

A Detailed Procedure for Harmonic Analysis of Three-Phase Diode Rectifiers Under Discontinuous Conduction Mode and Nonideal Conditions

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Abstract—Fast and detailed calculations of harmonics, generated by three-phase uncontrolled rectifiers with finite dc smoothing, can be performed by using detailed analytical procedures (DAPs). Several DAPs are developed for continuous- and discontinuous-conduction modes (CCM and DCM) when balanced operation of the system is assumed. The authors of this paper have recently proposed a new DAP (model 1a) in which nonideal conditions, such as unbalances and even harmonic voltages on the ac side, are taken into account. However, only CCM is assumed in model 1a. The main objective of this paper is to overcome this limitation of model 1a so that a new DAP (model 1b) for DCM is presented, which is capable of considering unbalanced operation. Moreover, the boundary of the DCM is also estimated for model 1b under nonideal conditions.

Index Terms—Discontinuous conduction mode (DCM), uncontrolled rectifiers, harmonics.

NOMENCLATURE

t	Time instant.
ω_1	Fundamental angular frequency.
τ	Variable angle defined as $\tau = \omega_1 t$.
R, G	Resistance (R) and conductance (G).
X	Inductive reactance at fundamental frequency.
B_c	Capacitive susceptance at fund. frequency.
u, e, i	Instantaneous voltages (u, e) and currents (i).
$\hat{U}, \hat{E}, \hat{I}$	Complex phasors for voltages and currents.
d	Subscript for denoting dc magnitudes.
ℓ	Subscript for denoting ac magnitudes.
ℓ	takes the values 1, 2, 3 for phases a, b, c.
$U_{\ell,0}, I_{\ell,0}$	DC components of ac magnitudes u_ℓ and i_ℓ .
U_d, I_d	DC components of dc magnitudes u_d and i_d .

k, m	Order of a generic harmonic.	36
n_h	Highest harmonic order in the ac sources.	37
α	Initial angle for a conduction subinterval.	38
δ	Final angle for a conduction subinterval.	39
γ	Conduction angle ($\gamma = \delta - \alpha$).	40
p	Index (1 to 6) for denoting a cond. subinterval.	41
$'$	First derivative of u_d or i_d with respect to τ .	42
$''$	Second derivative of u_d or i_d with respect to τ .	43
t	Subscript for the transient solution of a differential equation.	44
s	Subscript for the steady-state sol. of a differential equation.	46

I. INTRODUCTION

THREE-PHASE diode rectifiers with capacitive dc smoothing are widely extended in low voltage applications and they produce a considerable level of ac harmonic currents. Hence, a detailed modelling, representing the steady-state solution of these elements, is required in order to obtain a good harmonic estimation. Detailed methods can be implemented in time domain [1]–[2], in the harmonic domain [3] or methods which are based on a detailed analytical procedure (DAP) [4]–[6]. Finite capacitive dc smoothing and an accurate determination of the switching angles are considered in these detailed methods. The fastest results are obtained via DAPs as indicated in [6]. As reported in [6], other methods propose average modeling [7]–[9] or they assume certain simplifications in the determination of the switching angles [10]–[13] or they do not consider the ripple of the capacitor voltage [14]–[20]. Harmonic analysis of controlled and uncontrolled converters has been also proposed [21]–[25].

Discontinuous conduction mode (DCM) is considered in the DAPs of [4] and [5] under balanced operation. However, nonideal conditions, such as negative sequence voltage and second harmonic voltage on the ac side lead to a more complex behaviour of the converter. The DAP proposed in [6], and denoted as model 1a in this paper, provides exact results under nonideal conditions. However, model 1a only considers CCM in its normal formulation, namely, two or three diodes in conduction. This typical formulation is denoted as CCM-1 in [7], and it fits with the most practical cases [2]. However, as reported in [9], operation with only three diodes in conduction, denoted as CCM-2 in [7], is possible in particular situations [26].

Manuscript received December 5, 2016; revised June 26, 2017; accepted August 15, 2017. Date of publication; date of current version. This work was supported under a contract with EDF R&D. Paper no. TPWRD-01432-2016. (Corresponding author: Luis F. Beites.)

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Digital Object Identifier 10.1109/TPWRD.2017.2748384

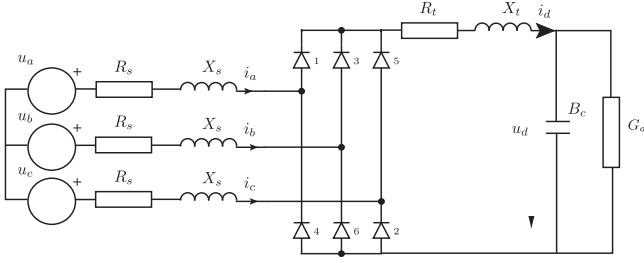


Fig. 1. Basic scheme for the analysis.

The object of this paper is to describe a new DAP (model 1b) for DCM, namely, two or no diodes in conduction. Model 1b, as model 1a, provides a systematic form of generating the nonlinear equations in order to calculate the switching angles in a full Newton formulation. Harmonics and dc components on the ac and dc sides can be evaluated by means a set of compact analytical expressions.

II. NOTATION AND MAIN RELATIONS

The scheme of Fig. 1 is the system to be studied. The notation of [6] is also here used, and hence, ac and dc instantaneous voltages and currents are functions of angle τ , and they are expressed by:

$$u_\ell = \sum_{m=0}^{n_h} \text{Im}\{\hat{U}_{\ell,m} \cdot e^{jm\tau}\}; \quad i_\ell = \sum_{m=0}^{\infty} \text{Im}\{\hat{I}_{\ell,m} \cdot e^{jm\tau}\} \quad (1)$$

$$u_d = \sum_{k=0}^{\infty} \text{Im}\{\hat{U}_{dk} \cdot e^{jk\tau}\}; \quad i_d = \sum_{k=0}^{\infty} \text{Im}\{\hat{I}_{dk} \cdot e^{jk\tau}\} \quad (2)$$

According to (1) and (2), dc components of ac and dc magnitudes are related to the harmonic phasors by:

$$\hat{U}_{\ell,0} = jU_{\ell,0}; \quad \hat{I}_{\ell,0} = jI_{\ell,0}; \quad \hat{U}_{d0} = jU_d; \quad \hat{I}_{d0} = jI_d \quad (3)$$

Six sections or intervals for the dc current i_d are considered. A generic section is denoted by s_p . Each section s_p can be divided in a conduction subinterval s_{px} (two diodes in conduction) and a discontinuity subinterval s_{py} (no diodes in conduction). The limits of these intervals are defined by using the basic (or switching) angles α and δ , so that:

$$s_p = [\alpha_p, \beta_p]; \quad s_{px} = [\alpha_p, \delta_p]; \quad s_{py} = [\delta_p, \beta_p] \quad (4)$$

$$\beta_p = \alpha_{p+1} \text{ (for } p \neq 6); \quad \beta_6 = \alpha_1 + 2\pi \quad (5)$$

It is assumed that dc current $i_d(\tau)$ exhibits a discontinuous behaviour in all subintervals s_{py} . This condition is given by:

$$i_d(\tau) = 0 \quad \forall \tau \in s_{py} \quad (6)$$

while current $i_d(\tau)$ is equal to zero in the limits of subintervals s_{px} , namely:

$$i_d(\alpha_p) = i_d(\delta_p) = 0 \quad (7)$$

Current $i_d(\tau)$ and basic angles are indicated in Fig. 2. A power flow from the ac side to the dc load takes place during the conduction subintervals. On the other hand, all diodes are

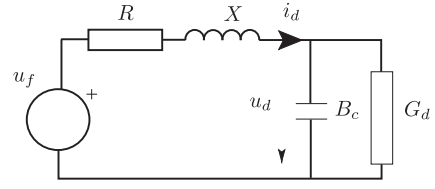
Fig. 2. Waveform of current i_d .

TABLE I
DETAILS OF CONDUCTION SUBINTERVALS FOR THE THREE-PHASE RECTIFIER

p	Diodes	Limits	$e_p(\tau)$
1	1, 6	α_1, δ_1	$e_1 = u_a - u_b$
2	1, 2	α_2, δ_2	$e_2 = u_a - u_c$
3	2, 3	α_3, δ_3	$e_3 = u_b - u_c$
4	3, 4	α_4, δ_4	$e_4 = u_b - u_a$
5	4, 5	α_5, δ_5	$e_5 = u_c - u_a$
6	5, 6	α_6, δ_6	$e_6 = u_c - u_b$

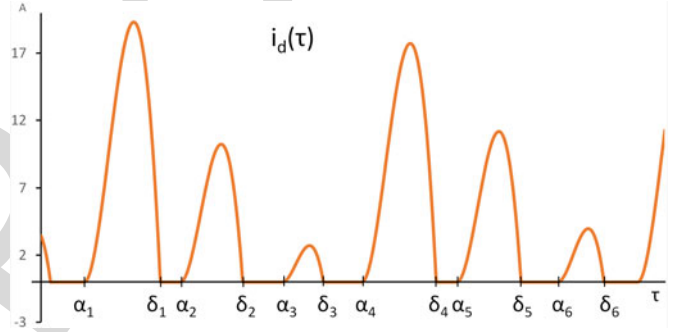


Fig. 3. DC side equivalent circuit.

blocked during the discontinuity subintervals, and the capacitor exhibits a certain discharge process. A key aspect of this analysis is to find basic (or switching) angles α_p and δ_p when the ac voltages and parameters R_s, X_s, R_t, X_t, B_c and G_d are known. Once the basic angles are determined, dc and ac harmonic components can be evaluated by means of a set of compact analytical expressions.

A. Conduction Subintervals

The conduction subintervals can be described by a loop in which only two AC phases and two diodes are involved, as shown in Table I and Fig. 3. Parameters R and X are:

$$R = 2R_s + R_t; \quad X = 2X_s + X_t \quad (8)$$

and voltage $u_f(\tau) = e_p(\tau)$ for $\tau \in s_{px}$. Voltage $e_p(\tau)$ is the ac source for a generic conduction loop p . Taking into account (1) and Table I, $e_p(\tau)$ can be expressed by:

$$e_p(\tau) = \sum_{m=0}^{n_h} \text{Im}\{\hat{E}_{p,m} \cdot e^{jm\tau}\}; \quad \hat{E}_{p,0} = jE_{p,0} \quad (9)$$

For instance, for $p = 1$, we have: $\hat{E}_{1,m} = \hat{U}_{a,m} - \hat{U}_{b,m}$ and $E_{1,0} = U_{a,0} - U_{b,0}$. On the other hand, dc magnitudes u_d and

121 i_d are related by the following two equations:

$$e_p = u_d + R \cdot i_d + X \cdot i'_d \quad (10)$$

$$i_d = G_d u_d + B_c u'_d; \quad G_d = 1/R_d \quad (11)$$

122 where $i'_d = di_d(\tau)/d\tau$ and $u'_d = du_d(\tau)/d\tau$. The elimination of
123 current i_d among (10) and (11) leads to the following differential
124 equation of second order for voltage u_d :

$$u''_d + 2\xi u'_d + k_o^2 u_d = f_p(\tau) \quad \text{for } \tau \in s_{px} \quad (12)$$

$$2\xi = \frac{R}{X} + \frac{G_d}{B_c}; \quad k_o^2 = \frac{RG_d + 1}{XB_c}; \quad f_p(\tau) = \frac{e_p(\tau)}{XB_c} \quad (13)$$

