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RESEARCH

Universal Deformations in Compressible Isotropic Cauchy Elastic Solids with Residual Stress

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Abstract
We investigate universal deformations in compressible isotropic Cauchy elastic solids with residual stress, without assuming any specific source for the residual stress. We show that universal deformations must be homogeneous, and the associated residual stresses must also be homogeneous. Since a non-trivial residual stress cannot be homogeneous, it follows that residual stress must vanish. Thus, a compressible Cauchy elastic solid with a non-trivial distribution of residual stress cannot admit universal deformations. These findings are consistent with the results of Yavari and Goriely (Proc. R. Soc. A 472(2196):20160690, 2016), who showed that in the presence of eigenstrains, universal deformations are covariantly homogeneous and in the case of simply-connected bodies the universal eigenstrains are zero-stress (impotent).

Keywords Universal deformation · Cauchy elasticity · Hyperelasticity · Green elasticity · Isotropic solids · Residual stress

Mathematics Subject Classification 35Q74 · 74B05 · 74B10 · 74E10

1 Introduction

A *universal deformation* is one that can be maintained in the absence of body forces for all materials within a given class, by the application of boundary tractions alone. While the required boundary tractions depend on the specific material, the deformation itself does not. The concept of universal deformations (sometimes also referred to as "controllable" or "general") was introduced by Ericksen in his seminal papers [16, 17], building on the earlier

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work of Rivlin [50–52]. Ericksen proved that in homogeneous compressible isotropic solids all universal deformations are homogeneous [17], while for homogeneous incompressible isotropic solids he discovered four nontrivial families of universal deformations [16]. Ericksen had conjectured that deformations with constant principal invariants must be homogeneous. This was later shown to be false by Fosdick [18], and subsequently a fifth family of universal deformations, consisting of inhomogeneous deformations with constant principal invariants, was discovered independently by Singh and Pipkin [59] and Klingbeil and Shield [32]. The question of whether there are additional inhomogeneous constant-principal invariant universal deformations remains open [19, 20, 31, 38–40].

Universal deformations have played an important role in nonlinear elasticity and related theories. They have been central to semi-inverse solution methods [26, 33, 62], to the design of experiments for determining constitutive equations [54, 55], and to the construction of exact solutions for distributed eigenstrains and defects [21–23, 67, 69, 74–79, 92]. They have been used as benchmark problems in computational mechanics [12, 14, 55, 58], and in deriving effective properties for nonlinear composites [24, 28, 37].

In linear elasticity, their counterparts are *universal displacements* [10, 27, 64, 90], whose dependence on material symmetry has been systematically studied. Isotropic solids admit the largest class of universal displacements, while triclinic solids admit the smallest. This analysis has been extended to inhomogeneous solids [83], linear Cauchy elasticity [88], and to linear anelasticity [82].

Extensions of Ericksen's analysis have included anisotropy [68, 81, 84], implicit elasticity [85], Cauchy elasticity [71], and anelasticity [25, 80]. Despite the more general constitutive structure of Cauchy elasticity, the universal deformations and universal inhomogeneities in this setting coincide with those of Green elasticity (hyperelasticity). In anelasticity, it has been shown that universal deformations are covariantly homogeneous [80], and in incompressible anelasticity the universal eigenstrain distributions corresponding to the six known families were identified in [25]. Recent studies have also considered accreting bodies [47, 87, 91] and liquid crystal elastomers [36, 44].

The role of other internal constraints has also received attention, particularly for fiber-reinforced solids. Homogeneous compressible isotropic solids reinforced with inextensible fibers were studied by Beskos [4], who examined whether the universal deformations of incompressible isotropic solids remain universal in this setting. A similar study for incompressible isotropic solids was carried out in [5]. Beatty [2, 3] considered homogeneous compressible isotropic solids reinforced with a single family of inextensible fibers and identified the fiber distributions for which homogeneous deformations are universal. He showed that only three such distributions exist, and in all three cases the fibers are straight lines in both the reference and deformed configurations. More recently, Yavari [70] studied universal displacements in compressible anisotropic linear elastic solids reinforced by a uniform distribution of inextensible straight fibers and characterized the corresponding sets for each compatible symmetry class. Yavari [72] presented the first systematic classification of universal deformations in compressible isotropic Cauchy elastic solids reinforced by a single family of inextensible fibers. For straight deformed fibers, he identified a new inhomogeneous non-isochoric family, denoted *Family* Z_1. He also showed that if all principal invariants are constant, only homogeneous universal deformations are possible. When fibers have non-vanishing curvature, the universality constraints become significantly more complex, and the existence of universal deformations in this case remains open. Other types of internal constraints have been studied, including the inexpansibility constraint [35], and in-plane rigidity [13].

Despite this extensive literature, the effect of *residual stress* on universality has not been systematically examined. Residual stresses, which are self-equilibrated in the absence of external loads, are ubiquitous in both natural and engineered solids, arising from growth, plastic deformation, thermal processes, or other anelastic mechanisms [6, 26, 29, 30, 42, 43, 57]. They modify the constitutive equations and raise the question of whether universal deformations can exist in their presence. In this paper, we study universal deformations in compressible isotropic Cauchy elastic solids with residual stress. We show that universal deformations are necessarily homogeneous and that universal residual stresses must also be homogeneous. Because nontrivial residual stresses are necessarily inhomogeneous, homogeneous residual stresses must vanish. Therefore, a compressible isotropic Cauchy elastic solid with a nontrivial residual stress distribution cannot admit universal deformations, consistent with the results of Yavari and Goriely [80] for eigenstrains, where universal eigenstrains were shown to be zero-stress (impotent).

