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Research

The Vault of the Chapel of the Presentation in Burgos Cathedral: "Divine Canon? No, Cordovan Proportion"

Abstract. The "Cordovan proportion" was discovered by chance in 1973, during an attempt to demonstrate the validity of the golden ratio in buildings in the city of Cordova. Far from being a local exception, later research have shown its sphere of application to be universal. However, few authors have researched the influence of the Cordovan triangle on the layout of architectural elements, and particularly on octagonal Gothic vaults. The present paper presents an analysis of the Cordovan proportion in the curved ribbing of Gothic vaults. Specifically, it consists of a geometric study of the octagonal vault of the Chapel of the Presentation in Burgos Cathedral. The results show that when the side of the octagon is taken as a modulus, the Cordovan triangles drawn by the layout of the vault form a geometric succession whose ratio is the Cordovan number.

Introduction

The Chapel of the Presentation in Burgos Cathedral

Burgos Cathedral in Spain, which was declared a World Heritage Building by UNESCO [1984], has four octagonal Gothic vaults: the Cimborio, the Constable Chapel, the Chapel of the Presentation and the old Chapter House. The best known vault is the one covering the Constable Chapel. However, the most complex vault regarding the layout of the ribs is the Chapel of the Presentation, due to the incorporation of curved ribbing.

The vault of the Chapel of the Presentation (fig. 1) was the work of the master mason Juan de Matienzo, between 1519 and 1522 [Gomez 1998], inspired by the work of Simón de Colonia and especially by the Constable Chapel. The vault in question comprises Gothic ribbing of a regular octagonal plant. The central severies are openwork, possibly influenced by Almoravid models, such as the Mosque of Tremecen in Algeria.

The stereotomy of the Gothic vault always has its origin in the horizontal projection [Palacios 2009; Rabasa 2000: 39]. The plan becomes totally relevant in the design of a vault right from the very beginning. Therefore, a geometric analysis must focus on the horizontal projection of the vault.

The Cordovan proportion: Cordovan rectangle vs. Cordovan triangle

A reprint of the essay entitled "The Cordovan Proportion" written in 1973 by the architect Rafael de la Hoz Arderius, has recently been announced, under the title of: "Divine Canon? No, Cordovan Proportion".[1] In the work, he demonstrated how, by chance, when attempting to demonstrate the validity of the golden ratio in buildings in the city of Cordova, a canon appeared, unknown until then, which was different from the harmonic rectangle. Suddenly, apparently anarchical distributions took on a logical composition and a pattern appeared which endowed the whole with order.

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Fig. 1. Vault of the Chapel of the Presentation in Burgos Cathedral

Therefore, since the golden ratio is the ratio of the side to the radius of a regular decagon [Bonell 2000: 25; Huntley 1970: 25], the Cordovan ratio is defined as the ratio of the side to the radius of a regular octagon (fig. 2). Its value is

$$c = \frac{r}{l} = \frac{1}{\sqrt{2-\sqrt{2}}} = 1.30656296487\ .$$[2]

A Cordovan rectangle is defined as that which maintains the Cordovan ratio between its longest and shortest side.

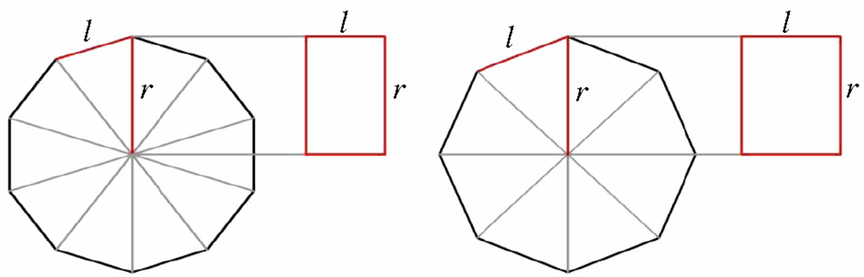


Fig. 2. left) Golden ratio, based on a decagon; right) Cordovan ratio, based on an octagon

It first it was thought to be a local exception, but various pieces of subsequent research have proven its scope of application to be wider. According to Hoz [2002], the Cordovan rectangle is not only to be found in the arcades of the Mosque of Cordoba but also in the following examples: Hispanic contexts, like the Aqueduct of Segovia and the Alcala Gate (Puerta de Alcalá) in Madrid; European contexts, like Agrippa's Pantheon in Rome, the Basilica of Maxentius, and in contexts even more distant from the point of discovery, like the Pyramids of Egypt, the Pyramid of the Moon in Teotihuacán or the Church of the Company of Jesus in Cordoba (Argentina).