125 where k_o is the normalized natural frequency as it is used in
126 [2] and ξ is a damping factor. The solution of (12) presents a
127 steady-state component u_{sp} and a transient component u_{tp} :

$$u_d = u_{sp} + u_{tp} \quad \text{for } \tau \in s_{px} \quad (14)$$

128 Voltage u_{sp} is obtained from:

$$u_{sp} = \sum_{m=0}^{n_h} \text{Im}\{\hat{U}_{sp,m} \cdot \varepsilon^{jm\tau}\}; \quad \hat{U}_{sp,m} = \underline{c}_m \cdot \hat{E}_{p,m} \quad (15)$$

$$\underline{c}_m = \frac{1}{1 + (R + jmX)(G_d + jmB_c)} \quad (16)$$

129 Voltage u_{tp} presents the following form when $\xi < k_o$:

$$u_{tp} = \text{Im}\{\hat{U}_{tp} \cdot \varepsilon^{\nu\tau}\}; \quad k_n = \sqrt{k_o^2 - \xi^2}; \quad \nu = -\xi + jk_n \quad (17)$$

130 The derivatives of components u_{sp} and u_{tp} are given by:

$$u'_{sp} = \sum_{m=0}^{n_h} \text{Im}\{\hat{U}'_{sp,m} \cdot \varepsilon^{jm\tau}\}; \quad \hat{U}'_{sp,m} = jm \cdot \underline{c}_m \cdot \hat{E}_{p,m} \quad (18)$$

$$u'_{tp} = \text{Im}\{\hat{U}'_{tp} \cdot \varepsilon^{\nu\tau}\}; \quad \hat{U}'_{tp} = \nu \cdot \hat{U}_{tp} \quad (19)$$

131 The steady-state component is completely determined by (15)
132 and (16). However, phasor $\hat{U}_{tp}$ of the transient component is
133 not known in advance, and it must be estimated from the initial
134 conditions $u_d(\alpha_p)$ and $u'_d(\alpha_p)$. The first condition is derived
135 from the crossing of ac and dc voltages for the initial instant
136 ($\tau = \alpha_p$). In accordance with the schemes of Figs. 1 and 3, the
137 first condition is given by:

$$u_d(\alpha_p) = e_p(\alpha_p) \quad (20)$$

138 Taking into account (7), (11) and (20), the second condition
139 can be expressed by:

$$u'_d(\alpha_p) = -\sigma \cdot u_d(\alpha_p) = -\sigma \cdot e_p(\alpha_p); \quad \sigma = G_d/B_c \quad (21)$$

140 Before the beginning of the conduction subinterval, the volt-
141 age drop on the line is zero since, under DCM operation, all
142 diodes are blocked in this previous state to the conduction.
143 Hence, (20) represents also the crossing of the ac and dc volt-
144 ages when the ac voltage is the voltage at the terminals of the
145 converter.

146 The idealized characteristic of the diode is assumed in this
147 paper, namely every diode is treated as a short circuit in conduc-
148 tion and as an open circuit in the blocking state. This assumption
149 is very often in harmonic analysis [4], [5], [27]. However, in the
150 conduction state, the diode could be represented by a series

combination of a small dc voltage V_0 and a small resistance R_0 151
[27]. In this last case, a quantity of $2R_0$ can be added to the 152
resistance R of (8) and a quantity of $-2V_0$ can be added to the 153
dc component $E_{p,0}$ of (9) in order to represent a nonideal diode 154
in conduction, while an open circuit is keep in the blocking state 155
of the diode. These additions do not change the formulation of 156
this paper. 157

The underdamped case ($\xi < k_o$) is assumed in this paper 158
since this condition is satisfied in most practical situations [2]. 159
However, the extension to the overdamped case ($\xi > k_o$) does 160
not involve any additional complexity [4]. In this last case, 161
the characteristic equation presents two different real roots: 162
 $\nu_1 = -\xi + \text{Im}\{k_n\}$ and $\nu_2 = -\xi - \text{Im}\{k_n\}$, and the transient 163
voltage is given by: $u_{tp} = \hat{U}_{t1p} \cdot \varepsilon^{\nu_1\tau} + \hat{U}_{t2p} \cdot \varepsilon^{\nu_2\tau}$, in where real 164
terms $\hat{U}_{t1p}$ and $\hat{U}_{t2p}$ play a similar role as complex phasor $\hat{U}_{tp}$ 165
in (17). The analysis of the overdamped case is omitted in this 166
paper for space reasons. 167

B. Discontinuity Subintervals 168

All diodes are blocked during the discontinuity subintervals 169
 s_{py} . The scheme of Fig. 3 can be also used in this case, and 170
voltage $u_f = u_d$ for $\tau \in s_{py}$, since current i_d is zero during 171
subintervals s_{py} . Voltage u_d is conditioned by the discharge 172
process of the capacitor during subintervals s_{py} , so that: 173

$$u_d(\tau) = u_d(\delta_p) \cdot \varepsilon^{-\sigma(\tau-\delta_p)} \quad \text{for } \tau \in s_{py} \quad (22)$$

where σ is defined in (21), and it is the normalized natural fre- 174
quency of first order circuit formed by the capacitor and conduc- 175
tance G_d . Therefore, we have for the final angle of subinterval 176
 s_{py} : 177

$$u_d(\beta_p) = u_d(\delta_p) \cdot \varepsilon^{-\sigma(\beta_p-\delta_p)} \quad (23)$$

Considering (5), (20) and (23) term $u_d(\delta_p)$ is equivalent to: 178

$$u_d(\delta_p) = e_{p+1}(\alpha_{p+1}) \cdot \varepsilon^{\sigma(\alpha_{p+1}-\delta_p)} \quad (24)$$

From (7), (11) and (24), term $u'_d(\delta_p)$ can be written as: 179

$$u'_d(\delta_p) = -\sigma \cdot u_d(\delta_p) = -\sigma \cdot e_{p+1}(\alpha_{p+1}) \cdot \varepsilon^{\sigma(\alpha_{p+1}-\delta_p)} \quad (25)$$

Finally, the combination of (22) and (24) leads to the expres- 180
sion for voltage $u_d(\tau)$ in discontinuity subinterval s_{py} : 181

$$u_d(\tau) = e_{p+1}(\alpha_{p+1}) \cdot \varepsilon^{\sigma(\alpha_{p+1}-\tau)} \quad \text{for } \tau \in s_{py} \quad (26)$$

For the last interval, namely, $p = 6$, we have: 182

$$e_7 = e_1; \quad \alpha_7 = \alpha_1 + 2\pi \quad (27)$$

Expressions (26) and (27) completely describe voltage $u_d(\tau)$ 183
during the discontinuity subintervals. 184

III. DETERMINATION OF THE BASIC ANGLES 185

The object of this analysis is to determine basic angles α_p and 186
 δ_p . To do that, four basic equations must be considered for each 187
interval p . They are obtained from the conditions indicated in 188
(20), (21), (24) and (25). Taking into account (14)–(19), these 189

conditions can be written as:

$$a_{1p} = e_p(\alpha_p) - u_{sp}(\alpha_p) = u_{tp}(\alpha_p) \quad (28)$$

$$a_{2p} = -\sigma e_p(\alpha_p) - u'_{sp}(\alpha_p) = u'_{tp}(\alpha_p) \quad (29)$$

$$a_{3p} = e_{p+1}(\alpha_{p+1}) \cdot \varepsilon^{\sigma(\alpha_{p+1} - \delta_p)} - u_{sp}(\delta_p) = u_{tp}(\delta_p) \quad (30)$$

$$a_{4p} = -\sigma e_{p+1}(\alpha_{p+1}) \cdot \varepsilon^{\sigma(\alpha_{p+1} - \delta_p)} - u'_{sp}(\delta_p) = u'_{tp}(\delta_p) \quad (31)$$

Functions a_{1p} to a_{4p} present a first equivalence with the steady-state component u_{sp} and voltages e_p and e_{p+1} . This first equivalence can be expressed in more detail by:

$$a_{1p} = \sum_{m=0}^{n_h} \text{Im} \left\{ (1 - \underline{c}_m) \cdot \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm\alpha_p} \right\} \quad (32)$$

$$a_{2p} = - \sum_{m=0}^{n_h} \text{Im} \left\{ (\sigma + jm\underline{c}_m) \cdot \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm\alpha_p} \right\} \quad (33)$$

$$a_{3p} = \sum_{m=0}^{n_h} \text{Im} \left\{ \begin{array}{l} \hat{\underline{E}}_{p+1,m} \cdot \varepsilon^{(\sigma+jm)\alpha_{p+1} - \sigma\delta_p} \\ - \underline{c}_m \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm\delta_p} \end{array} \right\} \quad (34)$$

$$a_{4p} = - \sum_{m=0}^{n_h} \text{Im} \left\{ \begin{array}{l} \sigma \hat{\underline{E}}_{p+1,m} \cdot \varepsilon^{(\sigma+jm)\alpha_{p+1} - \sigma\delta_p} \\ + jm\underline{c}_m \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm\delta_p} \end{array} \right\} \quad (35)$$

Functions a_{3p} and a_{4p} depend on basic angles α_{p+1} and δ_p as shown in (34) and (35), while functions a_{1p} and a_{2p} only depend on basic angle α_p as shown in (32) and (33).

Functions a_{1p} to a_{4p} of (28)–(31) present a second equivalence with the transient component u_{tp} and its derivative u'_{tp} . In accordance with (17) and (19), terms u_{tp} and u'_{tp} depend on phasor $\hat{\underline{U}}_{tp}$. However, phasor $\hat{\underline{U}}_{tp}$ of voltage u_{tp} is not known in advance. A key aspect of the analysis is the process by which phasor $\hat{\underline{U}}_{tp}$ is expressed as a combination of functions a_{1p} and a_{2p} , whose form is indicated in (32) and (33). To do that, conduction angle γ_p and auxiliary phasors $\underline{V}_p$ and $\underline{\lambda}_p$ are defined by:

$$\gamma_p = \delta_p - \alpha_p; \quad \underline{V}_p = \hat{\underline{U}}_{tp} \cdot \varepsilon^{\underline{\nu}\alpha_p}; \quad \underline{\lambda}_p = \varepsilon^{\underline{\nu}\gamma_p} \quad (36)$$

Taking into account (17), (19) and (36), voltages $u_{tp}(\alpha_p)$ and $u_{tp}(\delta_p)$, and their derivatives, are equivalent to:

$$u_{tp}(\alpha_p) = \text{Im}\{\underline{V}_p\}; \quad u_{tp}(\delta_p) = \text{Im}\{\underline{V}_p \cdot \underline{\lambda}_p\} \quad (37)$$

$$u'_{tp}(\alpha_p) = \text{Im}\{\underline{\nu} \cdot \underline{V}_p\}; \quad u'_{tp}(\delta_p) = \text{Im}\{\underline{\nu} \cdot \underline{V}_p \cdot \underline{\lambda}_p\} \quad (38)$$

Hence, (28) and (29) can be expressed by:

$$a_{1p} = \text{Im}\{\underline{V}_p\} = \frac{1}{j2} (\underline{V}_p - \underline{V}_p^*) \quad (39)$$

$$a_{2p} = \text{Im}\{\underline{\nu} \cdot \underline{V}_p\} = \frac{1}{j2} (\underline{\nu} \cdot \underline{V}_p - \underline{\nu}^* \cdot \underline{V}_p^*) \quad (40)$$

where $\underline{V}_p$ and $\underline{V}_p^*$ are the unknowns of this system of two equations, whose solution is:

$$\underline{V}_p = \underline{a}_{12p} = \frac{1}{k_n} (a_{2p} - \underline{\nu}^* a_{1p}) \quad (41)$$

Phasor $\underline{V}_p$ is equivalent to a complex function $\underline{a}_{12p}$, obtained as combination of functions a_{1p} and a_{2p} . Therefore, complex function $\underline{a}_{12p}$ depends only on angle α_p . Taking into account

(30), (31), (37), (38) and (41), functions a_{3p} and a_{4p} can be written as:

$$a_{3p} = \text{Im}\{\underline{a}_{12p} \cdot \underline{\lambda}_p\}; \quad a_{4p} = \text{Im}\{\underline{\nu} \cdot \underline{a}_{12p} \cdot \underline{\lambda}_p\} \quad (42)$$

It is evident that phasor $\hat{\underline{U}}_{tp}$ has been explicitly eliminated in both relations of (42) if functions a_{3p} , a_{4p} and $\underline{a}_{12p}$ are evaluated from (32) to (35) and (41).