This paper is organized as follows. In §2, Cauchy elasticity is briefly reviewed. The work of Yavari and Goriely [80] is extended to Cauchy anelasticity in §3. Universal deformations of compressible isotropic Cauchy elasticity with residual stress are characterized in §4. Conclusions are given in §5.

2 Cauchy Elasticity

Let us consider a body whose undeformed configuration is identified with an embedded submanifold $\mathcal{B}$ of the Euclidean ambient space $\mathcal{S}$. The flat metric of the ambient space is denoted by $\mathbf{g}$, and the induced metric on the reference configuration is $\mathbf{G} = \mathbf{g}\big|_{\mathcal{B}}$. A deformation is a smooth map $\varphi : \mathcal{B} \to \mathcal{C} \subset \mathcal{S}$, where $\mathcal{C} = \varphi(\mathcal{B})$ denotes the deformed configuration. The tangent map of φ defines the deformation gradient $\mathbf{F} = T\varphi$, a metric-independent linear map $\mathbf{F}(X) : T_X\mathcal{B} \to T_{\varphi(X)}\mathcal{C}$ at each material point $X \in \mathcal{B}$. In local coordinates $\{X^A\}$ on $\mathcal{B}$ and $\{x^a\}$ on $\mathcal{C}$, the deformation gradient has components $F^a{}_A = \partial\varphi^a/\partial X^A$ and the representation $\mathbf{F} = F^a{}_A \, \frac{\partial}{\partial x^a} \otimes dX^A$, where $\{\frac{\partial}{\partial x^a}\}$ and $\{dX^A\}$ are the coordinate bases for the tangent space $T_x\mathcal{C}$ and the cotangent space $T_X^*\mathcal{B}$, respectively [1]. With respect to a local coordinate chart $\{x^a\}$ on $\mathcal{C}$, g_{ab} are components of the spatial metric $\mathbf{g}$. The inverse spatial metric $\mathbf{g}^{-1} = \mathbf{g}^\sharp$ has components g^{ab} such that $g^{am} g_{mb} = \delta^a_b$.[1] Similarly, With respect to a local coordinate chart $\{X^A\}$ on $\mathcal{B}$, G_{AB} are components of the material metric $\mathbf{G}$. The inverse material metric $\mathbf{G}^{-1} = \mathbf{G}^\sharp$ has components G^{AB} such that $G^{AM} G_{MB} = \delta^A_B$.

The transpose of deformation gradient $\mathbf{F}^\mathsf{T}(X) : T_x\mathcal{C} \to T_X\mathcal{B}$ has components $(F^\mathsf{T})^A{}_a = g_{ab} \, F^b{}_B \, G^{AB}$. The right Cauchy-Green strain tensor is defined by $\mathbf{C} = \mathbf{F}^\mathsf{T}\mathbf{F}$, with components $C^A{}_B = (F^\mathsf{T})^A{}_a \, F^a{}_B$. This gives the standard expression

$$C_{AB} = (g_{ab} \circ \varphi) \, F^a{}_A \, F^b{}_B \,, \tag{2.1}$$

which shows that the right Cauchy-Green strain is the pull-back of the spatial metric, i.e., $\mathbf{C}^\flat = \varphi^*\mathbf{g} = \mathbf{F}^\star\mathbf{g}\mathbf{F}$, where the flat operator is defined via the reference metric $\mathbf{G}$, and $\mathbf{F}^\star$ is dual of the deformation gradient (a metric-independent two-point tensor) defined as $\mathbf{F}^\star(X) = F^a{}_A \, dX^A \otimes \frac{\partial}{\partial x^a} : T_x^*\mathcal{C} \to T_X^*\mathcal{B}$.

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[1] A metric $\mathbf{g}$ induces the isomorphism $\sharp : T^*\mathcal{C} \to T\mathcal{C}$, which is called the sharp operator and is used for raising indices. For example, given a 1-form (a co-vector) $\boldsymbol{\alpha} = \alpha_a \, dx^a$, $\boldsymbol{\alpha}^\sharp = g^{ab} \, \alpha_b \, \frac{\partial}{\partial x^a}$ is its corresponding vector. Similarly, $\mathbf{g}$ induces the isomorphism $\flat : T\mathcal{C} \to T^*\mathcal{C}$, which is called the flat operator and is used for lowering indices. For example, given a vector $\mathbf{u} = u^a \, \frac{\partial}{\partial x^a}$, $\mathbf{u}^\flat = g_{ab} \, u^b \, dx^a$ is its corresponding 1-form (co-vector).

The left Cauchy-Green strain tensor is defined as $\mathbf{B}^\sharp = \varphi^*(\mathbf{g}^\sharp) = \mathbf{F}^{-1}\mathbf{g}^\sharp\mathbf{F}^{-\star}$, with components $B^{AB} = F^{-A}{}_a\, F^{-B}{}_b\, g^{ab}$. The spatial counterpart of $\mathbf{C}^\flat$ is $\mathbf{c}^\flat = \varphi_*\mathbf{G} = \mathbf{F}^{-\star}\mathbf{G}\mathbf{F}^{-1}$, whose components are $c_{ab} = F^{-A}{}_a\, F^{-B}{}_b\, G_{AB}$. Likewise, the spatial analogue of $\mathbf{B}^\sharp$ is $\mathbf{b}^\sharp = \varphi_*(\mathbf{G}^\sharp) = \mathbf{F}\mathbf{G}^\sharp\mathbf{F}^\star$, with components $b^{ab} = F^a{}_A\, F^b{}_B\, G^{AB}$. Note that $\mathbf{b} = \mathbf{c}^{-1}$. For more details on the geometric formulation of elasticity see [41, 73].

