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A SIMPLIFIED PROCEDURE TO TRACK THE TENSION FORCE: NUMERICAL VALIDATION

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Abstract. *Recently, bridge collapses due to brittle fracture of grouted post-tensioning external tendons have been reported, highlighting the importance of structural health monitoring to prevent and predict these situations that strongly compromise safety. Non-destructive testing techniques based on the vibration response have proven their reliability in this aspect, as they allow continuous monitoring of the tendon modal parameters and the estimation of tension force. The tension force can be indirectly estimated from the modal properties of the tendon. A simplified method to track the tension force is proposed, which consists of calibrating the tendon model by obtaining the equivalent modal lengths for each natural frequency in such a way that general boundary conditions are considered for each mode. This study proposes a first numerical validation of this method through a detailed finite element model of a tendon. The finite element tendon model is subjected to a given stressing force; the bending stiffness and all other relevant parameters of the tendon are also known, subsequently, a modal analysis is performed to obtain the natural frequencies. The first twenty natural frequencies numerically calculated are then used to estimate the tension force based on the simplified method presented previously, the modal length calibration and optimization for each natural frequency is carried out for this example, and the estimated tension force and bending stiffness can be obtained. These results are compared with the finite element model of the tendon and then, the proposed tension force estimation procedure can be validated.*

1 INTRODUCTION

External post-tensioning systems are commonly used in the construction and rehabilitation of highway and railway bridges due to two main advantages: i) the ease of tendon replacement and ii) the possibility of the re-tensioning of tendons. In addition, this technology stands out for being economical and for its ease of installation, monitoring, inspection, and maintenance. However, bridge collapses have been reported due to corrosion under mechanical stress of the grouted external tendons. In this type of tendons, when a strand breaks, the bond between the strands and the grout leads to the re-achoring of the strand in the grout and a stress redistribution to the adjacent strands [1]. Thus, it is difficult to detect corrosion damage from the modal properties and the estimated tension force, since the degradation of the effective tension force of the bonded tendons is not proportional to the increase of damage.

Thus, reliable methods to continuously estimate the effective tension force considering general boundary conditions are needed since direct measurements in existing tendons are difficult to carry out. This paper proposes a simplified methodology to estimate the effective tension force from the modal properties of the tendon, taking advantage of the relationship between the natural frequencies and the tension force [2]. Thus, from the general semirigid solutions, equivalent modal lengths for each considered mode are derived and then, the effective tension force is estimated using all the considered modes. The main objective of this paper is to numerically validate the method for estimating the tension force through a detailed FE model of a tendon where all the relevant properties are known: i) the tension force T, ii) the bending stiffness EI, iii) the mass per unit length ρ, iv) the total length L, and v) the natural frequencies f_i. Hence, assuming that only the natural frequencies are known (which are actually measured) and ρ and L are also known, T and EI are estimated.

The outline of this paper is as follows. In Section 2, the FE model of the tendon used for the validation is described, as well as the analysis followed. Section 3 presents the methodology for the estimation of tension force and compares the results obtained with those of the FE model. Section 4 addresses some conclusions.

2 FE MODEL

2.1 FE model description

The model reproduces a 31-strand grouted external tendon with a total length of 12.7 m that has been stressed with a tension force of 4500 kN. This type of tendon consists of prestressing steel strands surrounded by a HDPE duct that is filled with a grout injection, which adherently covers all the strands and provides some degree of corrosion protection. The grout has a density of 2250 kg/m^3 and an elastic modulus of 25 GPa.

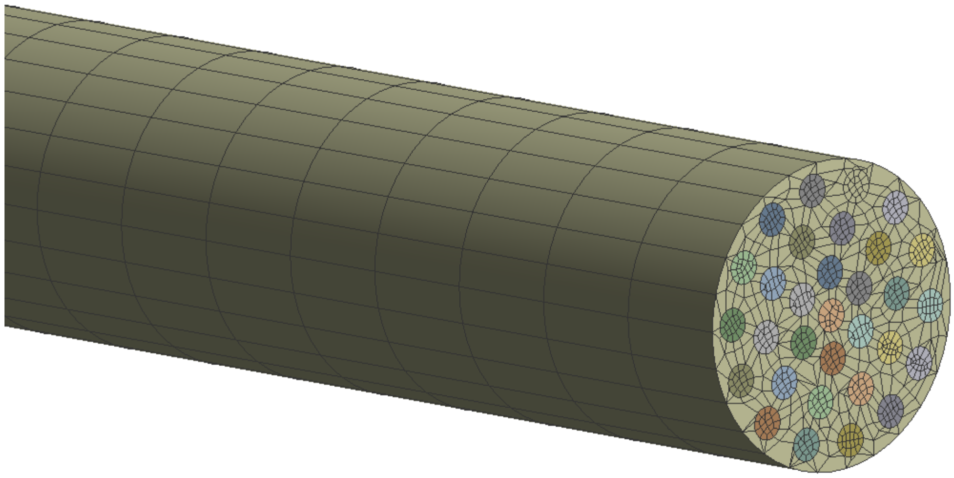

Figure 1: FE tendon used for the analysis.

