, \nabla \mathbf{U}_0^{ij}). \quad (3.22)$$

where $\hat{\mathbf{u}}_\omega^{ij}$, $\mathbf{U}_0^{ij}$ represent the average of linear and mean flow variables between nodes i and j . The gradient operator is computed as:

$$\nabla U_i = \frac{1}{\Omega_i} \sum_{j=1}^{n_{\text{edg}}} \frac{1}{2} (U_j - U_i) \mathbf{n}_{ij} \sigma_{ij} \quad (3.23)$$

while the fluxes, which need to be evaluated at the cell edge mid-point, are evaluated as:

$$\overline{\nabla U_{ij}} = \frac{1}{2}(\nabla U_i + \nabla U_j), \quad (3.24)$$

However, this approach leads to an odd-even decoupling that is not prevented by the artificial viscosity fluxes, especially in regions like boundary layers where physical viscous terms dominate. To prevent this, the edge-aligned gradient is replaced by the difference of variables along the edge [81]:

$$\nabla U_{ij} = \overline{\nabla U_{ij}} - (\overline{\nabla U_{ij}} \cdot \mathbf{l}_{ij} - \frac{U_j - U_i}{|\mathbf{x}_j - \mathbf{x}_i|}) \mathbf{l}_{ij} \quad (3.25)$$

where $\mathbf{l}_{ij}$ is the unit vector along the edge, defined as:

$$\mathbf{l}_{ij} = \frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|} \quad (3.26)$$

3.1.1.3. Temporal discretization

A modified low-storage Runge-Kutta 5 scheme, where the artificial viscosity and viscous terms are only evaluated in the odd stages [82], is employed to march in time until a converged steady state is achieved. One key advantage of frequency-domain solvers over time-domain solvers is that preconditioning techniques can be employed to reduce the convergence time. Accordingly, Eq. 3.13 is reformulated as:

$$\mathbf{P}^{-1} \frac{\partial \hat{\mathbf{u}}_\omega^i}{\partial t^*} = \frac{1}{\Omega_i} \left[(\hat{\mathbf{s}}_\omega^i + I \omega \hat{\mathbf{u}}_\omega^i) \Omega_i - \sum_{j=1}^{n_{\text{edg}}} (\hat{\mathbf{F}}_{ij} \cdot \mathbf{n}_{ij} - \hat{\mathbf{D}}_{ij} \cdot \mathbf{n}_{ij}) \sigma_{ij} \right] \quad (3.27)$$

where $\mathbf{P}^{-1}$ is the preconditioning matrix and t^* a pseudo-time. The matrix $\mathbf{P}^{-1}$ must be efficient to compute and apply, ensuring that the resulting system is better conditioned and easier to solve than the original formulation. This preconditioned equation, 3.27 is marched in time until the steady state is reached. Once convergence is achieved, $\mathbf{P}^{-1} \partial \hat{\mathbf{u}}_\omega / \partial t^* = 0$ meaning the preconditioner and the pseudo-time derivative no longer influence the solution. Local time-stepping and three levels of multi-grid are used to precondition the problem.

The local time step serves as a scalar preconditioner, dynamically adjusting the time step for each cell based on its local stability limit, determined by the eigenvalue spectrum of the discretized spatial operators. This approach is particularly beneficial in domains with a wide range of cell sizes, as is common in viscous problems where thin, stretched cells are necessary to accurately resolve the boundary layer.

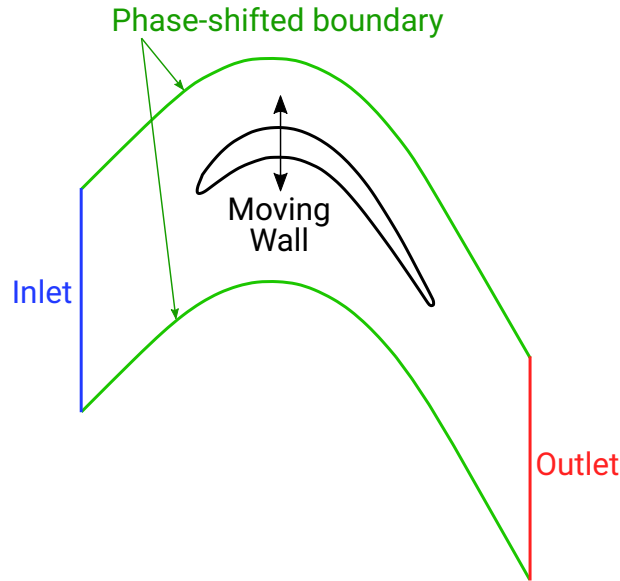


Figure 3.3.: COMPUTATIONAL DOMAIN AND BOUNDARY CONDITIONS EMPLOYED IN THE UNSTEADY LINEARISED ANALYSES.

Multigrid methods are extensively used in turbomachinery simulations to accelerate convergence to a steady state [83]. Most numerical schemes efficiently damp high-frequency errors in just a few iterations. However, low-frequency errors tend to decay much more slowly, requiring significantly more iterations. The multigrid strategy addresses this by transforming low-frequency errors into high-frequency ones using a hierarchy of progressively coarser meshes. On coarser grids, the longer wavelength modes become high-frequency errors, which are then more effectively damped by the numerical scheme. The process for constructing these coarser meshes, starting from the baseline grid, as well as the details of the multigrid formulation, are thoroughly discussed in [69, 84].

3.1.1.4. Boundary conditions

The Mu^2s^2T solver implements three primary types of boundary conditions: wall boundary conditions, phase-shifted periodicity, and inlet/outlet boundary conditions. These, along with the computational domain, are illustrated in Fig. 3.3. For solid walls, the boundary condition enforces a no-slip velocity in viscous cases or a velocity parallel to the wall for inviscid calculations. Adiabatic wall conditions are commonly applied in aeroelastic simulations.

A significant advantage of frequency-domain linearized Navier-Stokes solvers is their ability to simulate configurations where blades are not in-phase, while limiting the computational domain to a single blade. This is achieved by imposing modified periodic conditions on the lateral boundaries of the domain. Assuming the blades vibrate at a constant frequency, ω , and inter-blade phase angle, σ , the periodic boundary condition must account

for the phase-lag between the boundaries. This generalized periodicity is expressed as:

$$\mathbf{U}(\mathbf{x}, t) = \mathbf{U}(\mathbf{x}_{\Delta\theta}, t - \Delta T) \quad (3.28)$$

where $\mathbf{x} = (x, r, \theta)$ and $\mathbf{x}_{\Delta\theta} = (x, r, \theta + \theta_p)$, his indicates that the signal at one periodic boundary at time t corresponds to the signal at the opposite boundary $\theta + \theta_p$, where θ_p is the airfoil pitch, at a previous time instant, $t - \Delta T$,

$$\Delta T = \frac{\sigma}{\omega}. \quad (3.29)$$

For the unsteady perturbation flow, the complex periodicity condition is given as:

$$\hat{\mathbf{u}}(\mathbf{x}_{\Delta\theta}) = \hat{\mathbf{u}}(\mathbf{x})e^{I\sigma} \quad (3.30)$$

These periodic boundary conditions allow the computational domain to be reduced to a single blade passage, significantly decreasing the computational cost and time required for the simulation.

On the airfoil surface, the flow adheres to the no-slip condition, which requires that the fluid velocity matches the airfoil surface velocity. The position of the airfoil at any time, t is described by its position vector of the airfoil $\mathbf{x}_a(s, t)$, where s is the distance along the airfoil surface. Using a perturbation series, the position can be expressed as:

$$\mathbf{x}_a(s, t) = \mathbf{x}_{a,0}(s) + \hat{\mathbf{x}}'_a(s)e^{-I\omega t} \quad (3.31)$$

The first order approximation of the no-slip condition is then:

$$\hat{\mathbf{v}}(s) = I\omega\hat{\mathbf{x}}'_a(s) \quad (3.32)$$

where $\hat{\mathbf{v}}$ is the perturbation flow velocity on the airfoil surface.

For inlet and outlet boundaries, two-dimensional non-reflecting boundary conditions (NRBC), as proposed by Giles [85], are applied on a plane-by-plane basis to minimize the reflection of perturbations. This method involves decomposing the solution into azimuthal Fourier modes with wave number ζ , $\hat{\mathbf{u}}_\omega = \hat{\mathbf{u}}_{\omega,\zeta}e^{I\zeta\theta}$. Once the decomposition is obtained, each mode is treated independently, and the final solution is reconstructed using the Inverse Fourier Transform. This approach effectively reduces spurious reflections and ensures accurate boundary treatment.

M_{in}	α_{in}	M_e	α_e	Re
0.42	36.5°	0.7	60°	1.2×10^5

Table 3.1.: BOUNDARY CONDITIONS FOR THE DESIGN INTENT OF THE 70% SPAN ROTOR BLADE SECTION.

3.1.2. Aerodynamic characterization

The rotor blade analyzed in this study represents a typical low-speed, low-pressure turbine configuration commonly used in aeronautical applications. Its exit Mach number, Reynolds number, velocity triangles, and load distributions are characteristic of modern aeronautical LPT designs. However, certain aerodynamic refinements aimed at reducing secondary losses were omitted from the design to prioritize simplicity. As a result, the secondary flows in the tip region penetrate further into the mainstream than would be expected in a more optimized configuration. This design choice is shown to influence the evolution of the aerodynamic work with changes in rotational speed. Moreover, the aerodynamics near the hub is irrelevant from the aeroelastic point of view since this is the section with the lowest mobility.

Detailed design specifications for the 70% span section are provided in Table 3.1. The airfoil geometry, closely resembling the well-known T106 profile, is described in [59]. The total deflection reaches approximately 100 degrees, with a lift coefficient of around 1 for the blade sections. The exit Mach number lies within the high subsonic range. Aerodynamic sections across the span are nearly identical to the 70% span section and were stacked radially around their centers of mass. The rotor blade aspect ratio, $\Lambda = L/c_{ax}$, is 5.5, constrained by the minimum machinable chord length. The rotor consists of $N_b = 144$ blades.

Figure 3.4 a shows the isentropic Mach number, M_{is} , distribution for the 70% section at 90% of the design rotational speed, Ω , for an exit Mach number of $M_e = 0.675$. The section is located outside the secondary flow region, and operates at a slight positive incidence at this shaft speed, resulting in a front-loaded distribution. Figure 3.4 b shows, for the same location, the pressure coefficient, C_p , calculated as

$$C_p = \frac{P_0 - p_b}{P_0 - p_e} \quad (3.33)$$

where P_0 is the inlet total pressure, p_b is the pressure on the blade surface and p_e is the exit static pressure. The flow separates downstream of the maximum Mach number point. The bubble in the adverse pressure gradient region is very thin and hardly affects the damping characteristic of the airfoil. The RANS simulations were performed using Mu^2s^2T and set for aeroelastic analyses without transitional capability. The suction side separation

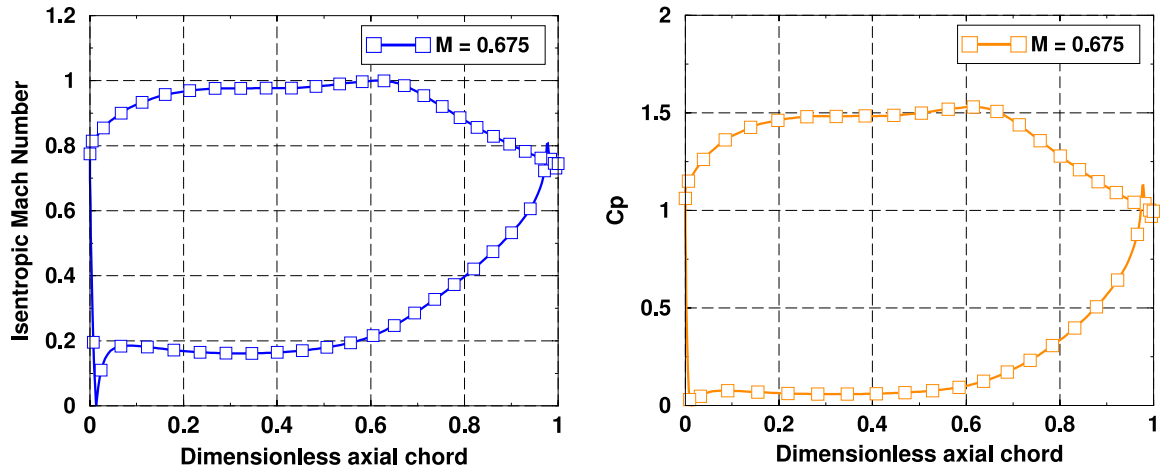


Figure 3.4.: LOADING OF THE BLADE AT 70% SPAN AND $\Omega = 90\%$: (A) ISENTROPIC MACH NUMBER DISTRIBUTION AND (B) PRESSURE COEFFICIENT.

bubble is hence no visible in the simulations. The design was entirely performed inside the ITP Aero design system.

3.1.3. Aeroelastic characterization

The bladed-disk studied in this work was specifically designed to replicate the aeromechanical behavior of an LPT in a real engine. To maintain reasonable experimental costs, the turbine rotor blade was scaled down from engine size, and testing was conducted in a subatmospheric pressure environment. The primary challenge in designing such an experiment lies in replicating all the relevant non-dimensional parameters characteristic of engine operation. While aerodynamic parameters like the Mach number, Reynolds number, and velocity triangles are commonly reproduced in cold flow rig testing, achieving representative non-dimensional aeroelastic parameters presents an additional level of complexity.

The rotor aeromechanical design was based on a reference rotor blade aerodynamics. A critical design objective was to create a controlled aeromechanical instability that would amplify blade vibrations enough for accurate measurement while remaining below levels that could compromise structural integrity during operation. The baseline cantilever configuration was chosen with a plain shroud and a thin blade tip to retain key similarities with conventional aeronautical LPT designs while minimizing the influence of clearance flow effects. Cantilever rotor blades were preferred as they favor flutter due to their lower natural vibration frequencies and also reduce uncertainties in predicting the natural frequencies and damping compared to blades with interlocked shrouds.

The bladed-disk under investigation was intentionally designed to have several unstable NDs in a wide range of operating points. The bladed-disk comprises a disk with sufficient

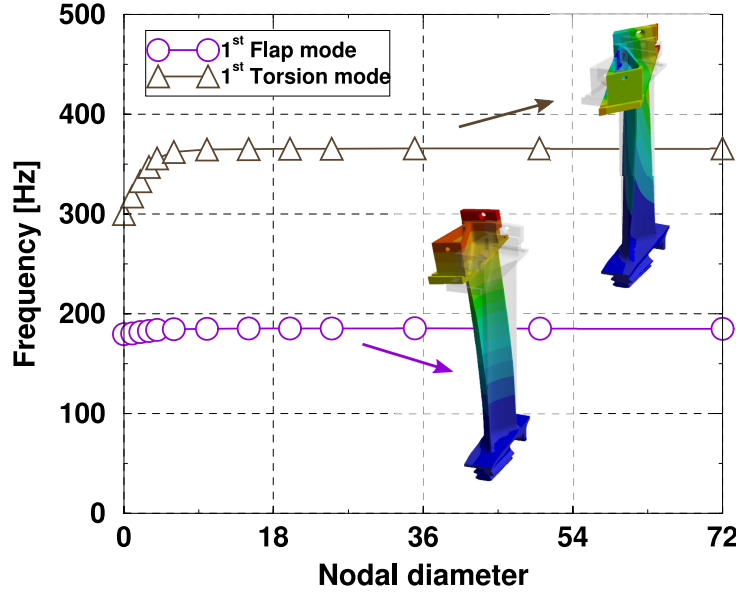


Figure 3.5.: NATURAL FREQUENCY AS A FUNCTION OF THE ND FOR THE 1st FLAP AND THE 1st TORSION MODES AT THE DESIGN OPERATION POINT ($\Omega = 100\%$).

stiffness, resulting in minimal disk participation, except in the first NDs, as illustrated in Fig. 3.5. The Campbell diagram, shown in Fig. 3.6 illustrates the frequency evolution with rotational speed based on Southwell coefficients derived from FEM analysis. In this diagram, $\Omega_M = 100\%$ represents the maximum rotational speed, while the design operating point is approximately 80% of Ω_M . This range was chosen to illustrate the typical intersections between excitation orders (EOs) and vibration frequencies throughout the engine's full operational range. However, throughout this thesis, the variable Ω is used to denote the rotational speed of the LPT, with $\Omega = 100\%$ corresponding to the design rotational speed.

The first flap mode presents the first crossing with the third EO in the operating range of the rig. The second mode, a torsion mode, is well-separated from the flap mode, preventing potential mode interactions. A supporting ring equipped with static magnets was specifically crafted to superimpose a forcing effect on the rotor flutter, although this aspect is not discussed in the present work. The natural frequency of the first mode is about 186 Hz at the design operation point.

The reduced frequency of the first mode, calculated as

$$k = \frac{\omega c_{ax}}{U_e} \quad (3.34)$$

based on the axial chord and exit velocity of the 70% section, at $\Omega = 90\%$ is $k = 0.09$, a value representative of real-engine LPT blades. Despite differences in size, material, and operating conditions between the engine and the rig blade, the reduced frequency dur-

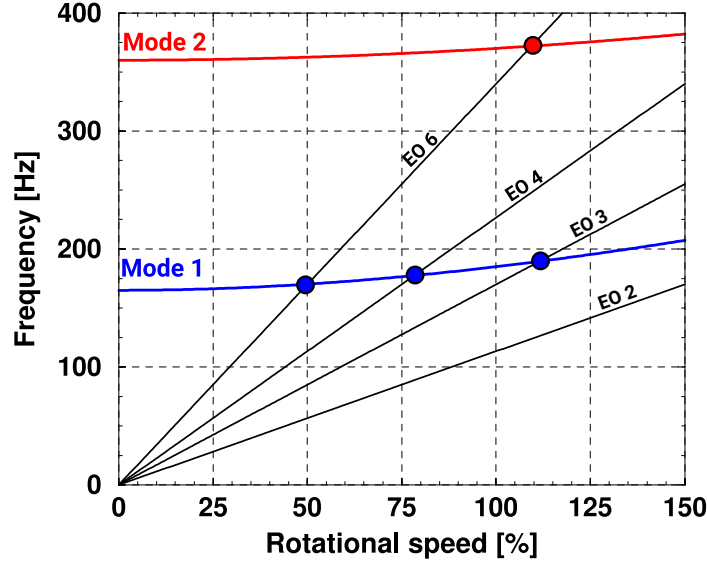


Figure 3.6.: ANALYTICAL CAMPBELL DIAGRAM OF THE BLADED-DISK.

ing testing was maintained close to the engine value ($k \approx 0.1$). Achieving this required artificially lowering the rig blade's natural frequency. Since $(\omega c)_{\text{Rig}} \simeq (\omega c)_{\text{Eng}}$ and M_e is constant, the change in temperature between the rig and engine modifies the reduced frequency, i.e. $k_{\text{Eng}}/k_{\text{Rig}} = \sqrt{T_{\text{Rig}}/T_{\text{Eng}}}$, indicating that the reduced frequency is inherently lower in the engine compared to the rig.

To address this, a significant mass was added to the tip shroud of the rig blade, reducing its frequency to match engine-like values, which is a much lower value than the corresponding to a pure geometrical scaling. The additional mass not only altered the blade's natural frequency and mode shape compared to a standard cantilever blade but also significantly affected the pull load and dry-friction damping. These geometric modifications, combined with changes in size, material, and operating conditions, resulted in a rig blade that effectively replicated the aeromechanical characteristics of an engine blade.

However, a key difference lies in the rig blade's higher capacity to absorb energy from the flow relative to its dissipation capability, as compared to the real-engine blade. This design choice intentionally induces an aeromechanical instability to ensure that vibration amplification is large enough to be measured. Additionally, the modified tip-shroud is used to include large fins to host mistuning masses, magnets, and ease BTT measurements. The pitch of the tip-shroud is sized in such a way that the clearance between two adjacent rotor blades is large enough to avoid shroud contacts when fluttering, which would have otherwise significantly and unrealistically modified the dynamics of the rotor blade.

The critical damping ratio (CDR) is expressed as a function of the dimensionless aerody-

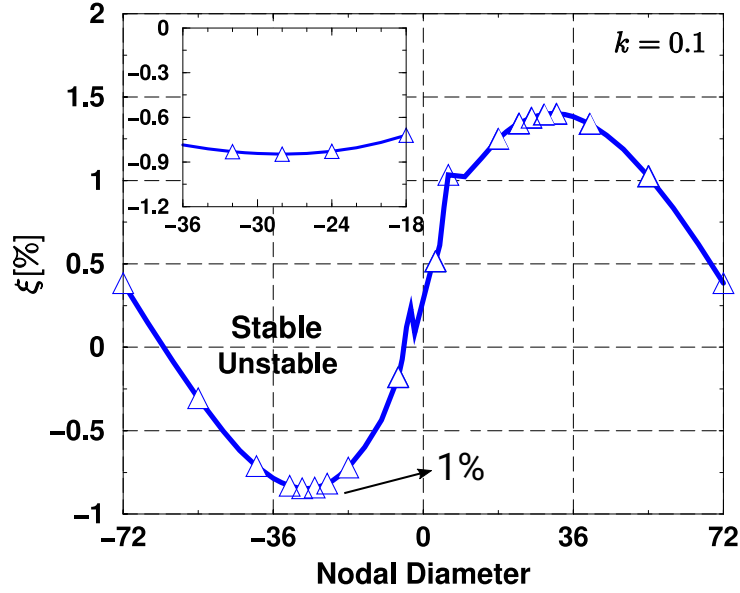


Figure 3.7.: CRITICAL DAMPING RATIO AS A FUNCTION OF THE ND WITH INSET OF THE MINIMUM DAMPING REGION AT $\Omega = 90\%$.

dynamic work as

$$\xi = \frac{\mu \tilde{W}_a}{k} \quad (3.35)$$

where $\mu = \rho_a S / m$ denotes the mass ratio, ρ_a represents the air density and m is the modal mass. The CDR serves as a measure of a system's capacity to damp forcing perturbations (positive sign) or to develop aeroelastic instability (negative sign). While it incorporates aerodynamic work and inertia effects, ξ is not a purely aerodynamic parameter. Figure 3.7 illustrates that the flap mode exhibits several highly unstable backward nodal diameters, a characteristic commonly observed in aeronautical cantilever LPT rotor blades. Furthermore, the figure emphasizes that the critical damping ratio for these blades is typically on the order of $\xi \sim 1\%$.

3.2. Effect of the Mach number

3.2.1. Aerodynamic work dependency with Mach number

The Mach number influences only the aerodynamic part of the model, while the mechanical behavior remains unaffected. Therefore, this subsection focuses on the Mach number's influence on the aerodynamic part of the model. Equation 2.3 establishes the relationship between the dimensional and dimensionless minimum aerodynamic work, highlighting the role of the Mach number in two distinct ways.

First, the Mach number contributes to the dynamic pressure, which scales the physical

M_e	0.500	0.675	0.775	0.825
k	0.123	0.092	0.080	0.075
C_L	1.200	1.195	1.182	1.180

Table 3.2.: REDUCED FREQUENCY k AND ZWEIFEL LIFT COEFFICIENT C_L FOR DIFFERENT EXIT MACH NUMBERS AT $\Omega = 90\%$.

aerodynamic work as $W_a \propto \mathcal{F}(M_e^2)$ (see Eqs. 2.3 and 2.4). Second, the dimensionless aerodynamic work, $\tilde{W}_a^{\min}[k(M_e, \Omega), C_L(M_e, \Omega)]$, depends on both the reduced frequency and the blade loading. However, within the operating Mach number range of $M_e = 0.5 - 0.8$, the influence of these parameters is negligible to a first-order approximation.

The reduced frequencies of the first mode based on the axial chord and exit velocity of the 70% section, at $\Omega = 90\%$, are detailed in Table 3.2. The slight decrease in the reduced frequency primarily stems from the escalation in the exit velocity. Nevertheless, the reduced frequency remains of the same order of magnitude ($k \sim 0.1$), making its impact on the dimensionless aerodynamic work significantly smaller than the effect of dynamic pressure.

The values of C_L , represented by the Zweifel coefficient, show minimal variation with the exit Mach number, indicating that blade loading is largely unaffected by the Mach number. To further understand how the loading is distributed along the blade, the isentropic Mach number M_{is} and pressure coefficient C_p are analyzed, as shown in Fig. 3.8. For higher Mach numbers (e.g. $M_e = 0.775$ and 0.825), both the M_{is} and C_p distributions exhibit an overshoot on the suction side, reflecting the fact that the rotor blade was originally designed for a lower Mach number ($M_e \simeq 0.7$). The C_p distribution on the 70% section shows that the adverse pressure gradient in the rear part of the blade remains unaltered to changes in the exit Mach number, which is typical in this kind of airfoils. However, since these variations in blade loading are relatively small, the influence of Mach number on $\tilde{W}_a^{\min}$ remains negligible.

The critical damping ratio, ξ , of the first mode for $\Omega = 90\%$, calculated using a linearized frequency domain solver [56], is presented in Fig. 3.9 (left) for different Mach numbers. It can be seen that ξ increases with the Mach number, highlighting that the critical damping ratio is not a dimensionless parameter designed to absorb aerodynamic variations but blade dynamics.

Figure 3.9 (right) illustrates the critical damping ratio, ξ , normalized by the inverse of the Mach number function, $\mathcal{F}(M_e^2)$, as expressed in Eq. 2.3, for different nodal diameters and various highly subsonic Mach numbers. Despite small alterations in the steady blade loading caused by the Mach number (Fig. 3.8) and variations in the reduced frequency, the four damping curves collapse into a single one. This is consistent with results reported

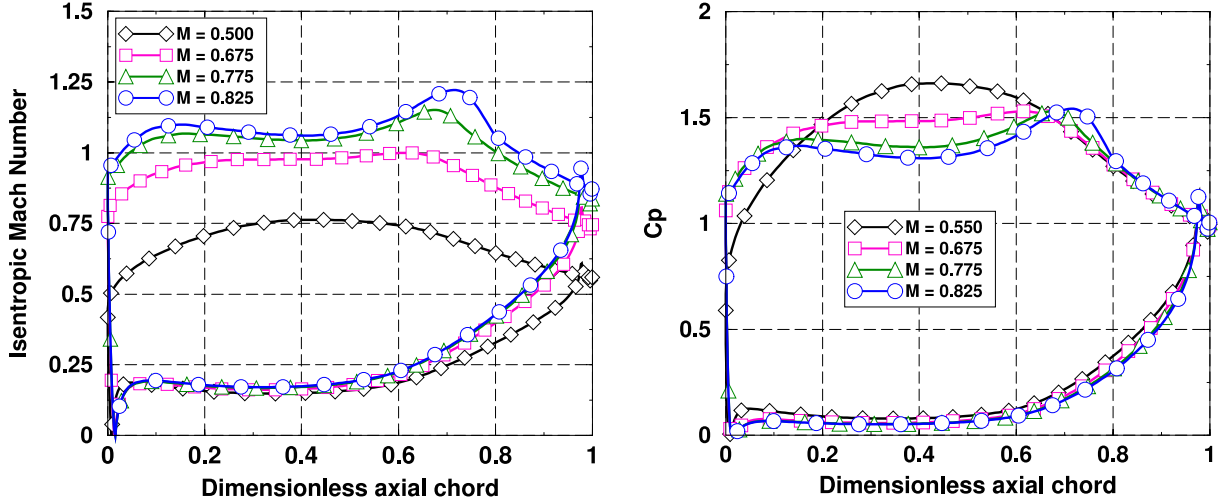


Figure 3.8.: LOADING OF THE BLADE AT 70% SPAN AND $\Omega = 90\%$ FOR DIFFERENT EXIT MACH NUMBERS: (A) ISENTROPIC MACH NUMBER DISTRIBUTION AND (B) PRESSURE COEFFICIENT.

by [59] but with a simpler scaling, where M_e^2 was used on the aerodynamic influence coefficients. The remarkable agreement between the four curves validates the expression 2.3 for scaling aerodynamic damping across different pressure ratios.

The variation of other aerodynamic parameters follows a pattern similar to the discussion on the Mach number. In test rigs, density is often adjusted to investigate the effects of the Reynolds number. According to Eq. 2.2, the primary impact of changing density on the aerodynamic damping component arises from the resulting change in dynamic pressure. From an aeromechanical perspective, altering the density level can also be interpreted as a change in the mass ratio. Consequently, the aerodynamic damping varies directly with the facility's density level, as demonstrated in previous studies [8].

