Numerical study on the dynamic response of external post-tensioning tendons under strand breakages

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ABSTRACT

Non-destructive testing (NDT) techniques based on vibration methods can be used to detect anomalies in external post-tensioning tendons. In this sense, a numerical study on the dynamic response of a tendon is carried out under strand breakages. In order to quantify the sensitivity of several performance indicators (PIs) to strand breakage. The indicators are: the natural frequencies, the ratios between them and the modal shape symmetry. Thus, firstly, the partial differential equation that governs the cable displacement including bending stiffness is solved. Secondly, a detailed 3D finite element (FE) model of a seven-strand tendon is developed. An elastic contact model between strands and grout has been assumed. Finally, the performance indicators together with the effective tension force are assessed for the simulation steps in such a way that their effectiveness as damage indicator tool for vibration-based NDT is assessed.

Keywords: Post-tensioned tendon, Finite element, Cable dynamics; Vibration based monitoring, Non-destructive testing.

1. INTRODUCTION

Recently, several failures have been detected in external post-tensioning tendons due to brittle breakage in stress corrosion cracking situations (interaction of corrosion and mechanical stress) [1]. Tendon failures can affect both serviceability and safety. Two examples of bridge collapses are: Viaducto O Castro A-6 (Lugo), 2022 and Morandi bridge, Genova (Italy), 2018. Thus, the use of non-destructive testing (NDT) techniques to detect potential tendon deterioration is essential to plan the tendon maintenance and anticipate damage. A vibration-based NDT technique can be used in intermittent and continuous monitoring for damage detection. This NDT technique relies on the sensitivity of some structural performance indicators (PIs) to damage, such as: the natural frequencies, the ratios between them and the modal shape symmetry. Structural damage can be represented by deviations of these indicator's normal values during the monitoring process.

2. ANALYTICAL MODEL

A tendon can be analysed as a cable with non-negligible bending stiffness. To quantify the PI sensitivity to tendon damage, an analytical model of a 10-meter cable with non-negligible bending stiffness is studied. Using taut spring theory, considering a uniform cable and constant tension force, the partial equation governing the tendon free response is:

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$$EI\,\frac{\partial^4 u(x,t)}{\partial x^4} - T\frac{\partial^2 u(x,t)}{\partial x^2} + m\frac{\partial^2 u(x,t)}{\partial t^2} = 0 \tag{1}$$

where $u(x,t)$ is the transverse displacement, $m(x)$ is the mass per unit linear length, *T(x)* is the tension force, E is Young's modulus and $I(x)$ is the moment of inertia of the cross-section [2]. This partial equation is solved considering three tendon segments to simulate damage by varying the bending stiffness (EI) or the tension (T) in the central section (Figure 1).

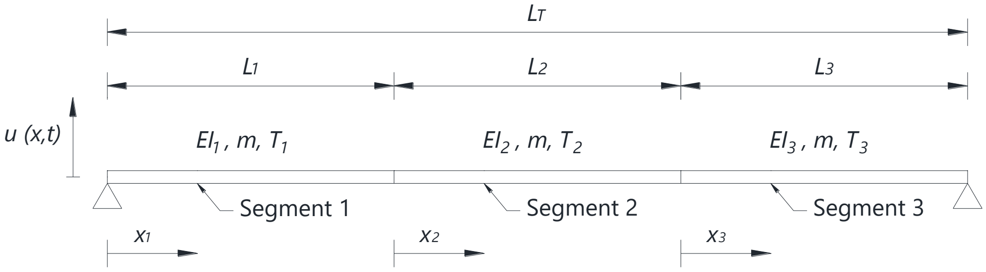


Figure 1. *Cable with non-negligible bending stiffness considering three different segments.*

Considering perfect continuity between segments, thus displacements, rotations, bending moments and shear forces are equal, the partial equation can be solved. The central part's length is considered as 10% of the total cable length, its parameters (EI_2 and T_2) are modified to simulate damage. Two analyses are carried out: i) bending stiffness reduction, ii) tension force reduction.

3. DETAILED FE MODEL

A detailed FE model of a seven-strand and horizontal tendon of 10 m length is developed considering the strand-grout interaction (an elastic contact model is assumed). Both, the tendon behaviour under strand breakages and the variations are analysed.