Moreover, the side and radius of a regular octagon can form an isosceles triangle with a longer base *l* and sides *r*, called a Cordovan triangle [Redondo, et al. 2008a, 2008b] (fig. 3). The rectangle circumscribing two Cordovan triangles touching at the vertex is known as a silver rectangle[3] (fig. 4).

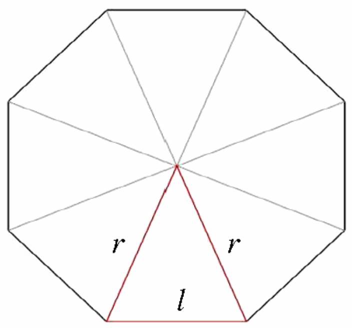


Fig. 3. Cordovan triangle

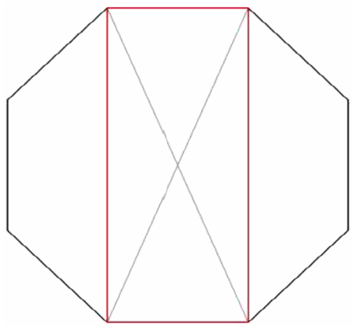


Fig. 4. Silver rectangle

Since architectural elements (mainly elevations) have been found that observe the ratio of the Cordovan rectangle, among which are those mentioned previously, there have been few authors who have researched the influence of the Cordovan triangle [Huylebrouck, et al. 2009], which, as will be seen in this research, is what defines the geometric layout of the studied vault.

Geometric Analysis

Data acquisition

Since we are dealing with a vault that is difficult to access, making direct measurements is highly complex. For this reason a photogrammetric method was used. Eight stations were chosen that were conveniently situated so as to avoid any areas of shadow in the vault, from which various shots were taken with a high resolution digital camera using a calibrated lens.

Using the *Photomodeler* computer program the same spatial points were referenced in different photographs.

They were then projected onto a horizontal plane in order to obtain the layout of the vault (fig. 5).

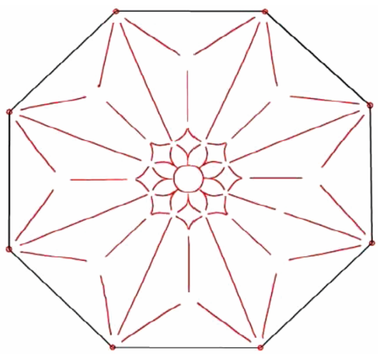


Fig. 5. Data obtained by the photogrammetric method

Layout of the vault

In an octagonal vault, the horizontal projection of the **polar keystone** usually coincides with the centre of the polygon. In this particular case, it is substituted by a circular layout rib to which the curved ribs are attached.

The **secondary keystones** are obtained by drawing the Cordovan triangles whose vertices are situated at the midpoint of the side of the octagon (fig. 6). These keystones are to be found at the midpoint of the base of the Cordovan triangle, which is denominated as T_1.