3.2.2. Discussion of the results

The energy balance equation 2.26 establishes a relationship between design and operational parameters and the dimensionless vibration amplitude, $\tilde{\delta}_a$. When considering variations only in the exit Mach number, the only affected term in Eq. 2.26 is $\Re^*$, which can be expressed as $\Re^* = \Re \mathcal{F}(M_e^2)$ for clarity. This formulation allows the effect of Mach number to be evaluated for a given dimensionless rotational speed $\tilde{\Omega}$, dimensionless minimum aerodynamic damping $\tilde{W}_a^{\min}$, parameter α and the remaining physical parameters grouped within $\Re$. The dimensionless parameters used in this analysis are summarized in Table 3.3, where the selected values are representative of an LPT rotor blade operating at its design point. It is worth noting that the dimensionless rotational speed $\tilde{\Omega} = 0.27$ aligns with the Campbell diagram in Fig. 3.6, where the first mode frequency at the

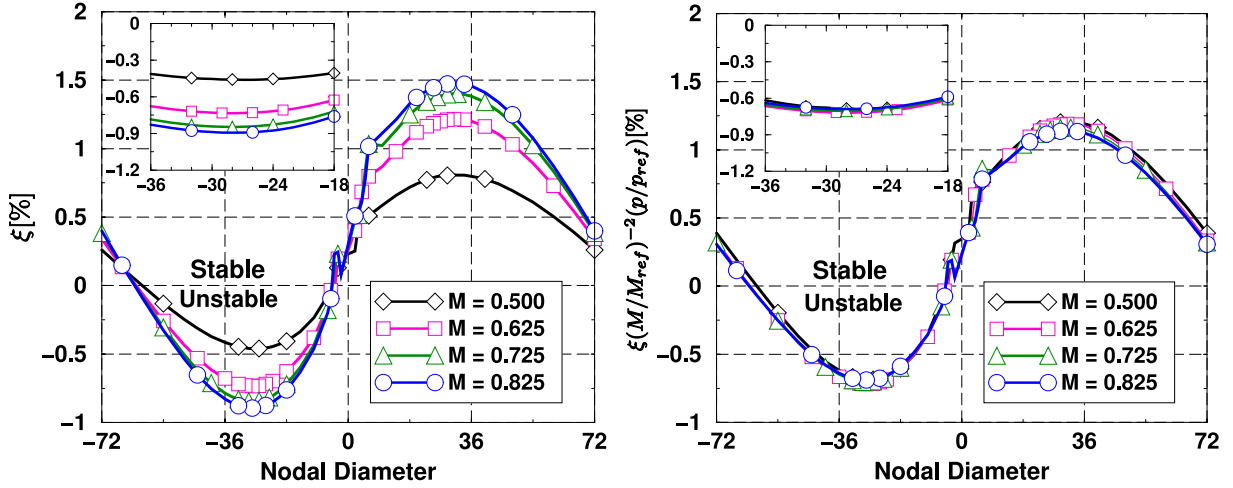


Figure 3.9.: CRITICAL DAMPING RATIO AS A FUNCTION OF THE ND AND THE MACH NUMBER AT $\Omega = 90\%$ WITH INSETS OF THE MINIMUM DAMPING REGION. LEFT: RAW DATA FROM THE LINEARISED RANS SOLVER. RIGHT: RESCALING TO $M_{ref} = 0.625$ USING EQ. 2.4.

$\Re$	$\tilde{\Omega}$	α	$\tilde{W}_a^{\min}$
1	0.27	1	0.8

Table 3.3.: PARAMETERS USED TO CALCULATE THE EVOLUTION OF THE DIMENSION-LESS VIBRATION AMPLITUDE WITH THE EXIT MACH NUMBER IN AN LPT ROTOR BLADE.

design point falls between the third and fourth EOs. Similarly, the minimum dimensionless aerodynamic work, $\tilde{W}_a^{\min} = 0.8$, is representative of high-loading low-reduced frequency blades [59–61].

The evolution of the flutter-induced dimensionless vibration amplitude is represented in Fig. 3.10 comparing results obtained using both the exact formulation (Eq. 2.26) and the approximate formulation (Eq. 2.28). Since $\Re \sim 1$ is constant, $\Re^*$ scales proportionally with $\mathcal{F}(M_e^2)$. Thus, this analysis is equivalent to studying the influence of $\Re^*$ on $\tilde{\delta}_a$ through the function $\mathcal{F}(M_e^2)$.

In the limit of low Mach numbers ($M_e \ll 1$), the vibration amplitude approaches zero, as the aerodynamic excitation becomes weak. This corresponds to the $\Re^* \ll 1$ limit, where the blade’s capacity to dissipate energy through dry friction exceeds the aerodynamic energy input from the flow. In this small-amplitude regime, both the exact and approximate formulations yield similar results, with $\tilde{\delta}_a$ following a linear dependence on $\mathcal{F}(M_e^2)$ due to its direct proportionality with $\Re^*$.

When the Mach number $M_e \sim 1$, the parameter $\Re^*$ also approaches unity, rendering the approximated formulation invalid. This breakdown occurs because the assumptions underlying the approximation—namely, that the dimensionless mechanical work scales

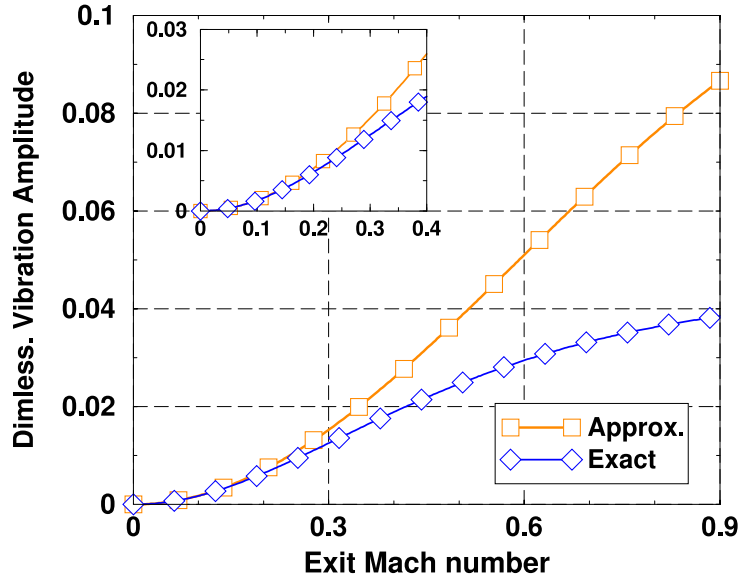


Figure 3.10.: EVOLUTION OF THE DIMENSIONLESS VIBRATION AMPLITUDE WITH THE EXIT MACH NUMBER USING THE EXACT ENERGY BALANCE FORMULATION (EQ. 2.26) AND THE APPROXIMATED FORMULATION (EQ. 2.28).

cubically with the dimensionless fir-tree displacement ($\tilde{W}_m \propto \tilde{\delta}^{*3}$) and hat a linear relationship exists between the dimensionless displacements ($\tilde{\delta}^* \propto \tilde{\delta}_a$)—are no longer valid in this regime. In this case, only the exact formulation remains applicable. As shown in Fig. 3.10, the evolution of $\tilde{\delta}_a$ with increasing M_e exhibits a moderate increase but deviates from the function $\mathcal{F}(M_e^2)$ due to the transition into a nonlinear friction regime.

3.3. Effect of the rotational speed

The rotational speed, Ω , influences the aerodynamic and the mechanical components of the model in different ways. This section first examines the impact of the rotational speed on the aerodynamic work, revealing an interesting result where the behavior of $\tilde{W}_a^{\min}$ at off-design conditions deviates from the predictions of the low-reduced frequency theory. Additionally, rotational speed affects the mechanical model through physical scaling effects and changes in the dimensionless hysteresis cycles at the contact interface. These effects are analyzed, followed by a discussion on the influence of Ω on the dimensionless vibration amplitude.

3.3.1. Aerodynamic work dependency with rotational speed

The influence of the rotational speed in the aerodynamic part of the model is through the minimum dimensionless aerodynamic work. This dependence arises because the two

Ω (%)	72	80	90	100	112	118
C_L	1.30	1.24	1.18	1.13	1.08	1.04
k	0.078	0.079	0.080	0.082	0.085	0.086

Table 3.4.: REDUCED FREQUENCY k AND ZWEIFEL LIFT COEFFICIENT C_L FOR DIFFERENT EXIT ROTATIONAL SPEEDS AT $M_e = 0.775$.

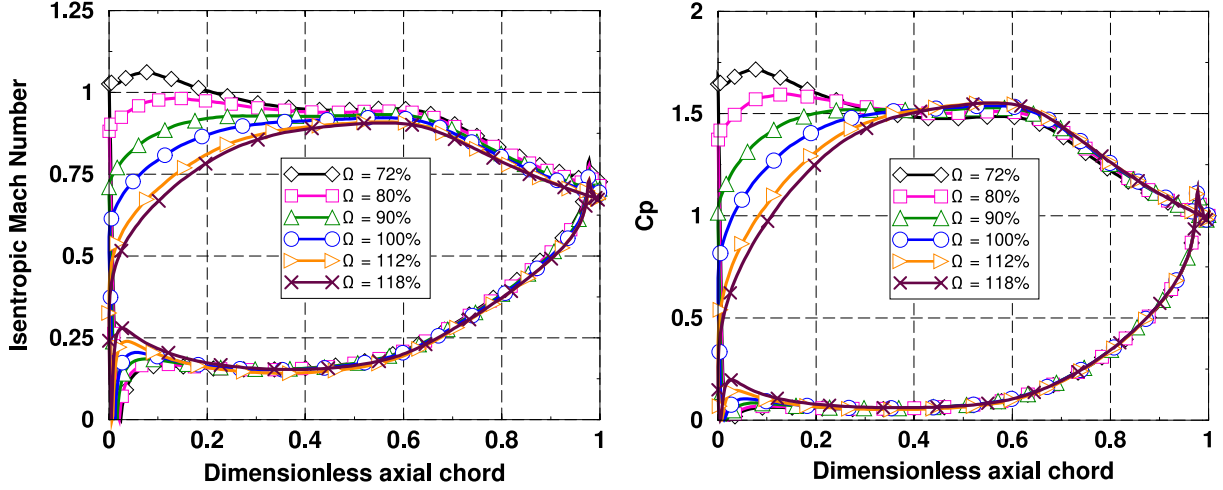


Figure 3.11.: LOADING OF THE BLADE AT 70% SPAN AND $M_e = 0.775$ FOR DIFFERENT ROTATIONAL SPEEDS. LEFT: ISENTROPIC MACH NUMBER DISTRIBUTION. RIGHT: PRESSURE COEFFICIENT.

key parameters governing $\tilde{W}_a^{\min}$, the reduced frequency and the blade loading, are both functions of Ω as indicated by Eq. 2.3.

The reduced frequencies of the first mode based on the axial chord and exit velocity of the 70% section, at $M_e = 0.775$, are detailed in Table 3.4. The slight increase in reduced frequency is primarily attributed to the stiffening effect induced by centrifugal forces, which grow with increasing rotational speed. According to low-reduced-frequency theory, an increase in reduced frequency reduces aerodynamic instability [59–61]. However, since the reduced frequency remains within the same order of magnitude ($k \sim 0.1$), its impact on the dimensionless aerodynamic work remains minimal.

Blade loading is characterized by the Zweifel coefficient, C_L , as presented in Table 3.4. Additionally, Fig. 3.11 illustrates the isentropic Mach number and pressure coefficient (computed using Eq. 3.33) for a representative two-dimensional section at 70% span, sufficiently far from secondary flow regions. These figures provide further insight into the loading distribution on the blade at different rotational speeds. As the rotational speed increases, blade loading decreases due to a reduction in the incidence angle. This change in incidence and loading translates into a decrease in the lift coefficient, C_L , with the shaft speed.

The critical damping ratio, ξ , of the first mode for $M_e = 0.775$ and different shaft speeds,

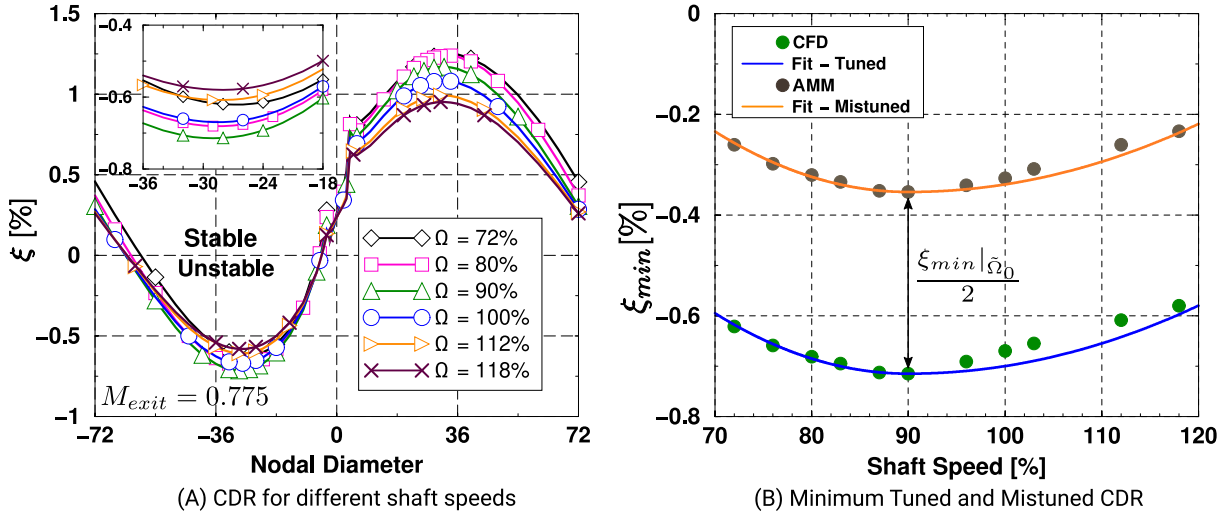


Figure 3.12.: CRITICAL DAMPING RATIO FOR DIFFERENT ROTATIONAL SPEEDS AT $M_e = 0.775$: (A) EVOLUTION WITH ND WITH INSET OF THE MINIMUM DAMPING REGION AND (B) EFFECT OF MISTUNING ON THE MINIMUM CDR.

is shown in Fig. 3.12 a. According to the low-reduced frequency theory of vibrating airfoils, the presented variations of reduced frequency and blade loading collectively lead to a moderate reduction in aerodynamic work with the shaft speed [62, 63]. Despite this, when looking at the evolution of the minimum damping ratio as a function of the shaft speed, see green dots in Fig. 3.12 b, this curve is not monotonic and has a minimum close to the nominal operating point. At high shaft speeds, the results match the low-reduced frequency theory since the aerodynamic loading decreases, and the minimum damping increases with the shaft speed. However, the trend differs from the predictions at low shaft speeds since the load increases but the minimum damping decreases. The same behavior is observed for $M_e = 0.675$ and $M_e = 0.825$.

Trying to explain this behaviour, the evolution of the isentropic Mach distribution with the shaft speed for different span heights is presented in Fig. 3.13 for the case with $M_e = 0.775$. At 70% (see Fig. 3.11a) and 90% span, the profile loading increases as the shaft speed reduces. These sections contribute to increasing the aerodynamic instability. However, at 99% span, the loading of the section decreases. The suction peak obtained increases when moving to a slower shaft speed but there is also unloading at about mid-chord which may be causing the reduction of aerodynamic work.

The presence of these secondary flows close to the tip is also seen when plotting the relative Mach number profiles as a function of the blade span half chord downstream the rotor, see Fig. 3.14. In these profiles is seen how the loading over 80% span decreases when decreasing the shaft speed, specially for the $\Omega = 72\%$ case.

The observed deviation from the expected behavior at low rotational speeds suggests that secondary flows near the blade tip play a significant role in this phenomenon. To further

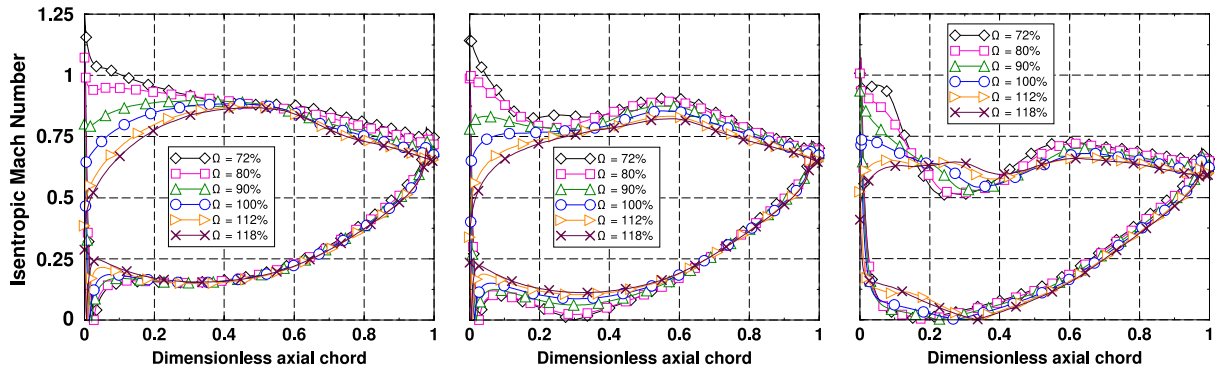


Figure 3.13.: EVOLUTION OF NUMERICAL ISENTROPIC MACH PROFILES WITH SHAFT SPEED FOR $M_e = 0.775$ AT DIFFERENT SPAN HIGHTS: LEFT: 90%, CENTRE: 95%, AND RIGHT: 99%.

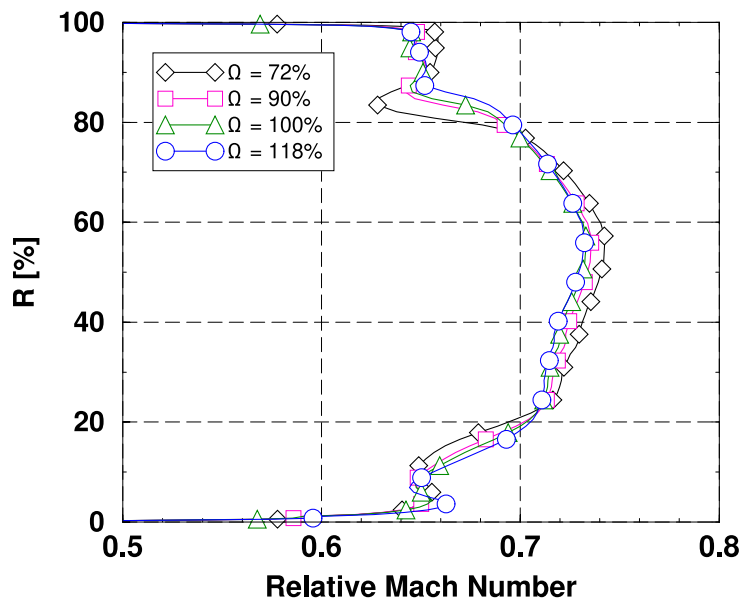


Figure 3.14.: NUMERICAL RELATIVE MACH NUMBER EVOLUTION FOR DIFFERENT SHAFT SPEEDS

investigate this effect, Fig. 3.15 presents flow streamlines plotted over the blade suction side for different shaft speeds. These plots indicate that as the shaft speed decreases, the penetration of secondary flows increases. This effect is driven by the horseshoe vortex formed at the blade tip due to the interaction between the boundary layer and the leading edge of the turbine blade. As the airflow approaches, the boundary layer along the endwall is deflected, with part of it wrapping around the leading edge. This forms pressure-side and suction-side vortices that curve around the blade, creating the characteristic horseshoe vortex. At lower rotational speeds, reduced centrifugal forces allow this vortex to expand radially. Additionally, at higher speeds, blades extract more energy from the flow, strengthening the pressure gradients that confine the vortex near the endwall. Weaker pressure gradients at lower speeds enable the vortex to spread more radially into

the passage.

The unsteady pressure characteristics over a vibration cycle in the tip region are shown in Fig. 3.15. At lower rotational speeds, the modulus of unsteady pressure exhibits stronger fluctuations, suggesting an increase in unsteady aerodynamic effects. Whether this increased unsteadiness stabilizes or destabilizes the blade depends on its phase relative to the blade's vibration, as outlined in Eq. 1.3.

To illustrate this, the phase distribution at different rotational speeds is shown in Fig. 3.15. The cases near the design operation point ($\Omega = 90\%$ and $\Omega = 103\%$) exhibit similar behavior, with the $\Omega = 90\%$ case showing a slightly larger negative phase region ($\psi < 0$) near the trailing edge. However, these regions have low unsteady pressure amplitudes, contributing minimally to system stability. The contribution of each part of the blade to the work-per-cycle in the flap direction, referred as prework, is also shown in Fig. 3.15. Prework analysis indicate that most of the blade suction surface destabilizes the system except for the regions with negative phase. Two regions of positive phase close to $\psi = \pi/2$, contribute to destabilization, but only the one near the tip has significant unsteady pressure amplitude, making it the primary driver of blade instability.

At low rotational speeds ($\Omega = 72\%$), a larger unstable region near the tip emerges in the prework analysis. This region, characterized by high unsteady pressure amplitude, tends to decrease blade stability. However, the stabilizing region on the rear suction side of the blade also expands, counteracting the instability. Since the increase in stabilization outweighs the destabilization effect, the overall stability of the blade slightly increases.

In conclusion, the $\Omega = 90\%$ case exhibits the highest instability, as it features a more extensive destabilizing region than the design point while having a smaller stabilizing region than the $\Omega = 72\%$ case. Given that the blade tip regions contribute most significantly to aerodynamic work, the localized unsteady effects of secondary flows provide a plausible explanation for the reduction in aerodynamic damping observed at lower rotational speeds.

Finally, it is well-known that mistuning couples the different tuned travelling waves, reducing the instability level of the bladed disk due to the averaging of the tuned aeroelastic modes [16, 86, 87]. For this particular rotor, the mistuning introduced in the manufacturing process of the system was measured in a ping test experiment. For each blade, the natural frequencies were measured by clamping the test specimen by the fir-tree attachment to a root block. The standard deviation of the frequency distribution was 1.1%, with the range of variation of the frequencies being about 6%, although the variation was concentrated in just a few blades [7, 8].

The influence of natural mistuning is retained using the AMM model [16], for 1% of natural mistuning. The critical damping ratio values obtained with the linear code (tuned case)

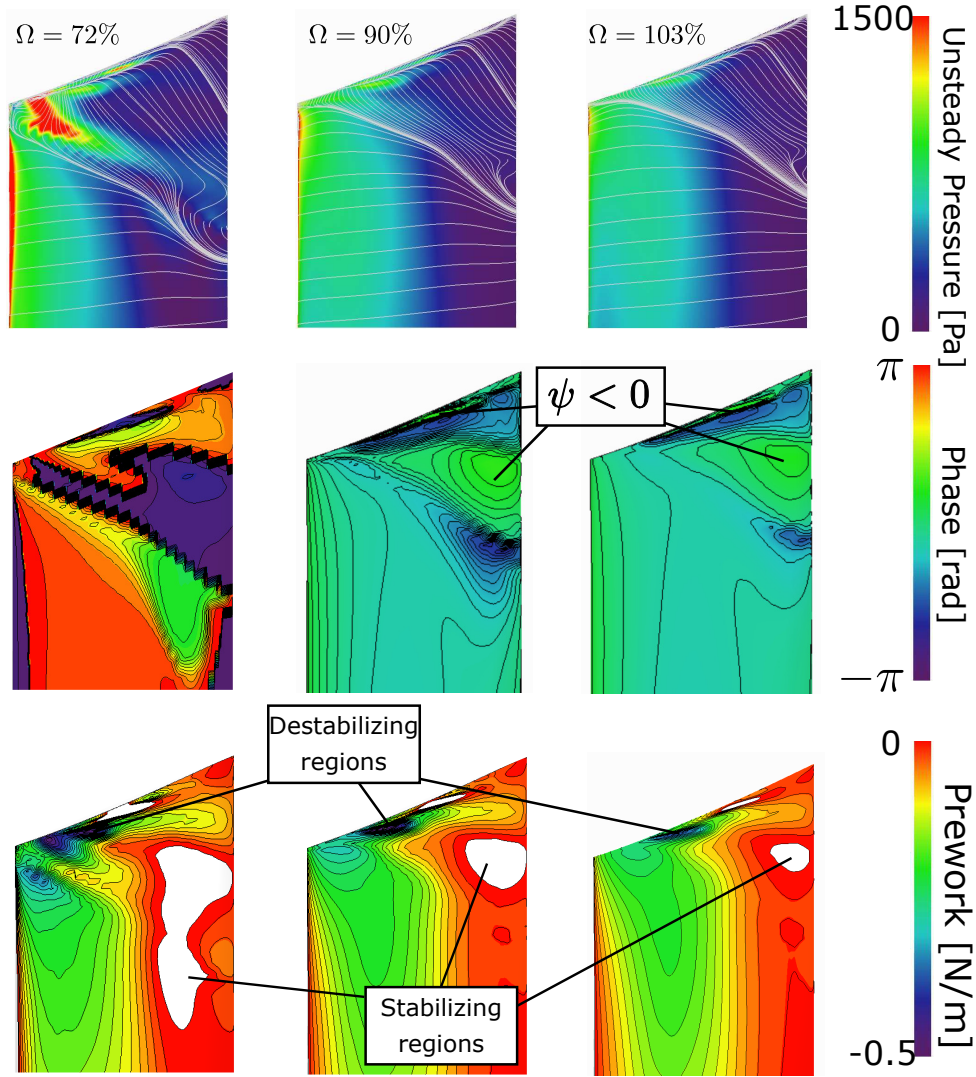


Figure 3.15.: CLOSE UP OF STREAM LINES, VIBRATION HARMONIC CONTENT OF THE MODULUS AND PHASE OF THE UNSTEADY PRESSURE AND AERODYNAMIC PREWORK-PER-CYCLE ON THE SUCTION SIDE OF THE BLADE TIP FOR $M_e = 0.775$ AT DIFFERENT SHAFT SPEEDS: LEFT: $\Omega = 72\%$. CENTRE: $\Omega = 90\%$ RIGHT: $\Omega = 103\%$.

have been transformed to the mistuned critical damping ratio using the AMM. For the operating point $\Omega = 90\%$ and $M_e = 0.775$, mistuning reduces the instability level by approximately 50%. This reduction of instability is shown in Fig. 3.12 b, where the grey dots represent the critical damping ratio values of the mistuned case. These results are coherent with the mistuning analyses reported in [8, 88].

The mistuning pattern is considered constant for the different operating conditions, however, the efficiency of this pattern is not. The variation in minimum damping is higher in the mistuned case (40%) compared to the tuned case (15%). Therefore, the minimum of the damping curve is more pronounced, thereby amplifying the aerodynamic effects. To include the damping variation with the rotational speed in the model, the minimum crit-

ical damping values of Fig. 3.12 b are fit a parabola and expressed in terms of aerodynamic work as

$$\tilde{W}_a^{\min} = \tilde{W}_a^{\min}|_{\tilde{\Omega}_0} + A(\tilde{\Omega} - \tilde{\Omega}_0)^2 \quad (3.36)$$

which reproduces the minimum found in the simulations. The dimensionless rotational speed corresponding to the minimum aerodynamic work is $\tilde{\Omega}_0$. Notice that the curve of Fig. 3.12 b is not symmetric with respect to the minimum, because the effects that reduce the instability on both sides are different. To capture this change of slope, the parameter A in Eq. 3.36 takes different values on the left A_L and right A_R of the maximum.

3.3.2. Mechanical work dependency with rotational speed

The mechanical work dissipated through friction in the fir-tree attachment depends on the rotational speed due to two reasons. According to Eq. 2.8, W_m can be separated in a physical scaling factor, $W_m \propto \mu N \delta_c$, and a dimensionless term which qualitatively represents the hysteresis cycle due to the relative displacement between the blade's fir-tree and the disk. It is worth noting that the evolution of both terms with the rotational speed is opposite. Additionally, this evolution is nonlinear in both cases. Consequently, it is key to understand how each of these terms affect the mechanical behaviour of the fir-tree and which is overall effect.

On the one hand, the physical scaling factor consists on the friction coefficient, μ , which is independent on the rotational speed, the normal force at the attachment, N , and the characteristic length of the microslip, δ_c . The two latter terms depend on the rotational speed in different ways. First, the normal force is directly related to the centrifugal force, as it is in charge of compensating the pull load of the blade, and it depends quadratically on the shaft speed, $N \propto \tilde{\Omega}^2$ according to Eq. 2.13. Notably, the rotational speed dependencies are given in dimensionless terms, i.e. $\tilde{\Omega}$, while the rest of the physical parameters are grouped in $\mathfrak{R}^*$ so the energy balance equation (Eq. 2.26) is expressed in dimensionless terms.

The next dependency with the rotational speed corresponds to the characteristic fir-tree displacement, δ_c . Taking into account the relation presented in Eq. 2.6, the characteristic fir-tree displacement is proportional to the contact penetration length, $\delta_c \propto \lambda$. Additionally, according to Eq. 2.7, λ is proportional to the normal stress, σ_n , as $\lambda \propto \sigma_n^\alpha$, where α is a constant which depends on the material surface [64, 65] and the elastic regime. Since the normal stress can be expressed as $\sigma_n = N/A_c$, a relation between the contact penetration length and the dimensionless rotational speed is derived as $\lambda \propto \tilde{\Omega}^{2\alpha}$. This expression states that the evolution of the penetration length is governed by the shaft speed and the exponent α . Since the normal stress in the contact is about two orders

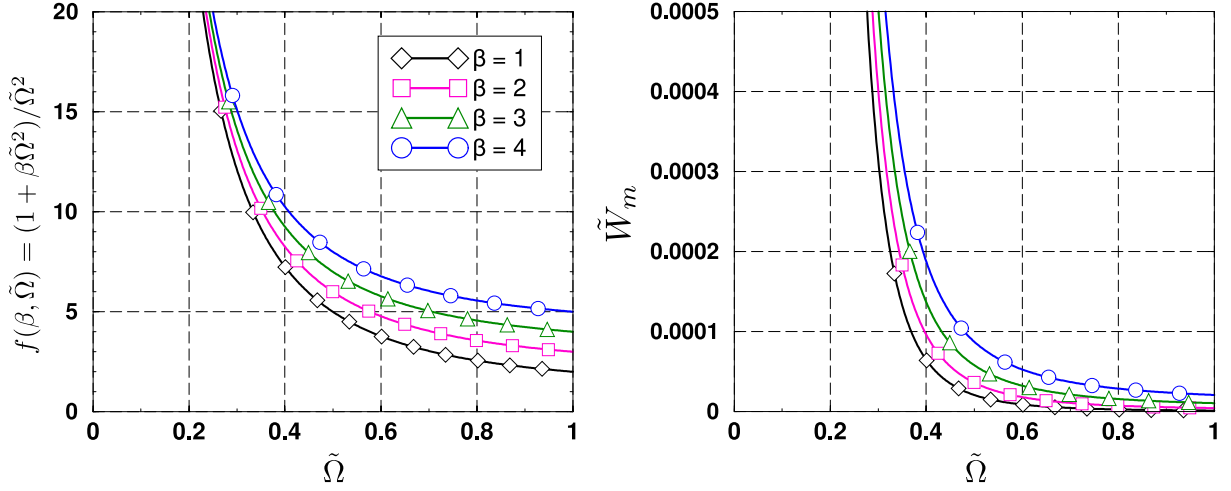


Figure 3.16.: EVOLUTION WITH THE DIMENSIONLESS ROTATIONAL SPEED $\tilde{\Omega}$ AND DIFFERENT VALUES OF THE SOUTHWELL COEFFICIENT β . LEFT: TANGENTIAL FORCE $\tilde{F}^*$. RIGHT: DIMENSIONLESS MECHANICAL WORK $\tilde{W}_m$ FOR A CONSTANT VALUE OF $\tilde{\delta}_a = 0.01$.

of magnitude below the elastic limit, a linear relation between the normal force and the penetration length is assumed within the range of operation, i.e. $\alpha = 1$. The rest of the physical parameters related with λ and δ_c are also grouped in the parameter $\mathfrak{R}^*$.