The tensors $\mathbf{C}$ and $\mathbf{b}$ share the same principal invariants I_1, I_2, and I_3, which are defined as follows [41, 46]:

$$I_1 = \operatorname{tr}\mathbf{b} = b^{ab}g_{ab}\,, \qquad I_2 = \frac{1}{2}\left(I_1^2 - \operatorname{tr}\mathbf{b}^2\right) = \frac{1}{2}\left(I_1^2 - b^{ab}b^{cd}g_{ac}g_{bd}\right)\,, \qquad I_3 = \det\mathbf{b}\,. \tag{2.2}$$

In Cauchy elasticity, the stress at a material point and at a given instant depends explicitly on the strain at that same point and time [11, 63, 65, 86]. However, the existence of an energy function is not guaranteed.[2] In terms of the first Piola-Kirchhoff stress tensor [46, 63, 65], one has

$$\mathbf{P} = \hat{\mathbf{P}}(X, \mathbf{F}, \mathbf{G}, \mathbf{g})\,. \tag{2.3}$$

Objectivity implies that the second Piola-Kirchhoff stress must take the form [65]

$$\mathbf{S} = \hat{\mathbf{S}}(X, \mathbf{C}^\flat, \mathbf{G})\,. \tag{2.4}$$

For isotropic materials, one arrives at the classical representation [7, 53, 66]

$$\mathbf{S} = \Lambda_0\mathbf{G}^\sharp + \Lambda_1\mathbf{C}^\sharp + \Lambda_{-1}\mathbf{C}^{-\sharp}\,, \tag{2.5}$$

where $\Lambda_i = \Lambda_i(X, I_1, I_2, I_3)$, $i = -1, 0, 1$, and $\sharp$ denotes the sharp operator associated with the metric $\mathbf{G}$.

The Cauchy stress for compressible isotropic Cauchy elastic materials admits the representation

$$\boldsymbol{\sigma} = \alpha\, \mathbf{g}^\sharp + \beta\mathbf{b}^\sharp + \gamma\mathbf{c}^\sharp\,, \tag{2.6}$$

where $\alpha = \alpha(I_1, I_2, I_3)$, $\beta = \beta(I_1, I_2, I_3)$, and $\gamma = \gamma(I_1, I_2, I_3)$ are arbitrary constitutive functions, where the principal invariants are given in (2.2).

2.1 Cauchy Elastic Solids with Distributed Eigenstrains

An anelastic body with eigenstrains is residually-stressed, in general. Deformation gradient is multiplicatively decomposed as $\mathbf{F} = \overset{e}{\mathbf{F}}\overset{a}{\mathbf{F}}$, where $\overset{e}{\mathbf{F}}$ and $\overset{a}{\mathbf{F}}$ are the elastic and anelastic distortions, respectively.[3] These distortions are not compatible, in general. Imagine that the body in its reference configuration is partitioned into a large number of small (infinitesimal) pieces. The anelastic distortion $\overset{a}{\mathbf{F}}$ maps each small piece to its local relaxed state. Let us consider an infinitesimal line element in the reference configuration. It is represented as

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[2] It is important to emphasize that Cauchy elasticity does not describe all elastic materials. A more general class of elastic solids are described by implicit constitutive relations of the form $\mathbf{f}(\boldsymbol{\sigma}, \mathbf{b}) = \mathbf{0}$ [45, 48, 49]. Cauchy elasticity is a special case within this broader class.

[3] See [56, 89] for more details on this decomposition and its history.

$\mathbf{N}dS$, where $\mathbf{N}$ is a unit vector. $\overset{a}{\mathbf{F}}\mathbf{N}dS$ is the relaxed line element. Suppose the Euclidean metric on the body is $\mathring{\mathbf{G}}$. The squared length of the relaxed line element is calculated as

$$\langle\!\langle \overset{a}{\mathbf{F}}\mathbf{N}dS, \overset{a}{\mathbf{F}}\mathbf{N}dS \rangle\!\rangle_{\mathring{\mathbf{G}}} = \langle\!\langle \mathbf{N}dS, \mathbf{N}dS \rangle\!\rangle_{\overset{a}{\mathbf{F}}{}^*\mathring{\mathbf{G}}}\,, \tag{2.7}$$

where $\mathbf{G} = \overset{a}{\mathbf{F}}{}^*\mathring{\mathbf{G}} = \overset{a}{\mathbf{F}}{}^\star\mathring{\mathbf{G}}\overset{a}{\mathbf{F}}$ is called the material metric. It is non-flat, i.e., it has non-vanishing Riemann curvature, in general. This idea goes back to Eckart [15] and Kondo [34].

In the presence of eigenstrains, the constitutive equations of a Cauchy anelastic solid have the form (2.3) or (2.4) when $\mathbf{G}$ is the material metric, see [86] for more details.