The steel strands consist of six external wires that helically surround a central wire, but a circle with equivalent mechanical and material properties is assumed. Considering that the effective area of a strand A_{strand} is 139 mm^2 provided by the manufacturer, the equivalent diameter, ϕ_{eq}, is:

$$A_{strand} = \frac{\pi \phi_{eq}^2}{4} \rightarrow \phi_{eq} = 13.3mm. \tag{1}$$

The density of the steel strands is 7850 kg/m^3 and the elastic modulus E_s is 195.5 GPa. The internal diameter of the duct is 14 cm, its thickness is 6 mm, and its density is 950 kg/m^3. However, the duct is not included in the model geometry for simplicity and its mechanical behaviour has been replaced by modelling a perfectly elastic grout with an equivalent density $\rho_{g,eq}$ of 2547.7 kg/m^3, obtained as shown in equation 2:

$$\rho_{g,eq} = \frac{A_g \rho_g + A_d \rho_d}{A_g}, \tag{2}$$

where A_g is the cross-sectional area of the grout, ρ_g is the density of the grout, A_d is the cross-sectional area of the duct, and ρ_d is the density of the duct. Then, the total mass per unit length ρ can be derived.

Tendon equivalent bending stiffness EI is calculated as follows:

$$EI = E_s I_s + E_g I_g, \tag{3}$$

where E_s is the elastic modulus of the steel strands, I_s is the moment of inertia of the steel cross-section, E_g is the elastic modulus of the grout, and I_g is the moment of inertia of the grout cross-section.

Thus, the relevant parameters of the FE model needed for the tension estimation are listed in Table 1.

Table 1: Relevant parameters for the validation of the tension force estimation procedure.

L [m]	T [kN]	EI [kNm^2]	ρ [kg/m]
12.7	4500	902.422	52.848

A bilinear isotropic hardening material model has been used for steel strands, as shown in Figure 2. Mechanical properties are listed in Table 2. The von Mises plasticity criterion with associative flow rule is assumed for the plasticity of the strands.

Table 2: Mechanical properties of steel strands.

E_s [GPa]	f_{py} [MPa]	ε_{py} [%]	f_{pu} [MPa]	ε_{pu} [%]	ρ_s [kg/m^3]
195.5	1760	0.9	1900	6	7850

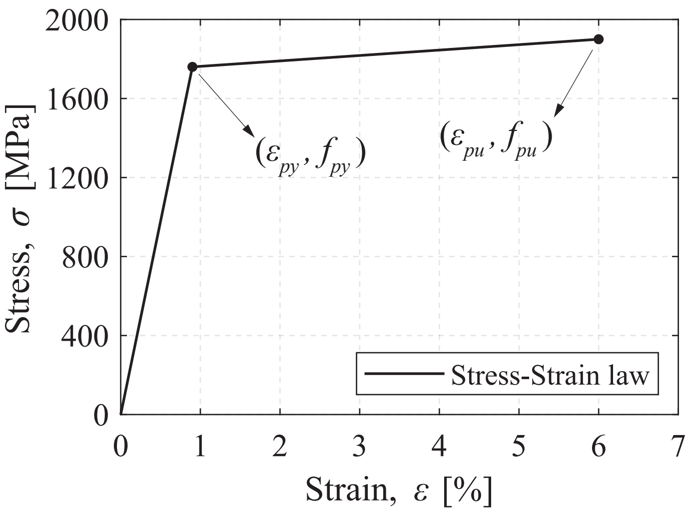

Figure 2: Steel strand stress-strain law.

The boundary conditions on the FE model are fixed supports at both ends of the strands, so displacements and rotations in all directions are constrained. The interface of steel and grout is modelled using a bonded contact, the nodes of the grout element are shared with those of the strand elements.