On the other hand, the dimensionless mechanical work $\tilde{W}_m$ presented in Eq. 2.17 depends on the dimensionless tangential force in the contact, $\tilde{F}^*$, which can be expressed as $\tilde{F}^* = f(\beta, \tilde{\Omega})\tilde{\delta}_a$, where $f(\beta, \tilde{\Omega}) = (1 + \beta\tilde{\Omega}^2)/\tilde{\Omega}^2$ according to Eq. 2.23. In this relation, the term $1 + \beta\tilde{\Omega}^2$ accounts for the stiffening effect on the vibration frequency, while the term $\tilde{\Omega}^2$ reflects the increase in normal force due to centrifugal forces associated with rotational speed.

The parameter $\tilde{F}^*$ represents the size of the hysteresis cycle, as depicted in Fig. 1.11. The evolution of $f(\beta, \tilde{\Omega})$ with $\tilde{\Omega}$ is presented in Fig. 3.16a, illustrating the dependency of the tangential force with the rotational speed. It is shown that $f(\beta, \tilde{\Omega})$ decreases with increasing $\tilde{\Omega}$, leading to a decrease of $\tilde{F}^*$ for a constant value of $\tilde{\delta}_a$. This behavior arises because the effect of higher rotational speeds on the normal force outweighs the stiffening effect on vibration frequency, leading to a tighter contact between the fir-tree and the disk due to centrifugal forces. The reduction in $\tilde{F}^*$ is more pronounced for lower values of β , where the stiffening effect is weaker. As a result, a tighter contact leads to a reduction in the dimensionless fir-tree displacement $\tilde{\delta}^*$, which in turn lowers the size of the hysteresis cycle, leading to a decrease in $\tilde{W}_m$. This is illustrated in Fig. 3.16b, where the evolution of $\tilde{W}_m$ for a constant $\tilde{\delta}_a = 0.01$, a typical value encountered in turbomachinery blades, is presented.

In summary, the physical scaling factor of the mechanical work varies with the rotational

speed as $W_m \propto \tilde{\Omega}^{2(1+\alpha)}$, which implies that the dissipated energy through dry friction increases as the rotational speed rises. However, the size of the hysteresis cycle decreases with increasing $\tilde{\Omega}$, following the Eq. 2.17 and the relation $\tilde{F}^* = f(\beta, \tilde{\Omega})\tilde{\delta}_a$. This reduction in $\tilde{F}^*$ leads to a decrease in the energy dissipated by dry friction as rotational speed increases, as illustrated in Fig. 3.16b.

3.3.3. Discussion of the results

The dimensionless rotational speed $\tilde{\Omega}$ influences both the aerodynamic and mechanical components of the model. Additionally, mistuning effects play a crucial role in the final response and must be considered. This section presents the model results, gradually incorporating different effects to increase the complexity of the analysis. First, the influence of rotational speed on the mechanical model is examined for both the tuned case (see Fig. 3.17a) and the mistuned case (see Fig. 3.17b). Next, the influence of the rotational speed on the aerodynamic model is included for the tuned case (see Fig 3.18a). Finally, mistuning effects are incorporated into the model (see Fig. 3.18b), which represents the final model configuration used throughout this thesis.

The mechanical model comprises two competing effects: at very low $\tilde{\Omega} \ll 1$, large hysteresis cycles lead to significant energy dissipation through friction. At the opposite limit, where $\tilde{\Omega} \sim 1$, the physical scaling of mechanical work increases sharply, enhancing dissipation. As a result, there exists an intermediate range of rotational speeds where these two drivers of the dry-friction dissipation are minimized, leading to a peak in vibration amplitude.

The isolated effect of rotational speed on vibration amplitude due to dry-friction dissipation behavior is illustrated in Fig. 3.17. The minimum aerodynamic work-per-cycle, $\tilde{W}_A^{\min}$, which represents the value of the most unstable nodal diameter at the most unstable rotational speed, remains constant and is set to $\tilde{W}_a^{\min} = \tilde{W}_a^{\min}|_{\tilde{\Omega}_0}^{\text{Tuned}} = 1.7$ in the tuned case. The evolution of $\tilde{\delta}_a$ with rotational speed for different values of $\mathfrak{R}^*$ is shown in Fig. 3.17a. Increasing $\mathfrak{R}^*$ leads to higher vibration amplitudes, as this parameter quantifies the blade's ability to absorb aerodynamic work relative to its frictional dissipation capacity. A maximum $\tilde{\delta}_a$ is observed within $\Omega \sim 140 - 220\%$ ($\tilde{\Omega} \sim 0.35 - 0.55$), with the rotational speed corresponding to this maximum increasing as $\mathfrak{R}^*$ grows. This occurs because a lower frictional dissipation capacity decreases the physical scaling of mechanical work, shifting the balance between mechanical scaling and hysteresis effects toward higher $\tilde{\Omega}$.

Figure 3.17b examines flutter-induced vibration as a function of shaft speed, incorporating mistuning effects by reducing $\tilde{W}_a^{\min}$ to $\tilde{W}_a^{\min}|_{\tilde{\Omega}_0}^{\text{Mist.}} = 0.85$. Compared to the tuned case, mistuning reduces $\tilde{\delta}_a$ for each value of $\mathfrak{R}^*$. Additionally, the shift in the position of the

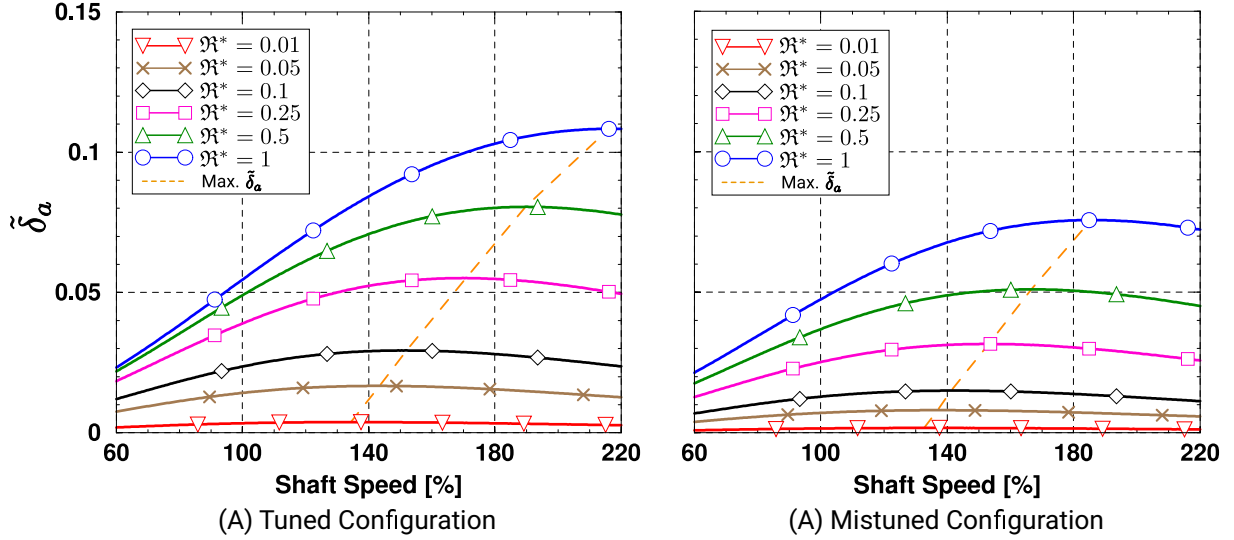


Figure 3.17.: EVOLUTION OF DIMENSIONLESS VIBRATION AMPLITUDE WITH THE ROTATIONAL SPEED FOR DIFFERENT VALUES OF $\Re^*$ FOR A CONSTANT VALUE OF THE MINIMUM AERODYNAMIC DAMPING $\tilde{W}_a^{\min}$: (A) TUNED CONFIGURATION AND (B) MISTUNED CONFIGURATION.

maximum $\tilde{\delta}_a$ is smaller in the mistuned case, as the lower aerodynamic instability level results in a smaller change in the blade's ability to absorb aerodynamic work relative to its frictional dissipation. Consequently, the physical scaling of mechanical work plays a more dominant role, requiring only a moderate increase in $\tilde{\Omega}$ to balance hysteresis size and mechanical scaling effects.

When accounting for the variation of aerodynamic work with rotational speed, $\tilde{W}_a^{\min}(\tilde{\Omega})$, as defined in Eq. 3.36, the vibration amplitude results are further modified. In this case, $\tilde{W}_a^{\min}$ at the most unstable remains is $\tilde{W}_a^{\min}|_{\tilde{\Omega}_0}^{\text{Tuned}} = 1.7$, but its value decreases for other rotational speeds, in a similar way than shown in Fig 3.11b with the critical damping ratio.

The vibration amplitude in the tuned case is presented in Fig. 3.18a, showing lower $\tilde{\delta}_a$ levels compared to the constant $\tilde{W}_a^{\min}$ case (noting the different axis limits in Figs. 3.17 and 3.18). This reduction results from lower aerodynamic instability at all rotational speeds except $\Omega = 90\%$, where both cases' $\tilde{W}_a^{\min}$ coincide. While the aerodynamic model shifts the peak $\tilde{\delta}_a$ toward lower $\tilde{\Omega}$, mechanical effects still push the maximum toward higher $\tilde{\Omega}$. As a result, the peak vibration amplitude occurs in the range $\Omega \sim 100 - 130\%$ ($\tilde{\Omega} \sim 0.27 - 0.35$).

Finally, incorporating mistuning effects alongside aerodynamic and mechanical dependencies on rotational speed results in a model where $\tilde{W}_a^{\min}$ follows Eq. 3.36 with $\tilde{W}_a^{\min}|_{\tilde{\Omega}_0}^{\text{Mist.}} = 0.85$ at the most unstable condition. As expected, aerodynamic instability is lower than in the tuned case, leading to smaller $\tilde{\delta}_a$ values. Furthermore, the variation in maximum

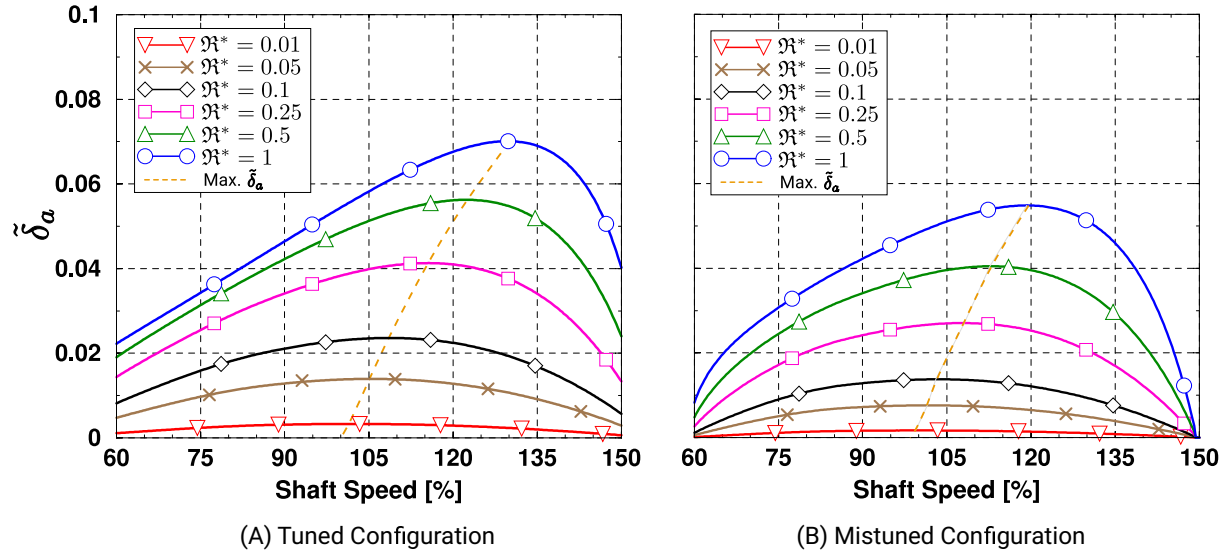


Figure 3.18.: EVOLUTION OF DIMENSIONLESS VIBRATION AMPLITUDE WITH THE ROTATIONAL SPEED FOR DIFFERENT VALUES OF $\mathfrak{R}^*$ CONSIDERING THE VARIATION OF THE MINIMUM AERODYNAMIC DAMPING WITH THE ROTATIONAL SPEED $\tilde{W}_A^{\text{MIN}}(\tilde{\Omega})$: (A) TUNED CONFIGURATION AND (B) MISTUNED CONFIGURATION.

amplitude with shaft speed is more constrained, appearing within $\Omega \sim 100 - 120\%$ ($\tilde{\Omega} \sim 0.27 - 0.32$). This final model configuration, incorporating all discussed effects, is used throughout this thesis.

It is interesting to analyze hysteresis cycles for the different operation points, which are presented in Fig. 3.19. The parameter $\mathfrak{R}^*$ governs the model's nonlinearity level: for $\mathfrak{R}^* \ll 1$, when the friction is in the microslip range, the vibration amplitude increases linearly with $\mathfrak{R}^*$, whereas the relationship becomes nonlinear when $\mathfrak{R}^* \sim 1$ and the friction regime moves towards macroslip. Additionally, the rotational speed at which the maximum vibration amplitude appears remains nearly constant regardless of $\mathfrak{R}^*$.

The hysteresis cycles for $\mathfrak{R}^* = 0.1$, $\mathfrak{R}^* = 0.5$ and $\mathfrak{R}^* = 1$ are presented in Fig. 3.19 at both low and high rotational speeds. These rotational speeds are those corresponding to a vibration amplitude that is 70% of the maximum $\tilde{\delta}_a$ for each $\mathfrak{R}^*$. In the low-speed case, the amplitude of the hysteresis cycle increases with $\mathfrak{R}^*$ since higher dimensionless vibration amplitudes result in a more significant displacement in the fir-tree contact.

The comparison between low and high rotational speeds is more interesting than that of different $\mathfrak{R}^*$ values. For the same $\mathfrak{R}^*$ value and dimensionless vibration amplitude, the low shaft speed case exhibits a larger hysteresis loop than the high shaft speed case. This occurs because the pull force is lower at low rotational speeds, the rotor blade is more loosely fitted into the disk attachment, and the mobility in the fir-tree contact is higher. Conversely, the hysteresis loop is smaller at high shaft speeds since the contact is more stuck than at low speeds. The high physical scaling of the mechanical work at high

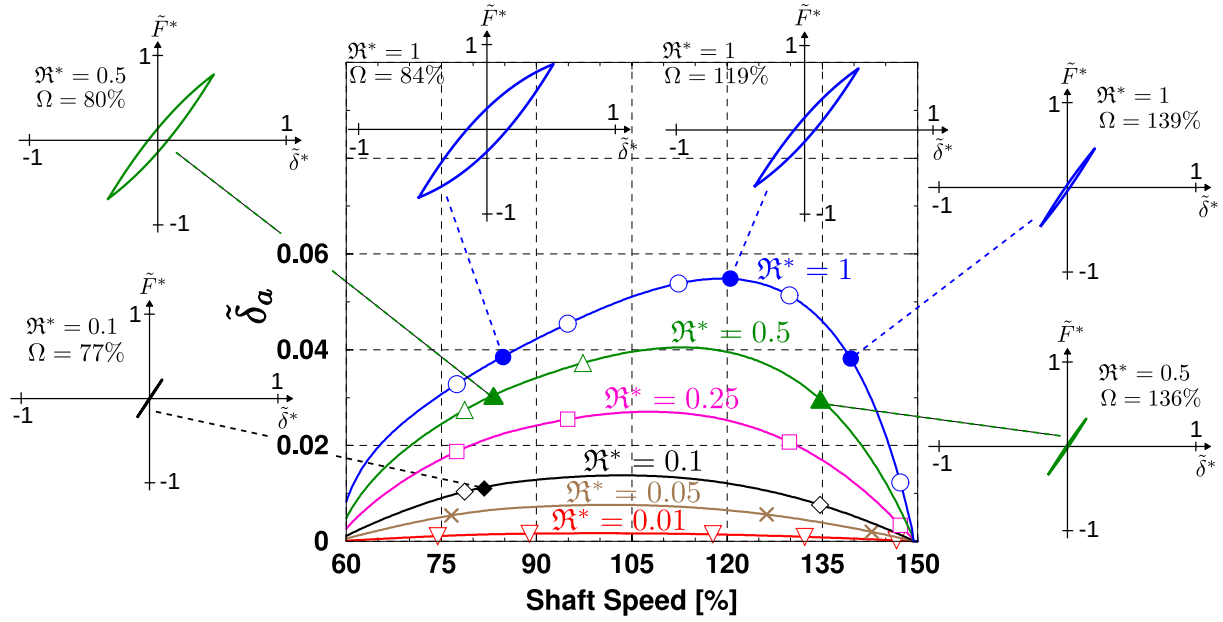


Figure 3.19.: RESULTS OF THE MODEL FOR DIFFERENT LEVELS OF AMPLITUDE $\mathfrak{R}^*$. THE HYSTERESIS CYCLES CORRESPONDING TO LOW SHAFT SPEED FOR $\mathfrak{R}^* = 0.1$ AND BOTH LOW AND HIGH SHAFT SPEED FOR $\mathfrak{R}^* = 0.5$ AND $\mathfrak{R}^* = 1$ ARE INCLUDED.

rotational speeds compensates the small size hysteresis cycles.

3.4. Sensitivity analysis of the model

A sensitivity analysis of the analytical model has been conducted to assess the relative importance of its dimensionless parameters. The energy balance equation 2.26, which governs the relationship between aerodynamic and mechanical work, identifies the key parameters influencing the evolution of the dimensionless vibration amplitude, $\tilde{\delta}_a$, with the dimensionless rotational speed, $\tilde{\Omega}$.

One of the critical parameters in this analysis is $\mathfrak{R}^*$, which includes various structural and operational factors derived from CFD and FEM simulations. However, $\mathfrak{R}^*$ also depends on the unknown parameters μ and λ_0 , meaning it must be determined either through calibration with experimental data or by comparison with a reference blade for which $\mathfrak{R}^*$ is known. The minimum aerodynamic work per cycle can be obtained through fluid-structure interaction (FSI) simulations using a CFD solver, while the Southwell coefficient, β is derived from structural analysis and is $\beta = 4$. Among all the parameters, the two sources of unavoidable uncertainty are the penetration length coefficient, α , and the friction coefficient, μ , the latter being essential for transforming dimensionless results into dimensional values.

The model's sensitivity to variations in the penetration length coefficient, α , was evaluated by comparing results obtained using the recommended value of $\alpha = 0.5$ from Sellgren and Olofsson [25] against the used value of $\alpha = 1$. This value linearly relates the penetration length to the contact force, and it has been chosen since the forces in the contact are far from the elastic limit. The observed changes are minor, with the rotational speed corresponding to the maximum vibration amplitude differing by less than 20%. This limited impact is attributed to the stabilizing effect of lower aerodynamic work values at higher rotational speeds, which compensates the reduced mechanical dissipation resulting from a higher penetration length. Thus, the influence of this parameter on the model's outcomes is limited.

Furthermore, moving from a dimensionless to a dimensional formulation requires the definition of the characteristic vibration amplitude in Eq. 2.24. The only source of uncertainty in this conversion is the friction coefficient μ . However, the limited variability of μ suggests that only limited deviations are expected in an actual case application.

This analysis highlights that, although the model depends on many variables, the sensitivity of its response is usually limited. Errors in evaluating the friction coefficient μ or the penetration length coefficient α result in minor inaccuracies in the model's outcome.

4. Experimental Validation

Contents

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This chapter presents a comparison between the model’s predictions of flutter-induced vibration amplitude for the previously presented LPT cantilever rotor blade and the corresponding experimental results. The study is based on an aeromechanical test campaign in which the blade was specifically designed to induce and measure flutter under controlled conditions. First, the experimental setup and testing procedure are introduced, followed by post-processing and analysis of the collected data. The experimental results are then utilized to calibrate the model’s parameter $\mathfrak{R}^*$. Finally, the model’s predicted trends are validated across a broad range of operating conditions, assessing its accuracy and applicability.

4.1. Experimental campaign

Over the past few decades, several European Union research projects have focused on aeroelastic phenomena in turbomachinery, including ADTurB I and II, as well as FUTURE. The most recent initiative in this series, launched in 2018, is the ARIAS project. Within ARIAS, one of the key research areas is the study of low-pressure turbine aeroelasticity, serving as a continuation of the corresponding work package from the FUTURE project.

The aeroelastic investigations on LPTs within ARIAS have been centered on three main topics. The first area of study explores the nonlinear interaction between amplitude-limited flutter and forced response [24, 67]. The second focuses on intentional mistuning strategies and their effect on both flutter and forced response behavior [26]. Lastly, the third and primary subject of this study examines the use of dry-friction mechanisms to limit the saturation amplitude of fluttering blades, with findings published in [49–51].

The ARIAS LPT cantilever rotor blade, characterized in the previous chapter, has undergone experimental testing at the high-speed wind tunnel facility of the Centro de Tecnologías Aeronáuticas (CTA) [47]. The blade was integrated into an isolated turbine bladed-disk, specifically designed to exhibit flutter under test conditions. Vibration data were acquired using tip-timing optical sensors and flush-mounted unsteady pressure transducers on the casing. To prevent unintended mistuning effects, no onboard instrumentation was installed on the rotor blades. The flutter response was assessed across different shaft speeds and air density conditions.

The blade was designed to ensure that its aerodynamic and aeroelastic dimensionless parameters closely resemble those of an actual engine LPT. Since replicating full-scale engine inlet conditions in a cold-flow test rig is not feasible, comparisons between engine and rig performance must be made in dimensionless terms. From an aerodynamic standpoint, the exit Mach number and Reynolds number were adjusted while the turbine geometry was scaled down to fit within the test facility. Additionally, preserving aeroelastic similarity required matching key dimensionless parameters, such as the reduced frequency and mode shape, to those of the engine blade. The rotor consisted of $N_b = 144$ blades, a number chosen for its compatibility with a wide range of simple mistuning patterns due to its highly versatile prime factorization ($2^3 \times 3^2$).

4.1.1. Wind-tunnel and Rig description

The experimental setup features an isolated bladed rotor subjected to the incoming conditions of the facility's Variable Inlet Guide Vanes (VIGVs), located more than ten chords upstream of the testing article. Consequently, the rotor blade is considered isolated except for the weak perturbations associated with the blade-passing frequency of the upstream VIGVs. These upstream vanes have a chord length approximately 50% greater than that of the rotor blade. The boundary layer generated at the inlet duct is bled one chord upstream of the rotor to avoid unrealistic effects in the secondary flow vortex system.

Figure 4.1 provides a general layout of the wind tunnel, illustrating the distribution of the instrumentation used during the test campaign. The facility control instrumentation includes total pressure and total temperature rakes positioned between the VIGVs and the rotor blades, as well as at the exit of the rig.

The facility's maximum mass flow is approximately 18 kg/s, contingent on the current pressure ratio and level. Equipped with two compressors—one positioned upstream and the other downstream of the test section—the facility enables independent control of the pressure level, thereby determining the Reynolds number and the pressure ratio across the rotor blade, setting the exit Mach number for a given shaft speed. Shaft energy

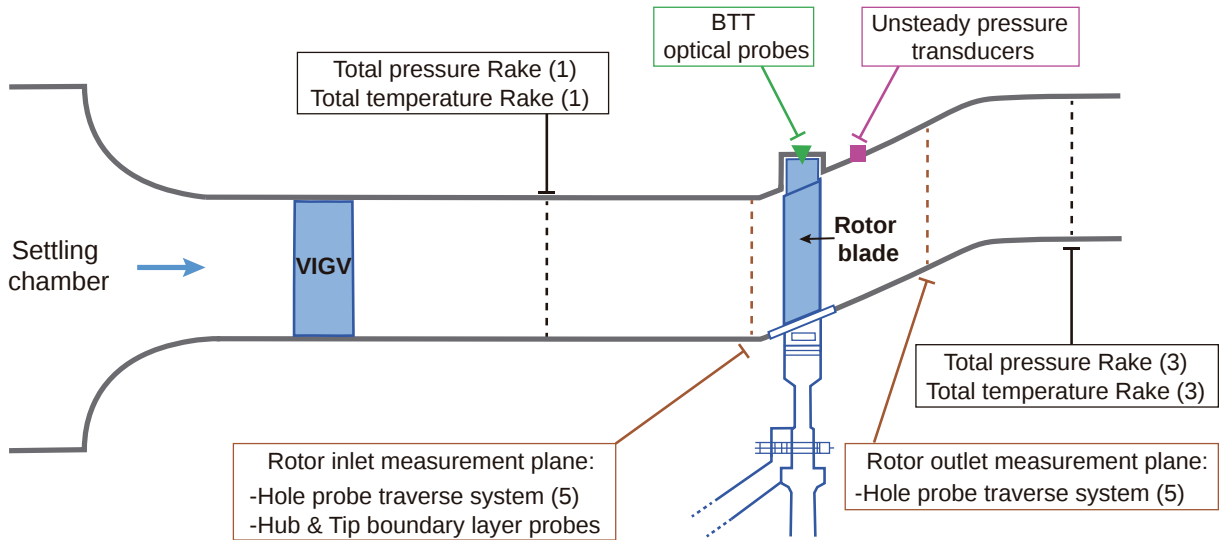


Figure 4.1.: LAYOUT OF THE CTA WIND-TUNNEL USED FOR THE EXPERIMENTAL CAMPAIGN. ▽: BTT PROBES. □: UNSTEADY PRESSURE TRANSDUCERS.

is dissipated using a water brake. The specific maximum pressure ratio and mass flow rates depend significantly on the chosen configuration. Over the years, numerous single and three-stage Low-Pressure Turbine (LPT) rigs have undergone testing in this facility [89–91].

4.1.2. Instrumentation

The instrumentation used to measure aerodynamics includes total pressure and total temperature rakes, which are placed at the rotor’s inlet and outlet to measure rig efficiency. The inlet and outlet boundary conditions are captured using fast-response five-hole miniaturized probes that are radially and azimuthally traversed across a distance equivalent to several rotor pitches. Figure 4.1 includes an schematic view of the steady aerodynamics instrumentation position in the rig. The details of the calibration of this instrumentation and the measurement procedure can be found in [92].

The instrumentation to measure flutter is divided in vibration amplitude and unsteady pressure sensors. The data acquisition system for the vibration amplitude is based on the tip-timing technique. Nine Blade Tip-Timing (BTT) optical probes are integrated into the outer casing, pointing radially towards the rotor to measure the vibration amplitude of every individual blade. All the probes were installed in the same axial position and unevenly circumferentially distributed in an 80-degree sector. The circumferential position of the BTT probes is illustrated in Fig. 4.2.

The unsteady pressure field created due to the rotor’s blade vibration is measured with a row of 27 unevenly distributed flush-mounted unsteady pressure transducers (Kulite

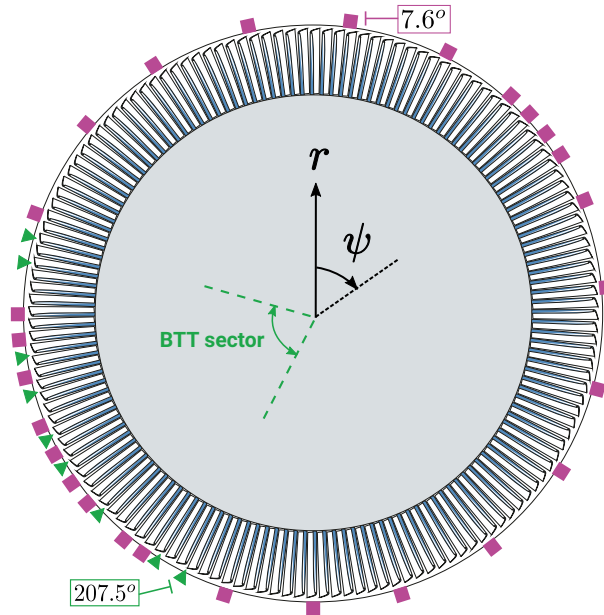


Figure 4.2.: DETAIL OF THE INSTRUMENTATION USED IN THE CTA WIND-TUNNEL FOR THE EXPERIMENTAL CAMPAIGN. ∇ : BTT PROBES. $\square$: UNSTEADY PRESSURE TRANSDUCERS.

XT-190M) in the outer casing, one chord downstream of the rotor blade. The rear part of the transducer is vented to a chamber at the same static pressure as the rotor exhaust duct flow. The pressure range of the transducers is 5 psi, and the calibration error is $\pm 0.1\%$ of the Full Scale Output Best Fit Straight Line. The sampling frequency is 25.6 kHz, well beyond the range of interest (500 to 2,000 Hz).