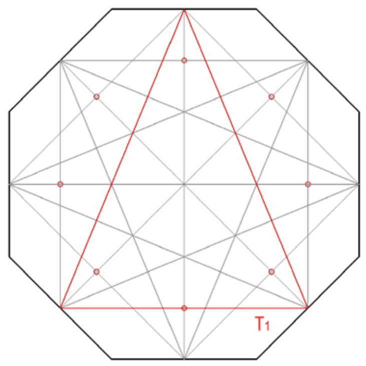


Fig. 6. Secondary keystones and Cordovan triangle T_1

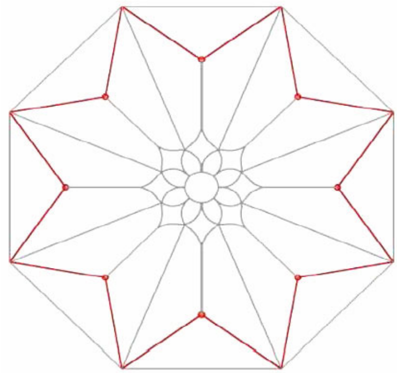


Fig. 7. Position of the secondary keystones and tierceron ribs of the vault

Having situated the secondary keystones, the tierceron ribs of the vault are now defined (fig. 7).

The position of the **tertiary keystones** will determine the layout of the diagonal and ridge ribs as well as the curved ones. There are sixteen keystones of this type positioned in two concentric circumferences. The eight keystones situated on the circumference of largest radius are called *outer tertiary keystones* (fig. 8), and it is to them that the ridge and curved ribs are attached; the remaining eight, situated on the circumference of smallest radius are called *inner tertiary keystones* (fig. 9), and to these are attached the diagonal and curved ribs.

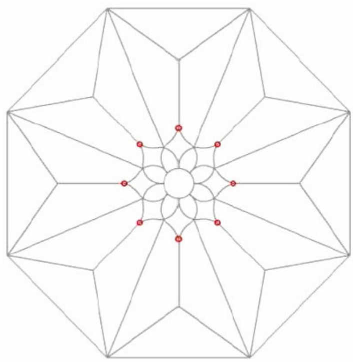


Fig. 8. Position of the outer tertiary keystones

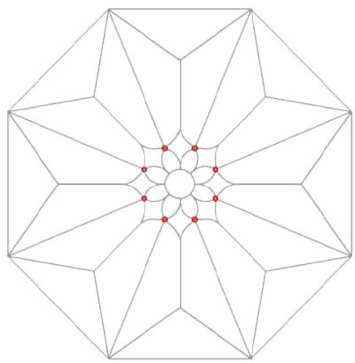


Fig. 9. Position of the inner tertiary keystones

The position of the *outer tertiary keystones* is determined by the construction shown in fig. 10. Their position coincides with the intersections of the longest sides of a Cordovan triangle, which is denominated as T_2, when rotated around the centre of the octagon.

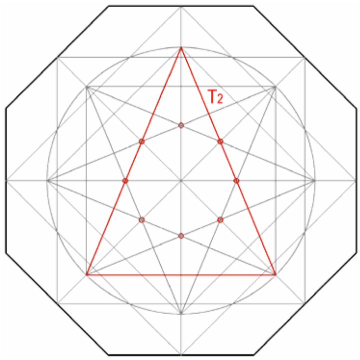


Fig. 10. Outer tertiary keystones and Cordovan triangle T_2

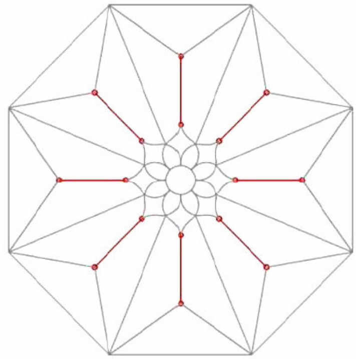


Fig. 11. Ridge ribs of the vault

Having defined the secondary keystones and the outer tertiary keystones, the ridge ribs of the vault are also determined (fig. 11).

The curved ribs are formed by arcs belonging to the two types of circumferences. Depending on the size of their radius they are called *greater circumferences* (fig. 12) and *lesser circumferences* (fig. 13). In order to define them, the centres and radii of each of the two types are determined.