Thus, the strands are designed with a split element in their central section to simulate strand breakages by deactivating the contact between the two sides of the strand (strand-strand contact), which initially are modelled with a bonded contact to guarantee its continuity before breakage. 3D Solid elements are used for this model. Sweep meshing method has been used so that the positions of the nodes match with those of the grout in each section. Both ends of the seven strands are modelled with fixed support. The stress states reached by stressing the strands are modelled by thermal loads. Two stressing states are considered: 40% and 70% of the minimum ultimate load of a prestressing strand [3]. The stressing forces considered are 775 kN and 1241 kN, respectively. A lineal elastic surface-to-surface contact model between the grout and the strands is assumed. Considering self-anchoring and the 30 cm transfer length [4] the contact stiffness adopted is $k = 1 \cdot 10^{15}$ N/m^2/m.

A non-linear analysis by load steps is then performed. A sparse direct solver and the Newton-Raphson algorithm with convergence in forces and displacements are used. The non-linearity is caused by the activation and deactivation of contacts, which is solved by the "Pure Penalty" methodology, whereas materials are kept elastic. The followed steps are: i) Stressing of strands by applying a thermal load, ii) Activation of strand-grout contacts to simulate the grout injection and iii) Successive breakage of strands by deactivating the strand-strand contacts (Figure 2). A modal analysis is carried out following each strand breakage.

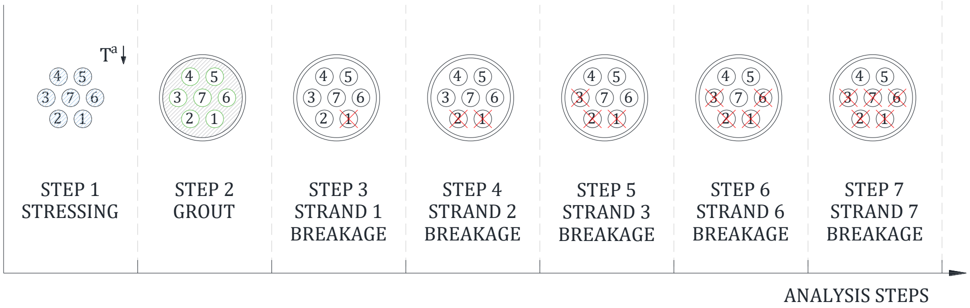
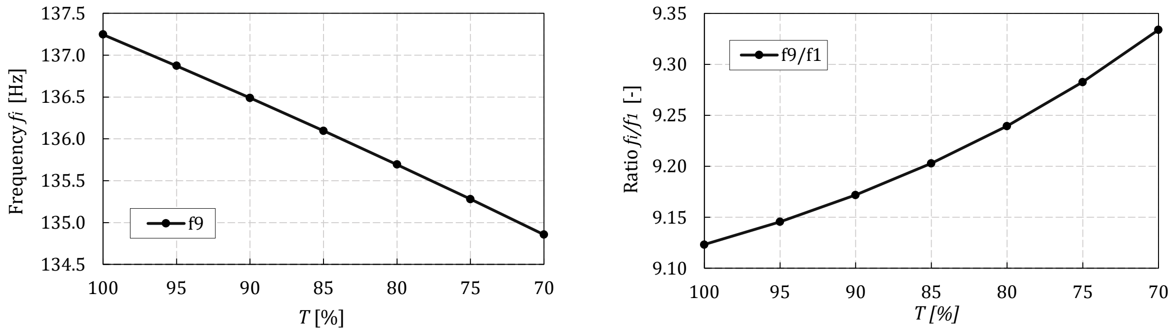


Figure 2. *FE analysis steps*

4. DISCUSSION OF RESULTS

4.1. Analytical model

The changes on PIs obtained by reducing the bending stiffness are negligible. Thus, the tension reduction results are discussed. Both the frequencies and the ratios are representative of damage presence as tension reduction occurs. Frequencies are the most sensitive indicator, especially the higher frequencies. Figure 3 shows the results for the ninth frequency.