4.1.3. Post-processing of experimental results

The BTT optic probes to determine vibration amplitude were provided and operated by Rolls-Royce. Two teams (Rolls-Royce and Siemens) performed BTT analysis using identical optical probes and data sets. BTT analysis of the test was challenging due to several reasons. In the first place, flutter signals involve more than 15 closely packed NDs at the same frequency (see Figs. 3.5 and 4.3), making standard algorithms unable to discriminate them based on the phasing between different unsteady pressure probes. The high number of rotor blades (144) makes the phasing difference between two consecutive NDs small, further difficulting the separation of their different contribution by the BTT. Secondly, the superimposition of flutter and forced response at the EO crossings further challenges the BTT algorithms that require making assumptions about the shape and frequency of the signals. Due to these limitations, BTT data could not be fully relied upon for detailed mode decomposition and were instead used to estimate a single vibration amplitude level for each rotational speed and frequency.

In contrast, unsteady pressure measurements provided a more effective method for identifying NDs, as they allowed spatial pattern recognition to be treated as a standard frequency analysis problem. Unlike the BTT data, the unsteady pressure analysis was performed within the scope of this study.

The post-processing of pressure signals followed a structured workflow. First, raw binary data were converted into readable formats, and pressure levels, initially recorded in decibels, were transformed into Pascals. The time evolution of each transducer's pressure signal was then segmented into one-second windows, after which the frequency content was extracted using the discrete Fourier Transform (DFT):

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi \frac{k}{N} n} \quad (4.1)$$

where X_k represents the harmonic content at frequency k , N is the number of samples in the time window and x_n are the signal samples. Following this, the individual spectra obtained from each window were time-averaged to produce a single frequency spectrum per transducer. Afterwards, the harmonic content is analyzed, and data from damaged transducers are excluded. Finally, the Fourier transforms of the distinct pressure transducers located at various circumferential positions are averaged to determine the representative harmonic content of the signal.

The experimental campaign encompasses a large and diverse test matrix. However, the first analysis concentrates on three different exit Mach numbers at various shaft speeds. Figure 4.3 illustrates the harmonic content of the unsteady pressure transducers for $\Omega = 90\%$, the baseline level of density ($\rho = 100\%$) and three different exit Mach numbers ($M_e = 0.675$, $M_e = 0.775$ and $M_e = 0.825$) for the baseline configuration in a wide range of frequencies.

The acoustic responses are categorized into five distinct families based on the excitation source, that sorted from low to high frequency can be described as (i) facility noise, (ii) flutter signals corresponding to various unstable nodal diameters, (iii) non-synchronous vibration with an unidentified source, (iv) forcing excitation with an unknown origin (potentially related to acoustic resonances) along with its harmonics (not shown in Fig. 4.3), and (v) the blade-passing frequency, not shown in Fig. 4.3 since it is located at 6,000 Hz.

The harmonic content of the pressure transducers includes frequencies much higher than those associated with the flutter signals due to their high resolution. However, given the thesis' focus on investigating the flutter peak evolution with the operating conditions, the frequency range displayed is confined to the flutter-relevant frequencies. These frequencies are then transformed to nodal diameters by Doppler shifting the frequencies from the

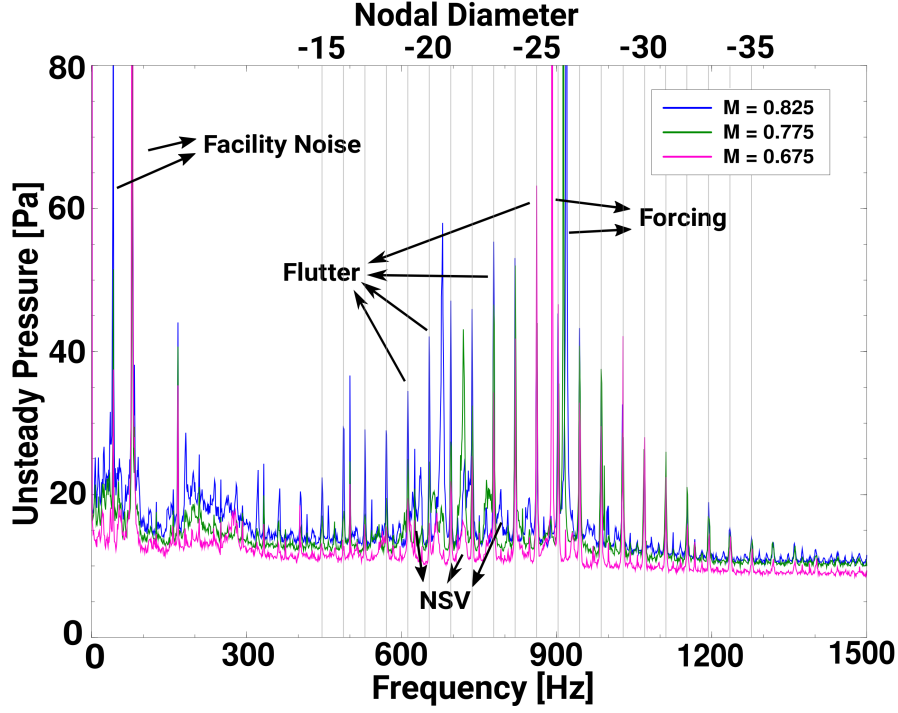


Figure 4.3.: TIME AND CIRCUMFERENTIAL AVERAGED FOURIER TRANSFORM OF THE UNSTEADY PRESSURE SIGNALS OBTAINED FOR $\Omega = 90\%$, $\rho = 100\%$ AND THREE DIFFERENT LEVELS OF EXIT MACH NUMBER ($M_e = 0.675$, $M_e = 0.775$ AND $M_e = 0.825$).

rotating to the stationary frame of reference. Subsequently, the vibration frequency in the stationary frame of reference, ω , can be expressed as

$$\omega = \omega_n + ND \Omega. \quad (4.2)$$

Since ω and Ω are measured, and the natural frequency, ω_n , can be obtained from FE model, Eq. 4.2 yields the ND associated with the flutter mode. In physical terms, the frequency separation between distinct flutter peaks is Ω , as shown in Fig. 4.3, with background lines marking the position of the flutter peaks corresponding to the unstable NDs. Hence, the flutter-induced peaks are discerned from other peaks as they appear at the frequencies given by Eq. 4.2.

In this work, flutter severity is measured using unsteady pressure transducers since they provide the vibration amplitude for each ND. It is therefore paramount to estimate the component of the unsteady pressure associated with flutter. In the low-reduced frequency limit, i.e. $k \ll 1$, the characteristic unsteady pressure on the airfoil can be estimated as

$$p'_c = B_p \rho_e U_e^2 (\delta_a / c) \quad (4.3)$$

where B_p is a constant which can be calibrated with experimental data. Once that

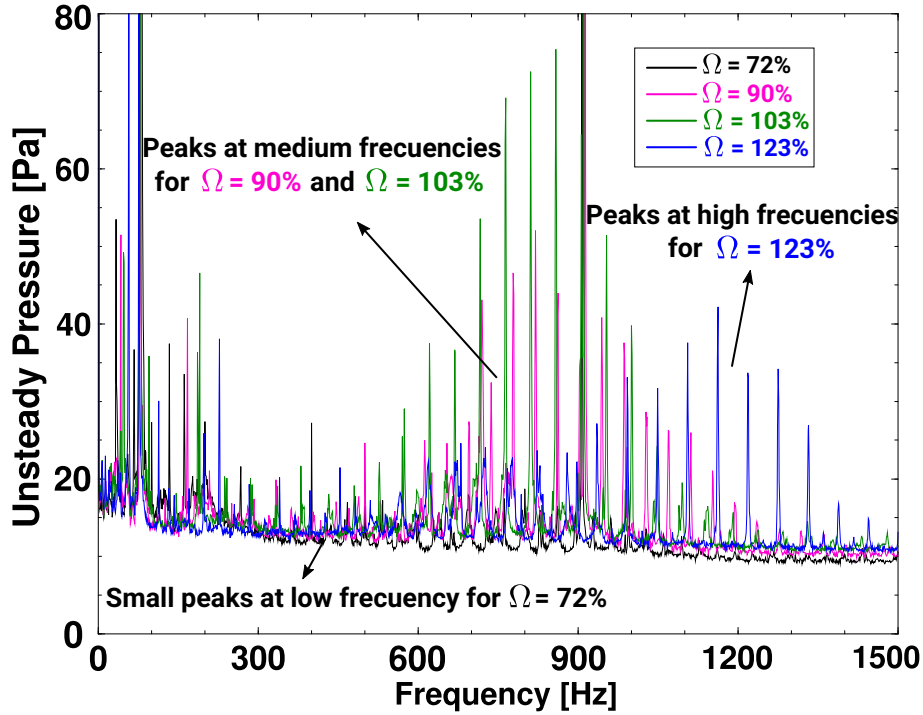


Figure 4.4.: TIME AND CIRCUMFERENTIAL AVERAGED FOURIER TRANSFORM OF THE UNSTEADY PRESSURE SIGNALS OBTAINED FOR $M_e = 0.775$, $\rho = 100\%$ AND FOUR DIFFERENT ROTATIONAL SPEEDS ($\Omega = 72\%$, $\Omega = 90\%$, $\Omega = 103\%$ AND $\Omega = 123\%$).

the vibration amplitude δ_a is obtained from Eq. 2.26, equation 4.3 is used to derive the unsteady pressure caused by the blade vibration. It is worth noting that the unknown constants B_p and B can be consolidated in a single one in the small vibration amplitude limit.

Another noteworthy observation is the variation of background noise levels with the Mach number. According to Eq. 4.3, pressure perturbations scale with $F(M_e^2)$, given that the dynamic pressure is expressed as $\rho_e U_e^2 = \gamma p_0 F(M_e^2)$. As a result, an increase in Mach number leads to a corresponding rise in background noise levels.

The variation of flutter peaks with rotational speed exhibits a different trend than that observed for Mach number effects. This behavior is illustrated in Fig. 4.4, where the harmonic content of the unsteady pressure transducers for $M_e = 0.775$, the baseline level of density ($\rho = 100\%$) and four different rotational speeds ($\Omega = 72\%$, $\Omega = 90\%$, $\Omega = 103\%$ and $\Omega = 123\%$) is represented.

According to Eq. 4.2, increasing the rotational speed shifts flutter peaks toward higher frequencies, further distinguishing them from other pressure perturbations within the harmonic spectrum. This effect is clearly observed when comparing the peaks at $\Omega = 103\%$ (located within the $f = 600 - 1000$ Hz range) with those at $\Omega = 123\%$ (which appear between $f = 900 - 1300$ Hz). Notably, at $\Omega = 72\%$ the flutter peaks are barely visible—an

observation that will be further explained in the next subsection.

In contrast, peaks associated with NSV remain within the same frequency range ($f = 600 - 900$ Hz) across all rotational speeds, as they are not driven by shaft speed variations.

One of the main advantages of unsteady pressure sensors on the stationary frame of reference over BTT measurements is that their frequency resolution allows identifying the nodal diameter of the source on the rotating frame of reference through their associated Doppler-shifted frequency. However, synchronous excitations on the rotor have an apparent frequency of zero in the static frame of reference, and therefore cannot be extracted from the spectra measured by the unsteady pressure sensors.

4.2. Analysis of flutter-induced signals

The experimental campaign was successful, as its primary objective—triggering flutter under most tested conditions and capturing its measurements—was achieved. The BTT probes provided vibration level data for all investigated test points. A typical response, such as the one shown in Fig. 4.5, reveals two distinct types of vibration behavior. First, a non-synchronous response, primarily due to flutter and potentially other unidentified sources, increases in frequency with shaft speed, following the trend of the first flap mode. This non-synchronous vibration is observed throughout the entire speed range.

Additionally, a strong synchronous response of the first flap mode occurs at EO4 near the design speed, while a weaker synchronous response at EO6 is detected only at lower shaft speeds. A minor resonance of the first mode with EO5 is observed at certain points, though it is challenging to distinguish from non-synchronous vibrations. In most test cases, the second mode's resonance with EO6 lies slightly above the maximum operational speed of the rig and could not be appropriately measured.

The test rig was designed to accommodate forced response experiments using magnets to induce synchronous excitations at EO4 and EO6. However, it is worth noting that even in the absence of these magnets, resonances at EO4 and EO6 are still identifiable in most cases. Specifically, the amplitude of the EO4 resonance remains of the same order of magnitude, regardless of whether the permanent magnets are installed. This suggests the presence of an unintended source of synchronous excitation within the rig, predominantly following EO4, though its origin remains unclear.

The unsteady pressure sensors recorded clean signals that were qualitatively consistent with the BTT results. The post-processed results shown in Figs. 4.3 and 4.4, where the unstable NDs are identified using the Doppler-shift transformation presented in Eq. 4.2 and the peaks not linked to fluttering NDs are filtered out for clarity, are presented

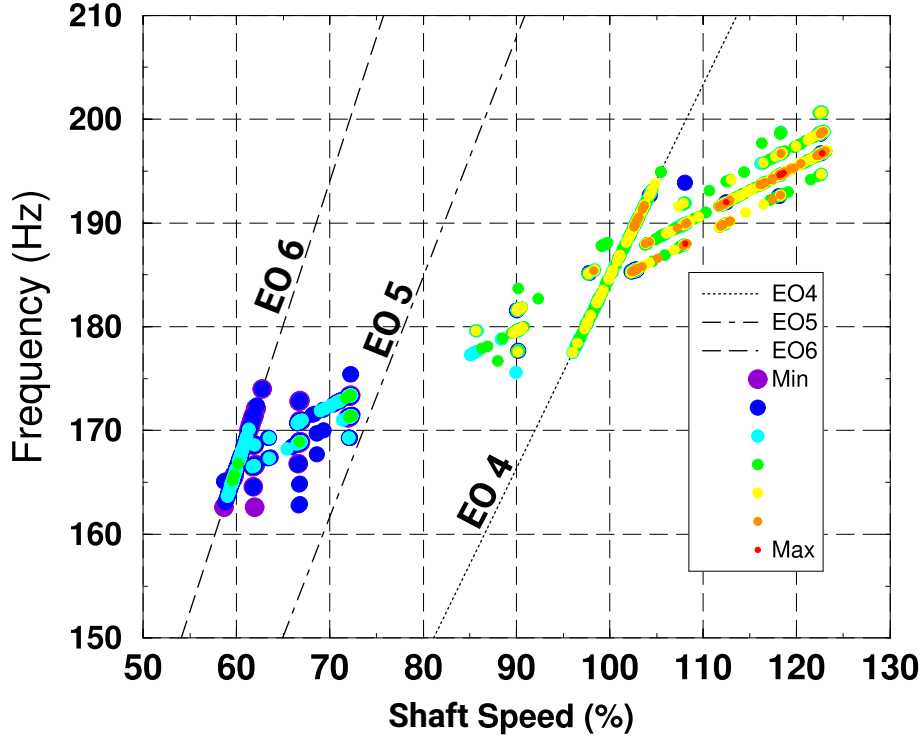


Figure 4.5.: VIBRATION AMPLITUDE OF THE ROTOR MEASURED BY BTT AS A FUNCTION OF THE SHAFT SPEED.

in Fig. 4.6. The resolution of the frequency spectra is high enough to differentiate the pressure content related to fluttering NDs from the peaks associated with other effects, thereby ensuring the flutter origin of the filtered peaks. Additionally, the background noise has been averaged, maintaining consistency with the previously established trend $p'_c \propto F(M_e^2)$.

The size of the window used during the post-processing of the signals influences the amplitude of the pressure peaks. Therefore, to assess the uncertainty of the pressure measurements, the standard deviation of each peak is calculated for different pressure transducers. The standard deviation of the majority of the peaks is around 15% of the peak average value, increasing up to 25% of the peak average value for the highest peaks. Figure 4.6 shows the standard deviation range of the peaks calculated for the $M_e = 0.775$ case corresponding to each shaft speed.

Figure 4.6 shows that most unsteady pressure signals monotonically increase with the exit Mach number. This behaviour is consistent with the Eq. 4.3, as the pressure perturbation follows the proportionality relation $p'_c \propto F(M_e^2)\delta_a$, while the vibration amplitude scales with $\Re^* \propto F(M_e^2)$. In the limit of small vibration amplitudes, where $\Re^* \ll 1$, Eq. 2.28 indicates that $\delta_a \propto F(M_e^2)$. Consequently, in this regime, the pressure perturbations follow a quadratic dependence on the Mach number, given by $p'_c \propto F(M_e^2)^2$.

The amplitude of the flutter-induced pressure peaks is coherent with the expression $p'_c \propto$

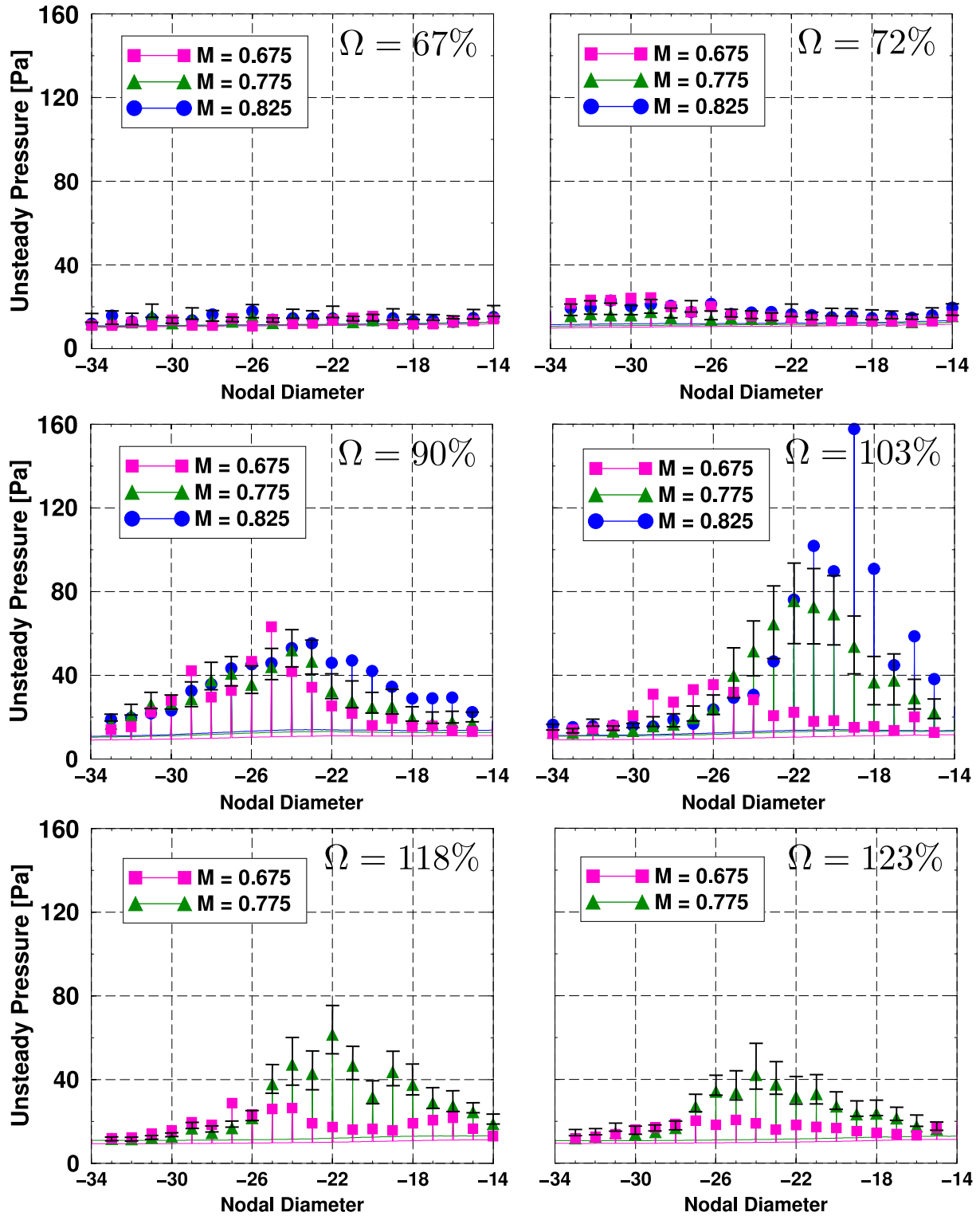


Figure 4.6.: TIME AND CIRCUMFERENTIAL AVERAGED FOURIER TRANSFORM OF THE FLASH-MOUNTED UNSTEADY PRESSURE PROBES FILTERED TO SHOW JUST THE FLUTTER PEAKS FOR DIFFERENT EXIT MACH NUMBERS.

$F(M_e^2)^2$ for the on-design operation point ($\Omega = 103\%$) and high shaft speeds ($\Omega = 118\%$ and $\Omega = 123\%$). However, the case with a shaft speed of $\Omega = 90\%$ does not follow this trend for certain nodal diameters (NDs from -15 to -25). In this case, the pressure content of the most unstable ND for $M_e = 0.675$ exceeds that corresponding to $M_e = 0.825$. The reason for this deviation from the trend remains unclear. Furthermore, the low-speed cases with $\Omega = 67\%$ and $\Omega = 72\%$ exhibit tiny peaks compared to the background noise due to the reduced vibration amplitude in these instances.

The highest peak is close to the predicted most unstable ND, approximately $ND = -26$, as depicted in Fig. 3.7. However, experimental results reveal a slight shift toward smaller NDs as the exit Mach number increases. This observation contrasts with linear uncoupled aerodynamic damping analyses, which suggest that the most unstable ND remains independent of Mach number.

One potential explanation considered is that natural mistuning drives this shift. Numerical studies have established that in a cantilever bladed disk with multiple unstable NDs, the final state is predominantly governed by the most unstable ND [6, 24]. The sidebands surrounding the most unstable ND in Fig. 4.7 are attributed to the inherent natural mistuning in the experiment [26]. The influence of mistuning depends on the relative magnitude of aerodynamic forces, meaning that variations in Mach number and shaft speed alter its effectiveness. However, as Mach number increases, aerodynamic forces grow stronger, reducing the relative significance of mistuning. In this case, flutter peaks would be expected to align more closely with predictions from linear uncoupled aerodynamic analyses, yet the experimental results show the opposite trend. This suggests that the observed shift is not caused by natural mistuning, and the underlying reason remains unclear.

The evolution of the flutter-induced pressure peaks with the rotational speed is illustrated in Fig. 4.7 for the three Mach numbers of the experiments. For the three different Mach numbers, it is observed that the pressure signals present a maximum at one of the intermediate rotational speeds. For the two highest Mach numbers, $M_e = 0.775$ and $M_e = 0.825$, the maximum response is reached at $\Omega = 103\%$, and this maximum is shifted towards a lower shaft speed, $\Omega = 90\%$, for the smallest Mach number $M_e = 0.675$. Note that the shaft speeds shown for the highest Mach number, $M_e = 0.825$, do not include $\Omega = 118\%$ and $\Omega = 123\%$, since there are no experiments for these shaft speeds.

This phenomenon was also observed in [8] though a complete explanation was not provided. However, by considering the exact expression for mechanical work (Eq. 2.9) instead of the approximation (Eq. 2.11), and incorporating the variation of aerodynamic damping with rotational speed, $\tilde{W}_a^{\min}(\tilde{\Omega})$, the presence of a maximum at intermediate rotational speeds can now be explained.

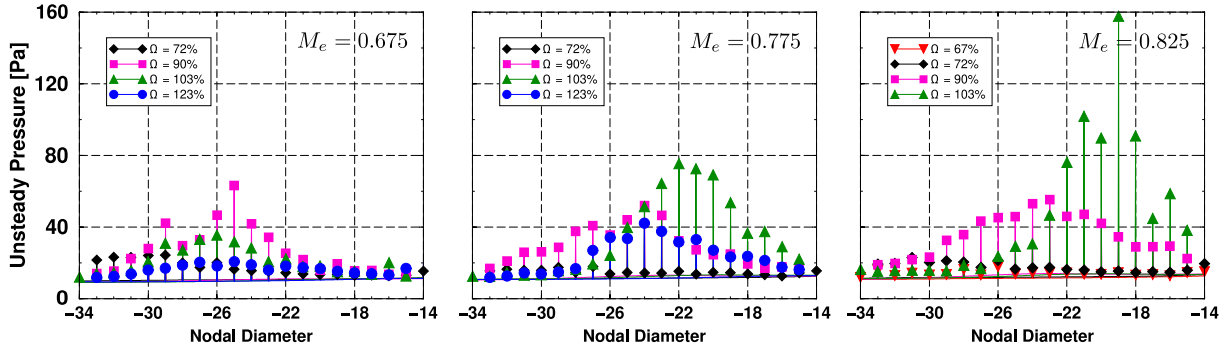


Figure 4.7.: TIME AND CIRCUMFERENTIAL AVERAGED FOURIER TRANSFORM OF THE FLASH-MOUNTED UNSTEADY PRESSURE PROBES FILTERED TO SHOW JUST THE FLUTTER PEAKS FOR DIFFERENT ROTATIONAL SPEEDS.

These results align with the BTT measurements shown in Fig. 4.5, where the highest vibration amplitudes—excluding EO resonances—occur within the rotational speed range of $\Omega = 100 - 110\%$. It is also important to note that all flutter modes are cut off, with decay rates varying across different NDs. However, due to the short distance between the rotor trailing edge and the transducers, the pressure amplitude decay is assumed to be negligible in this analysis.

4.3. Validation of the model

The analytical model presented earlier is calibrated and validated with the experimental results obtained with the unsteady pressure measurements. These measurements provide a frequency-based spectrum associated with fluttering nodal diameters using Equation 4.2. This simplifies the separation of flutter's influence from other factors, enabling the utilization of unsteady pressure data as a reference for evaluating analytical predictions.

The procedure consists of several steps: First, the experimental data is processed to estimate the pressure perturbation for each Mach number and rotational speed. Next, the calibration of $\mathfrak{R}^*$ (see Eq. 2.26) and B_p (see Eq. 4.3) is performed. Finally, the vibration amplitude trends predicted by the model are compared with experimental results that were not used during the calibration process.

The mean unsteady pressure for each operation point is computed by subtracting the background noise from the 15 highest measured frequency peaks to isolate the flutter-induced pressure signal and subsequently averaging them. The reason is that it has been considered that the flutter contribution to these peaks is superimposed to the background noise (see Fig. 4.7). This number of peaks ensures that at least 90% of the acoustic energy associated with the flutter-induced signals is included in the averaging process.

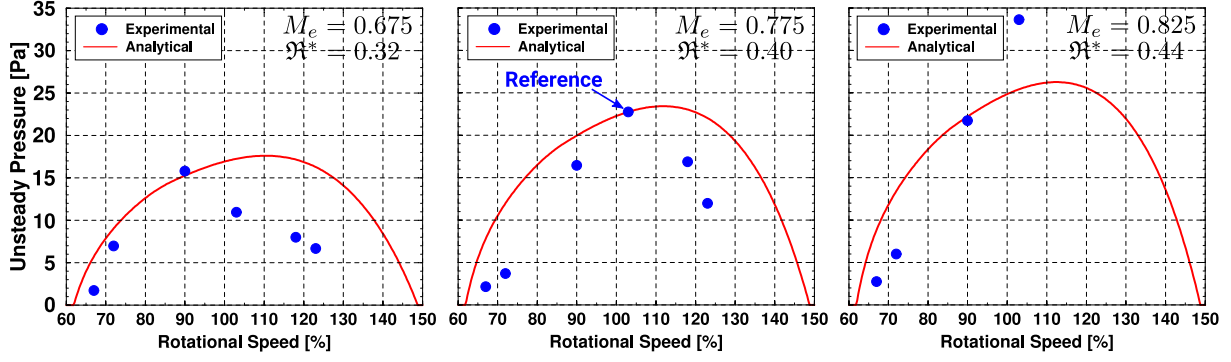


Figure 4.8.: MEAN UNSTEADY PRESSURE SIGNAL AS A FUNCTION OF THE ROTATIONAL SPEED COMPARED TO THE RESULTS OF THE ANALYTICAL MODEL FOR DIFFERENT MACH NUMBERS AND ITS CORRESPONDING VALUES OF $\mathfrak{R}^*$.

Figure 4.8 shows the averaged measured unsteady pressure and the model predictions as a function of the rotational speed for the three different exit Mach numbers tested. After subtracting the background noise, the averaged unsteady pressure is computed as the mean of the 15 highest peaks shown in Fig. 4.7. The calibration of $\mathfrak{R}^*$ is performed by assigning a value to this parameter such that the model's predicted maximum vibration amplitude aligns with the highest pressure peak observed in the experimental results. This is consistent with Eq. 4.3, where the unsteady perturbations p'_c are linear with the vibration amplitude δ_a .

The model calibration reference point is set at $M_e = 0.775$, where the highest unsteady pressure measured is located at $\Omega = 103\%$. However, the exact rotational speed at which the maximum unsteady pressure occurs is unclear due to the limited number of measured rotational speeds and has been estimated to be around $\Omega \approx 110\%$. Thus, the value of $\mathfrak{R}^*$ that aligns the maximum at this shaft speed is $\mathfrak{R}^* = 0.4$, completing the formulation of the dimensionless problem.

For the case with $M_e = 0.675$, the maximum on the experimental measurements is at a lower shaft speed than the model prediction, but the general trend is still qualitatively correct. Also, the vibration amplitudes of most of the operating points tested are qualitatively correct. The only exception may be the maximum response with $M_e = 0.825$, where the model predicts a vibration amplitude smaller than the results obtained in the experiment. Nevertheless, the trend in the experimental measurements and the model remains consistent in this case.