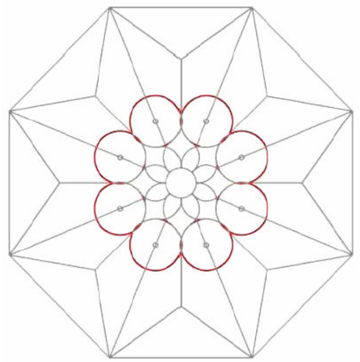


Fig. 12. Greater circumferences

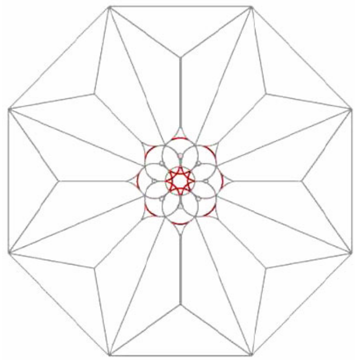


Fig. 13. Lesser circumferences

The centres of the greater circumferences are obtained using the construction performed in fig. 14. These coincide with the intersections of the longest sides of the Cordovan triangle T_1 and a Cordovan triangle, denominated as T_3, whose longest side is the radius of the octagon and whose shortest side is the side of the octagon.

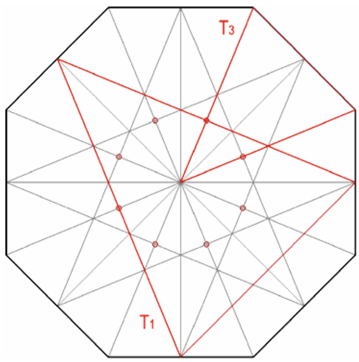


Fig. 14. Centres of the greater circumferences and Cordovan triangles T_1 and T_3

Another way of finding the centres is illustrated in fig. 15. The points looked for coincide with two of the vertices of a Cordovan triangle, denominated as T_4. The length of its sides is half the length of the sides of T_1 (fig. 16).

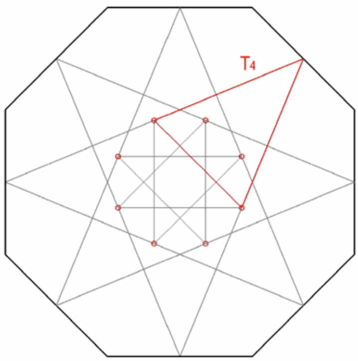


Fig. 15. Centres of the greater circumferences and Cordovan triangle T_4

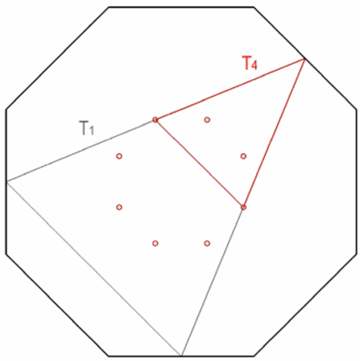


Fig. 16. Cordovan triangles T_1 and T_4

The radius of the circumference is determined by the distance between its centre and the closest outer tertiary keystones, found previously (fig. 17).

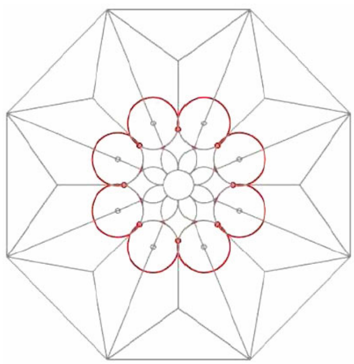


Fig. 17. Radius of the greater circumferences

The centre of the lesser circumferences is determined according to the construction set out in fig. 18. These centres coincide with the midpoint of the base of triangle T_4.

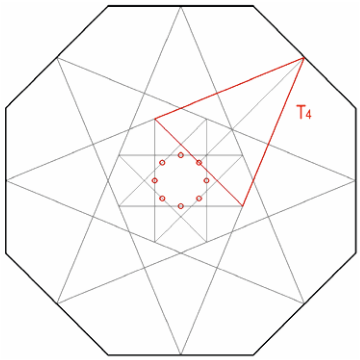


Fig. 18. Centres of the lesser circumferences and Cordovan triangle T_4

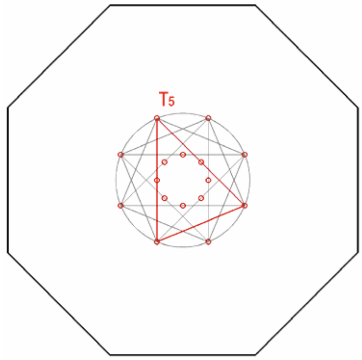


Fig. 19. Ratio of the greater to the lesser circumferences and Cordovan triangle T_5

The ratio of the greater to the lesser circumferences is shown in fig. 19. The vertices of a Cordovan triangle, denominated T_5, are the centres of the greater circumferences, and the midpoint of their longest sides is the centre of the circumferences of smaller radius.