A. Nonlinear Equations and Their Solution

The relations of (42) lead to the following nonlinear equations for interval p :

$$\Delta F_{a,p} = a_{3p} - \text{Im}\{\underline{a}_{12p} \cdot \underline{\lambda}_p\} = 0 \quad (43)$$

$$\Delta F_{b,p} = a_{4p} - \text{Im}\{\underline{\nu} \cdot \underline{a}_{12p} \cdot \underline{\lambda}_p\} = 0 \quad (44)$$

A system of 12 mismatch equations or residues, based on (43) and (44), must be built, where basic angles α_p and δ_p are treated as variables or unknowns. An iterative algorithm, such as the Newton method, is used, whose form can be written as:

$$\begin{bmatrix} \Delta F_{a,p} \\ \Delta F_{b,p} \end{bmatrix} = - \sum_{q=1}^{n_p} \begin{bmatrix} \frac{\partial \Delta F_{a,p}}{\partial \alpha_q} & \frac{\partial \Delta F_{a,p}}{\partial \delta_q} \\ \frac{\partial \Delta F_{b,p}}{\partial \alpha_q} & \frac{\partial \Delta F_{b,p}}{\partial \delta_q} \end{bmatrix} \begin{bmatrix} \Delta \alpha_q \\ \Delta \delta_q \end{bmatrix} \quad (45)$$

The application of this algorithm requires to obtain the partial derivatives of the residues with respect to angles α_p and δ_p . The analytical expressions of these derivatives are relatively simple due to the compact form of this procedure, and they are given in Appendix A. On the other hand, the structure of the corresponding Jacobian Matrix is relative sparse as shown in (46).

$$\begin{bmatrix} x & x & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ x & x & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & x & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & x & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x & x & x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x & x & x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x & x & x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x & x & x & 0 \\ x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x & x \\ x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x & x \end{bmatrix} \quad (46)$$

The nonzero elements of (46) are denoted by symbol x . For instance, the first two rows of (46) correspond to the terms of the first interval ($p = 1$), and they depend on angles α_1 , δ_1 and α_2 . The next two rows of (46) correspond to the terms of the second interval ($p = 2$), and they depend on angles α_2 , δ_2 and α_3 . It is evident from this scheme that the coupling among the first and second interval is due to the participation of angle α_2 in both intervals. The initial values for basic angles α_p and δ_p are obtained from the ideal behaviour of the converter [27]: balanced and sinusoidal ac voltages; infinite capacitive dc smoothing ($B_c = \infty$) and $R_s = R_t = 0$.

244 B. Determination of the Transient Component

245 Once the iterative process is performed, the final values of
246 basic angles α_p and δ_p are known. With these results, the final
247 values of functions $a_{1p}(\alpha_p)$, $a_{2p}(\alpha_p)$ and $\underline{a}_{12p}$ are obtained
248 from (32), (33) and (41) respectively. After that, and taking into
249 account (36) and (41), phasor $\hat{U}_{tp}$ is determined from:

$$\hat{U}_{tp} = \underline{a}_{12p} \cdot \varepsilon^{-\nu\alpha_p} \quad (47)$$

250 and voltage u_{tp} of (17) can be expressed by:

$$u_{tp}(\tau) = \frac{1}{j2} (\hat{U}_{tp} \varepsilon^{\nu\tau} - \hat{U}_{tp}^* \varepsilon^{\nu^*\tau}) \quad (48)$$

251 Voltage u_{sp} is known before performing the iterative process,
252 and according to (9) and (15), it can be written as:

$$u_{sp}(\tau) = \frac{1}{j2} \sum_{m=0}^{n_h} (\underline{c}_m \cdot \hat{E}_{p,m} \varepsilon^{jm\tau} - \underline{c}_m^* \cdot \hat{E}_{p,m}^* \varepsilon^{-jm\tau}) \quad (49)$$

253 The expressions of (48) and (49) are the basis for calculating
254 the harmonic currents, as shown in Appendix B.

255 IV. DC AND AC HARMONICS

256 The goal of this section is to provide analytical expressions
257 for harmonic components of the ac currents and of the dc mag-
258 nitudes (voltage and current). The dc components are also eval-
259 uated. To do that, harmonic currents $\hat{I}_{dk}$ of dc current $i_d(\tau)$ are
260 firstly evaluated from:

$$\hat{I}_{dk} = \frac{j}{\pi \cdot \rho_k} \int_{\alpha_1}^{\alpha_1+2\pi} i_d \cdot \varepsilon^{-jk\tau} d\tau = \sum_{p=1}^{n_p} \hat{I}_{dk,p} \quad (50)$$

261 where $\rho_k = 1$ for $k \neq 0$ and $\rho_k = 2$ for $k = 0$. Term $\hat{I}_{dk,p}$ is
262 the harmonic contribution of current i_d in subinterval s_{px} . The
263 analytical expression of this contribution is given in Appendix B.
264 Once dc harmonic currents are evaluated, dc harmonic voltages
265 $\hat{U}_{dk}$ are obtained from:

$$\hat{U}_{dk} = \frac{\hat{I}_{dk}}{G_d + jkB_c} \quad (51)$$

266 AC harmonic currents can be easily evaluated from harmonic
267 components $\hat{I}_{dk,p}$ taking into account that currents $i_a(\tau)$ and
268 $i_c(\tau)$ satisfy the following conditions:

$$i_a = \begin{cases} +i_d & (p = 1, 2) \\ -i_d & (p = 4, 5) \end{cases}; i_c = \begin{cases} +i_d & (p = 5, 6) \\ -i_d & (p = 2, 3) \end{cases} \quad (52)$$

269 The remaining current i_b is obtained from $i_b = -i_a - i_c$.
270 Taking into account (50) and (52), the harmonic components of
271 the ac currents are given by:

$$\begin{aligned} \hat{I}_{a,k} &= \hat{I}_{dk,1} + \hat{I}_{dk,2} - \hat{I}_{dk,4} - \hat{I}_{dk,5} \\ \hat{I}_{c,k} &= \hat{I}_{dk,5} + \hat{I}_{dk,6} - \hat{I}_{dk,2} - \hat{I}_{dk,3} \\ \hat{I}_{b,k} &= -\hat{I}_{a,k} - \hat{I}_{c,k} = \hat{I}_{dk,3} + \hat{I}_{dk,4} - \hat{I}_{dk,1} - \hat{I}_{dk,6} \end{aligned} \quad (53)$$

272 The dc components are derived from (3), (50), (51), and (53)
273 when $k = 0$.

V. BOUNDARY OF THE DCM

274

275 The operation of the DCM means that no more than two
276 diodes are in conduction. For instance, diodes 1 and 6 are in
277 conduction in subinterval s_{1x} ($p = 1$) and the next diode to
278 be fired, namely diode 2, presents an anode-cathode voltage
279 v_2 which satisfies the condition $v_2(\tau) < 0$ for $\tau \in s_{1x}$, and
280 hence $v_2(\delta_1) < 0$ for the operation under DCM. The condi-
281 tion $v_2(\delta_1) = 0$ is used in [15] to identify the boundary of the
282 DCM when an ideal scheme is assumed: balanced and sinu-
283 soidal ac voltages, infinite capacitive dc smoothing ($B_c = \infty$),
284 and $R_s = R_t = 0$. The scheme considered in this paper is much
285 more complex than the ideal scheme of [15]. Hence, instead
286 of identifying the boundary of the DCM, a verification of the
287 DCM is proposed in this paper which is based on checking
288 the conditions $v_2(\delta_1) < 0$ for diode 2, $v_3(\delta_2) < 0$ for diode 3
289 etc. In accordance with Fig. 1 and Table I, $u_b - u_c = e_3$ and
290 voltage $v_2(\delta_1)$ of diode 2 at the end of subinterval s_{1x} is given
291 by: $v_2(\delta_1) = R_s \cdot i_d(\delta_1) + X_s \cdot i_d'(\delta_1) + e_3(\delta_1)$. The voltages of
292 the remaining diodes present a similar form. In general, these
293 voltages can be expressed by:

$$v_{p+1}(\delta_p) = R_s \cdot i_d(\delta_p) + X_s \cdot i_d'(\delta_p) + e_{p+2}(\delta_p) \quad (54)$$

294 where $p = 1$ to 6, $v_7(\delta_6) = v_1(\delta_6)$, $e_7(\delta_5) = e_1(\delta_5)$ and $e_8(\delta_6)$
295 $= e_2(\delta_6)$. Taking into account (11), terms $i_d(\delta_p)$ and $i_d'(\delta_p)$ are
296 obtained from:

$$\begin{aligned} i_d(\delta_p) &= G_d \cdot u_d(\delta_p) + B_c \cdot u_d'(\delta_p) \\ i_d'(\delta_p) &= G_d \cdot u_d'(\delta_p) + B_c \cdot u_d''(\delta_p) \end{aligned} \quad (55)$$

297 According to (12), term $u_d''(\delta_p)$ can be calculated from $u_d(\delta_p)$
298 and $u_d'(\delta_p)$ by:

$$u_d''(\delta_p) = f_p(\delta_p) - 2\xi \cdot u_d'(\delta_p) - k_o^2 \cdot u_d(\delta_p) \quad (56)$$

299 Taking into account (14)–(19), voltage $u_d(\delta_p)$ and its deriva-
300 tive are estimated from:

$$\begin{aligned} u_d(\delta_p) &= u_{sp}(\delta_p) + u_{tp}(\delta_p) \\ u_d'(\delta_p) &= u_{sp}'(\delta_p) + u_{tp}'(\delta_p) \end{aligned} \quad (57)$$

301 In summary, once the iterative process is performed, the volt-
302 age $v_{p+1}(\delta_p)$ is calculated for each diode using (54)–(57). The
303 verification of the DCM requires that $v_{p+1}(\delta_p) < 0$ for all the
304 diodes. Otherwise we obtain a no coherent result. In this case, a
305 lower conductance G_d should be selected in order to satisfy the
306 condition $v_{p+1}(\delta_p) < 0$ for all the diodes.

307 A. Intermediate Conduction Mode (ICM)

308 Each section of model 1b (DCM) and model 1a (CCM) is
309 defined by two basic angles and it presents two subintervals:
310 conduction and discontinuity subintervals for DCM, commuta-
311 tion and conduction subintervals for CCM [6]. The transition
312 from DCM to CCM is not direct since an intermediate conduc-
313 tion mode (ICM) exists among DCM and CCM. Each section
314 with ICM is defined by four basic angles and it presents four
315 subintervals: conduction, commutation, conduction and discon-
316 tinuity subintervals.