It is important to highlight that unsteady pressure measurements exclusively capture peaks associated with flutter. As a result, the analytical model, which predicts vibration amplitude due to flutter, is calibrated using these flutter-induced pressure peaks. Once the parameter $\mathfrak{R}^*$ is calibrated, the constant B_p , which relates unsteady pressure perturbations to vibration amplitude, is calibrated using the same reference conditions

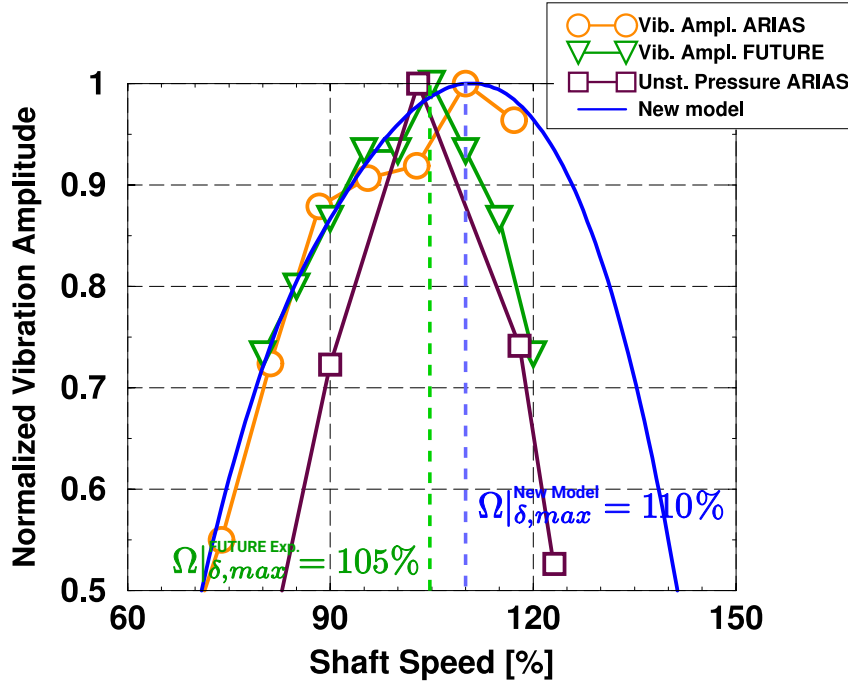


Figure 4.9.: NORMALIZED VIBRATION AMPLITUDE AND UNSTEADY PRESSURE MEASUREMENTS OF THE ARIAS AND FUTURE EXPERIMENTAL CAMPAIGNS AND NORMALIZED VIBRATION AMPLITUDE PREDICTED BY THE NEW MODEL (Eq. 2.26).

($M_e = 0.775$ and $\Omega = 103\%$).

Unsteady pressure measurements are chosen for calibration because the vibration amplitudes obtained from BTT in the FUTURE and ARIAS test campaigns are not suitable for this purpose. These BTT measurements are affected by EO4 synchronous response and other unidentified influences, likely arising from inlet distortions or acoustic resonances. Consequently, isolating the vibration amplitude associated purely with flutter in the ARIAS blade remains a challenge for BTT analysis.

When comparing the unsteady pressure measurements of the ARIAS blade with the vibration amplitudes recorded for the FUTURE blades, a qualitative agreement is observed. The peak unsteady pressure in the ARIAS tests occurs around $\Omega \approx 110\%$, whereas the maximum vibration amplitude in the FUTURE tests is found at $\Omega \approx 105\%$. This is illustrated in Fig. 4.9, where the normalized unsteady pressure and normalized vibration amplitude measured in the experiments and predicted by the model is shown. This consistency is expected since the ARIAS blade and test rig introduce only minor modifications compared to the FUTURE setup. However, while the measured vibration amplitudes are primarily driven by flutter, it cannot be guaranteed that they are entirely free from other excitation sources. Therefore, although the FUTURE vibration amplitudes cannot be used for calibrating $\mathfrak{R}^*$, the global trend provides a useful reference for validating the rotational speed at which maximum vibration occurs.

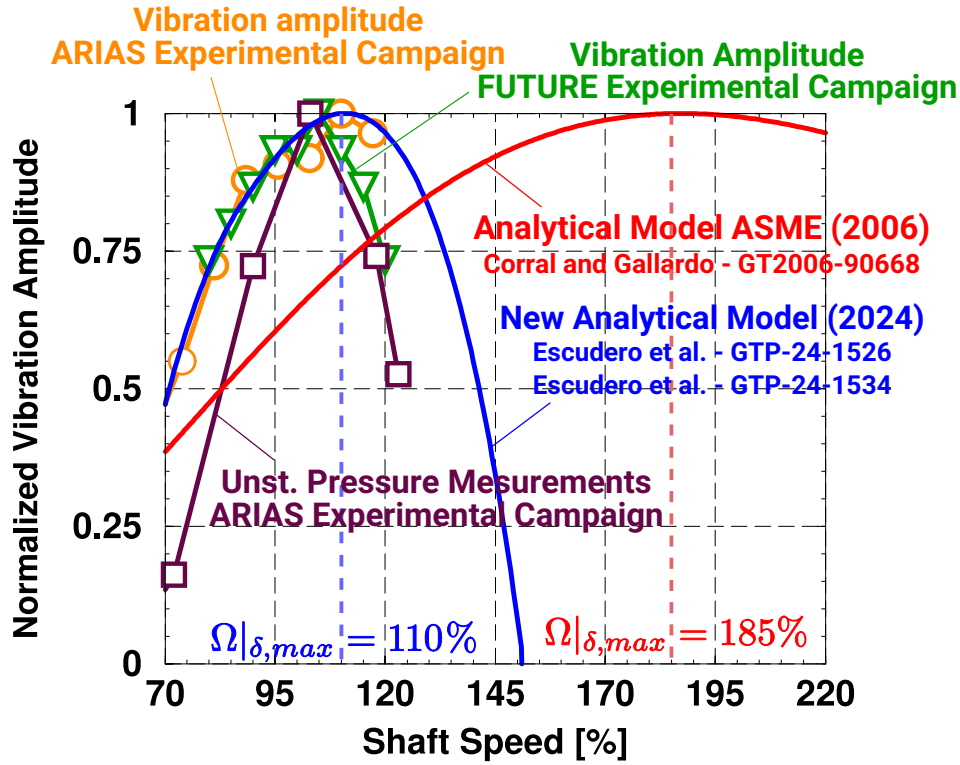


Figure 4.10.: COMPARISON OF THE NORMALIZED VIBRATION AMPLITUDE PREDICTED BY THE NEW MODEL (EQ. 2.28), THE OLD VERSION OF THE MODEL (EQ. 2.26) AND THE EXPERIMENTAL RESULTS.

Including the ARIAS blade vibration amplitudes in the comparison reveals a key discrepancy at higher rotational speeds, as shown in Fig. 4.9. Unlike the FUTURE measurements, where vibration amplitude decreases beyond the peak, the ARIAS blade exhibits consistently high vibration levels within the $\Omega = 100\text{--}120\%$ range. This behavior contradicts the unsteady pressure measurements, further confirming the presence of unidentified external influences contaminating the ARIAS vibration response. As a result, the ARIAS vibration amplitude data cannot be used for model calibration, nor can its trend be reliably employed for validation.

Finally, a comparison is made between the normalized vibration amplitudes predicted by both the original and revised versions of the analytical model, alongside the vibration measurements from the ARIAS and FUTURE blades and the normalized pressure data. Figure 4.10 shows that the revised model accurately predicts the peak vibration amplitude at $\Omega \approx 110\%$, closely matching the experimental results, which place the peak at $\Omega \approx 105\%$. In contrast, the original version of the model incorrectly predicted the maximum vibration at $\Omega \approx 185\%$, significantly deviating from the experimental observations. This improvement demonstrates that the modifications introduced in the new model, particularly in the dimensionless aerodynamic work formulation and the derivation of the exact mechanical work, lead to a significantly better alignment with experimental

data.

5. Industrial Application of the Model

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The analytical model, which was calibrated and validated in previous chapters using the cantilever LPT rotor blade from the ARIAS EU project, is now applied to predict the vibration amplitude of other turbomachinery blades. The chapter begins by outlining the methodology used, along with a structured roadmap of blade transformations considered for vibration amplitude estimation.

To build a foundation for understanding how various modifications influence the model parameters and aeromechanical behavior, a set of basic transformations is first introduced. These fundamental changes illustrate how simple geometric or operational variations alter the predicted vibration amplitude. The objective is to combine these transformations to extend the model's applicability to blades with different geometries, sizes, and operating conditions.

Following this, the ARIAS rig blade is systematically transformed into a Large Civil Engine (LCE) LPT blade. This process involves multiple modifications, each of which is analyzed in detail to assess its impact on the model and vibration amplitude. Finally, the vibration control strategy for fluttering blades is extended to fan applications. After discussing key aeroelastic characteristics of fans, two specific transformations are examined: the vibration amplitude prediction for a Small Civil Engine (SCE) fan blade and its corresponding scaled version for an LCE.

5.1. Methodology

The model is applied to read-across the flutter vibration amplitude predictions of different turbomachinery blades. This is done by systematically applying a set of simple transformations involving changes in scale, operation point, mechanical design, and material. These transformations provide new values of the parameters $\mathfrak{R}^*$, $\tilde{\Omega}$ and $\tilde{W}_a^{\min}$ of the model (see Eq. 2.26) and the characteristic vibration amplitude δ_{ac} (see Eq. 2.24). The latter is used to calculate the dimensional value of the vibration amplitude once the model provides the dimensionless vibration amplitude $\tilde{\delta}_a$.

The values of $\tilde{\Omega}$ and $\tilde{W}_a^{\min}$ are obtained for each operating point through CFD and structural simulations of the transformed blade. Meanwhile, δ_{ac} is influenced by geometric modifications and the rigidity constant B , which is derived from structural calculations. Since friction coefficient variability is minimal, a constant value of μ is assumed for the calculation of δ_{ac} .

The main limitation of the method is associated with the large number of physical parameters required, some of which are difficult to measure. In particular, those associated with the friction model, such as the penetration length, λ_0 , which is especially challenging to estimate. However, to deal with this, instead of calculate an absolute value of $\mathfrak{R}^*$ or calibrate this value with experimental tests of the new blade, the determination of $\mathfrak{R}^*$ for the transformed blade is performed by comparison against a reference blade. Once calibrated, the model can be used in an industrial environment both at the preliminary and detailed design phases. The most relevant part of the process is to use the method for similar rotor blades and vibration amplitude data of similar rotor blades. Corral and Gallardo [1,55] successfully used this method to predict the vibration amplitude of a family of LPT rotor blades corresponding to different engines and stages and varying aspect ratios. The technique was later used successfully to design new LCE LPTs.

One of the main challenges in this analysis is the scarcity of publicly available data on the vibration amplitude of fluttering blades saturated by friction. Most of this data are proprietary and cannot be disclosed. For this reason, the study starts with a rig blade as the reference point. The reference blade used in this study is the ARIAS LPT blade, whose $\mathfrak{R}^*$ value has been previously calibrated using experimental data. Additionally, the author's direct involvement in the ARIAS EU project, where this blade was extensively studied, provides valuable insights into the test campaign, measurement procedures, and overall analysis of results. It is important to note that while this study relies on rig data, Overall Engine Manufacturers (OEMs) possess real engine data that can replace the rig data used in this work.

Building upon this reference blade, a series of transformations are applied to predict the

vibration amplitudes of other turbomachinery blades. Although the strategy of stabilizing unstable blades through dry friction could theoretically be applied to any bladed disk, it has so far only been demonstrated in aeronautical LPT blades. High-Pressure Turbines (HPTs) are inherently stiff and operate at high reduced frequencies, where flutter is not a known issue. Similarly, compressor blisks, apart from exhibiting minimal material damping, lack significant friction mechanisms and must be explicitly designed to be flutter-free. Beyond LPT blades, only fan blades feature root attachments where nonlinear friction is present. However, an attempt has yet to be made to understand if flutter-induced vibration in fans can be limited by dry friction.

The roadmap of transformations is presented in Fig. 5.1. First, the vibration amplitude of the ARIAS LPT cold flow rig test [47] is calculated. Then, departing from this blade, the vibration amplitudes are scaled to engine-like size and conditions. This transformation involves modifications in geometry, operating conditions, scale, and material properties, with their isolated effects analyzed in Section 5.2.

Following this, the engine-like LPT blade is transformed into a fan blade representative of a Small Civil Engine (SCE). During this transformation the size is approximately constant since the height of SCE fan is about the same as that of the last stage LPT blade of a LCE. The geometry used for this step is based on the fan blade used in the ACAT EU Project (*AneCom AeroTest*), whose diameter ($D \simeq 850\text{mm}$) is about the same as that of a SCE fan. Finally, the SCE fan blade is scaled up to an LCE size fan blade, where the primary modification lies in the mechanical design, incorporating a hollow structure to reduce weight.

However, as noted earlier, each transformation shown in Fig. 5.1 consists of several fundamental modifications. Therefore, before delving into the full transformations, the set of basic transformations is first examined.

5.2. Basic transformations

This section examines how the model parameters $\mathfrak{R}^*$, $\tilde{W}_a^{\min}$ and $\tilde{\Omega}$ change under various basic transformations. The impact of changes in scale, operating conditions, mechanical design, and material properties on vibration amplitude is analyzed in detail. The rationale behind each transformation is illustrated in Fig. 5.2, which highlights the variations in model parameters and key physical properties for each transformation. Additionally, Table 5.1 provides a comparative overview of the ratio between the affected physical variables in each basic transformation.

Furthermore, the impact of these transformations on both the dimensionless vibration

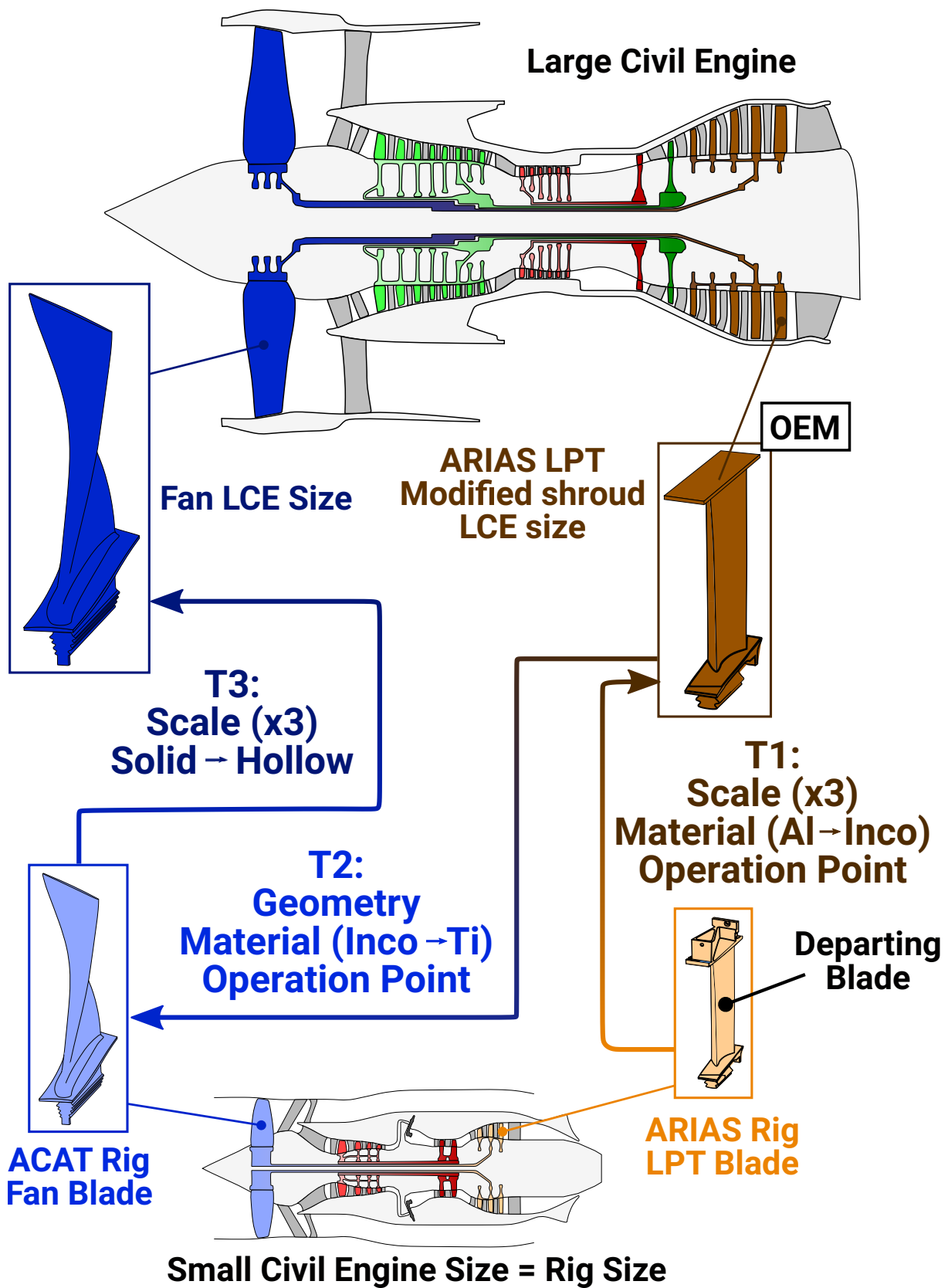


Figure 5.1.: ROADMAP OF THE TRANSFORMATIONS PRESENTED IN THE THESIS.

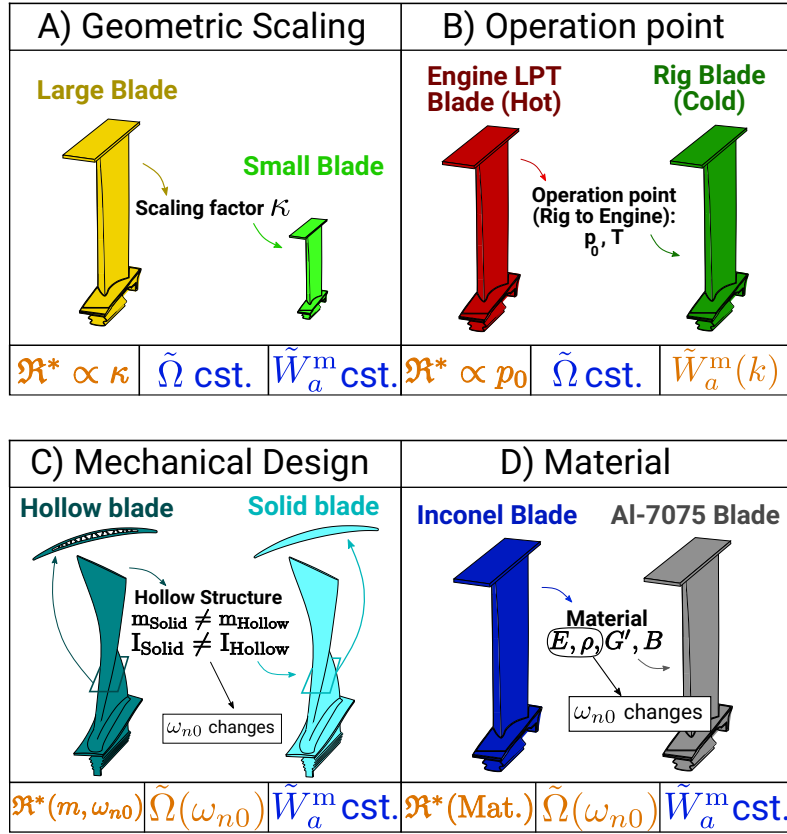


Figure 5.2.: SUMMARY OF THE BASIC BLADE TRANSFORMATIONS.

amplitude and its corresponding dimensional value is examined. The resulting changes in alternate stress on the blade, induced by vibration, are also assessed to provide a comprehensive understanding of how different modifications affect the blade's overall aeroelastic behavior.

5.2.1. Engine-to-rig Scale Effects

The impact on certain key parameters of a pure geometric scaling down of a Large-Engine-Size (LCE) rotor blade to a rig size is addressed in this subsection. Large civil engine fans and LPTs are typically scaled down one-third to fit into existing wind tunnels as shown in Fig. 5.2a. The ratio between the characteristic length of an LCE rotor blade, L_{LCE} , and a rig-size rotor blade L_{Rig} , is used to define the scaling factor κ as

$$\kappa = L_{\text{Rig}}/L_{\text{LCE}}. \quad (5.1)$$

This geometric scaling affects several structural parameters, including the fir-tree width, blade mean radius, and contact area. Additionally, maintaining consistent dimensionless aerodynamic parameters with engine conditions is crucial when scaling blades for test-

ing. Since the blade mean radius scales proportionally to the scaling factor, $R \propto \kappa$, the rotational speed must be adjusted inversely to maintain velocity triangles and tip Mach number, following $\Omega \propto \kappa^{-1}$.

The effects of scaling, together with the material change on the blade mass are given by

$$\frac{m_{\text{Rig}}}{m_{\text{LCE}}} = \kappa^3 \frac{\rho_{\text{Rig}}}{\rho_{\text{LCE}}} \quad (5.2)$$

where ρ is the material density and the blade volume is assumed to scale as $V \propto \kappa^3$. Similarly, the relationship between scaling, material properties, and natural frequency is given by

$$\frac{\omega_{\text{Rig}}}{\omega_{\text{LCE}}} = \frac{1}{\kappa} \sqrt{\frac{(EI/\rho)_{\text{Rig}}}{(EI/\rho)_{\text{LCE}}}}, \quad (5.3)$$

where E is the elastic modulus and I is the blade's area moment of inertia. Since a pure scaling transformation does not alter material properties or geometry, blade mass scales as $m_b \propto \kappa^3$, while the natural frequency remains inversely proportional to blade size, i.e. $\omega \propto \kappa^{-1}$. As a result, it is simple to verify that the $\mathfrak{R}^*$ parameter scales as

$$\mathfrak{R}^* \propto \kappa \quad (5.4)$$

The minimum dimensionless aerodynamic damping, $\tilde{W}_a^{\min}$, remains unchanged, as the blade loading (velocity triangles) and the reduced frequency $k = \omega c/v_e$ are preserved in the transformation. It is worth noting that aerodynamics that the exit velocity, v_e , is constant since the Mach number must be preserved in the test, and ωc is unaffected by the scale change. Lastly, the dimensionless rotational speed $\tilde{\Omega} = \Omega/\omega_{n0}$ also remains constant, since the reduction in rotational speed Ω is offset by the corresponding decrease in ω_{n0} .

With $\mathfrak{R}^*$ as the only changing parameter, solving the energy balance Eq. 2.26 yields the value of $\tilde{\delta}_a$. Since $\tilde{\delta}_a$ depends on $\mathfrak{R}^*$ in a nonlinear manner and $\mathfrak{R}^* \propto \kappa$, the evolution of $\tilde{\delta}_a$ with the scaling factor is also nonlinear. However, in the small displacement limit, where $\mathfrak{R}^* \ll 1$, this relation simplifies to $\tilde{\delta}_a \propto \mathfrak{R}^*$, as shown in Eq. 2.28, resulting in a linear dependency, $\tilde{\delta}_a \propto \kappa$. Overall, the dimensionless vibration amplitude increases with the scaling factor, following a linear relationship in the small displacement regime and a nonlinear trend when $\mathfrak{R}^* \sim 1$.

The reason lies in the scaling of mechanical work with the fir-tree displacement, directly related with the vibration amplitude. In the $\mathfrak{R}^* \ll 1$ limit, small variations in vibration amplitude correspond to relatively minor changes in mechanical work, as illustrated by the low slope of the mechanical work curve with respect to fir-tree displacement in Fig. 2.2 for small values of $\tilde{\delta}^*$. As a result, substantial increases in $\mathfrak{R}^*$ must be accompanied by

	p_0	L	a_{FT}	R	A_c	Ω	m_b	ω	k
Scaling	-	κ	κ	κ	κ^2	κ^{-1}	κ^3	κ^{-1}	-
Operation point	Rig/Eng	-	-	-	-	-	-	-	T
Mech. Design	-	-	-	-	-	-	$1 - \psi$	$\sqrt{I/m_b}$	$\sqrt{I/m_b}$
Material	-	-	-	-	-	-	ρ	$\sqrt{E/\rho}$	$\sqrt{E/\rho}$

Table 5.1.: PROPORTIONALITY OF BLADE PHYSICAL PARAMETERS TO BASIC TRANSFORMATIONS.

similarly significant increases in vibration amplitude to balance the aerodynamic work injected into the system. Conversely, in the nonlinear regime, the mechanical work rises sharply with even small variations in vibration amplitude, as indicated by the steeper slope of the mechanical work curve near $\tilde{\delta}^* \sim 1$ in Fig. 2.2. Consequently, in this regime, large increases in $\Re^*$ require only modest increases in vibration amplitude to achieve the necessary mechanical work compensation.

The characteristic vibration amplitude scales linearly with the scaling factor, $\delta_{ac} \propto \kappa$, according to Eq. 2.24. Since the friction coefficient remains constant due to unchanged material properties, and the rigidity constant B remains unaltered due to preserved geometry, the total vibration amplitude $\delta_a = \tilde{\delta}_a \delta_{ac}$ follows a direct proportionality to κ . In the small displacement regime, this relation simplifies to $\delta_a \propto \kappa^2$, whereas for $\Re^* \sim 1$ case, δ_a increases with κ in a nonlinear manner.

To further assess the blade's alternate stress, the Amplitude-Frequency factor (AF) is introduced, defined as $AF = \delta_a \omega / 2\pi$. To clarify the effect of the scaling factor on AF , the vibration amplitude is rewritten as $\delta_a = \tilde{\delta}_a \delta_{ac}$. Since the characteristic vibration amplitude scales as $\delta_{ac} \propto \kappa$ and the vibration frequency scales inversely with the scaling factor, $\omega \propto \kappa^{-1}$, these effects cancel each other out. As a result, AF is proportional to $\tilde{\delta}_a$, which, in the small displacement limit ($\Re^* \ll 1$) simplifies to the linear relation $AF \propto \kappa$. In contrast, for $\Re^* \sim 1$, AF follows the previously established nonlinear scaling behavior.

5.2.2. Operation Point Effects

Certain rig-to-engine blade transformations involve changes in operating conditions. In LPT testing, moving from an engine to a cold flow test results in lower pressure and temperature levels in the rig than in engine conditions, as illustrated in Fig. 5.2b. In fans, the primary impact of transitioning from rig to engine operation is only a geometric scaling. The pressure level is about the same for fans and low-pressure compressors.

To ensure aerodynamic similarity in rig testing, dimensionless parameters such as Mach number and velocity triangles remain unchanged. Since blade size is also constant, rotational speed does not vary. If the effect of temperature on vibration frequency is neglected,

$\tilde{\Omega}$ remains unchanged. In contrast, the parameter $\mathfrak{R}^*$ depends on the operation point since it is proportional to the pressure level, i.e.

$$\mathfrak{R}^* \propto p_0. \quad (5.5)$$

A key aspect of LPT testing is the impact of temperature changes on $\tilde{W}_a^{\min}$. Since sound speed scales with the temperature as $a_e \propto \sqrt{T}$, keeping the same Mach number in the rig as in the engine requires a lower flow velocity v_e in the rig than in the engine. Assuming the vibration frequency and chord length remain unchanged, the reduced frequency k increases. According to low-reduced frequency theory, a higher k leads to lower instability in highly loaded blades [60, 61, 93]. As a result, $\tilde{W}_a^{\min}(k)$ decreases in a cold flow test. The temperature difference between cold flow test rigs and engine LPT exceeds $1,000K$, significantly reducing the instability level of an LPT at rig conditions and difficulting the aeromechanic similarity with the engine.

The evolution of the $\mathfrak{R}^* \tilde{W}_a^{\min}(k)$ factor governs the dimensionless vibration amplitude in a manner similar to how $\mathfrak{R}^*$ does in scaling effects. In the transformation from engine to rig there is a reduction of $\mathfrak{R}^*$ due to the smaller pressure level in the rig compared to the engine and there is also a reduction of $\tilde{W}_a^{\min}(k)$ due to the higher k driven by the smaller flow velocity in the rig. Hence, both effects tend to decrease $\tilde{\delta}_a$. Additionally, the dimensional vibration amplitude and the corresponding alternate stress remain unaffected by additional modifications and evolve directly in accordance with $\tilde{\delta}_a$.

The combined effects of operational point shifts and blade downscaling introduce challenges in designing rig blades that accurately replicate the aeromechanical behavior of an LCE LPT rotor blade. Both geometric downscaling and cold flow conditions act to reduce the vibration amplitude of the LPT rotor blade. Typical engine-to-rig scaling factors are around $\kappa = 3$, while the pressure ratio between the last stage of an engine LPT and a rig is approximately $p_0^{\text{LCE}}/p_0^{\text{Rig}} \sim 2$, resulting in a $\mathfrak{R}^*$ value roughly six times smaller in the rig compared to the LCE. Additionally, the temperature difference leads to sound speed values in the rig that are approximately half of those in the engine, effectively doubling the reduced frequency of the rig blade. This stabilizes the blade and slightly reduces $\tilde{W}_a^{\min}(k)$, as shown in [61].

Since the typical vibration amplitude of an LCE LPT blade is below $\delta_a = 1$ mm, the combined effects of downscaling and cold flow conditions reduce the vibration amplitude of the rig blade to approximately $\delta_a \approx 0.1$ mm. This reduction poses significant measurement challenges during experimental testing.

5.2.3. Mechanical Design Effects

The mechanical design of a turbomachinery blade is heavily influenced by the engine component, its function, and the operating conditions it experiences. Certain components, which do not generally face flutter-related problems, are high pressure compressors, which are typically composed of solid metallic blades and high pressure turbines, which feature internal cooling cavities to inject low-temperature air extracted from the compressor. Among turbomachinery blades prone to flutter, the LPT blades are solid nickel alloy components whose mechanical design is primarily dictated by the high loads they endure. However, the operating conditions of fan blades allow for greater flexibility in modifying their mechanical design.