The radius of the lesser circumferences is the distance between their centres and the closest apothem of the octagon (fig. 20). The position of the *inner tertiary keystones* is fixed by the intersection of the lesser circumferences.

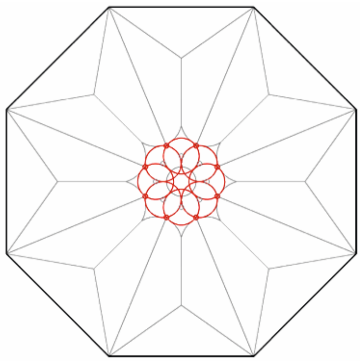


Fig. 20. Radius of the lesser circumferences and inner tertiary keystones

Once the inner tertiary keystones are known, the diagonal ribs (fig. 21) and the curved ribs (fig. 22) are also determined.

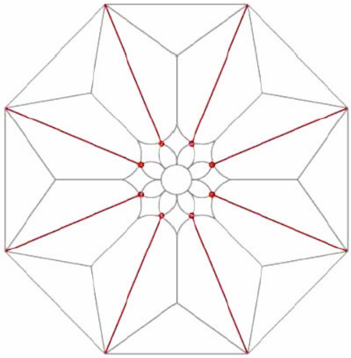


Fig. 21. Diagonal ribs of the vault

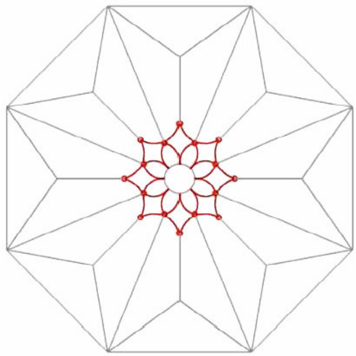


Fig. 22. Curved ribs of the vault

It is of particular interest that the greater circumferences do not pass through the inner tertiary keystones. As a result, the sections of curved ribs between the outer and inner tertiary keystones are formed by two arcs of circumference, secants of each other, and the point of intersection does not coincide with any keystone. This change of curvature can be easily appreciated in the following photograph (fig. 23).