ICM can be viewed as a variant of CCM in where a small discontinuity subinterval appears within the primitive full conduction subinterval of the ac currents (see Fig. 7 in [15]). ICM can be viewed too as a variant of DCM in where the dc current wave form contains, at the end of the primitive conduction subinterval, a commutation subinterval and a new conduction subinterval. For instance, and according to Table I, section s_1 ($p = 1$) presents the following four subintervals: conduction (1, 6), commutation (1, 2, 6), conduction (1,2) and discontinuity (no diodes).

The operating range of ICM is very small [5] in comparison with the operating range of DCM or CCM, and hence, its relevance and impact are more reduced. In order to verify this statement, the ideal scheme of [15] is considered in where μ is the overlap angle. The boundary DCM-ICM is obtained when $\mu = 0$, while $\mu = \mu_0 = 3.18^\circ$ defines the boundary CCM-ICM, namely, only a difference of 3.18° for the overlap angle exists for the ICM limits. The difference between limits is low for the dc voltage: $U_d(\mu = 0) = 1.0086U_d(\mu = \mu_0)$, while it is higher for the load: $G_d(\mu = \mu_0) = 1.444G_d(\mu = 0)$, but in any case this last difference is very moderate if it is compared with the difference between CCM limits: $G_d(\mu = 60^\circ) = 54.61G_d(\mu = \mu_0)$. These calculations confirm that most practical cases can be treated with model 1b (DCM) and model 1a (CCM).

Finally, if the coherence checking of model 1b is ignored, model 1b can be used in the operating range of ICM with acceptable errors for the first harmonics. This question is treated in Section VI-A. In any case, it is evident that a DAP with a detailed representation of the ICM would require a higher complexity than model 1b of this paper and model 1a described in [6], and hence, future developments are needed in this area.

VI. RESULTS

The system proposed in [3], [6] and [28] is also here used. It consists of a six pulse diode bridge of 400 V, 20 kVA, 50 Hz with the following parameters:

$$R_s = 0.04 \Omega; L_s = 0.51 \text{ mH}; C = 1.284 \text{ mF}; \\ R_t = 0.1 \Omega; L_t = 0.61 \text{ mH};$$

The accuracy of the proposed DAP is checked against PSCAD/EMTDC, using a simulation time of 0.8 s (the first 40 cycles). The steady-state solution with PSCAD is detected by no apparent changes in the harmonic results. The ideal model for the diode is selected in PSCAD.

A. Test 1: Balanced Operation

The phase to neutral ac voltages only present fundamental component with a positive sequence of $400/\sqrt{3}$ V, 0° . Two points of operation are defined. In the first one, a resistance of $R_d = 70 \Omega$ is used. With it, a clearly discontinuous conduction mode is obtained, as shown in Fig. 4. The convergence properties of the DAP are indicated in Table II and dc harmonic components are shown in Table III. A very good agreement with PSCAD is achieved as shown in Fig. 4 and Table III.

Using the method described in Section V, the boundary DCM-ICM in test 1 is reached with $R_d = 64.82 \Omega$. DC harmonic components for this operating point are given in Table IV, and



Fig. 4. Test 1: waveform for current i_a with $R_d = 70 \Omega$.

TABLE II
TEST 1: CONVERGENCE PROPERTIES WITH $R_d = 70 \Omega$

Iter	$\alpha_1 (^\circ)$	$\delta_1 (^\circ)$
0	43.67824	92.77824
1	43.61132	91.54128
2	43.58835	91.51915
3	43.58835	91.51925
4	43.58835	91.51925

also a very good agreement with PSCAD is obtained as shown in Table IV.

A third case is considered with $R_d = 41 \Omega$ in where the coherence checking of model 1b is ignored. Under this load, the converter operates clearly under ICM and very close to CCM, namely the worst case for model 1b without coherence checking. In spite of that, model 1b and PSCAD present similar waveforms for the ac current as shown in Fig. 5. Of course, model 1b is unable to reproduce the small waveform, similar to a triangle and marked in Fig. 5, in where three diodes are in conduction (from zero to the peak value of this triangle). The harmonics of current i_a are shown in Table V for this case in where the results from the DAP are indicated on the left and the results from PSCAD on the right in bracket. The discrepancies are due to the no coherent operation of model 1b when $R_d = 41 \Omega$. However, these discrepancies are relative low for the amplitudes and for the arguments of the first harmonics. A less critical case (for instance: $R_d = 50 \Omega$) provides lower discrepancies since it is more close to DCM.

B. Test 2: Unbalanced Operation with Negative Sequence Voltage at Fundamental Frequency

A negative sequence voltage of $6/\sqrt{3}$ V, 120° , at fundamental frequency, is added to the ac voltages considered in Test 1. Under this new condition, the boundary for the DCM is obtained when $R_d = 116.3 \Omega$. A value of $R_d = 117 \Omega$ is chosen for Test 2. Convergence properties are shown in Table VI and the final values for the main angles are indicated in Table VII. These angles exhibit a clear asymmetry due to the unbalanced operation. It leads, as shown in Tables VIII and IX, to a considerable

TABLE III
TEST 1: DC HARMONIC COMPONENTS WITH $R_d = 70 \Omega$

k	DAP				PSCAD (Time step $10 \mu s$)			
	$\hat{I}_{dk}(A)$	$\phi_{dk}(^\circ)$	$\hat{U}_{dk}(V)$	$\theta_{dk}(^\circ)$	$\hat{I}_{dk}(A)$	$\phi_{dk}(^\circ)$	$\hat{U}_{dk}(V)$	$\theta_{dk}(^\circ)$
0	7.75986	90	543.19026	90	7.76106	90	543.18972	90.00000
6	9.09465	14.92679	3.75762	-74.73502	9.09445	14.92749	3.75749	-74.73241
12	1.36776	-121.3781	0.28256	148.79104	1.36767	-121.38004	0.28258	148.79026
18	0.79427	58.34962	0.10939	-31.53765	0.79408	58.34907	0.10939	-31.54377
24	0.47139	-136.118	0.04869	133.96652	0.47119	-136.11197	0.04869	133.96048
30	0.27538	31.78962	0.02276	-58.14274	0.27522	31.80515	0.02276	-58.15461
36	0.17377	-153.1386	0.01197	116.91772	0.17363	-153.11923	0.01197	116.87244
42	0.1308	23.71833	0.00772	-66.23336	0.13059	23.74394	0.00775	-66.41348
48	0.10787	-164.8511	0.00557	105.1912	0.10770	-164.83808	0.00557	105.15920

TABLE IV
TEST 1: DC HARMONIC COMPONENTS WITH $R_d = 64.82 \Omega$

k	DAP				PSCAD (Time step $10 \mu s$)			
	$\hat{I}_{dk}(A)$	$\phi_{dk}(^\circ)$	$\hat{U}_{dk}(V)$	$\theta_{dk}(^\circ)$	$\hat{I}_{dk}(A)$	$\phi_{dk}(^\circ)$	$\hat{U}_{dk}(V)$	$\theta_{dk}(^\circ)$
0	8.36609	90.00000	542.29002	90.00000	8.36733	90.00000	542.28601	90.00000
6	9.60288	13.34235	3.96759	-76.29244	9.60380	13.33981	3.96795	-76.29437
12	1.39815	-130.90805	0.28884	139.27455	1.39822	-130.92765	0.28886	139.25641
18	0.79165	48.36727	0.10903	-41.51100	0.79158	48.34530	0.10903	-41.53212
24	0.48558	-148.15330	0.05016	121.93801	0.48552	-148.17736	0.05016	121.91589

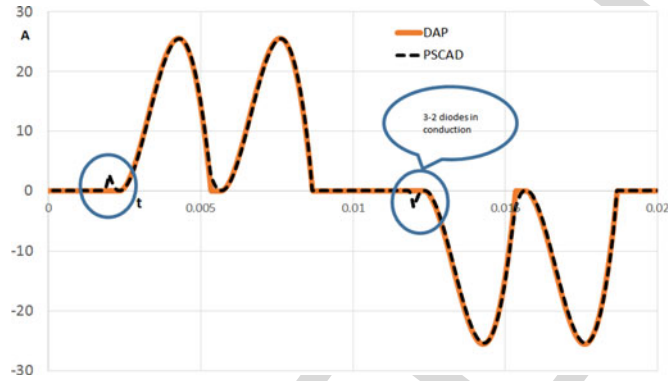


Fig. 5. Test 1: waveform for current i_a with $R_d = 41 \Omega$ in ICM.

TABLE V
TEST 1: AC HARMONIC CURRENTS WITH $R_d = 41 \Omega$ IN ICM
(TIME STEP $10 \mu s$)

k	$\hat{I}_{a,k}(A)$	$\phi_{a,k}(^\circ)$	k	$\hat{I}_{a,k}(A)$	$\phi_{a,k}(^\circ)$
1	14.827 (14.805)	-13.802 (-13.783)	17	0.567 (0.546)	104.388 (112.974)
5	9.416 (9.214)	108.655 (106.545)	19	0.470 (0.488)	-160.491 (178.818)
7	5.664 (5.693)	-104.744 (250.902)	23	0.340 (0.336)	-109.133 (266.285)
11	1.179 (1.111)	-52.758 (-50.074)	25	0.260 (0.271)	-9.071 (-39.449)
13	1.157 (1.183)	47.247 (35.941)	29	0.224 (0.230)	34.335 (57.558)

TABLE VI
TEST 2: CONVERGENCE PROPERTIES

Iter	$\alpha_1(^\circ)$	$\delta_1(^\circ)$
0	44.85869	88.15869
1	44.98046	87.38156
2	45.17384	86.45016
3	45.10902	86.49631
4	45.11187	86.49272
5	45.11187	86.49273
6	45.11187	86.49273

TABLE VII
TEST 2: FINAL VALUES FOR THE MAIN ANGLES

p	1	2	3	4	5	6
α_p	45.11187	110.93964	162.81625	225.11187	290.93964	342.81625
δ_p	86.49273	140.05373	212.18947	266.49273	320.05373	392.18947
γ_p	41.38086	29.11409	49.37322	41.38086	29.11409	49.37322

agreement with PSCAD is achieved as shown in Tables VIII and IX. 404 405

C. Test 3: Unbalanced Operation With a Second Harmonic Voltage of Positive Sequence 406 407

A second harmonic voltage of $0.8/\sqrt{3} V$, 0° , of positive sequence, is added to the ac voltages considered in Test 2. Under this new condition, the boundary for the DCM is obtained when $R_d = 121.49 \Omega$. A value of $R_d = 121.5 \Omega$ is chosen for Test 3. Convergence properties are shown in Table X. A full harmonic spectrum on the dc and ac sides can be seen as shown in Tables XI and XII. Significant levels of dc component are 408 409 410 411 412 413 414

level of non characteristic even harmonics on the dc side; to a significant level of negative sequence current at fundamental frequency; to the generation of triplen harmonics and to the generation of non characteristic sequences of the characteristic harmonics on the ac side. All odd harmonics of Table IX present quantities of positive (+) and negative (-) sequence. A very good