Fan blades used in SCE and rig tests, typically scaled down by a factor of one-third from LCE fan blades to fit within existing wind tunnels, are generally made of solid titanium. Conversely, modern LCE fan blades are made of hollow titanium blades to minimize weight, resulting in partially hollow sections in the thicker regions, as illustrated in Fig. 5.2c. This alteration in the mechanical design can be characterized by the fraction of volume, ψ , removed from the blade. It affects the blade's mass $m_b \propto (1 - \psi)$ and the area moment of inertia I , causing slight variations in its natural frequency, which follows the relation $\omega_{n0} \propto \sqrt{I/m_b}$, as expressed in Eq. 5.3. The reader must keep in mind that reducing the mass entails a reduction on the momentum of inertia as well.

The primary effect on the model arises from variations in the parameter $\mathfrak{R}^*$, driven by changes in mass and frequency, which follow $\mathfrak{R}^* \propto m_b^{-2} \omega_{n0}^{-4}$. The blade mass affects $\mathfrak{R}^*$ both directly and indirectly, as it also influences the natural frequency. However, these two effects cancel out, meaning that the variation in $\mathfrak{R}^*$ depends solely on changes in the area moment of inertia, leading to the relation

$$\mathfrak{R}^* \propto I^{-2} \tag{5.6}$$

The shift from a solid to a hollow blade typically results in $\sim 20 - 30\%$ change in natural frequency. While this frequency variation must be accounted for when computing $\tilde{\Omega}$, given its significant impact on vibration amplitude, it is too small to meaningfully affect aerodynamic damping, allowing $\tilde{W}_a^{\min}$ to be considered constant.

Following the example of downscaling an LCE fan blade to a rig fan blade, its mechanical design changes from hollow to solid. In this case, the area moment of inertia slightly increases, leading to a small decrease in $\mathfrak{R}^*$, which contributes to reducing $\tilde{\delta}_a$. The increase in I is typically smaller than the increase in m_b in the transition from hollow to solid, causing the natural frequency of the solid blade to be lower than that of the hollow blade. This frequency shift results in the most significant modification to the model, the

variation in $\tilde{\Omega}$, which increases due to the reduction in frequency.

This change in $\tilde{\Omega}$ affects $\tilde{\delta}_a$ differently depending on the dominant mechanism driving the mechanical work, as discussed in Subsection 3.3.2. For small values of $\tilde{\Omega}$, where hysteresis cycles are large but the physical scaling of mechanical work is minimal, an increase in $\tilde{\Omega}$ tightens the fir-tree attachment, reducing the relative displacement between the blade and the disk. Consequently, the size of hysteresis cycles decreases, leading to reduced mechanical work and an increase in $\tilde{\delta}_a$. Conversely, at high $\tilde{\Omega}$ values, where the physical scaling of the mechanical work dominates, an increase in $\tilde{\Omega}$ enhances this scaling effect, increasing mechanical work and thereby reducing $\tilde{\delta}_a$.

The overall effect of these modifications on vibration amplitude depends on the operational regime of $\tilde{\Omega}$. Since the characteristic vibration amplitude remains constant with mechanical design changes (rigidity constant B is assumed unchanged), the total vibration amplitude δ_a follows the same trend as $\tilde{\delta}_a$. Regarding alternate stress, two contributing factors must be considered: the vibration amplitude and the blade frequency. The decrease in $\Re^*$ leads to a reduction in δ_a through its effect on $\tilde{\delta}_a$, while the decrease in ω further reduces the AF factor. However, the final impact on AF depends on the $\tilde{\Omega}$ regime.

For large $\tilde{\Omega}$ values, where $\tilde{\delta}_a$ decreases as $\tilde{\Omega}$ increases, all factors contribute to reducing AF . In contrast, for small $\tilde{\Omega}$ values, $\tilde{\delta}_a$ increases as $\tilde{\Omega}$ does, leading to competing effects: the reduction in $\Re^*$ decreases $\tilde{\delta}_a$, while the increase in $\tilde{\Omega}$ raises it. Although the frequency reduction generally leads to a decrease in AF , no dominant effect clearly dictates how the alternate stress evolves in this case.

5.2.4. Material Effects

Material changes are a common feature in engine-to-rig transformations and in transitions between LPT and fan blades. Figure 5.2d illustrates an example of an LPT blade material change from Inconel, used in LCE LPT blades to aluminium, one of the materials used in experimental campaigns of LPT rotor blades. This transformation affects blade mass, frequency (slightly), rigidity (since the constant B depends on the material stiffness), and contact properties through the parameter G' .

The dependency of the parameter $\Re^*$ with the material properties can be expressed as

$$\Re^* \propto \frac{G'}{(\rho B)^2 \left[\sqrt{E/\rho} \right]^4}. \quad (5.7)$$

Among these factors, the most significant impact on $\Re^*$ arises from changes in the elastic modulus. The variations in B are relatively minor, and the effects of density on blade mass

and frequency offset each other. In metallic blades, assuming that $\sqrt{E/\rho}$ exhibit only minor variations, the density can be seen as the most influential factor in determining $\mathfrak{R}^*$. The parameter G' decreases when transitioning to lighter and less stiff materials, but this reduction is smaller than the decrease in E and ρ . Consequently, lighter and weaker materials result in higher $\mathfrak{R}^*$ due to their smaller ability to dissipate mechanical energy, which stems from the lower normal force associated with decreased mass. As with mechanical design modifications, small frequency variations must be accounted for in $\tilde{\Omega}$, while changes in $\tilde{W}_a^{\min}$ remain negligible.

The material transformation depicted in Fig. 5.2 involves replacing an Inconel LPT blade with an AL-7075 LPT blade. In this scenario, the aluminum blade exhibits a slightly higher elastic modulus-to-density ratio E/ρ , which increases its vibration frequency and, in turn, reduces the dimensionless rotational speed $\tilde{\Omega}$, compared to the LCE blade. Following the rationale outlined in the mechanical design subsection, a decrease in $\tilde{\Omega}$ leads to different effects depending on the operating regime. If the blade operates at low values of $\tilde{\Omega}$, where the hysteresis cycle size dominates mechanical work, a reduction in $\tilde{\Omega}$ results in a decrease in $\tilde{\delta}_a$. Conversely, if the blade operates at high $\tilde{\Omega}$, a lower $\tilde{\Omega}$ leads to an increase in $\tilde{\delta}_a$.

Assuming minimal changes in B , the characteristic vibration amplitude remains unchanged, leading to $\delta_a \propto \tilde{\delta}_a$. The combined effects of the proposed material transformation—namely, an increase in $\mathfrak{R}^*$ and a reduction in $\tilde{\Omega}$ —result in an increase in vibration amplitude within the high $\tilde{\Omega}$ regime. However, in the low $\tilde{\Omega}$ regime, the decrease in δ_a due to the reduction in $\tilde{\Omega}$ counteracts the increase in δ_a driven by the higher $\mathfrak{R}^*$, making it difficult to predict the final vibration amplitude definitively.

Regarding alternate stress, the increase in natural frequency caused by this material transformation suggests an overall increase in the AF factor. This effect is particularly clear in the high $\tilde{\Omega}$ regime, where the combined influence of $\mathfrak{R}^*$, $\tilde{\Omega}$ and ω all contribute to the same outcome, reinforcing the trend toward higher AF values.

5.3. Vibration amplitude of Low Pressure Turbine blades

The drive for lighter, more efficient engines has led to the development of LPT blades with shorter chords, resulting in high-aspect-ratio designs. However, these blades exhibit increased instability, posing significant challenges in managing flutter, particularly in highly loaded configurations with low reduced frequencies as those encountered in cantilever LPT blades. One approach adopted in the LPTs of the Rolls-Royce Trent Series engines is the

	p_0	L	R	a_{FT}	E	G	ρ	m_b	f	B
Rig LPT	62	0.1	0.38	0.008	72	27	2.81	0.031	186	0.5
	$[kPa]$	$[m]$	$[m]$	$[m]$	$[GPa]$	$[GPa]$	$[kg/m^3]$	$[kg]$	$[Hz]$	$[-]$

Table 5.2.: PHYSICAL PARAMETERS OF THE ARIAS LPT BLADE.

	$\mathfrak{R}^*$	$\tilde{\Omega}$	$\tilde{W}_a^{\min}$	β
Rig LPT	0.4	0.27	0.85	4

Table 5.3.: MODEL PARAMETERS OF THE ARIAS LPT BLADE.

use of friction to control vibration amplitude. Nevertheless, accurately determining the vibration amplitude is critical, as it directly influences blade life, as illustrated in Fig. 1.2.

The model introduced in this study has been calibrated and validated using experimental data from the ARIAS LPT blade. This blade serves as the departing point for performing a rig-to-engine transformation, allowing for the estimation of the vibration amplitude of an LCE LPT rotor blade, as illustrated in Fig. 5.1. The physical vibration amplitude of the LCE blade is computed, along with the fir-tree displacements and the corresponding alternate stress. Following this, the vibration amplitude and alternate stress levels for the LCE LPT rotor blade are analyzed and discussed.

5.3.1. Departing blade: ARIAS LPT blade

The ARIAS LPT blade is a rig-scale model designed to replicate the aeromechanics of a cantilever LCE LPT rotor blade. The aerodynamic, geometric, and structural parameters required to feed the model are summarized in Table 5.2. This lightweight blade, manufactured from aluminum alloy Al-7075, is approximately one-third the size of an LCE LPT rotor blade. The tip-shroud geometry has been modified, as shown in Fig. 5.3 differing from a typical LCE cantilever LPT blade for two key reasons. First, the vibration frequency was intentionally reduced to match the engine's reduced frequency. Second, the tip shroud was specifically designed to accommodate magnets and mistuning masses to facilitate BTT vibration measurements.

The dimensionless vibration amplitude of the ARIAS LPT blade, presented in Fig. 3.18b accounts for all model effects, including the exact formulation of mechanical work, as well as the influence of rotational speed and mistuning on aerodynamic work. These values are computed using the parameters listed in Table 5.3, where $\mathfrak{R}^* = 0.4$ has been calibrated, while the remaining parameters are derived from CFD and FEM simulations.

The characteristic vibration amplitude of this blade, $\delta_{ac} = 0.034$ m, is used to compute the physical vibration amplitude δ_a , whose evolution with rotational speed Ω is presented in

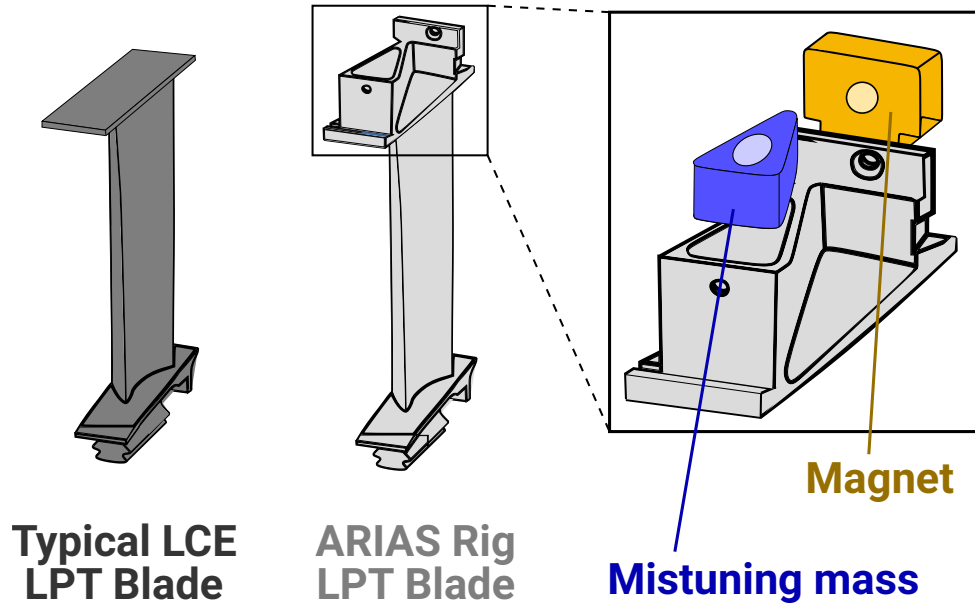


Figure 5.3.: COMPARISON BETWEEN TIP REGION OF A LCE LPT BLADE AND THE ARIAS LPT BLADE, WITH DETAILED VIEW OF THE MISTUNING MASS AND MAGNET INCLUDED IN THE TIP.

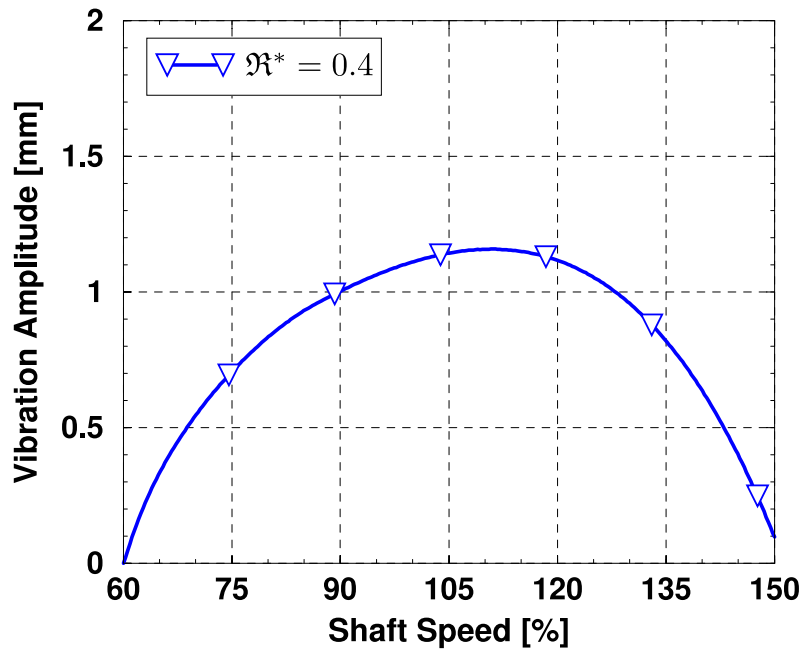


Figure 5.4.: EVOLUTION OF THE VIBRATION AMPLITUDE WITH THE ROTATIONAL SPEED FOR THE ARIAS LPT BLADE.

	$\tilde{\delta}_a$	δ_a	δ_a/L	$\tilde{\delta}^*$	AF
Rig LPT	0.035	1.2	1.2	0.3	0.2
	[-]	[mm]	[%]	[-]	[m Hz]

Table 5.4.: RESULTS OF THE ARIAS LPT BLADE: DIMENSIONLESS AND DIMENSIONAL VIBRATION AMPLITUDES $\tilde{\delta}_a$, δ_a , δ_a/L , DIMENSIONLESS FIR-TREE RELATIVE DISPLACEMENT SPEED $\tilde{\delta}^*$ AND ALTERNATE STRESS FACTOR AF .

Fig. 5.4. The peak vibration amplitude occurs at $\Omega \approx 110\%$, near the on-design operating point. As this condition represents the most severe case in terms of alternate stress, it serves as a reference for analyzing the ARIAS LPT blade's performance and comparing it to other turbomachinery blades.

Table 5.4 presents key results at $\Omega = 110\%$, including the dimensionless vibration amplitude $\tilde{\delta}_a$, dimensionless fir-tree displacement $\tilde{\delta}^*$, physical vibration amplitude δ_a and the alternate stress via the AF factor. Notably, the vibration amplitude, $\delta_a = 1.2$ mm, is remarkably high, considering the rig blade's smaller scale. This outcome aligns with the design intent to induce large flutter amplitudes for easier measurement. While BTT data could not be used to calibrate the model due to unidentified excitation sources, the predicted vibration amplitude is within the same order of magnitude as the BTT measurements.

The dimensionless fir-tree displacement, $\tilde{\delta}^* = 0.3$, corresponds to a dimensionless tangential force of $\tilde{F}^* = 0.45$, indicating a large hysteresis cycle. This highlights the LPT blade's significant capacity to dissipate mechanical work through dry friction. Finally, the AF factor is 0.2, which is slightly higher than typical values observed in LCE LPT blades (around $AF \approx 0.1 - 0.2$). However, this increase is justified by the high vibration amplitude observed in the rig blade.

5.3.2. Vibration amplitude of Large Civil Engine LPT blade

In this first transformation, the ARIAS LPT rotor blade is adapted to represent a large civil engine LPT blade, specifically the 5th rotor blade of a modern LCE. The transformation involves changes in size, material, and operating conditions.

Since the ARIAS blade's tip shroud differs from the standard cantilever LPT blades used in LCEs, the transformation is carried out in two sequential steps, as outlined in Fig. 5.5. First, the tip shroud of the ARIAS LPT blade was modified to fit a more standard LPT configuration to address these limitations. Afterwards, the blade is scaled up to engine-like size and the material and operating conditions are modified to reproduce those encountered in an LCE. The model was then applied to estimate the vibration amplitude

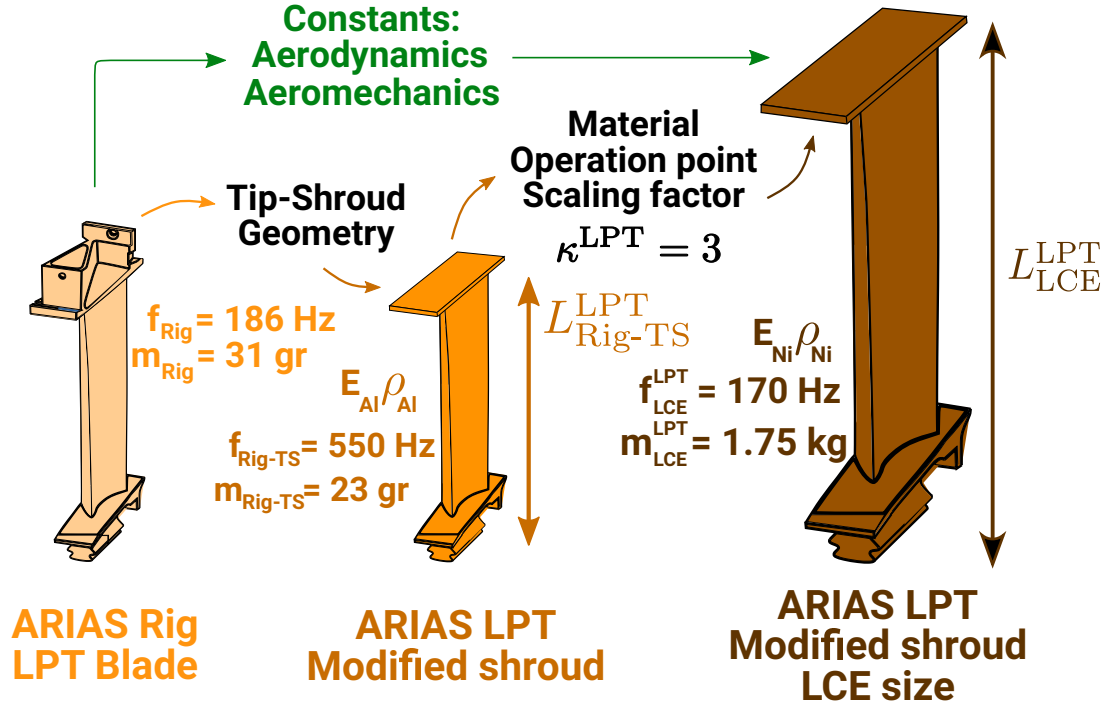


Figure 5.5.: SKETCH OF THE TRANSFORMATION 1: FROM THE ARIAS RIG LPT BLADE TO A LARGE CIVIL ENGINE SIZE LPT BLADE.

	p_0	L	R	a_{FT}	E	G	ρ	m_b	f	B
Rig LPT	62	0.1	0.38	0.008	72	27	2.81	0.031	186	0.5
Rig TS LPT	62	0.1	0.38	0.008	72	27	2.81	0.023	550	0.2
LCE LPT	150	0.3	0.68	0.024	174	68	8.19	1.75	170	0.25
	$[kPa]$	$[m]$	$[m]$	$[m]$	$[GPa]$	$[GPa]$	$[kg/m^3]$	$[kg]$	$[Hz]$	$[-]$

Table 5.5.: PHYSICAL PARAMETERS OF THE ARIAS LPT BLADE.

and stress experienced by an LCE LPT during operation. It is important to highlight that this transformation is not needed for engine manufacturers that have full access to the data. However, to depart from publicly available data, this is a necessary step.

The removal of the tip-shroud region, illustrated in Fig. 5.3, results in a new blade variant denoted as TS (Tip-Shroud modified blade), as depicted in Fig. 5.5. This modification removes approximately 30% of the blade's total mass from the tip region. As a result, the blade's mass is reduced to $m_b^{\text{Rig-TS}} = 0.023 \text{ kg}$, while its natural frequency increases to $f^{\text{Rig-TS}} = 550 \text{ Hz}$. Additionally, the rigidity constant is $B = 0.2$, which has been calculated from FEM. The remaining aerodynamic, geometric, and structural characteristics of the TS blade remain unchanged from the original ARIAS LPT blade, as previously presented in Table 5.5.

Consequently, blade's mass decreases, $m_b^{\text{Rig-TS}}/m_b^{\text{Rig}} \simeq 0.7$; blade's frequency increases, $\omega_{n0}^{\text{Rig-TS}}/\omega_{n0}^{\text{Rig}} \simeq 3$; and rotor blade's rigidity also decreases, $B^{\text{Rig-TS}}/B^{\text{Rig}} \simeq 0.4$. Hence, the

	$\mathfrak{R}^*$	$\tilde{\Omega}$	$\tilde{W}_a^{\min}$	β
Rig LPT	0.4	0.27	0.85	4
Rig TS LPT	0.05	0.1	0.42	4
LCE LPT	0.07	0.26	0.85	4

Table 5.6.: MODEL PARAMETERS OF THE ARIAS LPT BLADE.

value of $\mathfrak{R}^*$ decreases from the calibrated value of the ARIAS LPT rotor rig blade, $\mathfrak{R}^{*\text{Rig}} = 0.4$, to the tip-shroud modified LPT blade, $\mathfrak{R}^{*\text{Rig-TS}} = 0.05$. The dimensionless rotational speed $\tilde{\Omega}^{\text{Rig-TS}}$ decreases due to the increase in natural frequency and the $\tilde{W}_{a,\text{Rig-TS}}^{\min}$ is corrected due to the change in reduced frequency [63]. The final model parameters used for the analysis are summarized in Table 5.6. This results in a small dimensionless vibration amplitude $\tilde{\delta}_a^{\text{Rig-TS}}$, as well a small dimensional vibration amplitude of $\delta_a^{\text{Rig-TS}} = 0.1$ mm, as shown in Table 5.7. This confirms that a direct geometric scaling of a large LPT blade to a cold flow rig leads to a substantial reduction in vibration amplitude.

In the second part of the transformation, where the aeroelastic rig-to-engine scaling is analysed, several key factors must be addressed. Engine aerodynamics (Mach and Reynolds numbers), reduced frequency, k , and mode-shapes are preserved in the rig, but reproducing the vibration amplitude requires modelling and experimental data [1, 55], in particular, the $\mathfrak{R}^*$ parameter. The rig-to-engine geometric scaling factor, derived from fundamental data (Fan diameter, rotor aspect ratio, etc.) from an engine with an entry into service in the early 00's is $\kappa^{\text{LPT}} = L^{\text{LCE}}/L^{\text{Rig-TS}} = 3$. The aspect ratio of the turbine is between six and seven, corresponding to a reverse cut-off design for noise abatement [94].

The geometric scaling factor together the change in material properties, summarized in Table 5.5, affect the blade mass and the frequency following Eqs. 5.2 and 5.3. Then, the blade mass is $m_b^{\text{LCE}} = 1.75$ kg and the blade frequency is $f^{\text{LCE}} = 170$ Hz. The Inconel (LCE) to aluminium (rig) density ratio is about $\rho_{\text{LCE}}/\rho_{\text{Rig-TS}} \simeq 3$, and the $\sqrt{E/\rho}$ ratio $(\sqrt{E/\rho})_{\text{LCE}}/(\sqrt{E/\rho})_{\text{Rig-TS}} \simeq 0.9$. This actually mean that if the rig-to-engine scaling factor is $\kappa^{\text{LPT}} = 3$, the engine blade is about 80 times heavier and the rotor blade frequency three times lower than in the rig. The change in material also influences the contact properties and the blade rigidity.

The rig-to-engine scaling of the vibration amplitude also requires considering the change in operating conditions. LPT rigs, such as the one in which the rotor was tested, operate at sub-ambient pressure to keep constant the Reynolds number. Last stages in the engine operate at much higher pressures and temperatures. Since the pressure level affects the work-per-cycle and is involved in the $\mathfrak{R}^*$ parameter a typical pressure ratio of 1.5 has been chosen.

Despite the differences in size, material, and operating conditions, several dimensionless

parameters remain constant between the ARIAS rig and engine blades. The exit Mach number $M_e \approx 0.6 - 0.7$ is kept constant to reproduce the aerodynamics. The reduced frequency during the test was kept similar to that of the engine ($k = \omega c/v_e \approx 0.1$) since it was an aeromechanics test [8, 47]. However, this objective was achieved at the expense of artificially lowering the blade frequency in the rig. Since $(\omega c)_{\text{Rig-TS}} \simeq (\omega c)_{\text{LCE}}$ (see Eqs. 5.2 and 5.3) and M_e is constant, the change in temperature between the rig and engine modifies the reduced frequency, i.e. $k_{\text{LCE}}/k_{\text{Rig-TS}} = \sqrt{T_{\text{Rig-TS}}/T_{\text{LCE}}}$, meaning that the reduced frequency is much smaller in the engine than in the rig. This fact was compensated in the rig rotor blade by lowering the frequency to similar values as in the engine. The ARIAS rotor blade's frequency was much lower than that of a pure geometrical scaling because of the large mass of its tip-shroud design. The effect of these changes on the parameter $\mathfrak{R}^*$ can be expressed as

$$\frac{\mathfrak{R}^{*\text{LCE}}}{\mathfrak{R}^{*\text{Rig-TS}}} = \kappa \frac{p_0^{\text{LCE}}}{p_0^{\text{Rig-TS}}} \frac{G^{\text{LCE}}}{G^{\text{Rig-TS}}} \left(\frac{\rho B^{\text{Rig-TS}}}{\rho B^{\text{LCE}}} \right)^2 \left[\frac{(\sqrt{E/\rho})_{\text{Rig-TS}}}{(\sqrt{E/\rho})_{\text{LCE}}} \right]^4 \quad (5.8)$$

First, the difference in operating conditions increases $\mathfrak{R}^{*\text{LCE}}/\mathfrak{R}^{*\text{Rig-TS}}$ due to the total pressure ratio $p_0^{\text{LCE}}/p_0^{\text{Rig-TS}} \simeq 2$. The change of material modifies the contact properties, the natural frequency and the rigidity. The shear modulus of the contact relation, $G^{\text{LCE}}/G^{\text{Rig-TS}} \simeq 2$ increases the relation $\mathfrak{R}^{*\text{LCE}}/\mathfrak{R}^{*\text{Rig-TS}}$. The natural frequency change due to the change from aluminium (Rig) to Inconel (LCE) decreases the ratio $\mathfrak{R}^{*\text{LCE}}/\mathfrak{R}^{*\text{Rig-TS}}$ since $(\sqrt{E/\rho})_{\text{LCE}}/(\sqrt{E/\rho})_{\text{Rig-TS}} \simeq 0.9$. The density change, $\rho_{\text{LCE}}/\rho_{\text{Rig-TS}} \simeq 3$, has a strong influence in $\mathfrak{R}^{*\text{LCE}}/\mathfrak{R}^{*\text{Rig-TS}}$, decreasing this relation. Additionally, the change of material also modifies the rigidity, $B^{\text{LCE}}/B^{\text{Rig-TS}} \simeq 1.25$, which reduces $\mathfrak{R}^{*\text{LCE}}/\mathfrak{R}^{*\text{Rig-TS}}$.

If these relations between the physical parameters are injected in Eq. 5.8, the ratio $\mathfrak{R}^{*\text{LCE}}/\mathfrak{R}^{*\text{Rig-TS}} \simeq 1.4$ can be calculated, which leads to $\mathfrak{R}^{*\text{LCE}} \simeq 0.07$ since $\mathfrak{R}^{*\text{Rig-TS}} = 0.05$ has been obtained from the previous comparison with the ARIAS rig LPT blade.

Using the parameters summarized in Table 5.6, the dimensionless vibration amplitude $\tilde{\delta}_a$ is calculated by solving the energy balance (Equation 2.26), and the results are presented in Table 5.7. The LCE blade presents a lower $\tilde{\delta}_a$ compared to the ARIAS blade, as a smaller $\mathfrak{R}^*$ reduces the blade's capacity to absorb aerodynamic work relative to its ability to dissipate mechanical work through friction. While the dimensional vibration amplitudes are comparable, the scaled vibration amplitude with the blade length is smaller for the large civil engine blade due to its larger size.

A notable difference arises in the dimensionless relative displacement within the fir-tree region $\tilde{\delta}^*$. For the large civil engine blade, $\tilde{\delta}^*$ is significantly smaller than for the ARIAS rotor blade. This is attributed to the much lower $\mathfrak{R}^*$, which requires a smaller hysteresis

	$\tilde{\delta}_a$	δ_a	δ_a/L	$\tilde{\delta}^*$	AF
Rig LPT	0.035	1.2	1.2	0.3	0.2
Rig TS LPT	0.001	0.1	0.1	0.05	0.05
LCE LPT	0.010	1.4	0.5	0.08	0.25
	[-]	[mm]	[%]	[-]	[m Hz]

Table 5.7.: RESULTS OF THE ARIAS LPT BLADE, THE MODIFIED TIP-SHROUD ARIAS BLADE AND THE THE LARGE CIVIL ENGINE LPT BLADE: DIMENSIONLESS AND DIMENSIONAL VIBRATION AMPLITUDES $\tilde{\delta}_a$, δ_a , δ_a/L , DIMENSIONLESS FIR-TREE RELATIVE DISPLACEMENT SPEED $\tilde{\delta}^*$ AND ALTERNATE STRESS FACTOR AF .

cycle to achieve vibration amplitude saturation. Despite these variations, the resulting stress values for the two blades are similar, even though the large civil engine blade is three times larger. The amplitude-frequency factor $AF = \delta_a \omega_n / 2\pi$ of the LPT blade is $AF_{LCE} = 0.25$. These findings align with current strategies in modern large civil engine LPT blade designs.