2.2 Cauchy Elastic Solids with Residual Stress

Let us consider a body composed of a compressible Cauchy elastic solid that, in the absence of residual stress, is isotropic. The second Piola-Kirchhoff and Cauchy stress tensors are denoted by $\mathbf{S}$ and $\boldsymbol{\sigma}$, respectively. We assume that the body is residually stressed and denote the second Piola-Kirchhoff residual stress by $\mathring{\mathbf{S}}$. Residual stress is self-equilibrated in the absence of body forces and boundary tractions; thus, $\operatorname{Div}\mathring{\mathbf{S}} = \mathbf{0}$ in $\mathcal{B}$ and $\mathring{\mathbf{S}}\mathbf{N} = \mathbf{0}$ on $\partial\mathcal{B}$, where $\mathbf{N}$ is the unit normal to the undeformed boundary, and Div is the divergence operator with respect to the material metric $\mathbf{G}$. It should be emphasized that the divergence with respect to $\mathbf{G}$ arises solely in the context of a residual stress field. When writing the equilibrium equations in terms of the second Piola-Kirchhoff stress, the divergence operator is taken with respect to the metric $\mathbf{C}^\flat = \varphi^*\mathbf{g}$. However, for $\mathring{\mathbf{S}}$ one has $\mathring{\varphi} = \iota : \mathcal{B} \hookrightarrow \mathcal{S}$, where ι is the inclusion map with $\iota^*\mathbf{g} = \mathbf{G}$. We denote the push-forward of $\mathring{\mathbf{S}}$ to the current configuration by $\mathring{\boldsymbol{\sigma}}$, i.e., $\mathring{\boldsymbol{\sigma}} = \varphi_*\mathring{\mathbf{S}} = \mathbf{F}\mathring{\mathbf{S}}\mathbf{F}^\star$. Although the expression for $\mathring{\boldsymbol{\sigma}}$ resembles that of the Kirchhoff stress, it is not the actual Kirchhoff stress arising from a constitutive law. It is simply the spatial representation of a pre-existing residual stress field, independent of the deformation.

Remark 2.1 Consider the inclusion map $\iota : \mathcal{B} \hookrightarrow \mathcal{S}$. The residual stress $\mathring{\mathbf{S}}$ induces a corresponding *Cauchy residual stress* defined by $\boldsymbol{\sigma}_0 = \iota_*\mathring{\mathbf{S}}$. With a slight abuse of notation, we identify these two tensors and simply work with $\mathring{\mathbf{S}}$.

The second Piola-Kirchhoff stress $\mathbf{S}$ is an isotropic function of $\mathbf{C}$ and $\mathring{\mathbf{S}}$. For a symmetric second-order tensor $\mathbf{S}$ that depends isotropically on two symmetric tensors, $\mathsf{A} = \mathbf{C}^\sharp$ and $\mathsf{B} = \mathring{\mathbf{S}}$, its representation can be expressed as [8, 9, 60]

$$\mathbf{S} = \sum_{j=1}^{m} \gamma_j\, \mathbf{G}_j\,, \qquad \gamma_j = \gamma_j(I_1, \ldots, I_n)\,, \tag{2.8}$$

where $\{I_1, \ldots, I_n\}$ is a set of polynomial scalar invariants that is assumed to be irreducible, i.e., no I_k in the set can be written as a polynomial function of the remaining invariants, and $\{\mathbf{G}_1, \ldots, \mathbf{G}_m\}$ is a set of generating tensors. For $\mathbf{S} = \mathbf{S}(\mathsf{A}, \mathsf{B})$, an irreducible integrity basis is

$$\begin{aligned} &I_1 = \operatorname{tr}\mathsf{A}\,, && I_2 = \operatorname{tr}\mathsf{A}^2\,, && I_3 = \operatorname{tr}\mathsf{A}^3\,, && I_4 = \operatorname{tr}\mathsf{B}\,, && I_5 = \operatorname{tr}\mathsf{B}^2\,, \\ &I_6 = \operatorname{tr}\mathsf{B}^3\,, && I_7 = \operatorname{tr}(\mathsf{AB})\,, && I_8 = \operatorname{tr}\left(\mathsf{A}^2\mathsf{B}\right), && I_9 = \operatorname{tr}\left(\mathsf{AB}^2\right), && I_{10} = \operatorname{tr}\left(\mathsf{A}^2\mathsf{B}^2\right), \end{aligned} \tag{2.9}$$

and a set of generating tensors is

$$\left\{\mathbf{G}^\sharp, \mathsf{A}, \mathsf{A}^2, \mathsf{B}, \mathsf{B}^2, \mathsf{AB} + \mathsf{BA}, \mathsf{A}^2\mathsf{B} + \mathsf{BA}^2, \mathsf{AB}^2 + \mathsf{B}^2\mathsf{A}\right\}. \tag{2.10}$$

Therefore, we have the following representation for $\mathbf{S}$:

$$\mathbf{S} = \gamma_0 \mathbf{G}^\sharp + \gamma_1 \mathsf{A} + \gamma_2 \mathsf{A}^2 + \gamma_3 \mathsf{B} + \gamma_4 \mathsf{B}^2 + \gamma_5 \left(\mathsf{AB} + \mathsf{BA}\right) + \gamma_6 \left(\mathsf{A}^2\mathsf{B} + \mathsf{BA}^2\right) + \gamma_7 \left(\mathsf{AB}^2 + \mathsf{B}^2\mathsf{A}\right) . \tag{2.11}$$