TABLE VIII
TEST 2: DC HARMONIC COMPONENTS

k	DAP				PSCAD (Time step $1 \mu s$)			
	$\hat{I}_{dk} (A)$	$\phi_{dk} (^\circ)$	$\hat{U}_{dk} (V)$	$\theta_{dk} (^\circ)$	$\hat{I}_{dk} (A)$	$\phi_{dk} (^\circ)$	$\hat{U}_{dk} (V)$	$\theta_{dk} (^\circ)$
0	4.69642	90.00000	549.48075	90.00000	4.70096	90.00000	549.48073	90.00000
2	4.46089	39.87167	5.52907	-49.52135	4.45792	39.86732	5.52906	-49.52138
4	3.20248	63.75408	1.98475	-25.94242	3.20344	63.80122	1.98475	-25.94240
6	5.72400	21.69209	2.36500	-68.10558	5.72554	21.68634	2.36500	-68.10549
8	1.87167	-56.17520	0.57999	-146.02345	1.87071	-56.10017	0.57999	-146.02337
10	0.37627	-61.38765	0.09328	-151.26625	0.37897	-61.35962	0.09328	-151.26479
12	0.83921	-75.89204	0.17337	-165.79088	0.83934	-76.07417	0.17337	-165.79012
14	0.81307	129.52176	0.14397	39.60847	0.81234	129.60626	0.14397	39.60883
16	0.23758	119.38362	0.03681	29.45950	0.24059	119.30939	0.03681	29.46439
18	0.27148	86.38100	0.03739	-3.55155	0.27287	85.82876	0.03739	-3.54644
20	0.42138	-27.99462	0.05223	-117.93392	0.42113	-28.04941	0.05223	-117.93187
22	0.14546	-125.58524	0.01639	144.46994	0.14702	-124.48907	0.01639	144.48178
24	0.11019	-138.14783	0.01138	131.90276	0.11280	-138.67891	0.01138	131.91699

TABLE IX
TEST 2: AC HARMONIC SEQUENCE CURRENTS

k	DAP				PSCAD (Time step $1 \mu s$)			
	$\hat{I}_{k+} (A)$	$\phi_{k+} (^\circ)$	$\hat{I}_{k-} (A)$	$\phi_{k-} (^\circ)$	$\hat{I}_{k+} (A)$	$\phi_{k+} (^\circ)$	$\hat{I}_{k-} (A)$	$\phi_{k-} (^\circ)$
1	5.35232	-11.10762	2.64792	142.89774	5.35195	-11.12448	2.64799	142.88667
3	2.42630	-63.55436	2.13546	166.57343	2.42651	-63.60203	2.13526	166.51317
5	1.53198	-39.47313	3.86558	123.56321	1.53170	-39.56775	3.86551	123.47863
7	2.72659	-80.60730	1.36155	53.99653	2.72671	-80.72611	1.36190	53.87275
9	0.83895	-169.96274	0.37105	68.58471	0.83933	189.87440	0.37074	68.42754
11	0.17986	152.65820	0.78022	38.46301	0.17962	152.39908	0.78053	38.27491
13	0.29571	158.41147	0.48269	-113.67144	0.29600	158.20937	0.48307	246.10527
15	0.44527	14.76059	0.18799	-131.37728	0.44565	14.50033	0.18775	228.28799
17	0.08133	4.24814	0.19491	-161.18517	0.08105	3.81108	0.19504	198.58954
19	0.11167	-29.09047	0.27430	91.54894	0.11181	-29.30115	0.27469	91.21312
21	0.22206	-147.79659	0.08382	-13.33553	0.22243	211.84089	0.08403	-13.91303
23	0.07344	119.18350	0.06266	-20.91137	0.07363	118.51819	0.06248	-21.12156

TABLE X
TEST 3: CONVERGENCE PROPERTIES

It	1	2	3	4	5	6
0 α	44.99118	104.99118	164.99118	224.99118	284.99118	344.99118
0 δ	87.89118	147.89118	207.89118	267.89118	327.89118	387.89118
1 α	45.09627	107.66448	163.50356	225.06258	287.37569	343.31879
1 δ	86.94984	144.64077	210.9351	267.36291	325.27137	391.14875
2 α	45.3224	111.0962	162.9055	225.16597	290.34069	342.4241
2 δ	85.76235	139.70785	212.0829	266.76397	321.52514	392.53322
3 α	45.27363	111.65587	163.26849	225.08954	290.71904	342.70788
3 δ	85.7659	138.35184	211.67179	266.84448	320.79185	392.19117
4 α	45.27594	111.67286	163.27733	225.09359	290.7262	342.71312
4 δ	85.76305	138.25189	211.65761	266.83924	320.76709	392.18224
5 α	45.27594	111.67288	163.27731	225.0936	290.7262	342.71312
5 δ	85.76305	138.25173	211.65764	266.83924	320.76715	392.18225

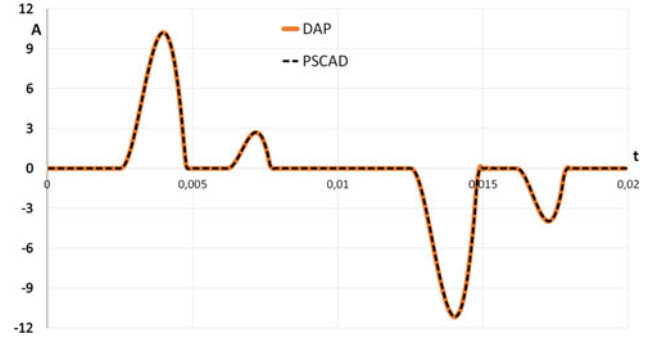


Fig. 6. Test 3: waveform for current i_a with $R_d = 121.49 \Omega$.

present in the ac currents ($I_{a,0} = -0.16 A$, $I_{b,0} = -0.072 A$ and $I_{c,0} = 0.23 A$) and non characteristic harmonics are present on the dc side (Table XI) (mainly third harmonic) and on the ac side (Table XII) (mainly second harmonic). A very good agreement with PSCAD is achieved as shown in Fig. 6 and Tables XI and XII.

A time step of $10 \mu s$ (instead of $1 \mu s$) provides also acceptable results for the harmonic analysis of test 3 via PSCAD and its use increases the simulation speed. A CPU time of 8 s is required

to reach the steady-state solution via PSCAD with a time step of $10 \mu s$. However, the CPU time is less than 0.016 s when the DAP is implemented in C, and running in Windows, namely, the simulation speed is 500 times higher with the proposed DAP.

D. Discussion and Application

If an identical representation of the diode is used for model 1b and PSCAD, the harmonic analysis by means of model 1b (with coherence checking) is more accurate than by means of PSCAD since the accuracy with PSCAD depends on the time step to

TABLE XI
TEST 3: DC HARMONIC COMPONENTS

k	DAP				PSCAD (Time step 1 μ s)			
	$\hat{I}_{dk}$ (A)	ϕ_{dk} ($^\circ$)	$\hat{U}_{dk}$ (V)	θ_{dk} ($^\circ$)	$\hat{I}_{dk}$ (A)	ϕ_{dk} ($^\circ$)	$\hat{U}_{dk}$ (V)	θ_{dk} ($^\circ$)
0	4.52583	90.00000	549.84293	90.00000	4.52764	90.00000	549.84291	90.00000
1	0.40163	120.34274	0.99545	31.51172	0.40087	120.76210	0.99545	31.51169
2	4.33075	39.44878	5.36779	-49.96667	4.33164	39.46018	5.36779	-49.96667
3	0.26502	44.68297	0.21899	-44.92732	0.26564	44.79794	0.21899	-44.92785
4	3.11460	64.95581	1.93028	-24.75190	3.11389	64.93827	1.93028	-24.75190
5	0.26451	-21.14909	0.13115	-110.91526	0.26184	-20.97510	0.13114	-110.91561
6	5.55443	22.14077	2.29494	-67.66438	5.55430	22.15250	2.29494	-67.66432
7	0.20319	57.90538	0.07196	-31.92760	0.20605	57.70042	0.07196	-31.92531
8	1.86259	-54.98148	0.57718	-144.83534	1.86382	-54.99418	0.57718	-144.83526
9	0.09537	-121.98594	0.02627	148.14397	0.09682	-121.01324	0.02627	148.14211
10	0.37023	-54.37746	0.09178	-144.26054	0.36909	-54.42606	0.09178	-144.26019
11	0.07755	-160.65120	0.01748	109.45508	0.07615	-159.62847	0.01748	109.45859
12	0.83870	-74.05487	0.17326	-163.95744	0.83946	-74.00497	0.17327	-163.95708
13	0.04808	-22.36811	0.00917	-112.27818	0.04669	-25.44275	0.00917	-112.28483
14	0.78264	132.99241	0.13859	43.07592	0.78401	132.98244	0.13859	43.07634
15	0.08688	95.20446	0.01436	5.28240	0.08966	95.99331	0.01436	5.28663

TABLE XII
TEST 3: AC HARMONIC SEQUENCE CURRENTS

k	DAP				PSCAD (Time step 1 μ s)			
	I_{k+} (A)	ϕ_{k+} ($^\circ$)	I_{k-} (A)	ϕ_{k-} ($^\circ$)	I_{k+} (A)	ϕ_{k+} ($^\circ$)	I_{k-} (A)	ϕ_{k-} ($^\circ$)
1	5.15881	-11.03714	2.56702	142.35052	5.15857	-11.05410	2.56680	142.33441
2	0.22323	19.06813	0.17136	150.44203	0.22344	19.03852	0.17184	150.32943
3	2.36052	-63.84074	2.07235	167.61794	2.36037	-63.88955	2.07224	167.56774
4	0.13001	-62.54149	0.17712	84.64989	0.13035	-62.78287	0.17708	84.47330
5	1.49477	-38.07669	3.74267	123.92894	1.49477	-38.16141	3.74271	123.84311
6	0.12770	-128.54481	0.14170	157.59673	0.12752	231.20343	0.14177	157.44079
7	2.65328	-80.06695	1.34707	54.67088	2.65347	-80.18881	1.34717	54.55660
8	0.09441	-41.74618	0.05425	4.58904	0.09442	-41.95762	0.05457	4.08130
9	0.83844	-168.00835	0.37225	72.78936	0.83862	191.84595	0.37235	72.62637
10	0.05988	118.04875	0.05374	-42.08478	0.06024	117.61448	0.05342	-42.59650
11	0.16388	159.25617	0.77455	39.73591	0.16400	159.06373	0.77481	39.53244
12	0.03803	80.71940	0.03380	88.26153	0.03771	79.98212	0.03377	87.91614
13	0.29526	161.19303	0.46506	-109.93115	0.29551	160.94557	0.46523	249.86461
14	0.02523	-136.02886	0.05661	-151.27920	0.02521	223.50874	0.05679	208.12821
15	0.43006	18.16551	0.17977	-129.19351	0.43027	17.92328	0.17994	230.51389

estimate the switching instants and to calculate the harmonic components via FFT. Both limitations do not exist in model 1b since the basic angles are determined with very high resolution and the harmonic components are estimated using analytical expressions.

It is important to note that the harmonic analysis with the proposed DAP of model 1b is several hundred times faster than with the conventional time domain simulation via PSCAD. This interesting property makes very attractive the integration of model 1b as nonlinear load into the iterative harmonic analysis (IHA) or into the harmonic power flow (HPF) [29]–[33] in where model 1b must be solved several times with different ac harmonic voltages sources within the iterative process. The linear network of the IHA or of the HPF can present any complexity, and hence filters, connected on the ac side as it is indicated in [8], can be directly included into the linear network. Moreover, an easy and direct integration of model 1b into the IHA or into the HPF is possible if the harmonic interaction converter-linear network is solved by means of the Gauss algorithm.

The average modeling of [9] has been recently proposed for the analysis of the first characteristic harmonics with DCM, CCM-1 and CCM-2, and balanced operation. This procedure is about three times faster than the conventional time domain simulation [9]. Moreover, the analysis of noncharacteristic harmonics, due to nonideal conditions, have not been considered in [9]. However, the average modeling of [9] allows to simulate the dynamic response of the converter, for instance, the transient response from DCM to CCM due to a sudden change in the load R_d . Models 1b and 1a provide the state state response in DCM and CCM-1 operation with non noncharacteristic harmonics. However, the transient response from DCM to CCM-1 is not possible with models 1b and 1a since they are only oriented to the harmonic analysis under steady-state operation.