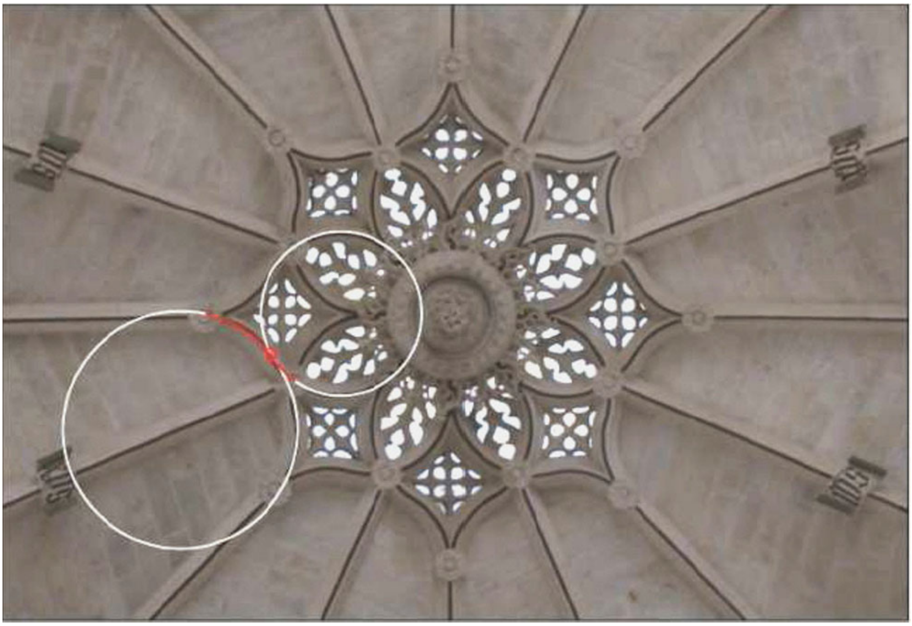


Fig. 23. Detail of the curved ribs of the vault

Having defined the position of the keystones and the geometry of the ribs, the layout of the vault is also determined. However, as fig. 1 illustrates, there are two **decorative elements** in its ribbing that remain to be situated: *false keystones* in the tierceron ribs (fig. 24) and some *bosses* on the diagonal ribs (fig. 25).

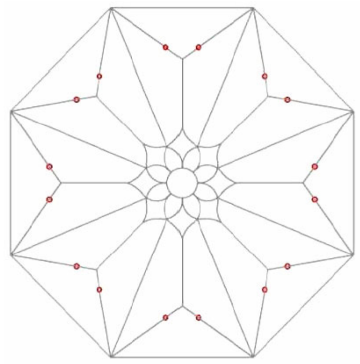


Fig. 24. Position of the *false keystones* on the tierceron ribs

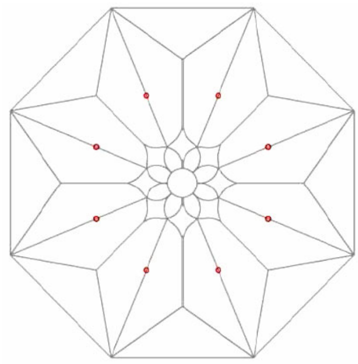


Fig. 25. Position of the *bosses* on the diagonal ribs

The position of the *false keystones* on the tierceron ribs is determined by the construction shown in fig. 26. That is, the intersections of the longest sides of the Cordovan triangle T_1 with the tierceron ribs define the points that are sought.

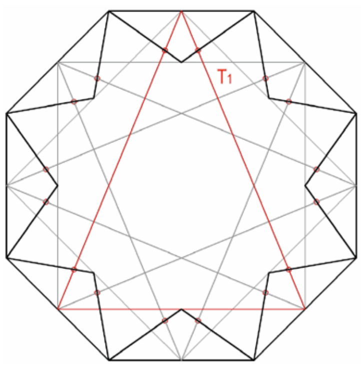


Fig. 26. *False keystones* in the tierceron ribs and Cordovan triangle T_1

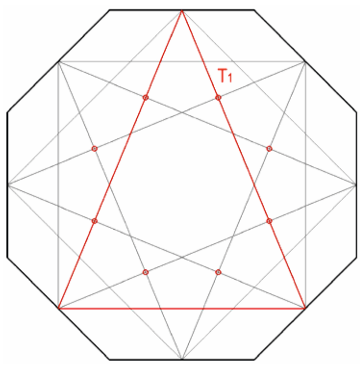


Fig. 27. *Bosses* on the diagonal ribs

The position of the *bosses* on the diagonal ribs is obtained by intersecting the longest sides of the Cordovan triangles T_1 (fig. 27).

Results

The Cordovan triangles denominated as T_1, T_2, T_3, T_4 and T_5, which serve as a pattern (fig. 28) for finding the layout of the vault of the Chapel of the Presentation, are interrelated. To be precise, the relationship between each with the previous one is the Cordovan number *c*.

As pointed out previously, the ratio of the longest to the shortest side of the Cordovan triangle is *c*. The relationship between the shortest "l_i" (or longest "L_i") sides of the cited triangles is also the Cordovan number; that is, the shortest side of one of them is the longest side of another and so on successively, as fig. 29 shows.