2.2 Analysis description

A non-linear static analysis divided into different load steps has been performed. During the analysis, the tensioning of the strands and the grouting of the tendon are modelled in different steps. Different factors cause the non-linearity: i) the non-linear constitutive law of steel, ii) the consideration of large deformations, and iii) the activation of grout elements to reproduce grout injection. A sparse direct solver and the Newton-Raphson algorithm with convergence in forces and displacement have been used. The steps of the analysis are the following:

1. Stressing of the strands by applying a thermal load: The stressing force of the tendon F_{pu} is 4500 kN, which corresponds to the 55% of the strands ultimate limit strength f_{pu}. This stress state has been achieved in the model by applying a thermal load to the steel strands (before the grout elements have been activated) with a temperature decrease ΔT equal to:

$$\Delta T = \frac{F_{pu}}{E_s A_s \alpha}, \tag{4}$$

where A_s is the steel area of the tendon and α is the thermal expansion coefficient, taken as $1.2 \cdot 10^{-5}$ ºC^{-1}.

2. Grout activation to simulate the injection.

Then a modal analysis is performed to obtain the frequencies used for the estimation procedure of the tension force assessed in Section 3.

The results obtained from the FE analysis are the first 20 natural frequencies (which are expected to be measured experimentally), which are shown in Table 3. These results are the input for the tension estimation procedure, whose methodology and results obtained are presented in Section 3.

Table 3: Frequencies obtained from the FE analysis.

Modal order	Frequency f_i [Hz]	Modal order	Frequency f_i [Hz]
1	12.815	11	208.25
2	26.04	12	237.84
3	40.054	13	269.43
4	55.186	14	302.97
5	71.704	15	338.45
6	89.815	16	375.83
7	109.67	17	415.06
8	131.39	18	456.11
9	155.03	19	498.92
10	180.64	20	543.45

3 TENSION FORCE ESTIMATION

3.1 Methodology

The methodology for estimating the tension force consists of calibrating the general analytical model of the tendon considering several natural frequencies. That is, assuming a tendon with rotational stiffness at both ends as shown in Figure 3. The equation that governs the transverse displacement of a taut tendon with bending stiffness is as follows:

$$EI\frac{\partial^4 v(x,t)}{\partial x^4} - T\frac{\partial^2 v(x,t)}{\partial x^2} + \rho\frac{\partial^2 v(x,t)}{\partial t^2} = 0, \tag{5}$$

with the following general boundary conditions:

$$v(0) = 0, \tag{6}$$

$$v(L) = 0, \tag{7}$$

$$EIv''(0) = k_{r1}v'(0), \tag{8}$$

$$EIv''(L) = k_{r1}v'(L), \tag{9}$$

where $v(x,t)$ [m] is the transversal displacement. Numerical methods are required to solve this equation since it has no analytical solution. Therefore, the solution of this equation cannot be used directly to estimate in-line and continuously (of a monitoring system) the effective tension force. An optimisation algorithm has to be executed to carry out the estimation of the effective tension force, which becomes inoperative. Thus, the proposed methodology calibrates an equivalent modal length for each mode and the analytical solution for the simply supported taut cable applies.

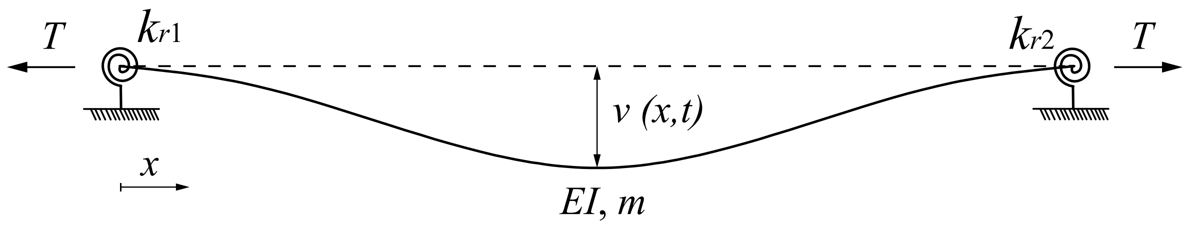

Figure 3: Tendon with bending stiffness and rotational springs at both ends.

The proposed tension estimation procedure is as follows:

1. The frequencies obtained from the FE analysis f_i, and shown in Table 3, are used as an input to the estimation procedure based on the calibration of the continuous model (Equations 5-9). The results obtained from this calibration are: i) the tension force T_c, ii) the bending stiffness EI_c, iii) the stiffness of the one-end rotational spring $k_{r1,c}$, and iv) the stiffness of the rotational spring at the other end $k_{r2,c}$. A genetic algorithm programmed in Matlab is used to carry out the calibration, where the objective function is the difference between the frequencies obtained from the estimated tension and the frequencies obtained from the FE model (Table 3). The calibration is performed a certain number of times, in this case, 8 times, to obtain weighted averages of these values as a function of the inverse of the minimised objective function, so that more importance is given to the result that leads to a lower value of the objective function.