VII. CONCLUSION

Following the method proposed in [6], a DAP for calculating ac and dc harmonic components of a diode rectifier under non-ideal conditions is presented. Discontinuous conduction mode

(DCM) is considered in this paper, and a procedure for detecting the boundary of DCM is also proposed.

A set of twelve analytical non linear equations allows an easy calculation of switching angles, which become the only variables to be updated in this DAP. Once they are determined, characteristic and non characteristic harmonic components can be easily estimated by a set of compact analytical expressions.

The switching angles are calculated by a full Newton formulation, and the accuracy of this DAP has been tested by comparison with time domain simulation via PSCAD-EMTDC. The steady-state solution with PSCAD requires a considerable computer effort while the simulation speed with the proposed DAP is very high. That is a clear advantage in order to include this DAP in harmonic power flows. This DAP has been implemented in the low-level programming language C, and included in the EDF ExpertEC program as a plug in.

The procedure for determining the boundary of DCM; the accuracy of this DAP under nonideal conditions; the systematic form of defining and solving the nonlinear equations and the set of analytical expressions for harmonic calculations are additions to previous DAPs.

APPENDIX A

TERMS OF THE JACOBIAN MATRIX

According to (36), the derivatives of term $\underline{\lambda}_p$ are given by:

$$\frac{\partial \underline{\lambda}_p}{\partial \alpha_p} = -\frac{\partial \underline{\lambda}_p}{\partial \delta_p} = -\underline{\nu} \cdot \underline{\lambda}_p \quad (58)$$

Functions a_{1p} , a_{2p} and $\underline{a}_{12p}$ only depend on angle α_p , and they are defined in (32), (33) and (41). Hence, the corresponding derivatives are:

$$\frac{\partial a_{1p}}{\partial \alpha_p} = \sum_{m=0}^{n_h} \text{Im} \left\{ jm \cdot (1 - \underline{c}_m) \cdot \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm \alpha_p} \right\} \quad (59)$$

$$\frac{\partial a_{2p}}{\partial \alpha_p} = - \sum_{m=0}^{n_h} \text{Im} \left\{ jm \cdot (\sigma + jm \cdot \underline{c}_m) \cdot \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm \alpha_p} \right\} \quad (60)$$

$$\frac{\partial \underline{a}_{12p}}{\partial \alpha_p} = \frac{1}{k_n} \left(\frac{\partial a_{2p}}{\partial \alpha_p} - \underline{\nu}^* \frac{\partial a_{1p}}{\partial \alpha_p} \right) \quad (61)$$

Functions a_{3p} and a_{4p} only depend on angles α_{p+1} and δ_p , and they are defined in (34) and (35). Hence, the corresponding derivatives are:

$$\frac{\partial a_{3p}}{\partial \alpha_{p+1}} = \sum_{m=0}^{n_h} \text{Im} \left\{ (\sigma + jm) \hat{\underline{E}}_{p+1,m} \cdot \varepsilon^{(\sigma + jm) \alpha_{p+1} - \sigma \delta_p} \right\} \quad (62)$$

$$\frac{\partial a_{4p}}{\partial \alpha_{p+1}} = - \sum_{m=0}^{n_h} \text{Im} \left\{ \sigma (\sigma + jm) \hat{\underline{E}}_{p+1,m} \cdot \varepsilon^{(\sigma + jm) \alpha_{p+1} - \sigma \delta_p} \right\} \quad (63)$$

$$\frac{\partial a_{3p}}{\partial \delta_p} = - \sum_{m=0}^{n_h} \text{Im} \left\{ \sigma \hat{\underline{E}}_{p+1,m} \cdot \varepsilon^{(\sigma + jm) \alpha_{p+1} - \sigma \delta_p} + jm \underline{c}_m \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm \delta_p} \right\} \quad (64)$$

$$\frac{\partial a_{4p}}{\partial \delta_p} = \sum_{m=0}^{n_h} \text{Im} \left\{ \sigma^2 \hat{\underline{E}}_{p+1,m} \cdot \varepsilon^{(\sigma + jm) \alpha_{p+1} - \sigma \delta_p} + m^2 \underline{c}_m \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm \delta_p} \right\} \quad (65)$$

Residues $\triangle F_{a,p}$ and $\triangle F_{b,p}$ depend on angles α_p , δ_p and α_{p+1} , and they are defined in (43) and (44). Hence, the corresponding terms of the Jacobian matrix present the following expressions:

$$\frac{\partial \triangle F_{a,p}}{\partial \alpha_p} = -\text{Im} \left\{ \frac{\partial \underline{a}_{12p}}{\partial \alpha_p} \underline{\lambda}_p + \underline{a}_{12p} \frac{\partial \underline{\lambda}_p}{\partial \alpha_p} \right\} \quad (66)$$

$$\frac{\partial \triangle F_{b,p}}{\partial \alpha_p} = -\text{Im} \left\{ \underline{\nu} \frac{\partial \underline{a}_{12p}}{\partial \alpha_p} \underline{\lambda}_p + \underline{\nu} \cdot \underline{a}_{12p} \frac{\partial \underline{\lambda}_p}{\partial \alpha_p} \right\} \quad (67)$$

$$\frac{\partial \triangle F_{a,p}}{\partial \delta_p} = \frac{\partial a_{3p}}{\partial \delta_p} - \text{Im} \left\{ \underline{a}_{12p} \cdot \frac{\partial \underline{\lambda}_p}{\partial \delta_p} \right\} \quad (68)$$

$$\frac{\partial \triangle F_{b,p}}{\partial \delta_p} = \frac{\partial a_{4p}}{\partial \delta_p} - \text{Im} \left\{ \underline{\nu} \cdot \underline{a}_{12p} \cdot \frac{\partial \underline{\lambda}_p}{\partial \delta_p} \right\} \quad (69)$$

$$\frac{\partial \triangle F_{a,p}}{\partial \alpha_{p+1}} = \frac{\partial a_{3p}}{\partial \alpha_{p+1}}; \quad \frac{\partial \triangle F_{b,p}}{\partial \alpha_{p+1}} = \frac{\partial a_{4p}}{\partial \alpha_{p+1}} \quad (70)$$

We can observe that the terms of the Jacobian matrix present simple analytical expressions.

APPENDIX B

HARMONIC CURRENTS

In accordance with (11) and (14), current $i_d(\tau)$ can be obtained from voltage $u_d(\tau)$, and this voltage can be expressed in function of components u_{tp} and u_{sp} . Hence, current i_d can be also split into two components i_{tp} and i_{sp} , so that:

$$i_d = i_{tp} + i_{sp}; \quad \begin{cases} i_{tp} = G_d \cdot u_{tp} + B_c \cdot u'_{tp} \\ i_{sp} = G_d \cdot u_{sp} + B_c \cdot u'_{sp} \end{cases} \quad (71)$$

Taking into account (48) and (49), components i_{tp} and i_{sp} can be written as:

$$i_{tp}(\tau) = \frac{1}{j2} (\underline{d}_t \cdot \hat{\underline{U}}_{tp} \varepsilon^{\underline{\nu} \tau} - \underline{d}_t^* \cdot \hat{\underline{U}}_{tp}^* \varepsilon^{\underline{\nu}^* \tau}) \quad (72)$$

$$i_{sp}(\tau) = \frac{1}{j2} \sum_{m=0}^{n_h} (\underline{d}_m \cdot \hat{\underline{E}}_{p,m} \varepsilon^{jm \tau} - \underline{d}_m^* \cdot \hat{\underline{E}}_{p,m}^* \varepsilon^{-jm \tau}) \quad (73)$$

$$\underline{d}_t = G_d + \underline{\nu} B_c; \quad \underline{d}_m = \underline{c}_m (G_d + jm \cdot B_c) \quad (74)$$

Harmonic contributions of terms i_{tp} and i_{sp} are defined as:

$$\hat{\underline{I}}_{dk,tp} = \frac{j}{\pi \rho_k} \int_{\alpha_p}^{\delta_p} i_{tp} \cdot \varepsilon^{-jk \tau} d\tau; \quad \hat{\underline{I}}_{dk,sp} = \frac{j}{\pi \rho_k} \int_{\alpha_p}^{\delta_p} i_{sp} \cdot \varepsilon^{-jk \tau} d\tau \quad (75)$$

$$\hat{\underline{I}}_{dk,p} = \hat{\underline{I}}_{dk,tp} + \hat{\underline{I}}_{dk,sp}; \quad \rho_k = \begin{cases} 1 & \text{for } k \neq 0 \\ 2 & \text{for } k = 0 \end{cases} \quad (76)$$

Taking into account (72) and (73), the harmonic contributions present the following compact form:

$$\hat{\underline{I}}_{dk,tp} = \underline{\beta}'_{k,tp} \cdot \underline{d}_t \cdot \hat{\underline{U}}_{tp} + \underline{\beta}''_{k,tp} \cdot \underline{d}_t^* \cdot \hat{\underline{U}}_{tp}^* \quad (77)$$

$$\hat{\underline{I}}_{dk,sp} = \sum_{m=0}^{n_h} (\underline{\beta}'_{km,sp} \cdot \underline{d}_m \cdot \hat{\underline{E}}_{p,m} + \underline{\beta}''_{km,sp} \cdot \underline{d}_m^* \cdot \hat{\underline{E}}_{p,m}^*) \quad (78)$$

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$$\underline{\beta}'_{k,tp} = \frac{\varepsilon^{(\nu-jk)\delta_p} - \varepsilon^{(\nu-jk)\alpha_p}}{2\pi\rho_k(\nu-jk)} \quad (79)$$

$$\underline{\beta}''_{k,tp} = -\frac{\varepsilon^{(\nu^*-jk)\delta_p} - \varepsilon^{(\nu^*-jk)\alpha_p}}{2\pi\rho_k(\nu^*-jk)} \quad (80)$$

$$\underline{\beta}'_{km,sp} = j \frac{\varepsilon^{-j(k-m)\delta_p} - \varepsilon^{-j(k-m)\alpha_p}}{2\pi\rho_k(k-m)} \quad (81)$$

$$\underline{\beta}''_{km,sp} = -j \frac{\varepsilon^{-j(k+m)\delta_p} - \varepsilon^{-j(k+m)\alpha_p}}{2\pi\rho_k(k+m)} \quad (82)$$

$$\underline{\beta}'_{kk,sp} = \frac{\delta_p - \alpha_p}{2\pi\rho_k} \quad \text{for } m = k \quad (83)$$

$$\underline{\beta}''_{00,sp} = -\underline{\beta}'_{00,sp} \quad \text{for } m = k = 0. \quad (84)$$

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A Detailed Procedure for Harmonic Analysis of Three-Phase Diode Rectifiers Under Discontinuous Conduction Mode and Nonideal Conditions

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Abstract—Fast and detailed calculations of harmonics, generated by three-phase uncontrolled rectifiers with finite dc smoothing, can be performed by using detailed analytical procedures (DAPs). Several DAPs are developed for continuous- and discontinuous-conduction modes (CCM and DCM) when balanced operation of the system is assumed. The authors of this paper have recently proposed a new DAP (model 1a) in which nonideal conditions, such as unbalances and even harmonic voltages on the ac side, are taken into account. However, only CCM is assumed in model 1a. The main objective of this paper is to overcome this limitation of model 1a so that a new DAP (model 1b) for DCM is presented, which is capable of considering unbalanced operation. Moreover, the boundary of the DCM is also estimated for model 1b under nonideal conditions.

