EACWE 2025 Trondheim

Estimation of mean value and variance of velocity from pressure measured by a pressure transducer

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SUMMARY:
Pressure measurements are commonly used to determine mean velocity in Pitot tubes and surface pressure in pressure taps. However, it is often assumed that the temporal variations recorded by pressure transducers directly reflect those at the measurement point, without accounting for the attenuation and phase shift introduced by the tube. This study has two main objectives: first, to propose a simplified analytically derived formulation for obtaining the transfer function of tubes; and second, to develop a methodology for calculating the mean value and variance of velocity based on the statistical properties of the measured pressure. These formulations aim to improve the accuracy of experimental measurements in pressure systems, addressing the limitations of current techniques.

Keywords: pressure measurement, experimental aerodynamics, transfer function, mean velocity calculation

1. INTRODUCTION

There are several experimental techniques for determining fluid variables, such as velocity or pressure at specific points in the flow field. Among these, pressure measurements using pressure transducers are one of the most widely used methods. This technique is employed not only to measure pressure (via pressure taps) but also to calculate mean wind velocity using Pitot tubes. In both cases, the measurement point is connected by a pneumatic soft tube with a pressure transducer. However, it is often assumed that the temporal variations recorded by the transducers precisely reflect the conditions at the measurement point, which is located at the opposite end of the pneumatic tube. This assumption overlooks important details of the pressure's unsteady behavior, such as its variance. In reality, the tube introduces both attenuation and phase shifts effects that depend on the frequency of the disturbances, together with the consequences of friction effects and the presence of the tube end. Several authors have attempted to derive the transfer function that relates the real and measured pressures in tubes under different assumptions. In 1950, Iberall proposed a theoretical formulation that accounts for various effects, such as fluid compressibility, modeling pressure transducers as air-filled reservoirs (Iberall, 1950). Later studies built on this formulation, extending it to systems where multiple tubes are interconnected through reservoirs (Bergh and Tijdeman, 1965; Davies, 1988). Classical texts on acoustics also address the propagation of acoustic waves in ducts (Morse and Ingard, 1968). Recent contributions and technical notes, consolidated much of the current knowledge regarding the transfer function in tubes (Kovarek et al., 2018).

When calculating mean flow velocity from pressure measurements, several approaches are available. The most accurate method involves deriving the velocity time series at the tube's end from the pressure time series measured by the transducer using the aforementioned transfer function. However, implementing this method can be challenging. Factors such as tube attenuation (leading to a lack of signal) or harmonic amplification (resulting in excess noise) can introduce significant deviations from the actual values. This study has two main objectives: first, to propose a simplified alternative formulation for obtaining the transfer function of the pressure signals across the pneumatic lines; and second, to develop a methodology for calculating the mean and variance of velocity based on the mean and variance of pressure. These developments aim to improve the accuracy of determining the pressure time series and, consequently, the mean and variance of velocity at the tube's end based on the pressure measurements recorded by the transducer.

2. MATHEMATICAL FORMULATION AND METHODOLOGY

Depending on the required outcome of the pressure measurements, alternative formulations may be used, of decreasing complexity as the outcomes simplifies. If the complete time series are required, the so-called *exact method* should be used. However, if the temporal variations are not needed, but the variance of the data is of relevance, the here denominated *theoretical method* may be employed. Finally, if only mean values are required, a simplified method can be used.

2.1. Exact Method: Provides the complete time series

Pressure transducers typically provide the time series of the pressure difference, $\Delta p(t) = p_t - p(t)$, between the reference pressure, which is typically the total pressure, p_t, and the static pressure, $p(t)$. From this difference, the *scaled pressure* measured by the transducer can be calculated as $\alpha_{trans}(t) = 2\Delta p(t)/\rho$, where ρ is the air density. The tube's transfer function, assumed to be known, is defined as $\mathrm{TF}(\omega) = \tilde{\alpha}_r(\omega)/\tilde{\alpha}_{trans}(\omega)$, where $\tilde{\alpha}(\omega)$ represents the Fourier Transform of the scaled pressure time series in the frequency domain, evaluated at the transducer and at the opposite end of the tube (*i.e.*, $\tilde{\alpha}_{trans}(\omega)$ and $\tilde{\alpha}_r(\omega)$, respectively). For the transfer function, the classic expressions may be used (Kovarek et al., 2018), or alternatively:

$$\mathrm{TF}(\omega) = \frac{\tilde{\alpha}_r(\omega)}{\tilde{\alpha}_{trans}(\omega)} = \cosh\phi, \quad \text{with } \phi = \mathrm{i}\tilde{\omega}\sqrt{\frac{1-\mathrm{i}\frac{8\Lambda^2}{\tilde{\omega}\mathrm{Re}_L}}{1+\mathrm{i}\frac{N\tilde{\omega}}{\mathrm{Re}_L}}}, \tag{1}$$

which is a novel expression analytically derived by neglecting the volume inside the transducer compared to the volume within the tube, and assuming there are no joints along the tube. In Eq. (1), $\tilde{\omega} = \omega L/c$, $\mathrm{Re}_L = cL/\nu$, $\Lambda = L/R$, and $N = \tilde{\nu}/\nu$, where L is the tube length, c the speed of sound in the tube, ν the kinematic viscosity, $\tilde{\nu}$ the volumetric kinematic viscosity, and R the tube radius. Using the chosen expression for the transfer function, the scaled pressure at the tube's end can be expressed as $\tilde{\alpha}_r(\omega) = \mathrm{TF}(\omega)\tilde{\alpha}_{trans}(\omega)$. By applying the inverse Fourier Transform, the time-domain pressure $\alpha_r(t)$ can be obtained. Finally, the velocity may be calculated as $u_r(t) = \sqrt{\alpha_r(t)}$, assuming steady, inviscid, incompressible flow. Based on these definitions, the mean values of both variables are defined as $\overline{\alpha}_r = \overline{\alpha_r(t)}$ and $\overline{u}_r = \overline{u_r(t)}$, while their deviations from the mean are given by $\alpha_r'(t) = \alpha_r(t) - \overline{\alpha}_r$ and $u_r'(t) = u_r(t) - \overline{u}_r$. The standard deviations can then be calculated as $\sigma_{\alpha,r} = \sqrt{\overline{\alpha_r'(t)}}$ and $\sigma_{u,r} = \sqrt{\overline{u_r'(t)}}$. In order to characterize the signal variance, the turbulence intensities, defined as $I_{\alpha,r} = \sigma_{\alpha,r}/\overline{\alpha}_r$ and $I_{u,r} = \sigma_{u,r}/\overline{u}_r$, may be used.

2.2. Theoretical Method: Provides mean values and variances

If only the standard deviations and mean values of the pressure at the measurement position (and velocity, if applicable) are required, an alternative formulation may be employed. This approach is more robust than the exact method, as it eliminates the need for transformations between the time and frequency domains, thereby reducing the potential introduction of numerical noise. Consequently, a more robust formulation is proposed, based solely on the mean values and variances of the variables α and u. From the transducer's time series $\alpha_{trans}(t)$, the mean value $\overline{\alpha}_{trans}$, variance $\sigma^2_{\alpha,trans}$, and power spectral density $\text{PSD}_{\alpha,trans}(\omega)$ are determined. Using the transfer function, the mean value at the tube's end (where $\overline{\alpha}_r$ is shown to be equivalent to $\overline{\alpha}_{trans}$) and the power spectral density $\text{PSD}_{\alpha,r}(\omega) = |\text{TF}(\omega)|^2 \text{PSD}_{\alpha,trans}(\omega)$ are derived. Finally, the variance at the tube's end can be calculated as follows:

$$\sigma^2_{\alpha,r} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \text{PSD}_{\alpha,r}(\omega)\mathrm{d}\omega - \overline{\alpha}_r^2. \tag{2}$$

Once the mean value and variance of α at the tube's end are determined, the mean value and variance of u can be calculated using the following expressions:

$$\overline{u}_{r,theo} = \sqrt{\overline{\alpha}_r}\left(1 - \frac{1}{8}I^2_{\alpha,r}\right), \text{ and } I^2_{u,theo} = \frac{1}{4}I^2_{\alpha,r}. \tag{3}$$