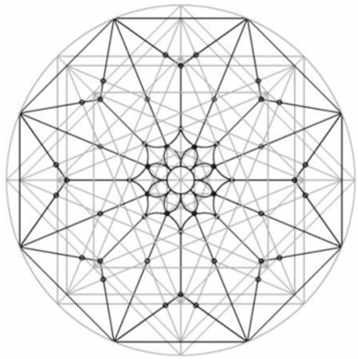
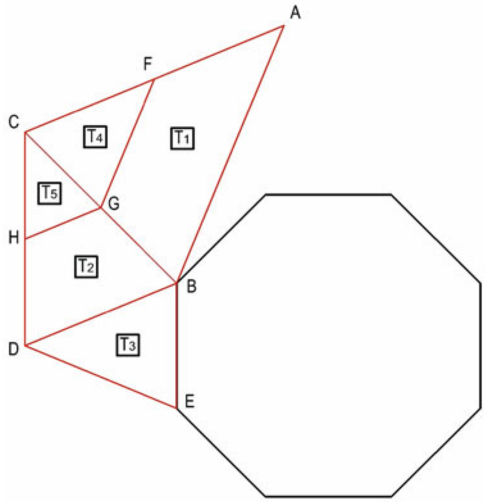


Fig. 28. Cordovan triangles that generate the layout of the vault



$$L_1 = AB = AC$$

$$l_1 = L_2 = BC = CD$$

$$l_2 = L_3 = BD = DE$$

$$l_3 = BE$$

$$L_4 = \frac{L_1}{2} = CF = FG$$

$$l_4 = L_5 = \frac{l_1}{2} = CG = CH$$

$$l_5 = \frac{l_2}{2} = GH$$

Fig. 29. Geometric relation between Cordovan triangles

In the above figure, it can be seen that they not only are the triangles related to one another but that, in addition, the length of their sides depends on the side of the octagon l, exactly in proportion to the Cordovan number:

$$l_3 = l$$

$$l_2 = L_3 = l_3 \times c = l \times c$$

$$l_1 = L_2 = l_2 \times c = l \times c \times c = l \times c^2$$

$$L_1 = l_1 \times c = l \times c^2 \times c = l \times c^3$$

$$l_5 = \frac{l_2}{2} = \frac{l}{2} \times c$$

$$l_4 = L_5 = l_5 \times c = \frac{l}{2} \times c \times c = \frac{l}{2} \times c^2$$

$$L_4 = l_4 \times c = \frac{l}{2} \times c^2 \times c = \frac{l}{2} \times c^3$$

The sides of the triangles form two geometric successions *a* and *b*, the ratio of which coincides with the Cordovan number *c*:

$$a_n = l \times c^n \text{ and } b_n = \frac{l}{2} \times c^n$$

(where *l* is the side of the octagon), as table 1 shows.

			T_1	
			shortest side	longest side
		T_2		
		shortest side	longest side	
	T_3			
	shortest side	longest side		
$a_n = l \times c^n$	$l \times c^0$	$l \times c^1$	$l \times c^2$	$l \times c^3$
$b_n = \frac{l}{2} \times c^n$	$\frac{l}{2} \times c^0$	$\frac{l}{2} \times c^1$	$\frac{l}{2} \times c^2$	$\frac{l}{2} \times c^3$
		shortest side	longest side	
		T_5		
			shortest side	longest side
			T_4	

Table 1. Geometric successions of ratio *c*

The positions of the centres of the circumferences defining the layout of the curved ribs are also interrelated. The circumference whose diameter coincides with the apothem of the octagon passes through the centres of the greater circumferences (fig. 30). The circumference whose diameter is equal to the distance between the centre of the polygon and the centre of the greater circumference passes through the centres of the lesser circumferences (fig. 31).

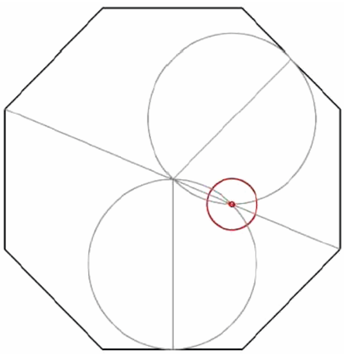


Fig. 30. Relationship between the centre of the greater circumferences and the octagon

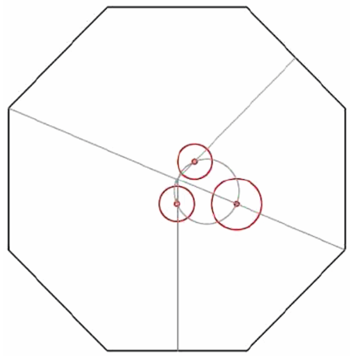


Fig. 31. Relationship between the centres of the greater and lesser circumferences