Index Terms—Discontinuous conduction mode (DCM), uncontrolled rectifiers, harmonics.

NOMENCLATURE

t	Time instant.
ω_1	Fundamental angular frequency.
τ	Variable angle defined as $\tau = \omega_1 t$.
R, G	Resistance (R) and conductance (G).
X	Inductive reactance at fundamental frequency.
B_c	Capacitive susceptance at fund. frequency.
u, e, i	Instantaneous voltages (u, e) and currents (i).
$\hat{U}, \hat{E}, \hat{I}$	Complex phasors for voltages and currents.
d	Subscript for denoting dc magnitudes.
ℓ	Subscript for denoting ac magnitudes.
ℓ	takes the values 1, 2, 3 for phases a, b, c.
$U_{\ell,0}, I_{\ell,0}$	DC components of ac magnitudes u_ℓ and i_ℓ .
U_d, I_d	DC components of dc magnitudes u_d and i_d .

k, m	Order of a generic harmonic.	36
n_h	Highest harmonic order in the ac sources.	37
α	Initial angle for a conduction subinterval.	38
δ	Final angle for a conduction subinterval.	39
γ	Conduction angle ($\gamma = \delta - \alpha$).	40
p	Index (1 to 6) for denoting a cond. subinterval.	41
$'$	First derivative of u_d or i_d with respect to τ .	42
$''$	Second derivative of u_d or i_d with respect to τ .	43
t	Subscript for the transient solution of a differential equation.	44
s	Subscript for the steady-state sol. of a differential equation.	46

I. INTRODUCTION

THREE-PHASE diode rectifiers with capacitive dc smoothing are widely extended in low voltage applications and they produce a considerable level of ac harmonic currents. Hence, a detailed modelling, representing the steady-state solution of these elements, is required in order to obtain a good harmonic estimation. Detailed methods can be implemented in time domain [1]–[2], in the harmonic domain [3] or methods which are based on a detailed analytical procedure (DAP) [4]–[6]. Finite capacitive dc smoothing and an accurate determination of the switching angles are considered in these detailed methods. The fastest results are obtained via DAPs as indicated in [6]. As reported in [6], other methods propose average modeling [7]–[9] or they assume certain simplifications in the determination of the switching angles [10]–[13] or they do not consider the ripple of the capacitor voltage [14]–[20]. Harmonic analysis of controlled and uncontrolled converters has been also proposed [21]–[25].

Discontinuous conduction mode (DCM) is considered in the DAPs of [4] and [5] under balanced operation. However, non-ideal conditions, such as negative sequence voltage and second harmonic voltage on the ac side lead to a more complex behaviour of the converter. The DAP proposed in [6], and denoted as model 1a in this paper, provides exact results under nonideal conditions. However, model 1a only considers CCM in its normal formulation, namely, two or three diodes in conduction. This typical formulation is denoted as CCM-1 in [7], and it fits with the most practical cases [2]. However, as reported in [9], operation with only three diodes in conduction, denoted as CCM-2 in [7], is possible in particular situations [26].

Manuscript received December 5, 2016; revised June 26, 2017; accepted August 15, 2017. Date of publication; date of current version. This work was supported under a contract with EDF R&D. Paper no. TPWRD-01432-2016. (Corresponding author: Luis F. Beites.)

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Digital Object Identifier 10.1109/TPWRD.2017.2748384

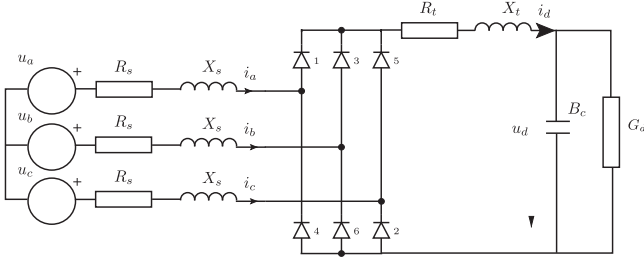


Fig. 1. Basic scheme for the analysis.

The object of this paper is to describe a new DAP (model 1b) for DCM, namely, two or no diodes in conduction. Model 1b, as model 1a, provides a systematic form of generating the nonlinear equations in order to calculate the switching angles in a full Newton formulation. Harmonics and dc components on the ac and dc sides can be evaluated by means a set of compact analytical expressions.

II. NOTATION AND MAIN RELATIONS

The scheme of Fig. 1 is the system to be studied. The notation of [6] is also here used, and hence, ac and dc instantaneous voltages and currents are functions of angle τ , and they are expressed by:

$$u_\ell = \sum_{m=0}^{n_h} \text{Im}\{\hat{U}_{\ell,m} \cdot e^{jm\tau}\}; \quad i_\ell = \sum_{m=0}^{\infty} \text{Im}\{\hat{I}_{\ell,m} \cdot e^{jm\tau}\} \quad (1)$$

$$u_d = \sum_{k=0}^{\infty} \text{Im}\{\hat{U}_{dk} \cdot e^{jk\tau}\}; \quad i_d = \sum_{k=0}^{\infty} \text{Im}\{\hat{I}_{dk} \cdot e^{jk\tau}\} \quad (2)$$

According to (1) and (2), dc components of ac and dc magnitudes are related to the harmonic phasors by:

$$\hat{U}_{\ell,0} = jU_{\ell,0}; \quad \hat{I}_{\ell,0} = jI_{\ell,0}; \quad \hat{U}_{d0} = jU_d; \quad \hat{I}_{d0} = jI_d \quad (3)$$

Six sections or intervals for the dc current i_d are considered. A generic section is denoted by s_p . Each section s_p can be divided in a conduction subinterval s_{px} (two diodes in conduction) and a discontinuity subinterval s_{py} (no diodes in conduction). The limits of these intervals are defined by using the basic (or switching) angles α and δ , so that:

$$s_p = [\alpha_p, \beta_p]; \quad s_{px} = [\alpha_p, \delta_p]; \quad s_{py} = [\delta_p, \beta_p] \quad (4)$$

$$\beta_p = \alpha_{p+1} \text{ (for } p \neq 6); \quad \beta_6 = \alpha_1 + 2\pi \quad (5)$$

It is assumed that dc current $i_d(\tau)$ exhibits a discontinuous behaviour in all subintervals s_{py} . This condition is given by:

$$i_d(\tau) = 0 \quad \forall \tau \in s_{py} \quad (6)$$

while current $i_d(\tau)$ is equal to zero in the limits of subintervals s_{px} , namely:

$$i_d(\alpha_p) = i_d(\delta_p) = 0 \quad (7)$$

Current $i_d(\tau)$ and basic angles are indicated in Fig. 2. A power flow from the ac side to the dc load takes place during the conduction subintervals. On the other hand, all diodes are

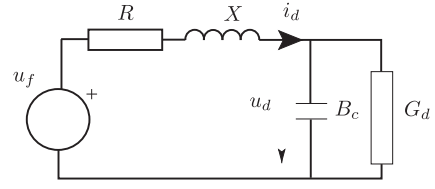
Fig. 2. Waveform of current i_d .

TABLE I
DETAILS OF CONDUCTION SUBINTERVALS FOR THE THREE-PHASE RECTIFIER

p	Diodes	Limits	$e_p(\tau)$
1	1, 6	α_1, δ_1	$e_1 = u_a - u_b$
2	1, 2	α_2, δ_2	$e_2 = u_a - u_c$
3	2, 3	α_3, δ_3	$e_3 = u_b - u_c$
4	3, 4	α_4, δ_4	$e_4 = u_b - u_a$
5	4, 5	α_5, δ_5	$e_5 = u_c - u_a$
6	5, 6	α_6, δ_6	$e_6 = u_c - u_b$

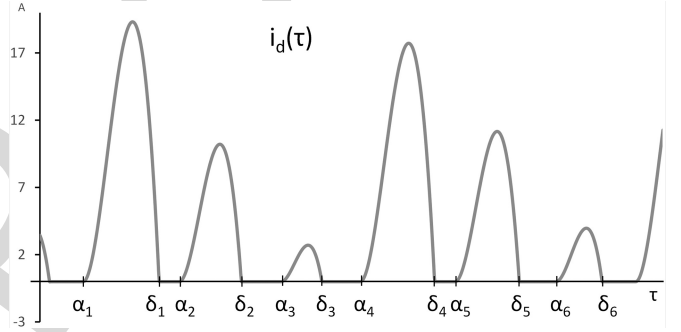


Fig. 3. DC side equivalent circuit.

blocked during the discontinuity subintervals, and the capacitor exhibits a certain discharge process. A key aspect of this analysis is to find basic (or switching) angles α_p and δ_p when the ac voltages and parameters R_s, X_s, R_t, X_t, B_c and G_d are known. Once the basic angles are determined, dc and ac harmonic components can be evaluated by means of a set of compact analytical expressions.

A. Conduction Subintervals

The conduction subintervals can be described by a loop in which only two AC phases and two diodes are involved, as shown in Table I and Fig. 3. Parameters R and X are:

$$R = 2R_s + R_t; \quad X = 2X_s + X_t \quad (8)$$

and voltage $u_f(\tau) = e_p(\tau)$ for $\tau \in s_{px}$. Voltage $e_p(\tau)$ is the ac source for a generic conduction loop p . Taking into account (1) and Table I, $e_p(\tau)$ can be expressed by:

$$e_p(\tau) = \sum_{m=0}^{n_h} \text{Im}\{\hat{E}_{p,m} \cdot e^{jm\tau}\}; \quad \hat{E}_{p,0} = jE_{p,0} \quad (9)$$

For instance, for $p = 1$, we have: $\hat{E}_{1,m} = \hat{U}_{a,m} - \hat{U}_{b,m}$ and $E_{1,0} = U_{a,0} - U_{b,0}$. On the other hand, dc magnitudes u_d and

121 i_d are related by the following two equations:

$$e_p = u_d + R \cdot i_d + X \cdot i'_d \quad (10)$$

$$i_d = G_d u_d + B_c u'_d; \quad G_d = 1/R_d \quad (11)$$

122 where $i'_d = di_d(\tau)/d\tau$ and $u'_d = du_d(\tau)/d\tau$. The elimination of
123 current i_d among (10) and (11) leads to the following differential
124 equation of second order for voltage u_d :

$$u''_d + 2\xi u'_d + k_o^2 u_d = f_p(\tau) \quad \text{for } \tau \in s_{px} \quad (12)$$

$$2\xi = \frac{R}{X} + \frac{G_d}{B_c}; \quad k_o^2 = \frac{RG_d + 1}{XB_c}; \quad f_p(\tau) = \frac{e_p(\tau)}{XB_c} \quad (13)$$

125 where k_o is the normalized natural frequency as it is used in
126 [2] and ξ is a damping factor. The solution of (12) presents a
127 steady-state component u_{sp} and a transient component u_{tp} :

$$u_d = u_{sp} + u_{tp} \quad \text{for } \tau \in s_{px} \quad (14)$$

128 Voltage u_{sp} is obtained from:

$$u_{sp} = \sum_{m=0}^{n_h} \text{Im}\{\hat{U}_{sp,m} \cdot \varepsilon^{jm\tau}\}; \quad \hat{U}_{sp,m} = \underline{c}_m \cdot \hat{E}_{p,m} \quad (15)$$

$$\underline{c}_m = \frac{1}{1 + (R + jmX)(G_d + jmB_c)} \quad (16)$$

129 Voltage u_{tp} presents the following form when $\xi < k_o$:

$$u_{tp} = \text{Im}\{\hat{U}_{tp} \cdot \varepsilon^{\nu\tau}\}; \quad k_n = \sqrt{k_o^2 - \xi^2}; \quad \nu = -\xi + jk_n \quad (17)$$