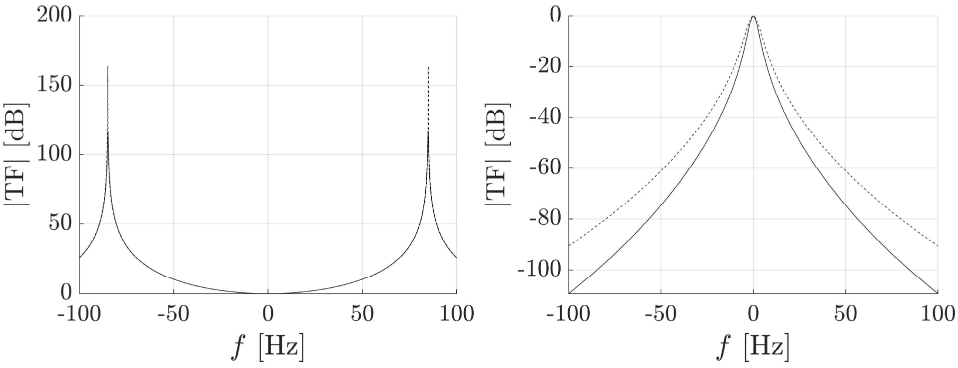
2.3. Approximated Method: Only provides mean values

Finally, when only the mean values are of interest, from Eq. (1), it can be noted that the mean pressure values at both ends of the tube are identical (since $\text{TF}(\omega = 0) = 1$). Additionally, for calculating the mean velocity, one can refer to:

$$\overline{u}_{r,approx} = \sqrt{\overline{\alpha}_{trans}}. \tag{4}$$

3. RESULTS

For each of the three methods presented in Section 2, the amount of information obtained from the measurement point decreases with the simplicity of the calculations. This raises the question of how reliable the simpler methods are compared to the more complex ones. Although simpler methods may lead to some loss of information, one would expect their results to closely approximate those obtained using more sophisticated approaches. Regarding the transfer function calculation, Figures 1-(a) and (b) compare the transfer functions from Kovarek et al., 2018 with Eq. (1), highlighting the influence of the geometric relation $\Lambda = L/R$ on the transfer function's magnitude. For long tubes, friction effects dominate and cause attenuation, while for short tubes, the tube's end amplifies certain frequencies. The results from both methods are quite similar, although the expression given by Eq. (1) is much simpler than that provided in Kovarek et al., 2018. The error in the mean velocity is defined as $\varepsilon_{\overline{u},i} = |\overline{u}_{r,i}/\overline{u}_r - 1|$, where i refers to either the theoretical ($i = theo$) or approximate ($i = approx$) method. Similarly, the error in the variance is defined as $\varepsilon_{I^2_u} = \left|I^2_{u,theo} - \sigma^2_{u,r}\right|$. In the conducted research, it has been confirmed that, although both errors increase with I_α, their values remain below 0.1% ($\varepsilon_{\overline{u},i}$) and 1% ($\varepsilon_{I^2_u}$) for turbulence intensities, I_α, less than 10%. Therefore, the differences between the three methods are negligible for most applications. For this reason, using more sophisticated methods over simpler ones is only justified when additional information is required, such as when measuring unsteady flows where the full time series (or at least variances) of both pressure and velocity may be needed.



(a) **(b)**

Figure 1. Transfer function module, |TF|, as a function of the frequency, f, given by Kovarek et al., 2018 (continuous lines) and by Eq. (1) (dashed lines) for $\Lambda = 10^1$ (a) and $\Lambda = 10^4$ (b).

4. CONCLUSIONS

Pressure measurements are utilized not only to determine pressure values via pressure taps but also to calculate mean wind velocity using Pitot tubes. A new, simplified formulation for the transfer function has been introduced to derive the pressure at the tube's end from transducer readings. Three methods for calculating mean flow velocity from the measured pressure time series have been presented. The results show that as the signal variance decreases, all three methods yield similar outcomes; however, the simpler methods offer less detail compared to the more sophisticated approaches. This indicates the importance of considering which variables—such as full time series data, variances, or only mean values—are relevant to the specific test. Notably, the exact method requires applying the inverse Fourier Transform to recover the time series, which can introduce significant errors influenced by signal characteristics. Consequently, when only mean values and variances are needed, the theoretical method might be more robust and reliable than the exact method.

ACKNOWLEDGEMENTS

This work is part of the project TED2021-130541B-C21, funded by MCIN/AEI/10.13039/ 501100011033, the European Union "NextGenerationEU"/PRTR and project PID 2022-137630OB-C21 financed by MCIN/AEI/10.13039/ 501100011033/FEDER, UE. The work of Carlos Carbajosa has been supported by the FPU grant FPU23/00716 from the Spanish Ministry of Science and Innovation (Ministerio de Ciencia, Innovación y Universidades).

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