From the Cayley-Hamilton theorem, the square of a symmetric second-order tensor can be expressed as a linear combination of the tensor itself, its inverse, and the inverse metric tensor $\mathbf{G}^\sharp$. This follows because, for a second-order tensor $\mathbf{A}$, the Cayley-Hamilton theorem implies that $\mathbf{A}^2$ lies in the span of $\{\mathbf{G}^\sharp, \mathbf{A}, \mathbf{A}^{-1}\}$, with coefficients depending only on the principal invariants of $\mathbf{A}$. Therefore, the following is a set of generating tensors for $\mathbf{S}$

$$\left\{\mathbf{G}^\sharp, \mathbf{C}^\sharp, \mathbf{B}^\sharp, \mathring{\mathbf{S}}, \mathring{\mathbf{S}}^2, \mathbf{C}\mathring{\mathbf{S}} + \mathring{\mathbf{S}}\mathbf{C}, \mathbf{B}\mathring{\mathbf{S}} + \mathring{\mathbf{S}}\mathbf{B}, \mathbf{C}\mathring{\mathbf{S}}^2 + \mathring{\mathbf{S}}^2\mathbf{C}\right\} . \tag{2.12}$$

Therefore,

$$\mathbf{S} = \beta_0\, \mathbf{G}^\sharp + \beta_1 \mathbf{C}^\sharp + \beta_2\, \mathbf{B}^\sharp + \beta_3\, \mathring{\mathbf{S}} + \beta_4\, \mathring{\mathbf{S}}^2 + \beta_5 \big(\mathbf{C}\, \mathring{\mathbf{S}} + \mathring{\mathbf{S}}\, \mathbf{C}\big) + \beta_6 \big(\mathbf{B}\mathring{\mathbf{S}} + \mathring{\mathbf{S}}\mathbf{B}\big) + \beta_7 \big(\mathbf{C}\, \mathring{\mathbf{S}}^2 + \mathring{\mathbf{S}}^2\, \mathbf{C}\big) , \tag{2.13}$$

where the scalar-valued response functions $\beta_i = \beta_i(I_1, \ldots, I_{10})$ depend smoothly on the ten functionally independent invariants defined as [61]

$$\begin{aligned}
I_1 &= \operatorname{tr} \mathbf{C} = C_{AB}\, G^{AB} , \\
I_2 &= \frac{1}{2}\left(I_1^2 - \operatorname{tr} \mathbf{C}^2\right) = \frac{1}{2}\left(I_1^2 - C_{AB} C_{CD}\, G^{AC}\, G^{BD}\right) , \\
I_3 &= \det \mathbf{C} , \\
I_4 &= \operatorname{tr} \mathring{\mathbf{S}} = \mathring{S}^{AB} G_{AB} , \\
I_5 &= \operatorname{tr}(\mathring{\mathbf{S}}^2) = \mathring{S}^{AC}\, \mathring{S}_C{}^B\, G_{AB} , \\
I_6 &= \operatorname{tr}(\mathring{\mathbf{S}}^3) = \mathring{S}^{AD}\, \mathring{S}_D{}^C\, \mathring{S}_C{}^B\, G_{AB} , \\
I_7 &= \operatorname{tr}(\mathbf{C}\, \mathring{\mathbf{S}}) = C^A{}_C\, \mathring{S}^{CB}\, G_{AB} , \\
I_8 &= \operatorname{tr}(\mathbf{C}^2\, \mathring{\mathbf{S}}) = C^{AD}\, B_{DC}\, \mathring{S}^{CB}\, G_{AB} , \\
I_9 &= \operatorname{tr}(\mathbf{C}\, \mathring{\mathbf{S}}^2) = C^A{}_C\, \mathring{S}^{CD}\, \mathring{S}_D{}^B\, G_{AB} , \\
I_{10} &= \operatorname{tr}(\mathbf{C}^2\, \mathring{\mathbf{S}}^2) = C^A{}_C\, C^C{}_D\, \mathring{S}^D{}_E\, \mathring{S}^E{}_B\, G^{BA}
\end{aligned} \tag{2.14}$$

The Cauchy stress $\boldsymbol{\sigma}$ is an isotropic function of $\mathbf{b}$ and $\mathring{\boldsymbol{\sigma}}$. It has the following representation

$$\boldsymbol{\sigma} = \alpha_0\, \mathbf{g}^\sharp + \alpha_1 \mathbf{b}^\sharp + \alpha_2 \mathbf{c}^\sharp + \alpha_3 \mathring{\boldsymbol{\sigma}} + \alpha_4 \mathring{\boldsymbol{\sigma}}^2 + \alpha_5 \big(\mathbf{b}\mathring{\boldsymbol{\sigma}} + \mathring{\boldsymbol{\sigma}}\mathbf{b}\big) + \alpha_6 \big(\mathbf{c}\, \mathring{\boldsymbol{\sigma}} + \mathring{\boldsymbol{\sigma}}\, \mathbf{c}\big) + \alpha_7 \big(\mathbf{b}\, \mathring{\boldsymbol{\sigma}}^2 + \mathring{\boldsymbol{\sigma}}^2\, \mathbf{b}\big) , \tag{2.15}$$

where the scalar-valued response functions $\alpha_i = \alpha_i(I_1, \ldots, I_{10})$ depend smoothly on the ten functionally independent invariants, which are equivalently defined as