The relationship between both constructions becomes apparent in fig. 32. In this figure, two right triangles are marked ("arcs capables" of the circumferences). The vertices of the largest triangle coincide with: the centre of the octagon, the midpoint of the polygon side and the centre of the greater circumference. The vertices of the smallest triangle are: the centre of the octagon, the centre of the greater circumference and the centre of the lesser circumference. Thus, both triangles are related to one another since the hypotenuse of the smaller one coincides with one of the catheti of the larger one. The relationship between the sides of both triangles is not arbitrary, but is exactly *2c*.

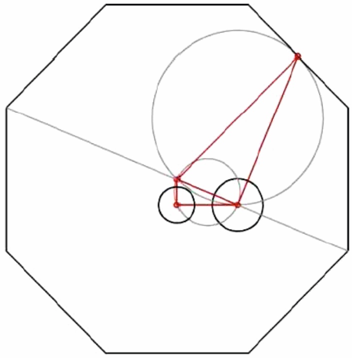


Fig. 32. Relationship between the centres of the circumferences and the octagon

Conclusions

As is clearly shown in this work, the canon of the octagonal vault of the Chapel of the Presentation of the Cathedral of Burgos is the Cordovan proportion. The superposition of the proposed layout and that obtained using data acquisition (fig. 33) shows the small differences existing between them (possible deformations and/or imprecisions during the construction).

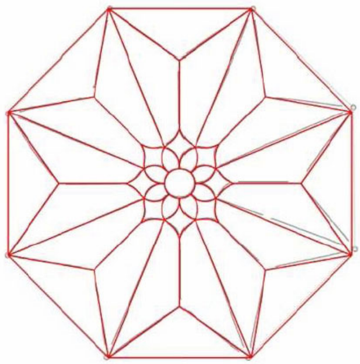


Fig. 33. Superposition of the proposed layout (red) and that obtained using data acquisition (grey)

The layout of the analysed vault was obtained from a Cordovan triangle, whose shortest side is the side of the octagon, together with other similar, whose sides are multiple of the Cordovan number *c*. The superposition of these triangles determines not only the position of the keystones but also the geometry of their ribbing and even the layout of the curved ribs formed by two arcs of circumference.

Although it is not known if the master mason of this vault knew the Cordovan proportion, this research is the first to prove its compliance with the layout of the curved ribbing.

Acknowledgments

The author would like to express his gratitude to the Metropolitan Chapter of Burgos Cathedral and the staff of the cathedral's Historic Archives for their kind assistance.

Notes

1. The article "¿Canon divino? No, cordobés" ("Divine Canon? No, Cordovan") was published in the Spanish newspaper *ABC* in February 2010 [Roso 2010].
2. Given the very slight difference between the Cordovan number (1.3065…) and the sesquitercian number (1.3333…) [Pacioli 1987: 81], extremely high precision is required in its study in order not to commit any errors.
3. A rectangle is said to be a "silver rectangle" if on cutting two equal squares whose sides are equal to the shortest side of the rectangle, the resulting polygon is a rectangle similar to the original. The ratio of its longest to its shortest side is $1+\sqrt{2}$.

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About the author

Tomás Gil-López earned his Ph. D. in Architecture in 2003. He is a Professor at the Madrid School of Architecture in Madrid Polytechnic University, Spain. He is an instructor in courses on Descriptive Geometry, CAD and Technological Innovation in Construction at the bachelor and master level in Madrid Polytechnic University (2004-2011). Since the late 1990s, he has devoted his time to research in descriptive geometry and its applications in architecture and education. His contributions at international congress are focused on the domain of geometric analysis in Gothic vaults. He is the author of *Superficies Cuádricas y sus Combinaciones (I). Superficies de Curvatura Simple* (Madrid, 2006) and *Influencia de la Configuración del Borde Público-Privado. Parámetros de Diseño* (Madrid, 2007). He has also completed several works of architecture and urban design.