5.4. Vibration amplitude of Fan blades

The increasing demand for lower fuel consumption and reduced emissions has driven advancements in aeronautical engine technologies, influencing both LPT development and the design of fan blades with higher bypass ratios to enhance propulsion efficiency. Modern designs often feature lightweight, hollow titanium or composite materials, and these highly swept blades are increasingly prone to aeroelastic instabilities that risk structural failure. Reducing blade count and using lightweight materials in these configurations result in decreased solidity and mass ratios. At the same time, the coupling of structural and aerodynamic modes introduces complex aeroelastic behaviours. Fan designs are rigorously tailored to mitigate these risks, operating free from flutter-induced failures. As a result, aerodynamic and mechanical constraints are imposed during fan design to maintain stability, which can limit their aerodynamic performance.

Beyond conventional blade-to-blade aerodynamic damping effects, other mechanisms can contribute to instability in aeronautical fans. Notably, stall flutter, caused by flow separation [95], and acoustic flutter, driven by fan-intake interactions [96] are key contributors. Acoustic reflections at the engine intake can interact with fan blade vibrations, leading to a feedback loop known as flutter bite. This phenomenon occurs when fan blades oscillating in their first mode generate acoustic waves that propagate upstream. Some of this energy radiates outward at the intake highlight, while the remainder reflects back toward the fan. If the returning wave is misaligned with the blade motion, it can amplify vibrations and trigger instability, particularly near the stall line, expanding the flutter

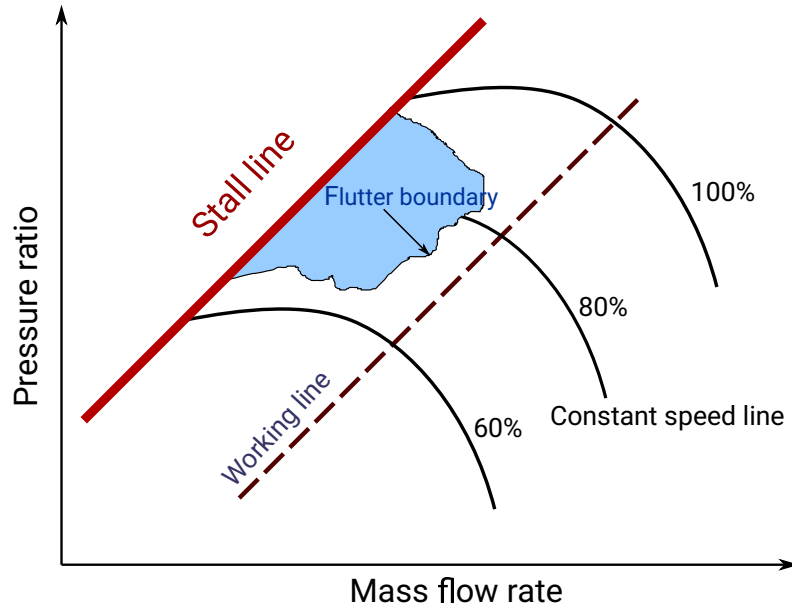


Figure 5.6.: FAN FLUTTER BITE BOUNDARY.

boundary, as illustrated in Fig. 5.6.

Flutter bite manifests when two conditions coincide. First, the aerodynamic damping of the blades must be low enough that acoustic perturbations can destabilize the system. This typically occurs near the stall line due to the altered aerodynamic flow conditions at this off-design point, where the blade operates at high angle of attack and radial flow migration can arise. Second, the instability reaches its most severe state when the reflected wave lags by 90 degrees the wave emitted by the fan. Conversely, the most stabilizing condition occurs when the reflected wave leads the emitted wave by 90 degrees. The severity of the instability depends on the amplitude of the reflected wave upon re-encountering the fan.

Fan blade stability is often quantified using the critical damping ratio, ξ , which measures system's capacity to damp forcing perturbations (positive sign) or to develop aeroelastic instability (negative sign). While fan blades are typically designed to operate with $\xi > 0$ to ensure stability, the aforementioned flutter-triggering mechanisms can push this value into the negative range, reaching a typical critical damping ratio of $\xi^{\text{Fan}} = -0.1\%$, significantly smaller than that of LPTs, where $\xi^{\text{LPT}} = -1\%$.

Given that fan instability levels are an order of magnitude lower than those observed in LPTs, this section examines the potential of using fir-tree friction mechanisms to control vibration amplitudes in aeronautical fans building on the successful strategies employed in LPTs. The objective is to assess whether fans' the design and operational conditions allow for applying similar strategies to saturate flutter-induced vibrations.

To investigate this, the LCE LPT blade presented before is transformed into a fan blade, as shown Fig. 5.1. For this purpose, the ACAT rig fan is used as the reference configuration. This is an aerodynamically representative SCE fan, which has a size comparable to that of an LCE LPT blade. The aerodynamic and aeroelastic characteristics of the transformed fan blade are first examined. Subsequently, the vibration amplitude and alternating stress of the ACAT fan blade are calculated, yielding values that differ from those encountered in LPT blades. The design and operational factors responsible for these differences are analyzed. Finally, this study is extended to LCE fan blades, evaluating their ability to control vibration amplitudes using friction at the root blade attachment.

5.4.1. Characterization of the Fan blade

The ACAT fan is a transonic fan rig blade designed to replicate the aerodynamic and aeroelastic characteristics of an aeronautical fan. This particular fan was selected not only for its engine-representative design but also because previous noise simulations [97] and acoustic-driven aeroelastic analyses [98] have been conducted within our research group. In its original configuration, the ACAT fan consists of 20 blades with a tip diameter of $D = 865$ mm. Located downstream are the bypass stream with 44 Outlet Guide Vanes (OGV) and the core stream with 44 Engine Section Stators (ESS) vanes. At design speed, the fan operates with a total pressure ratio of $\pi_{1,2} = 1.42$, a relative tip Mach number of $M_{\text{Tip}} = 0.9$ and Reynolds number of $\text{Re} = 2.2 \cdot 10^6$. The blade sections exhibit a lift coefficient of approximately $c_L = 0.7$, and the pressure distribution at 90% span, shown in Fig. 5.7 reveals the presence of a shock wave on the suction side, forming a low-pressure region over the first 15% of the chord. Additional details on the aerodynamic design of the fan can be found in [98].

The fan's Campbell diagram, presented in Fig. 5.8a, illustrates the evolution of its first and second modes, corresponding to flap and torsion modes, respectively. The frequency of both modes increases with rotational speed due to centrifugal stiffening, in accordance with Southwell's law [66]. The first flap mode has its first EO crossing within the operating range at EO2, which aligns with standard fan design principles, where the first mode is located slightly above the maximum rotational speed to avoid interactions with 1EO.

Chennuru et al. [98] investigated the aeroelastic behavior of this fan blade and the influence of intake acoustics on its flutter characteristics. Two intake configurations were analyzed: an infinitely long intake ($L/D = \infty$) and a finite intake ($L/D = 0.5$). The infinitely long intake configuration eliminates acoustic reflections, isolating the aerodynamic damping effects driven solely by the blade's flow. By comparing the aerodynamic damping between these two cases, the study assessed the impact of intake acoustics on flutter stability.

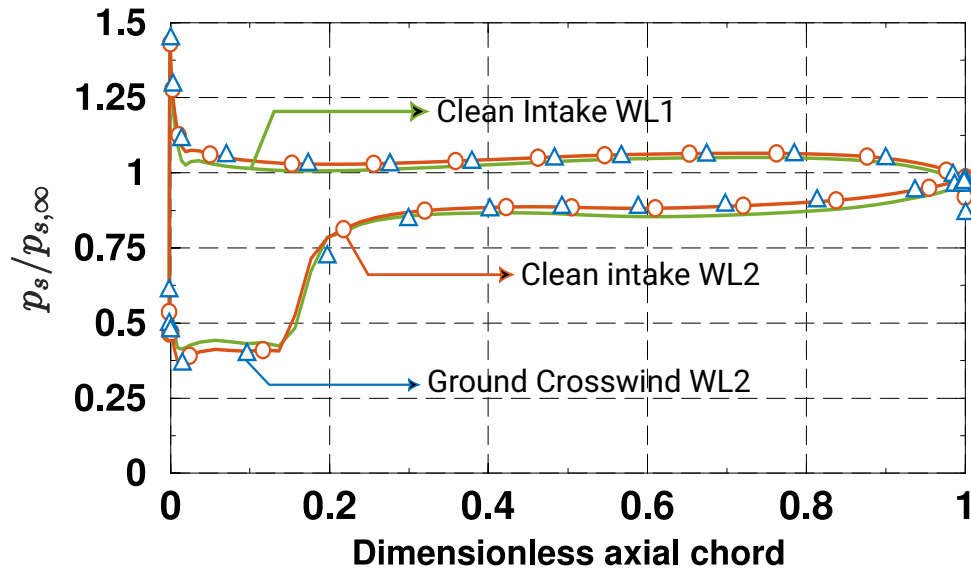


Figure 5.7.: FAN BLADE LOADING.

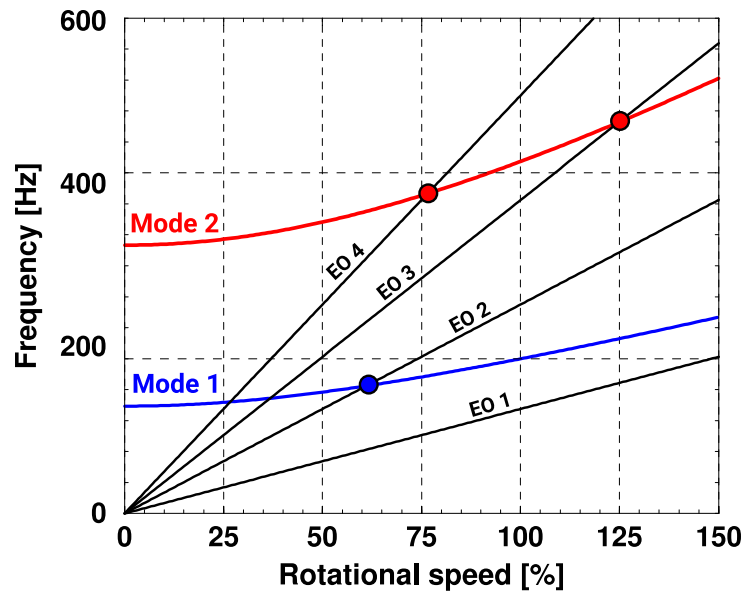


Figure 5.8.: ANALYTICAL CAMPBELL DIAGRAM OF THE ACAT FAN BLADE WITH THE FIRST AND SECOND VIBRATION MODES.

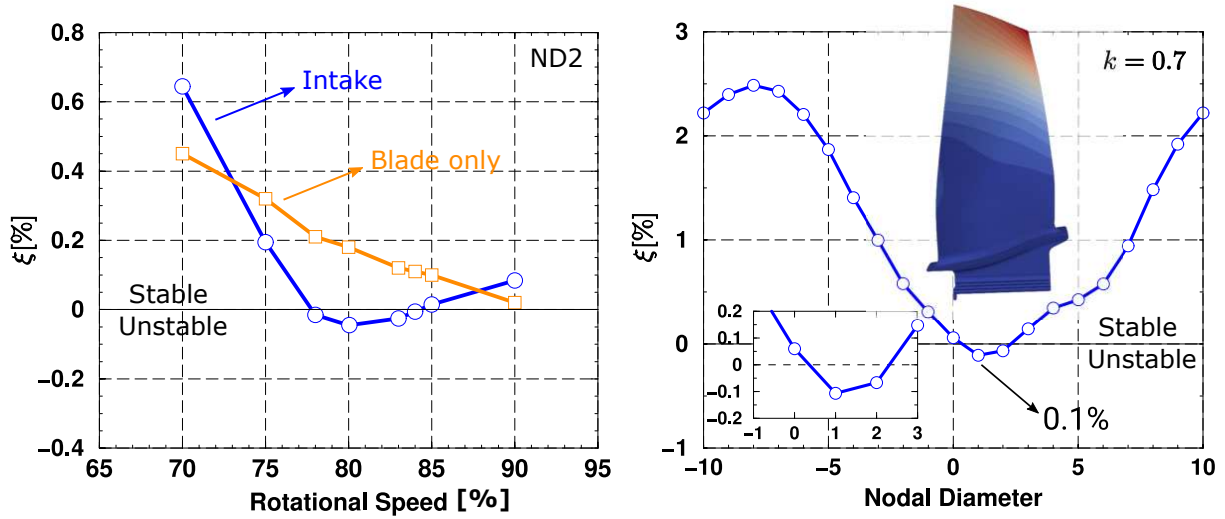


Figure 5.9.: AEROELASTIC CHARACTERIZATION OF THE ACAT FAN BLADE: (A) EVOLUTION OF THE ND2 CRITICAL DAMPING RATIO WITH THE ROTATIONAL SPEED. (B) CRITICAL DAMPING RATIO AS A FUNCTION OF THE ND.

The ND2 critical damping ratio for both configurations is displayed in Fig. 5.9a. In the infinitely long intake case, damping decreases with increasing speed due to 3D separation and radial flow migration on the suction side. In contrast, for the intake with $L/D = 0.5$, representative of a real-engine intake configuration, the damping reaches a minimum around $\Omega = 80\%$ and exhibits a region of negative damping between $\Omega = 78\% - 83\%$. At this speed, the first flap mode attains a frequency of $f = 182$ Hz and a reduced frequency of $k = 0.7$. The discrepancy between the damping trends for the blade-alone and intake-inclusive cases highlights the influence of intake acoustics, which introduces alternating regions of stabilizing and destabilizing contributions depending on the rotational speed.

The evolution of the critical damping ratio across different nodal diameters is depicted in Fig. 5.9b. The results show a cosine-like distribution, with only the first and second NDs exhibiting instability, an expected characteristic of blades operating outside the low-reduced frequency regime [60, 61]. This behavior fundamentally differs from that of LPT blades, where reduced frequencies are typically in the range of $k \sim 0.1$ and present multiple unstable NDs.

Since the ACAT fan with the $L/D = 0.5$ intake, representative of an actual SCE fan-intake configuration, presents a negative critical damping ratio at $\Omega = 80\%$, this operating condition is selected as the reference point for evaluating the feasibility of vibration amplitude saturation through fir-tree friction in SCE fan blades.

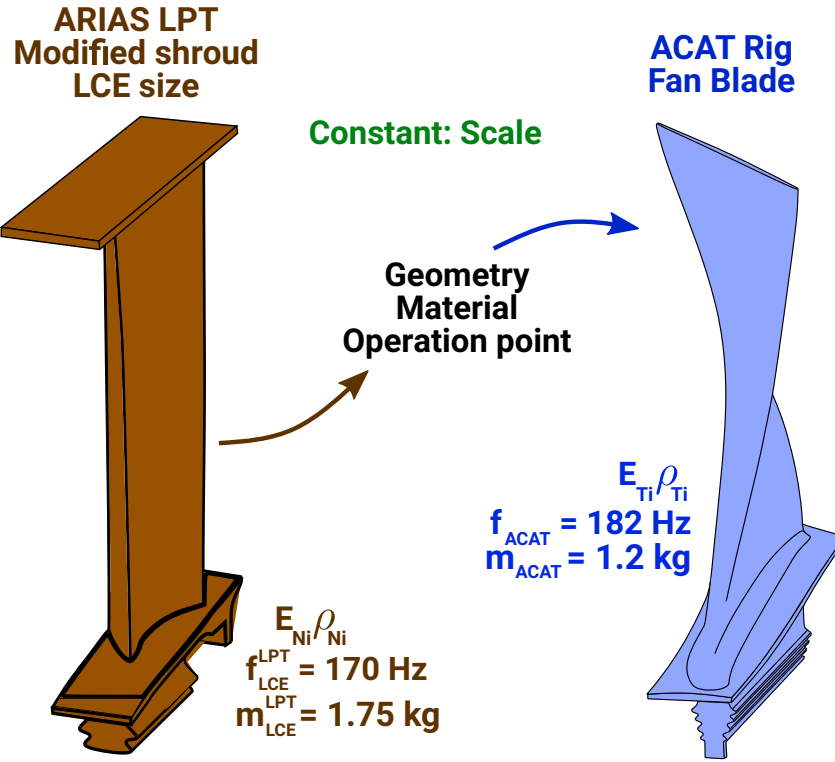


Figure 5.10.: SKETCH OF THE TRANSFORMATION 2: FROM THE LCE LPT BLADE TO A THE ACAT BLADE, AN SMALL CIVIL ENGINE FAN BLADE.

	p_0	L	R	a_{FT}	E	G	ρ	m_b	f	B
LCE LPT	150	0.3	0.68	0.024	174	68	8.19	1.75	170	0.25
SCE Fan	100	0.3	0.29	0.016	114	44	4.4	1.2	182	0.1
LCE Fan	100	0.91	0.87	0.049	114	44	4.4	10	55	0.1
	[kPa]	[m]	[m]	[m]	[GPa]	[GPa]	[kg/m ³]	[kg]	[Hz]	[-]

Table 5.8.: PHYSICAL PARAMETERS OF THE LCE LPT BLADE AND THE FAN BLADES.

5.4.2. Vibration amplitude of Small Civil Engine Fan blade

The transformation from the LCE LPT blade, described in Subsection 5.3.2, to the ACAT fan blade is outlined in Fig. 5.10. This step involves changes in geometry, material, and operating conditions. However, there is no pure scaling effect, as both the rig-to-engine and LPT-to-fan scale factors are typically $\kappa \approx 3$. As a result, the blade length remains comparable between a typical LCE LPT and a typical SCE fan blade, at approximately $L_{LCE}^{LPT} \approx L_{Rig}^{Fan} \approx 300$ mm.

Geometric differences between the LCE LPT and SCE fan blades influence the parameter $\mathfrak{R}^*$ in multiple ways, as summarized in Table 5.8. The SCE fan blade features a larger chord length due to its aspect ratio, typically $\Lambda \sim 2 - 3$, compared to the higher aspect ratio of modern LPT blades, which ranges from $\Lambda \sim 5 - 7$. Additionally, the narrower fir-tree width of the SCE fan blade reduces both $\mathfrak{R}^*$ and the characteristic vibration

amplitude δ_{ac} , leading to a decrease in the overall vibration amplitude. Conversely, the fan's larger contact area A_c increases $\mathfrak{R}^*$. The fan blade also has a smaller mean radius compared to the LPT blade, which does not directly affect $\mathfrak{R}^*$ under the assumption that $\alpha = 1$, but it influences δ_{ac} through Eq. 2.24, thereby reducing δ_a .

These geometric changes also influence blade mass, natural frequency, and rigidity (represented by constant B), altering the blade's structural properties. The most significant impact is an increase in $\mathfrak{R}^*$ due to the fan's lower B than the LPT. The geometric changes modify the aerodynamic shape of the airfoil, the SCE fan blade loading decreases compared to the LCE LPT blade and, including the effect of the operation point, the reduced frequency increases, shifting from $k \approx 0.1$ in the LPT to $k \approx 0.5$ in the fan. This shift reduces the minimum dimensionless aerodynamic work $\tilde{W}_a^{\min}$, further differentiating the fan's aeroelastic behavior from that of the LPT. Moreover, fans operate at lower pressure levels than LPT blades, which further reduces $\mathfrak{R}^*$.

Material changes also play an important role. The LPT blade is made of the nickel alloy Inconel, whereas the ACAT fan blade uses the titanium alloy Ti-6Al-4V. This material change increases the contact's effective Young's modulus, E' and shear modulus G' , increasing the parameter $\mathfrak{R}^*$. The blade mass is strongly influenced by the difference in material density, though the effect of the material change on the natural frequency and rigidity remain is small since the Young's modulus-to-density ratio, E/ρ , remains approximately constant. This is due to centrifugal stiffening in LPT rotor blades, which causes their natural frequency to increase more rapidly. In summary, the combined effects of geometric and material changes result in a blade mass reduction of approximately one-third, which increases $\mathfrak{R}^*$, while the slight increase in vibration frequency leads to a marginal decrease in $\mathfrak{R}^*$.

The overall effects of the geometric, material and operation point on $\mathfrak{R}^*$ are summarized in Table 5.9. The larger $\mathfrak{R}^*$ in the SCE fan compared to the LCE LPT indicates that fan blades are less efficient at dissipating aerodynamic energy through friction, as $\mathfrak{R}^*$ quantifies the balance between aerodynamic work absorption and mechanical work dissipation via dry friction.

Operational differences between LPT and fan blades also influence their rotational speed for a given blade size L . The modification of the rotational speed directly modifies the parameter $\tilde{\Omega}$, a parameter which plays a particularly critical role in defining the blade's dynamic response. For the fan, $\tilde{\Omega}^{\text{Fan}} = 0.6$, corresponding to an operational rotational speed of $\Omega = 80\%$ of the maximum speed, where flutter bite is typically observed. The fan's natural frequency is $f^{\text{Fan}} = 182$ Hz, aligning with aeroelastic design principles for fans. Specifically, the first vibration mode's natural frequency slightly exceeds the 1EO at maximum rotational speed, to avoid resonances with the 1EO within the operational

	$\Re^*$	$\tilde{\Omega}$	$\tilde{W}_a^{\min}$	β
LCE LPT	0.07	0.26	0.85	4
SCE Fan	0.32	0.6	0.5	2
LCE Fan	20.8	0.7	0.5	2

Table 5.9.: MODEL PARAMETERS OF THE LCE LPT BLADE AND THE FAN BLADES.

limit of the engine. In contrast, the LPT operates at a dimensionless rotational speed of $\tilde{\Omega}^{\text{LPT}} = 0.27$, corresponding to a rotational speed around $\Omega = 100\%$ of the maximum speed, where flutter is typically detected with higher vibration amplitudes.

The dimensionless rotational speed influences the mechanical work in two distinct ways, as described in equation 2.8. First, it serves as a scaling factor that affects the normal force N and the characteristic length of the microslip δ_c , which is proportional to the penetration length λ , itself dependent on the normal force. Consequently, a higher dimensionless rotational speed increases the blade's capacity to dissipate work through friction in dimensional terms.

However, the dimensionless rotational speed also impacts the dimensionless mechanical work, $\tilde{W}_m$, as expressed in equation 2.25. As the rotational speed rises, the centrifugal forces acting on the blade increase, enhancing the normal force at the blade-to-disk interface. This increased normal force results in a tighter fit, reducing the relative motion between the blade and the disk. This results in a smaller microslip size, counteracting the capacity to dissipate work. Thus, the dimensionless rotational speed has opposing effects on the blade's saturated vibration amplitude.

Additionally, the dimensionless mechanical work increases with the Southwell coefficient β . This effect is more pronounced in the LPT, where $\beta^{\text{LPT}} = 2$, compared to the fan, where $\beta^{\text{Fan}} = 1$. The reason lies in the centrifugal stiffening of the LPT rotor blades, which causes their natural frequency to increase more rapidly.

The overall effect of the geometry, material and operation point modifications on the model parameters are summarized in Table 5.9. Using these values, the energy balance equation 2.26 is solved, and the resulting values are presented in table 5.10. The fan benefits from lower $\tilde{W}_a^{\min}$, and its higher $\tilde{\Omega}$ increases the scaling factor of the mechanical work, contributing to a reduction in vibration amplitude. However, this same increase in $\tilde{\Omega}$ also reduces the hysteresis cycle size, significantly limiting friction dissipation. Additionally, the larger $\Re^*$ further amplifies the blade's capacity to absorb aerodynamic work over its dissipation capacity. These latter effects dominate, ultimately leading to a greater overall vibration amplitude in the fan compared to the LPT.

The dimensionless vibration amplitude, $\tilde{\delta}_a$, is more than five times higher in the fan compared to the LPT. This behaviour is mainly due to the microslip regime: the fan's

	$\tilde{\delta}_a$	δ_a	δ_a/L	$\tilde{\delta}^*$	AF
LCE LPT	0.010	1.4	0.5	0.08	0.25
SCE Fan	0.058	8.0	2.6	0.12	1.45
LCE Fan	Not saturated by friction				
	$[-]$	$[mm]$	$[\%]$	$[-]$	$[m Hz]$

Table 5.10.: RESULTS OF THE LCE LPT BLADE AND THE FAN BLADES: DIMENSIONLESS AND DIMENSIONAL VIBRATION AMPLITUDES $\tilde{\delta}_a$, δ_a , δ_a/L , DIMENSIONLESS FIR-TREE RELATIVE DISPLACEMENT SPEED $\tilde{\delta}^*$ AND ALTERNATE STRESS FACTOR AF .

tighter blade-to-disk contact requires a higher $\tilde{\delta}_a$ to reach a relative displacement $\tilde{\delta}^* \approx 0.1$, where both blades become saturated. In dimensional terms, the LPT vibration amplitude previously presented was $\delta_a = 1.4$ mm, which is considered high for an LCE LPT. However, it should be noted that this blade was derived from a transformation using the ARIAS LPT blade as a reference, an experimental blade intentionally designed to exhibit large vibration amplitudes for measurement purposes. In contrast, the vibration amplitude of the SCE fan blade reaches $\delta_a = 8$ mm, a significantly large value compared to typical SCE fan blades.

The vibration amplitudes can be converted to amplitude-frequency (AF) to estimate alternating stresses on the blades. The LPT reaches $AF = 0.25$, which is in the upper limit of an LPT rotor, indicating that its vibration amplitude is sufficiently low to ensure HCF endurance. In contrast, the ACAT fan reaches $AF = 1.5$, exceeding the AF threshold of an aeronautical fan that is in the order of $AF \approx 1$.

The primary strength of the presented model lies not in its precise prediction of the final vibration amplitude but in its ability to identify trends and highlight behavioural differences between blades and operating conditions. Therefore, the AF values shown in table 5.10 should be validated with more detailed analysis.

Since the AF value for the fan is near the critical limit, further analysis is needed to evaluate the viability of using fir-tree friction to stabilize small-sized fans like the ACAT fan. These fans typically have diameters up to $D = 1$ m., characteristic of certain civil engines in the 10,000 lbf thrust range, commonly used in business jets. This size is also representative of military engines capable of delivering up to 20,000 lbf of thrust with afterburners, as found in some fighter jets.

5.4.3. Vibration amplitude of Large Civil Engine Fan blade

The results from previous Subsection 5.4.2 suggest that unstable small metallic fan blades for business jets are on the verge of being saturated by fir-tree friction. Building on this, the study is extended LCE fan blades to assess flutter-induced vibration amplitude at the

other end of the fan size range. This transformation evaluates the differences in size and mechanical design between a small metallic fan blade, such as the ACAT blade presented before, and modern titanium large fan blades. The transformation process is illustrated in Fig. 5.11.

The ACAT fan blade is scaled using a factor of $\kappa^{\text{Fan}} = L_{\text{LCE}}^{\text{Fan}}/L_{\text{Rig}}^{\text{Fan}} \approx 3$, selecting a length representative of a modern LCE fan blade. This scaling affects the blade's size, mass, and frequency. The mechanical design of fan blades varies with size: while small and rig fan blades are solid titanium, LCE fan blades incorporate hollow sections in their lower span to reduce weight. These hollow sections are confined to thicker regions where material removal does not compromise structural integrity. This design change not only affects blade mass but also alters frequency by modifying the area moment of inertia.

To estimate the blade mass and frequency of the LCE fan, standard design principles are applied. The first vibration mode frequency is set to ensure that its crossing with the first engine order (1EO) remains outside the operating range with a sufficient safety margin. Given that the low-speed spool of modern aeronautical engines rotate at a maximum speed of approximately $\Omega \approx 3000$ RPM, the highest-frequency 1EO perturbation occurs at $f_{\text{1EO}} \approx 50$ Hz. Thus, this value is used as a reference for setting the first-mode frequency at $f_{\text{LCE}}^{\text{Fan}} = 55$ Hz, ensuring a 10% safety margin above the first engine order. The blade mass is set at $m_b = 10$, which is characteristic of an LCE fan blade. Since the material remains unchanged and the hollow sections do not affect the fir-tree attachment, contact properties are assumed to be constant.

Table 5.8 presents the modifications in the LCE fan blade's physical parameters due to scaling and mechanical design transformations. The resulting model parameters, computed using these values, are listed in Table 5.9. The analysis is conducted at $\Omega = 80\%$, corresponding to a dimensionless rotational speed of $\tilde{\Omega} = 0.7$, slightly higher than the ACAT fan's $\tilde{\Omega} = 0.6$, as the LCE design prioritizes efficiency by reducing the frequency margin above the first engine order. The dimensionless aerodynamic damping remains constant since the aerodynamic surface is unchanged and the small modification of the reduced frequency is negligible in $\tilde{W}_a^{\text{min}}$ since the exit velocity and the product ωc barely change. The Southwell coefficient β is assumed to remain constant within the operating range. However, the key parameter $\mathfrak{R}^*$ increases by two orders of magnitude in the LCE fan, reaching $\mathfrak{R}^* \approx 20$. This increase is attributed to the linear dependence of $\mathfrak{R}^*$ on the scaling factor κ^{Fan} and the reduction in blade mass and frequency due to the hollow structure.