$$I_1 = \operatorname{tr} \mathbf{b}^\sharp = b^{ab}\, g_{ab} ,$$

$$\begin{aligned}
I_2 &= \frac{1}{2}\left(I_1^2 - b^{ab} b^{cd}\, g_{ac}\, g_{bd}\right) , \\
I_3 &= \det \mathbf{b} , \\
I_4 &= \operatorname{tr} \mathring{\boldsymbol{\sigma}} = \mathring{\sigma}^{ab} g_{ab} , \\
I_5 &= \operatorname{tr}(\mathring{\boldsymbol{\sigma}}^2) = \mathring{\sigma}^{ac}\, \mathring{\sigma}_c{}^b\, g_{ab} , \\
I_6 &= \operatorname{tr}(\mathring{\boldsymbol{\sigma}}^3) = \mathring{\sigma}^{ad}\, \mathring{\sigma}_d{}^c\, \mathring{\sigma}_c{}^b\, g_{ab} , \\
I_7 &= \operatorname{tr}(\mathbf{b} \cdot \mathring{\boldsymbol{\sigma}}) = b^a{}_c\, \mathring{\sigma}^{cb}\, g_{ab} , \\
I_8 &= \operatorname{tr}(\mathbf{b}^2 \cdot \mathring{\boldsymbol{\sigma}}) = b^{ad}\, b_{dc}\, \mathring{\sigma}^{cb}\, g_{ab} , \\
I_9 &= \operatorname{tr}(\mathbf{b} \cdot \mathring{\boldsymbol{\sigma}}^2) = b^a{}_c\, \mathring{\sigma}^{cd}\, \mathring{\sigma}_d{}^b\, g_{ab} , \\
I_{10} &= \operatorname{tr}(\mathbf{b}^2 \cdot \mathring{\boldsymbol{\sigma}}^2) = b^a{}_c\, b^c{}_d\, \mathring{\sigma}^d{}_e\, \mathring{\sigma}^e{}_b\, g^{ba} .
\end{aligned} \tag{2.16}$$

In coordinates, the Cauchy stress is written as

$$\begin{aligned}
\sigma^{ab} &= \alpha_0\, g^{ab} + \alpha_1\, b^{ab} + \alpha_2\, c^{ab} + \alpha_3\, \mathring{\sigma}^{ab} + \alpha_4\, \mathring{\sigma}^{ac}\, \mathring{\sigma}_c^b \\
&\quad + \alpha_5 \left(b_c^a\, \mathring{\sigma}^{cb} + \mathring{\sigma}^{ac}\, b_c^b\right) + \alpha_6 \left(c_c^a\, \mathring{\sigma}^{cb} + \mathring{\sigma}^{ac}\, c_c^b\right) \\
&\quad + \alpha_7 \left(b_c^a\, \mathring{\sigma}^{cd}\, \mathring{\sigma}_d^b + \mathring{\sigma}^{ac}\, \mathring{\sigma}_c^d\, b_d^b\right) .
\end{aligned} \tag{2.17}$$

3 Universal Deformations and Eigenstrain in Compressible Isotropic Cauchy Elasticity

In this section we briefly discuss universal deformations and eigenstrains in Cauchy anelasticity and extend the work of Yavari and Goriely [80] to Cauchy anelasticity.

When body forces are absent, the equilibrium equations take the form $\sigma^{ab}{}_{|b} = 0$.[4] Using the fact that the Riemannian metric is compatible with its Levi-Civita connection, i.e., $g^{ab}{}_{|c} = 0$, and therefore $g^{ab}{}_{|b} = 0$, we obtain

$$\sigma^{ab}{}_{|b} \;=\; \beta\, b^{ab}{}_{|b} \;+\; \gamma\, c^{ab}{}_{|b} \;+\; \alpha_{,b}\, g^{ab} \;+\; \beta_{,b}\, b^{ab} \;+\; \gamma_{,b}\, c^{ab} \;=\; 0 . \tag{3.1}$$

Note that

$$\begin{aligned}
\alpha_{,b} &= \frac{\partial \alpha}{\partial I_1}\, I_{1,b} + \frac{\partial \alpha}{\partial I_2}\, I_{2,b} + \frac{\partial \alpha}{\partial I_3}\, I_{3,b} , \\
\beta_{,b} &= \frac{\partial \beta}{\partial I_1}\, I_{1,b} + \frac{\partial \beta}{\partial I_2}\, I_{2,b} + \frac{\partial \beta}{\partial I_3}\, I_{3,b} , \\
\gamma_{,b} &= \frac{\partial \gamma}{\partial I_1}\, I_{1,b} + \frac{\partial \gamma}{\partial I_2}\, I_{2,b} + \frac{\partial \gamma}{\partial I_3}\, I_{3,b} .
\end{aligned} \tag{3.2}$$

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[4] $(.)_{|a}$ denotes the covariant derivative with respect to $\frac{\partial}{\partial x^a}$. In Cartesian coordinates this reduces to a partial derivative. For any scalar field f, one has $f_{|a} = f_{,a}$.