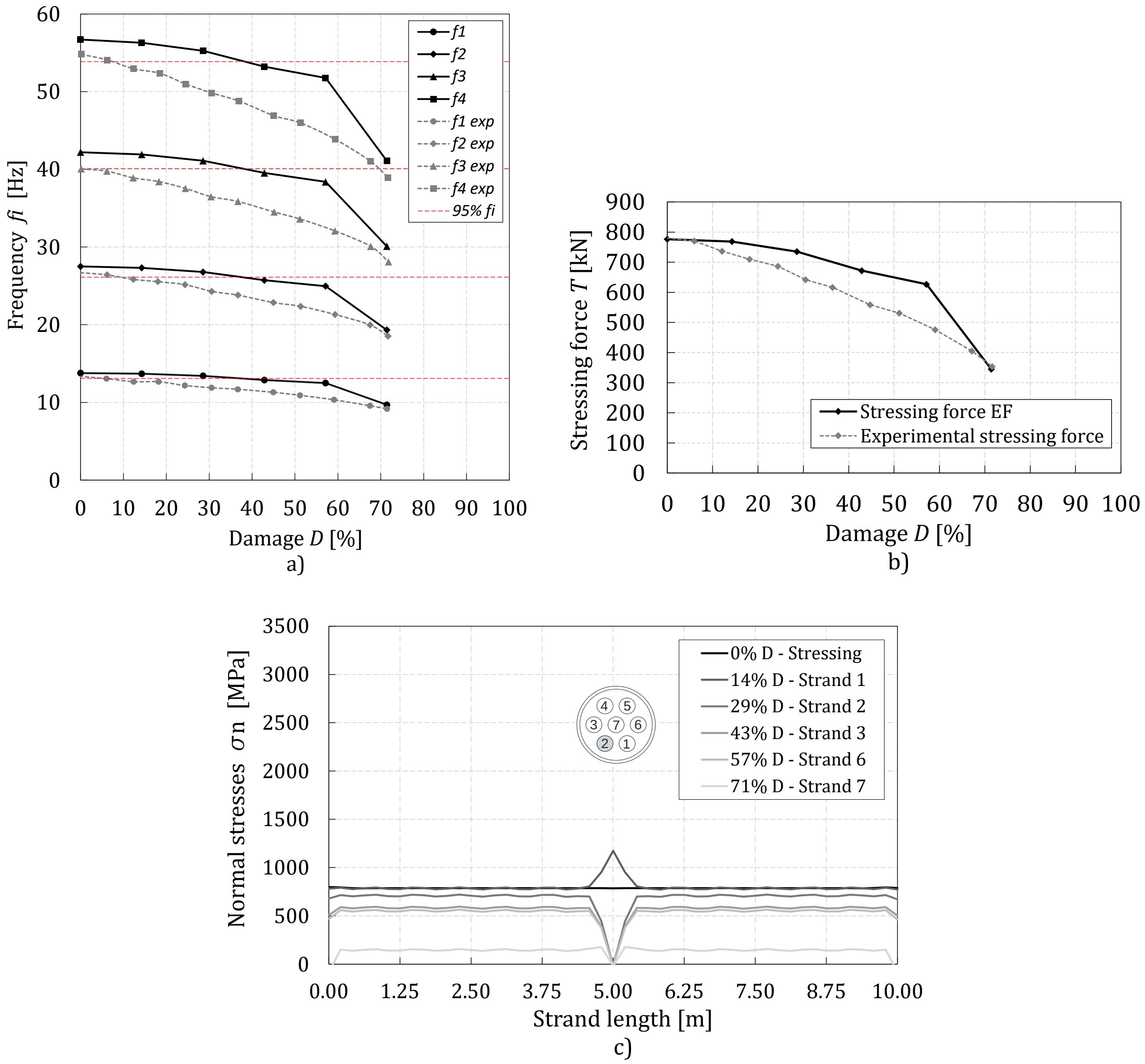
a) b)

Figure 3. *Analytical model tension reduction results. a) Frequency b) Ratios*

4.2. FE model

The frequencies and stressing force values are compared with the experimental results obtained in [3]. The results follow similar trends; however, differences arise in the damage intermediate levels mainly due to the high stiffness contact and the purely elastic materials assumed. The results show that a tendon anomaly is detected, as both the frequencies and the stressing force are lower as the damage increases (Figure 4a and Figure 4b).

The normal stresses along the strand 2 with successive strand breakages (strands 1,2,3,6 and 7) is shown in Figure 4c. The results show that stress of a strand increases when a neighbour strand breaks. When breakage occurs, the broken strand normal stress is only lost in the surroundings of the broken section because of the self-anchorage phenomena. Due to the high stiffness of the elastic grout-strand contact assumed, the relative sliding only takes place in the transfer length, the remaining length approximately keeps its stresses. The FE model reproduces with some limitations the mechanical behaviour of a prestressed tendon under successive breakages. Self-anchoring is fundamental to understanding why stressing forces do not decrease proportionally with the area of steel lost in the strand. The high stiffness contact may not be the most suitable, leading to an overestimation of the results compared to reality. The non-consideration of plasticity causes the stress redistribution when a strand breaks to be non-uniform, which seems to conclude that the stress redistribution in the strand is governed by the plastic nature of the steel.



a)

b)

c)

Figure 4. *FE results. a) Frequency b) Stressing force c) Normal stress along the strand 2*

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