2. Taking into account that the tension force can be written as [3]:

$$T = 4\rho L_i^2 \left(\frac{f_i}{i}\right)^2 - \frac{EI}{L_i^2} i^2 \pi^2, \tag{10}$$

   for each mode i. Minimizing the error between T and T_c, obtained from calibrating the general tendon (equation 10), L_i for each considered mode is extracted. Thus, in what follows to the stimated tension force $\widehat{T}$ will be derived from equation 10.

3. Applying equation 10, 20 values of T will be derived so that the estimated value will be the mean value.

4. Applying equation 10, to two frequencies, an estimated value of $\widehat{EI}$ can be derived [3]:

$$EI = \frac{\rho L^4}{\pi^4} \left[\left(\frac{2\pi f_r}{r\gamma_r} \right)^2 - \left(\frac{2\pi f_s}{s\gamma_s} \right)^2 \right] \frac{1}{r^2\gamma_r^2 - s^2\gamma_s^2}, \tag{11}$$

$\widehat{EI}$ is the average value.

5. Finally, using equation 10, the first 20 natural frequencies $\widehat{f}_i$ are estimated using $\widehat{T}$ and $\widehat{EI}$. The root mean square error between the calculated natural frequencies and the frequencies obtained from the FE model is calculated as:

$$RME = \sqrt{\frac{1}{20} \sum \left(\frac{f_i - \widehat{f}_i}{f_i} \right)^2}. \tag{12}$$

The RME should be small enough. It does mean that L_i calculated are correct.

3.2 Results

The first step for the tension force estimation is the calibration of the mathematical model using a genetic algorithm. The input parameters are the frequencies f_i obtained from the FE analysis and presented in Table 3, the length of the tendon L and the mass per unit length ρ, which are listed in Table 2, are considered to be known. The results obtained are the tension force T_c calibrated, the bending stiffness EI_c calibrated, and the stiffness of the rotational springs at both ends $k_{r1,c}$ and $k_{r2,c}$, and are shown in Table 4.

Table 4: Results of the mathematical model calibration.

T_c [kN]	EI_c [kNm^2]	$k_{r1,c}$ [N/m^2/m]	$k_{r2,c}$ [N/m^2/m]
4870	809	$3.5066 \cdot 10^{14}$	$5.5234 \cdot 10^{14}$

Then, the values of L_i are obtained using T_c and EI_c, so the tension force $\widehat{T}$ and bending stiffness $\widehat{EI}$ can be estimated using the equations 10 and 11. These results are shown in Table 5. As can be seen, the estimated results coincide with the results of the mathematical model calibration.

Table 5: Results of $\widehat{T}$ and $\widehat{EI}$ estimation.

$\widehat{T}$ [kN]	$\widehat{EI}$ [kNm^2]
4870	809

Then, equation 5 is solved using numerical methods (as it does not have an analytical solution) to obtain the frequencies $f_{i,num}$ from the estimated tension force T and estimated bending stiffness EI. These results are shown in Table 6. These numerical frequencies ($f_{i,num}$) are compared with those obtained from the FE analysis (f_i) to evaluate the error and determine whether it is acceptable. Hence, the root mean square error is obtained from equation 12 is $7.37 \cdot 10^{-5}$, a sufficiently low value to consider that the estimation performed is correct.

Table 6: Frequencies obtained numerically from the estimated T and EI.

Modal order	Frequency $f_{i,num}$ [Hz]	Modal order	Frequency $f_{i,num}$ [Hz]
1	12.836	11	207.90
2	26.06	12	237.94
3	40.036	13	270.22
4	55.085	14	304.77
5	71.481	15	341.62
6	89.448	16	380.78
7	109.13	17	422.28
8	130.77	18	466.12
9	154.38	19	512.32
10	180.06	20	560.88

However, as mentioned before, this procedure presents an important drawback related to resolution time. In this case, the genetic algorithm has taken 1h 27min to complete the calibration of the mathematical model.

4 CONCLUSIONS

A simplified procedure to estimate the effective tension force in external post-tensioning tendons has been proposed. A FE model of a tendon has been created as a specimen. Thus, the natural frequencies of this model are the inputs to the simplified procedure, which makes use of a preliminary calibration of equivalent model lengths considering general boundary conditions. The application of this method avoids to solve the general partial differential equation of the tendon with bending stiffness, which cannot be solved in line since it has not analytical solution. The next step is to validate the proposed procedure in real tendons in a bridge.

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