Equivalently,

$$\begin{aligned}
\alpha_{,b} &= \alpha_1\, I_{1,b} + \alpha_2\, I_{2,b} + \alpha_3\, I_{3,b}\,,\\
\beta_{,b} &= \beta_1\, I_{1,b} + \beta_2\, I_{2,b} + \beta_3\, I_{3,b}\,,\\
\gamma_{,b} &= \gamma_1\, I_{1,b} + \gamma_2\, I_{2,b} + \gamma_3\, I_{3,b}\,,
\end{aligned} \tag{3.3}$$

where

$$\alpha_i = \frac{\partial\alpha}{\partial I_i}\,, \qquad \beta_i = \frac{\partial\beta}{\partial I_i}\,, \qquad \gamma_i = \frac{\partial\gamma}{\partial I_i}\,, \qquad i = 1,2,3\,. \tag{3.4}$$

Substituting (3.3) into (3.1), one finds

$$\begin{aligned}
&\beta\, b^{ab}{}_{|b} + \gamma\, c^{ab}{}_{|b}\\
&+ \left(I_{1,b}\, g^{ab}\, \alpha_1 + I_{2,b}\, g^{ab}\, \alpha_2 + I_{3,b}\, g^{ab}\, \alpha_3\right)\\
&+ \left(I_{1,b}\, b^{ab}\, \beta_1 + I_{2,b}\, b^{ab}\, \beta_2 + I_{3,b}\, b^{ab}\, \beta_3\right)\\
&+ \left(I_{1,b}\, c^{ab}\, \gamma_1 + I_{2,b}\, c^{ab}\, \gamma_2 + I_{3,b}\, c^{ab}\, \gamma_3\right) = 0\,.
\end{aligned} \tag{3.5}$$

Since α, β, and γ are arbitrary functions, their derivatives are independent. Therefore, for equilibrium to hold for any compressible isotropic Cauchy anelastic solid, each coefficient must vanish separately:

$$\begin{cases}
b^{ab}{}_{|b} = c^{ab}{}_{|b} = 0\,, & (3.6)\\
g^{ab}\, I_{1,b} = g^{ab}\, I_{2,b} = g^{ab}\, I_{3,b} = 0\,, & (3.7)\\
b^{ab}\, I_{1,b} = b^{ab}\, I_{2,b} = b^{ab}\, I_{3,b} = 0\,, & (3.8)\\
c^{ab}\, I_{1,b} = c^{ab}\, I_{2,b} = c^{ab}\, I_{3,b} = 0\,. & (3.9)
\end{cases}$$

The universality constraints (3.7) imply that I_1, I_2, I_3 are constant. Consequently, the conditions (3.8) and (3.9) are automatically satisfied. In summary,

$$I_1, I_2, I_3 \text{ are constant}, \qquad b^{ab}{}_{|b} = c^{ab}{}_{|b} = 0\,. \tag{3.10}$$

These are exactly the universality conditions obtained by Yavari and Goriely [80] for hyperanelasticity. Recall that the principal invariants explicitly depend on eigenstrains through the material metric $\mathbf{G}$; for instance, $I_1 = \operatorname{tr}\mathbf{b} = b^{ab} g_{ab} = F^a{}_A\, F^b{}_B\, G^{AB} g_{ab}$. It was shown in [80] that (3.10) imply that universal deformations are covariantly homogeneous, and for a simply-connected body this means the universal eigenstrains are zero-stress (impotent). Here we observe that the same conclusion holds when, in the absence of eigenstrains, the body is composed of a Cauchy elastic solid.

4 Universal Deformations and Residual Stresses in Compressible Isotropic Cauchy Elasticity

For a residually-stressed Cauchy elastic body, equilibrium equations in the absence of body forces read $\operatorname{div}\boldsymbol{\sigma} = \mathbf{0}$. Substituting (2.15) into the equilibrium equations, one obtains

$$\begin{aligned}\operatorname{div}\boldsymbol{\sigma} &= \nabla\alpha_0 + \alpha_1 \operatorname{div}\mathbf{b}^\sharp + \alpha_2 \operatorname{div}\mathbf{c}^\sharp + \mathbf{b}^\sharp\cdot\nabla\alpha_1 + \mathbf{c}^\sharp\cdot\nabla\alpha_2 + \alpha_3 \operatorname{div}\mathring{\boldsymbol{\sigma}} + \mathring{\boldsymbol{\sigma}}\cdot\nabla\alpha_3 \\ &\quad + \alpha_4 \operatorname{div}\mathring{\boldsymbol{\sigma}}^2 + \mathring{\boldsymbol{\sigma}}^2\cdot\nabla\alpha_4 + \alpha_5 \operatorname{div}\big(\mathbf{b}\,\mathring{\boldsymbol{\sigma}} + \mathring{\boldsymbol{\sigma}}\,\mathbf{b}\big) + \big(\mathbf{b}\,\mathring{\boldsymbol{\sigma}} + \mathring{\boldsymbol{\sigma}}\,\mathbf{b}\big)\cdot\nabla\alpha_5 \\ &\quad + \alpha_6 \operatorname{div}\big(\mathbf{c}\,\mathring{\boldsymbol{\sigma}} + \mathring{\boldsymbol{\sigma}}\,\mathbf{c}\big) + \big(\mathbf{c}\,\mathring{\boldsymbol{\sigma}} + \mathring{\boldsymbol{\sigma}}\,\mathbf{c}\big)\cdot\nabla\alpha_6 \\ &\quad + \alpha_7 \operatorname{div}\big(\mathbf{b}\,\mathring{\boldsymbol{\sigma}}^2 + \mathring{\boldsymbol{\sigma}}^2\,\mathbf{b}\big) + \big(\mathbf{b}\,\mathring{\boldsymbol{\sigma}}^2 + \mathring{\boldsymbol{\sigma}}^2\,\mathbf{b}\big)\cdot\nabla\alpha_7 = \mathbf{0}\,.\end{aligned} \tag{4.1}$$

Notice that, in general, $\operatorname{div}\mathring{\boldsymbol{\sigma}} \neq \mathbf{0}$.