By solving the energy balance equation 2.26 with these parameters, the results reveal that for the LCE fan blade, the mechanical work dissipated by fir-tree friction is insufficient to counteract the aerodynamic work at any vibration amplitude. Consequently, the vibration

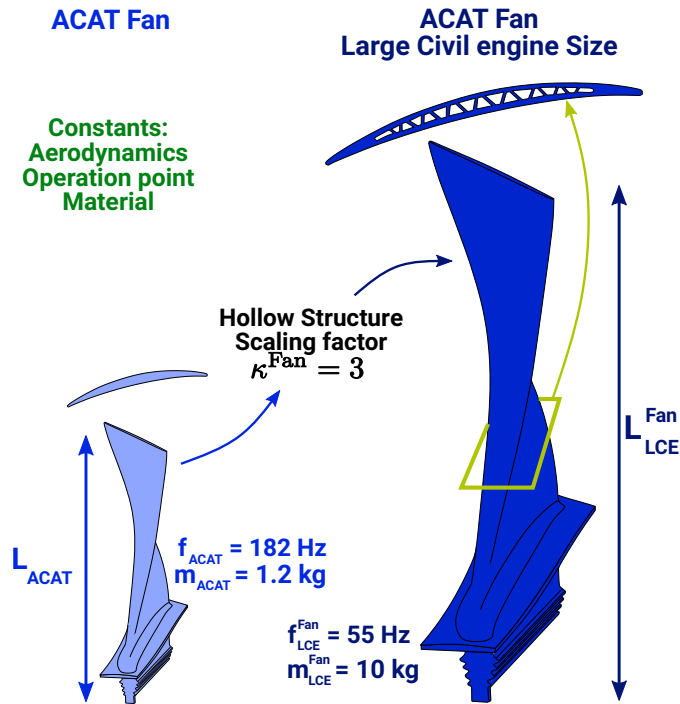


Figure 5.11.: SKETCH OF THE TRANSFORMATION 3: FROM THE ACAT BLADE, AN SMALL CIVIL ENGINE FAN BLADE, TO A LARGE CIVIL ENGINE FAN.

amplitude grows uncontrollably in the absence of a saturation mechanism, ultimately leading to blade failure.

Two main factors drive this outcome. First, the increased $\tilde{\Omega}$ in the LCE fan tightens the blade-to-disk contact, reducing the size of the hysteresis cycle and thus limiting frictional dissipation. Second, the dominant factor is the drastic rise in $\mathfrak{R}^*$, indicating that the hollow LCE fan blade has a significantly greater capacity to absorb aerodynamic work compared to its ability to dissipate it. This is primarily due to the combined effects of scaling and mass reduction caused by the hollow sections, which decrease the physical scaling of mechanical work.

These findings demonstrate that the proposed strategy to control the vibration amplitude of unstable turbomachinery blades through fir-tree friction is not applicable to modern hollow titanium fan blades used in large civil engines.

6. Discussion

The findings presented in this thesis have been thoroughly analyzed in the previous chapters. However, according to Universidad Politécnica de Madrid's Doctoral Thesis requirements, this chapter revisits the key results and summarizes the main discussions.

Modern turbofan engines are designed for higher bypass ratios, with the fan generating over 80% of the engine's thrust and LPTs accounting for 25-30% of its total weight. While LPT efficiency exceeds 90%, further advancements focus on reducing weight keeping or increasing turbine efficiency. One common approach involves designing highly loaded rotor blades with shorter chords, leading to high-aspect-ratio configurations. However, this increases the risk of flutter, which can be controlled through friction-based mechanisms such as shrouds, dampers, and fir-tree attachments.

The vibration amplitude of fluttering blades increases until either structural failure occurs or an energy dissipation mechanism is activated. In turbomachinery, this dissipation is typically governed by nonlinear dry friction, resulting in a saturated vibration response. Various scenarios are illustrated in Figure 6.1. If the blade is stable, the vibration amplitude remains zero, as all modes are stable and perturbations decay.

When unstable modes are present, different outcomes are possible. If the instability is severe and cannot be effectively mitigated by nonlinear mechanisms, the amplitude may become large enough to cause rotor blade failure. In an intermediate scenario, the amplitude saturates at a finite level. If this amplitude, and consequently the alternating stresses, are low enough, the blade may only be able to sustain vibration indefinitely, making flutter acceptable since the stresses correspond to infinite life on the Goodman diagram. Alternatively, if the vibration amplitude is high, the blade may only sustain the vibration for a finite time, resulting in a limited lifespan.

Quantifying the saturated vibration amplitude is a critical aspect of turbomachinery blade design. While high-fidelity contact simulations are essential for accurately capturing limit cycle oscillations (LCOs), low-fidelity models remain valuable for identifying global trends. Corral and Gallardo [1,55] previously estimated the vibration amplitude of unstable cantilevered LPT rotor blades using a simplified friction model, which assumed small hysteresis loops in the fir-tree attachment. Building on this approach, this thesis presents an en-

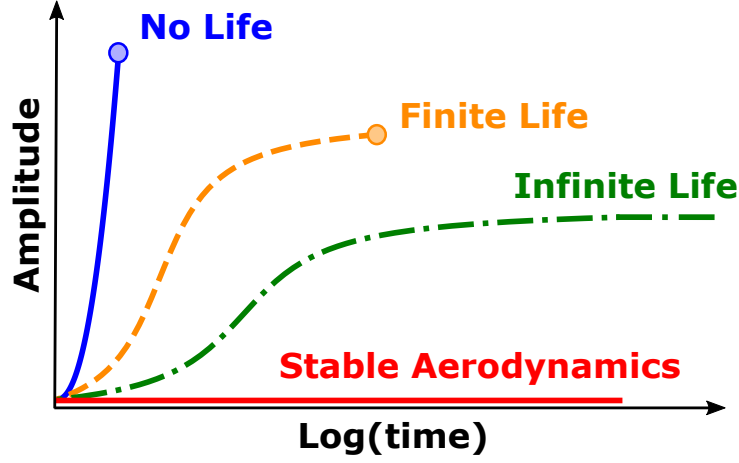


Figure 6.1.: EVOLUTION OF A FLUTTERING BLADE VIBRATION AMPLITUDE WITH TIME FOR DIFFERENT AERODYNAMICS AND DISSIPATION REGIMES.

hanced version of the model that incorporates additional effects and is validated using publicly available LPT rotor blade data.

The model relies on the equilibrium condition in which the aerodynamic work-per-cycle, W_a , is balanced by the dry friction dissipation in the blade attachment, W_m , such that

$$W_a = W_m. \quad (6.1)$$

The aerodynamic work is determined using Eq. 2.2, while the mechanical work is calculated with Eq. 2.8. This formulation accounts for the physical scaling of the mechanical work, which follows the relation $W_m \propto \mu N \delta_c$, as well as a dimensionless function $\tilde{W}_m$, which represents the size of the hysteresis cycles in the fir-tree caused by relative displacement between the blade root and the disk. The hysteresis cycle size is computed using Eq. 2.9, but in cases of small relative displacement ($\tilde{\delta}^* \ll 1$), it can be approximated using Eq. 2.11. Figure 6.2 illustrates how an LPT blade absorbs aerodynamic work from the flow and dissipates it through friction in the fir-tree attachment.

The vibration amplitude related to the aerodynamic damping, δ_a , (see Eq. 2.2), and fir-tree displacement associated with the mechanical work, δ^* , (see Eq. 2.9) differ not only in their physical interpretation, but also in magnitude, often by several orders. To connect these scales and fully define the problem, an approximate relationship is established, introducing an undetermined constant that is later calibrated through FEM simulations.

The blade vibration amplitude is determined by balancing aerodynamic and mechanical work, solving the nonlinear equation:

$$\Re^* \tilde{\Omega}^{-2(1+\alpha)} \tilde{W}_a^{\min}(\tilde{\Omega}) \tilde{\delta}_a^2 = \tilde{W}_m \left[f(\beta, \tilde{\Omega}) \tilde{\delta}_a \right] \quad (6.2)$$

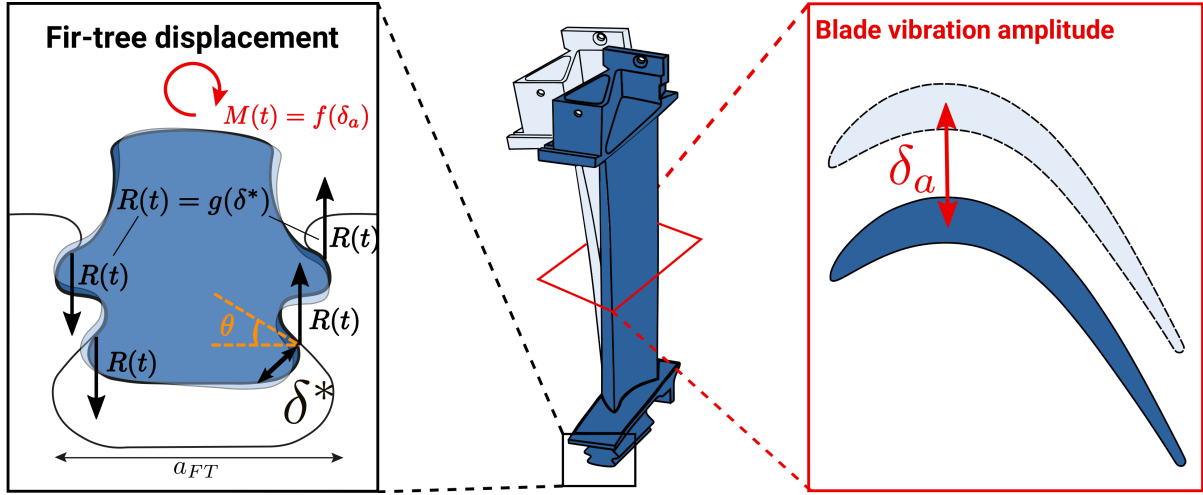


Figure 6.2.: SKETCH OF A VIBRATING LPT BLADE WHERE THE DISPLACEMENT IN THE FIRTREE REGION BALANCES THE AERODYNAMIC WORK INJECTED IN THE SYSTEM

where $\mathfrak{R}^*$ is a dimensionless parameter that characterizes the rotor blade's ability to absorb aerodynamic work relative to its dissipation capacity through dry friction in the fir-tree attachment. This parameter, defined as:

$$\mathfrak{R}^* = \frac{\frac{1}{2}\gamma P_0 F(M_e^2) a_{FT}^2 \mu}{m_b L \omega_{n_0}^2 B^2 \sin^2 \theta} \frac{4G'}{E'} \frac{R}{\lambda_0} \left(\frac{E' A_c}{m_b \omega_{n_0}^2 R} \right)^\alpha \quad (6.3)$$

depends on operational conditions, material properties, blade size, and geometry.

This model is applied to predict the vibration amplitude of the ARIAS LPT blade, a test-rig blade intentionally designed to flutter. According to Eq. 6.2, the dimensionless vibration amplitude $\tilde{\delta}_a$ depends on the parameter $\mathfrak{R}^*$, the dimensionless rotational speed $\tilde{\Omega}$, the minimum dimensionless aerodynamic work $\tilde{W}_a^{\min}$, and the parameters α and β , which account for penetration regime effects and centrifugal stiffening, respectively.

Figure 6.3 illustrates the evolution of $\tilde{\delta}_a$ with rotational speed for $\alpha = 1$ and $\beta = 4$. The effect of the parameter $\mathfrak{R}^*$ is first discussed: higher values indicate increased aerodynamic work absorption, leading to higher $\tilde{\delta}_a$. In the small displacement limit ($\tilde{\delta}^* \ll 1$, i.e. $\mathfrak{R}^* \ll 1$), $\tilde{\delta}_a$ grows linearly with $\mathfrak{R}^*$, following the approximation presented in Eq. 2.28. However, far from the small displacement limit, where $\tilde{\delta}^* \sim 1$, $\tilde{\delta}_a$ continues to increase with $\mathfrak{R}^*$ but in a nonlinear manner, as dictated by Eq. 6.2.

Rotational speed also plays a crucial role in determining vibration amplitude and hysteresis cycle size. At low rotational speeds, the fir-tree contact is loose, leading to larger hysteresis cycles and higher $\tilde{W}_m$, but the mechanical work scaling remains low. Conversely, at high rotational speeds, the tighter contact reduces the hysteresis cycle size, though the mechanical work scaling increases.

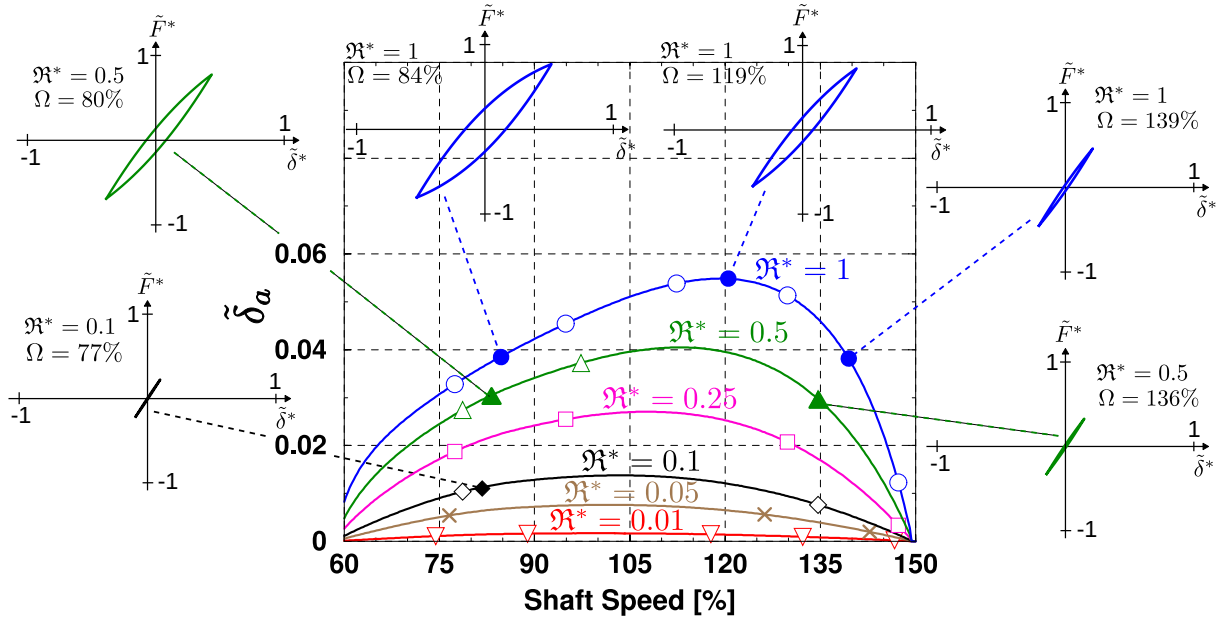


Figure 6.3.: RESULTS OF THE MODEL FOR DIFFERENT LEVELS OF AMPLITUDE $\mathfrak{R}^*$. THE HYSTERESIS CYCLES CORRESPONDING TO LOW SHAFT SPEED FOR $\mathfrak{R}^* = 0.1$ AND BOTH LOW AND HIGH SHAFT SPEED FOR $\mathfrak{R}^* = 0.5$ AND $\mathfrak{R}^* = 1$ ARE INCLUDED.

As a result, $\tilde{\delta}_a$ exhibits a peak at an intermediate rotational speed where neither dissipation mechanism dominates. When considering only mechanical work effects, as shown in Fig. 3.17b (including natural mistuning), the maximum $\tilde{\delta}_a$ occurs at $\Omega = 160\%$ for $\mathfrak{R}^* = 0.5$. However, since aerodynamic instability peaks at $\Omega = 90\%$, as illustrated in Fig. 3.12b, the combined effect results in a maximum $\tilde{\delta}_a$ around $\Omega = 110\%$ for $\mathfrak{R}^* = 0.5$.

The model, calibrated with experimental data from the ARIAS LPT blade ($\mathfrak{R}^* = 0.4$) is now applied to predict the vibration amplitude of other turbomachinery blades. This is achieved by systematically modifying key parameters—scale, operating conditions, mechanical design, and material properties—resulting in updated values for $\mathfrak{R}^*$, $\tilde{\Omega}$ and $\tilde{W}_a^{\min}$ (see Eq. 2.26). Additionally, the characteristic vibration amplitude δ_{ac} (see Eq. 2.24) allows conversion from dimensionless to dimensional vibration amplitude.

For Large Civil Engine LPT blades, the predicted vibration amplitude is similar to that of the ARIAS LPT rig blade. This highlights two points: first, the ARIAS LPT blade exhibits high vibration amplitudes despite its small size due to its large $\mathfrak{R}^*$. Second, while an LCE LPT blade has a lower $\mathfrak{R}^*$, its larger scale ($\kappa = 3$) results in a comparable vibration amplitude of $\delta_a \sim 1$ mm. than the rig blade studied, which is a reasonable number given the scaling factor $\kappa = 3$ between the rig and the LCE LPT blades. Both cases share a dimensionless rotational speed of $\tilde{\Omega} = 0.27$, leading to large hysteresis cycles for small vibration amplitudes and enhancing dissipation. As a result, alternate stress values of $AF = 0.20 - 0.25$, are obtained, ensuring the blade can withstand HCF

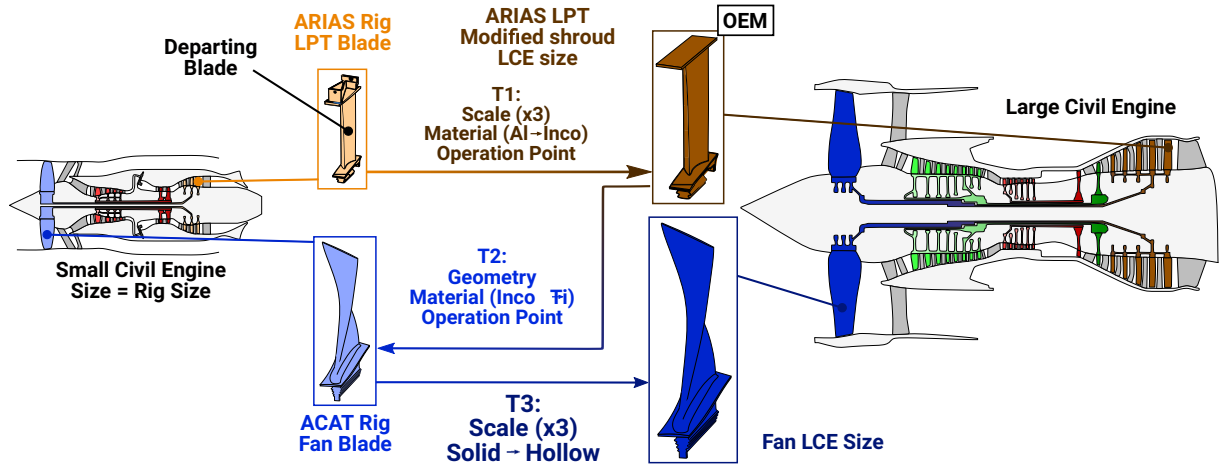


Figure 6.4.: ROADMAP OF THE TRANSFORMATIONS PRESENTED IN THE THESIS.

throughout its lifespan.

Small Civil Engine fan blades, which are similar in size to LCE LPT blades, operate at higher $\Re^*$ values and $\tilde{\Omega} \approx 0.6$, reducing hysteresis cycle size and energy dissipation capacity. In the ACAT fan blade studied here, the predicted alternate stress is $AF = 1.45$, exceeding the typical fan limit of $AF \sim 1$. While the model provides valuable insight into trends, its accuracy is limited. Therefore, further high-fidelity simulations are recommended to assess the viability of fir-tree friction as a mechanism for limiting fan blade vibration amplitudes and ensuring HCF durability.

Finally, the strategy is tested on a modern LCE fan blade, which features hollow sections for weight reduction and operates at a slightly increased vibration frequency to optimize efficiency. These design choices lead to the model parameters $\Re^* \approx 20$, $\tilde{\Omega} = 0.6$ and $\tilde{W}_a^{\min} = 0.5$, preventing vibration amplitude saturation, leading to uncontrolled growth and eventual failure. These results indicate that fir-tree friction, while effective for LPTs, is not a feasible vibration control mechanism for modern hollow titanium fan blades in large civil engines.

In summary, the study expands from the ARIAS LPT blade (a rig blade representative of an SCE LPT) to three additional cases: (1) an LCE LPT blade, where the effects of size, material, and operating conditions on vibration amplitude and alternate stress are examined; (2) the ACAT fan blade (a rig blade representative of an SCE fan), where geometric, material, and operational differences relative to the LCE LPT are analyzed; and (3) a modern LCE fan blade, where the feasibility of fir-tree friction for vibration control is assessed. These transformations are summarized in Fig. 6.4.

7. Conclusions

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This thesis develops a model to predict the limit-cycle oscillations of unstable turbomachinery rotor blades, specifically estimating the vibration amplitude of fluttering blades stabilized by dry friction in the fir-tree attachment. Building on the foundational work of Corral and Gallardo [1], this enhanced model has been proposed, validated and applied in an industrial setting within this thesis, contributing to multiple publications [49–51].

The original model [1] employed dimensional analysis to estimate vibration amplitudes but lacked accuracy, particularly in predicting the rotational speed at which maximum vibration occurs. Two key assumptions limited its applicability: First, it presumed minimal displacement in the fir-tree attachment, contradicting recent experimental findings [8, 47] that indicate significantly larger vibration amplitudes in fluttering LPT blades. Second, it assumed constant aerodynamic work-per-cycle across rotational speeds, failing to capture off-design operational effects. This research addresses these limitations by incorporating these critical factors into the model.

The primary objective of this research is to extend and validate a predictive model for assessing the impact of design parameters and operating conditions on turbomachinery blade vibration amplitudes. The updated model successfully predicts the rotational speed at which maximum vibration occurs and how it varies with changes in design and operation. The ARIAS LPT rotor blade serves as a reference for validation and calibration before applying the model to assess the feasibility of dry-friction mechanisms for vibration control in various turbomachinery components.

First, the vibration amplitude of the ARIAS LPT rotor blade, characteristic of a large civil engine, has been analysed. The effects of size, geometry, materials and operating conditions on blade vibration amplitude are included in the model. The LPT rotor blade achieved a saturated state at low-stress levels, ensuring its ability to withstand HCF. Next, the approach was extended to an LCE LPT, demonstrating that modifications in size, material, geometry, and operating point still resulted in a blade capable of sustaining HCF throughout its lifespan.

The same analysis strategy was then applied to a small metallic fan blade with a size typical of a business jet, which was analysed. The model predicted a vibration amplitude near the stress limit for HCF endurance, suggesting that this strategy could control the blade's vibration amplitude. However, due to the model's limited accuracy, these results must be confirmed through more detailed analyses. Finally, a fan blade characteristic of a large civil engine is analysed, including the scaling and mechanical design effects on blade vibration amplitude. The fan blade did not reach saturation at any vibration amplitude. Three primary factors explain this difference:

- The dimensionless parameter $\mathfrak{R}^*$, which combines the physical factors influencing the model, represents the blade's ability to absorb energy from the flow relative to its capacity to dissipate it through friction. Fan blades exhibit significantly higher $\mathfrak{R}^*$ than LPT blades, leading to higher vibration amplitudes and AFs.
- The minimum dimensionless aerodynamic work $\tilde{W}_a^{\min}$ is of the same order of magnitude for both blades. However, the minimum critical damping ratio, ξ , is an order of magnitude higher for the LPT due to its lower reduced frequency. At first glance, this might suggest that the LPT would face greater challenges in saturating vibration amplitude through friction. However, the dimensionless form of the critical damping ratio is not relevant for flutter, just for forced response. In contrast, the model relies on the dimensionless aerodynamic work $\tilde{W}_a^{\min}$, a purely aerodynamic parameter, to quantify the aerodynamic work. This parameter is crucial for determining the mechanical work that must be dissipated to balance the aerodynamic energy input and achieve a saturated state. When measured in this parameter, the fan and LPT dimensionless aerodynamic work are comparable.
- The dimensionless rotational speed $\tilde{\Omega}$ of fan blades is approximately twice that of LPT blades, resulting in higher normal forces and tighter fir-tree contact between the blade and disk. Consequently, the relative displacement in the fir-tree region is smaller for fan blades, reducing the size of the hysteresis cycle and, therefore, their dissipation capacity.

In summary, while the dry-friction strategy effectively controls the vibration amplitude of cantilever LPT rotor blades, this mechanism cannot sustain LCO in modern large civil engine fan blades due to their size and physical and operational characteristics.

7.1. Future work

While this study has provided valuable insights, there are still some open questions, topics that deserve deeper analysis and new possible areas of further research to refine the model

and expand its industrial applications. Key areas for future investigation include:

- **Aerodynamic Work Characterization:** The evolution of minimum aerodynamic work with rotational speed has been analyzed using an LPT rig blade geometry. However, secondary flow patterns may differ from those in an actual engine LPT blade. Additionally, this study employed a linear Navier-Stokes solver, which has demonstrated high accuracy at on-design conditions but remains uncertain at off-design conditions. A more precise characterization using an engine-representative LPT blade and a nonlinear solver could enhance the accuracy of aerodynamic work predictions.
- **Model Validation Across Multiple Cases:** The model has been validated using experimental data from a single blade, providing a solid reference for comparison. However, extending the validation process to other turbomachinery blades and incorporating medium- to high-fidelity computational models would enhance the model's reliability and broader applicability.
- **High-Fidelity Simulations for SCE Fan Analysis:** The predicted alternate stress for the SCE fan slightly exceeds the threshold required for sustaining HCF throughout its lifespan. While the model excels at capturing trends and explaining underlying phenomena, its accuracy remains limited. When results are on the verge of saturation with sufficiently small alternate stress, high-fidelity simulations are necessary to determine the final vibrational state and ensure reliable conclusions with greater precision.
- **Expansion to Other Vibration Sources and Dissipation Mechanisms:** The current model focuses solely on flutter saturation via dry friction in the fir-tree attachment. Future work could explore additional vibration sources, such as forced response and non-synchronous vibrations (NSV). Moreover, investigating the interaction of multiple dissipation mechanisms—such as shrouds and under-platform dampers—could provide a more comprehensive understanding of vibration control strategies in turbomachinery blades.
- **Apply the Model to Other Flutter-Prone Components:** The model could be applied to predict the final saturated state of turbine last stages of gas and steam turbines. The last stages of land-based large power turbines are long, slender, and prone to flutter. It is expected that the dry friction in the fir-tree attachment cannot sustain LCO if the blade is unstable, but this point needs to be analysed.
- **Improve Didactics and Engineering Understanding:** The dimensionless parameters used in the model group many physical parameters highly compacting the information. However, it would be desirable to further illustrate the model's consequences and scaling laws in a more physical form.

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A. Derivation of dimensionless mechanical work-per-cycle

The dimensionless mechanical work-per-cycle due to the dry friction in the fir-tree is calculated based on reference [44], resulting in Eq. 2.9. The area enclosed within the loop shown in Fig. 1.11 represents the energy dissipation during the micro-slip cycle. The dimensionless tangential force, $\tilde{F}$, is integrated throughout a vibration cycle. This cycle can be subdivided into loading and the unloading phases, where the sign of Eq. 2.5 changes. The expression to compute $\tilde{W}_m$ is then expressed as

$$\tilde{W}_m = \tilde{W}_L - \tilde{W}_U = \int_{-\tilde{\delta}^*}^{+\tilde{\delta}^*} \tilde{F}_L - \tilde{F}_U d\tilde{\delta} \quad (\text{A.1})$$

where $\tilde{F}_L$ is the force during the loading phase and $\tilde{F}_U$ is the force during unloading. The dimensionless mechanical work-per-cycle of the loading branch is calculated as:

$$\tilde{W}_L = \int_{-\tilde{\delta}^*}^{+\tilde{\delta}^*} -\tilde{F}^* + 2 \left[1 - \left(1 - \frac{|\tilde{\delta} - \tilde{\delta}^*|}{2} \right)^{5/2} \right] d\tilde{\delta} \quad (\text{A.2})$$

which results in

$$\tilde{W}_L = 2(2 - \tilde{F}^*)\tilde{\delta}^* + \frac{8}{7} \left[(1 - \tilde{\delta}^*)^{7/2} - 1 \right]. \quad (\text{A.3})$$

As the cycle is symmetric, i.e.: $\tilde{W}_L = -\tilde{W}_U$, only one branch of the hysteresis loop is computed. Finally, by substituting Eq. A.3 into Eq. A.1, the dimensionless mechanical work is derived.

B. Dimensionless mechanical work-per-cycle approximation for $\tilde{\delta}^* \ll 1$

The approximation of the dimensionless mechanical work-per-cycle for small fir-tree displacement, $\tilde{\delta}^* \ll 1$ is provided, which corresponds to a small dimensionless tangential force, $\tilde{F}^* \ll 1$ (see Eq. 2.10). The dimensionless mechanical work is calculated throughout the vibration amplitude range using Eq. 2.9, which can be expressed as a function of $\tilde{\delta}^*$ as

$$\tilde{W}_m = 4 \left[2 - \left(1 - (1 - \tilde{\delta}^*)^{5/2} \right) \right] \tilde{\delta}^* + \frac{16}{7} \left[(1 - \tilde{\delta}^*)^{7/2} - 1 \right]. \quad (\text{B.1})$$

In the case of $\tilde{\delta}^* \ll 1$, the exponential expressions of $\tilde{\delta}^*$ are approximated using third order Taylor approximations as

$$\tilde{W}_m = 4 \left[2 - \left(1 - \left(1 - \frac{5}{2}\tilde{\delta}^* + \frac{15}{8}\tilde{\delta}^{*2} + \frac{5}{16}\tilde{\delta}^{*3} \right) \right) \right] \tilde{\delta}^* + \frac{16}{7} \left[\left(1 - \frac{7}{2}\tilde{\delta}^* + \frac{35}{8}\tilde{\delta}^{*2} - \frac{35}{16}\tilde{\delta}^{*3} \right) - 1 \right]. \quad (\text{B.2})$$

First and second order terms in Eq. B.2 vanish, resulting in the third order expression presented in Eq. 2.11, where fourth and higher-order terms have been disregarded

$$\tilde{W}_m \simeq \frac{15}{6}\tilde{\delta}^{*3}$$