We aim to characterize all deformations that can be maintained in the absence of body forces. That is, we seek pairs of deformations and residual stresses $(\varphi, \mathring{\boldsymbol{\sigma}})$ for which the equilibrium equations admit a solution for arbitrary strain-energy density functions. We note that such pairs of deformations and residual stresses solve the equilibrium equations for any isotropic Cauchy elastic solid. This implies that the constitutive response functions—and their derivatives—can be chosen arbitrarily. In particular, we must have $\nabla\alpha_0 = \mathbf{0}$. But observe that

$$\nabla\alpha_0 = \sum_{j=1}^{10} \frac{\partial\alpha_0}{\partial I_j}\,\nabla I_j = \mathbf{0}\,. \tag{4.2}$$

Therefore, the arbitrariness of the partial derivatives of α_0 with respect to the ten invariants implies that

$$I_1, \ldots, I_{10} \ \text{ are constant}\,. \tag{4.3}$$

Now the equilibrium equations (4.1) are simplified to read

$$\begin{aligned}&\alpha_1 \operatorname{div}\mathbf{b}^\sharp + \alpha_2 \operatorname{div}\mathbf{c}^\sharp + \alpha_3 \operatorname{div}\mathring{\boldsymbol{\sigma}} + \alpha_4 \operatorname{div}\mathring{\boldsymbol{\sigma}}^2 + \alpha_5 \operatorname{div}\big(\mathbf{b}\,\mathring{\boldsymbol{\sigma}} + \mathring{\boldsymbol{\sigma}}\,\mathbf{b}\big) \\ &\quad + \alpha_6 \operatorname{div}\big(\mathbf{c}\,\mathring{\boldsymbol{\sigma}} + \mathring{\boldsymbol{\sigma}}\,\mathbf{c}\big) + \alpha_7 \operatorname{div}\big(\mathbf{b}\,\mathring{\boldsymbol{\sigma}}^2 + \mathring{\boldsymbol{\sigma}}^2\,\mathbf{b}\big) = \mathbf{0}\,.\end{aligned} \tag{4.4}$$

In (4.1), as α_1 and α_2 can be chosen arbitrarily, their coefficients must vanish, and hence $\operatorname{div}\mathbf{b}^\sharp = \operatorname{div}\mathbf{c}^\sharp = \mathbf{0}$. These together with knowing that I_1, I_2, and I_3 are constant imply that universal deformations must be homogeneous [17]. Knowing that α_3 can be chosen arbitrarily, its coefficient must vanish, i.e., $\operatorname{div}\mathring{\boldsymbol{\sigma}} = \mathbf{0}$. Constancy of I_4, I_5, and I_6 and knowing that $\operatorname{div}\mathring{\boldsymbol{\sigma}} = \mathbf{0}$ implies that $\mathring{\boldsymbol{\sigma}}$ is homogeneous [85, Lemma 3.1]. Knowing that $\mathbf{b}^\sharp$, $\mathbf{c}^\sharp$, and $\mathring{\boldsymbol{\sigma}}$ are homogeneous, equilibrium equations (4.4) are now trivially satisfied.

Recall that $\mathring{\mathbf{S}} = \mathbf{F}^{-1}\mathring{\boldsymbol{\sigma}}\mathbf{F}^{-\star}$. Knowing that both $\mathbf{F}$ and $\mathring{\boldsymbol{\sigma}}$ are homogeneous, one concludes that the residual stress field $\mathring{\mathbf{S}}$ is homogeneous. As mentioned earlier, a residual stress field must be divergence-free and satisfy traction-free boundary conditions. These requirements preclude uniformity, implying that non-trivial universal residual stresses cannot exist. Thus, we have proved the following result.

Proposition 4.1 *For homogeneous compressible isotropic Cauchy elastic solids with residual stress, the set of universal deformations consists precisely of all homogeneous deformations. Moreover, there are no non-trivial universal residual stresses.*

Remark 4.2 Yavari and Goriely [80] studied universal deformations in compressible hyperelastic bodies with finite eigenstrains, assuming isotropy in the absence of eigenstrains. In two dimensions, they showed that the material manifold must be flat, implying that universal eigenstrains must be zero-stress. In three dimensions, they proved that the material manifold must be a Riemannian symmetric space and, under the assumption of simple connectivity, flat. Thus, in dimensions two and three, the only universal eigenstrains in simply-connected bodies are zero-stress, and all universal deformations are homogeneous. In this paper, we do not specify the source of residual stress. However, our result is consistent with that of Yavari and Goriely [80]: universal deformations are homogeneous, and no non-trivial universal residual stresses exist.

5 Conclusions

In this paper we investigated universal deformations in compressible isotropic Cauchy elastic solids with residual stress, without specifying the source of the residual stress. We showed that universal deformations are homogeneous, and the associated residual stresses must also be homogeneous. Since a non-trivial residual stress cannot be homogeneous, it follows that residual stress must vanish. Thus, a compressible Cauchy elastic solid with a non-trivial distribution of residual stress cannot admit universal deformations. This result is consistent with Yavari and Goriely [80], who proved that in the presence of eigenstrains, universal deformations are covariantly homogeneous and the universal eigenstrains are zero-stress (impotent).

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Data Availability No datasets were generated or analysed during the current study.

Declarations

Competing Interests The authors declare no competing interests.

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