130 The derivatives of components u_{sp} and u_{tp} are given by:

$$u'_{sp} = \sum_{m=0}^{n_h} \text{Im}\{\hat{U}'_{sp,m} \cdot \varepsilon^{jm\tau}\}; \quad \hat{U}'_{sp,m} = jm \cdot \underline{c}_m \cdot \hat{E}_{p,m} \quad (18)$$

$$u'_{tp} = \text{Im}\{\hat{U}'_{tp} \cdot \varepsilon^{\nu\tau}\}; \quad \hat{U}'_{tp} = \nu \cdot \hat{U}_{tp} \quad (19)$$

131 The steady-state component is completely determined by (15)
132 and (16). However, phasor $\hat{U}_{tp}$ of the transient component is
133 not known in advance, and it must be estimated from the initial
134 conditions $u_d(\alpha_p)$ and $u'_d(\alpha_p)$. The first condition is derived
135 from the crossing of ac and dc voltages for the initial instant
136 ($\tau = \alpha_p$). In accordance with the schemes of Figs. 1 and 3, the
137 first condition is given by:

$$u_d(\alpha_p) = e_p(\alpha_p) \quad (20)$$

138 Taking into account (7), (11) and (20), the second condition
139 can be expressed by:

$$u'_d(\alpha_p) = -\sigma \cdot u_d(\alpha_p) = -\sigma \cdot e_p(\alpha_p); \quad \sigma = G_d/B_c \quad (21)$$

140 Before the beginning of the conduction subinterval, the volt-
141 age drop on the line is zero since, under DCM operation, all
142 diodes are blocked in this previous state to the conduction.
143 Hence, (20) represents also the crossing of the ac and dc volt-
144 ages when the ac voltage is the voltage at the terminals of the
145 converter.

146 The idealized characteristic of the diode is assumed in this
147 paper, namely every diode is treated as a short circuit in conduc-
148 tion and as an open circuit in the blocking state. This assumption
149 is very often in harmonic analysis [4], [5], [27]. However, in the
150 conduction state, the diode could be represented by a series

combination of a small dc voltage V_0 and a small resistance R_0 151
[27]. In this last case, a quantity of $2R_0$ can be added to the 152
resistance R of (8) and a quantity of $-2V_0$ can be added to the 153
dc component $E_{p,0}$ of (9) in order to represent a nonideal diode 154
in conduction, while an open circuit is keep in the blocking state 155
of the diode. These additions do not change the formulation of 156
this paper. 157

The underdamped case ($\xi < k_o$) is assumed in this paper 158
since this condition is satisfied in most practical situations [2]. 159
However, the extension to the overdamped case ($\xi > k_o$) does 160
not involve any additional complexity [4]. In this last case, 161
the characteristic equation presents two different real roots: 162
 $\nu_1 = -\xi + \text{Im}\{k_n\}$ and $\nu_2 = -\xi - \text{Im}\{k_n\}$, and the transient 163
voltage is given by: $u_{tp} = \hat{U}_{t1p} \cdot \varepsilon^{\nu_1\tau} + \hat{U}_{t2p} \cdot \varepsilon^{\nu_2\tau}$, in where real 164
terms $\hat{U}_{t1p}$ and $\hat{U}_{t2p}$ play a similar role as complex phasor $\hat{U}_{tp}$ 165
in (17). The analysis of the overdamped case is omitted in this 166
paper for space reasons. 167

B. Discontinuity Subintervals 168

All diodes are blocked during the discontinuity subintervals 169
 s_{py} . The scheme of Fig. 3 can be also used in this case, and 170
voltage $u_f = u_d$ for $\tau \in s_{py}$, since current i_d is zero during 171
subintervals s_{py} . Voltage u_d is conditioned by the discharge 172
process of the capacitor during subintervals s_{py} , so that: 173

$$u_d(\tau) = u_d(\delta_p) \cdot \varepsilon^{-\sigma(\tau-\delta_p)} \quad \text{for } \tau \in s_{py} \quad (22)$$

where σ is defined in (21), and it is the normalized natural fre- 174
quency of first order circuit formed by the capacitor and conduc- 175
tance G_d . Therefore, we have for the final angle of subinterval 176
 s_{py} : 177

$$u_d(\beta_p) = u_d(\delta_p) \cdot \varepsilon^{-\sigma(\beta_p-\delta_p)} \quad (23)$$

Considering (5), (20) and (23) term $u_d(\delta_p)$ is equivalent to: 178

$$u_d(\delta_p) = e_{p+1}(\alpha_{p+1}) \cdot \varepsilon^{\sigma(\alpha_{p+1}-\delta_p)} \quad (24)$$

From (7), (11) and (24), term $u'_d(\delta_p)$ can be written as: 179

$$u'_d(\delta_p) = -\sigma \cdot u_d(\delta_p) = -\sigma \cdot e_{p+1}(\alpha_{p+1}) \cdot \varepsilon^{\sigma(\alpha_{p+1}-\delta_p)} \quad (25)$$

Finally, the combination of (22) and (24) leads to the expres- 180
sion for voltage $u_d(\tau)$ in discontinuity subinterval s_{py} : 181

$$u_d(\tau) = e_{p+1}(\alpha_{p+1}) \cdot \varepsilon^{\sigma(\alpha_{p+1}-\tau)} \quad \text{for } \tau \in s_{py} \quad (26)$$

For the last interval, namely, $p = 6$, we have: 182

$$e_7 = e_1; \quad \alpha_7 = \alpha_1 + 2\pi \quad (27)$$

Expressions (26) and (27) completely describe voltage $u_d(\tau)$ 183
during the discontinuity subintervals. 184

III. DETERMINATION OF THE BASIC ANGLES 185

The object of this analysis is to determine basic angles α_p and 186
 δ_p . To do that, four basic equations must be considered for each 187
interval p . They are obtained from the conditions indicated in 188
(20), (21), (24) and (25). Taking into account (14)–(19), these 189

conditions can be written as:

$$a_{1p} = e_p(\alpha_p) - u_{sp}(\alpha_p) = u_{tp}(\alpha_p) \quad (28)$$

$$a_{2p} = -\sigma e_p(\alpha_p) - u'_{sp}(\alpha_p) = u'_{tp}(\alpha_p) \quad (29)$$

$$a_{3p} = e_{p+1}(\alpha_{p+1}) \cdot \varepsilon^{\sigma(\alpha_{p+1} - \delta_p)} - u_{sp}(\delta_p) = u_{tp}(\delta_p) \quad (30)$$

$$a_{4p} = -\sigma e_{p+1}(\alpha_{p+1}) \cdot \varepsilon^{\sigma(\alpha_{p+1} - \delta_p)} - u'_{sp}(\delta_p) = u'_{tp}(\delta_p) \quad (31)$$

Functions a_{1p} to a_{4p} present a first equivalence with the steady-state component u_{sp} and voltages e_p and e_{p+1} . This first equivalence can be expressed in more detail by:

$$a_{1p} = \sum_{m=0}^{n_h} \text{Im} \left\{ (1 - \underline{c}_m) \cdot \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm\alpha_p} \right\} \quad (32)$$

$$a_{2p} = - \sum_{m=0}^{n_h} \text{Im} \left\{ (\sigma + jm\underline{c}_m) \cdot \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm\alpha_p} \right\} \quad (33)$$

$$a_{3p} = \sum_{m=0}^{n_h} \text{Im} \left\{ \begin{array}{l} \hat{\underline{E}}_{p+1,m} \cdot \varepsilon^{(\sigma+jm)\alpha_{p+1} - \sigma\delta_p} \\ - \underline{c}_m \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm\delta_p} \end{array} \right\} \quad (34)$$

$$a_{4p} = - \sum_{m=0}^{n_h} \text{Im} \left\{ \begin{array}{l} \sigma \hat{\underline{E}}_{p+1,m} \cdot \varepsilon^{(\sigma+jm)\alpha_{p+1} - \sigma\delta_p} \\ + jm\underline{c}_m \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm\delta_p} \end{array} \right\} \quad (35)$$

Functions a_{3p} and a_{4p} depend on basic angles α_{p+1} and δ_p as shown in (34) and (35), while functions a_{1p} and a_{2p} only depend on basic angle α_p as shown in (32) and (33).

Functions a_{1p} to a_{4p} of (28)–(31) present a second equivalence with the transient component u_{tp} and its derivative u'_{tp} . In accordance with (17) and (19), terms u_{tp} and u'_{tp} depend on phasor $\hat{\underline{U}}_{tp}$. However, phasor $\hat{\underline{U}}_{tp}$ of voltage u_{tp} is not known in advance. A key aspect of the analysis is the process by which phasor $\hat{\underline{U}}_{tp}$ is expressed as a combination of functions a_{1p} and a_{2p} , whose form is indicated in (32) and (33). To do that, conduction angle γ_p and auxiliary phasors $\underline{V}_p$ and $\underline{\lambda}_p$ are defined by:

$$\gamma_p = \delta_p - \alpha_p; \quad \underline{V}_p = \hat{\underline{U}}_{tp} \cdot \varepsilon^{\underline{\nu}\alpha_p}; \quad \underline{\lambda}_p = \varepsilon^{\underline{\nu}\gamma_p} \quad (36)$$

Taking into account (17), (19) and (36), voltages $u_{tp}(\alpha_p)$ and $u_{tp}(\delta_p)$, and their derivatives, are equivalent to:

$$u_{tp}(\alpha_p) = \text{Im}\{\underline{V}_p\}; \quad u_{tp}(\delta_p) = \text{Im}\{\underline{V}_p \cdot \underline{\lambda}_p\} \quad (37)$$

$$u'_{tp}(\alpha_p) = \text{Im}\{\underline{\nu} \cdot \underline{V}_p\}; \quad u'_{tp}(\delta_p) = \text{Im}\{\underline{\nu} \cdot \underline{V}_p \cdot \underline{\lambda}_p\} \quad (38)$$

Hence, (28) and (29) can be expressed by:

$$a_{1p} = \text{Im}\{\underline{V}_p\} = \frac{1}{j2} (\underline{V}_p - \underline{V}_p^*) \quad (39)$$

$$a_{2p} = \text{Im}\{\underline{\nu} \cdot \underline{V}_p\} = \frac{1}{j2} (\underline{\nu} \cdot \underline{V}_p - \underline{\nu}^* \cdot \underline{V}_p^*) \quad (40)$$

where $\underline{V}_p$ and $\underline{V}_p^*$ are the unknowns of this system of two equations, whose solution is:

$$\underline{V}_p = \underline{a}_{12p} = \frac{1}{k_n} (a_{2p} - \underline{\nu}^* a_{1p}) \quad (41)$$

Phasor $\underline{V}_p$ is equivalent to a complex function $\underline{a}_{12p}$, obtained as combination of functions a_{1p} and a_{2p} . Therefore, complex function $\underline{a}_{12p}$ depends only on angle α_p . Taking into account

(30), (31), (37), (38) and (41), functions a_{3p} and a_{4p} can be written as:

$$a_{3p} = \text{Im}\{\underline{a}_{12p} \cdot \underline{\lambda}_p\}; \quad a_{4p} = \text{Im}\{\underline{\nu} \cdot \underline{a}_{12p} \cdot \underline{\lambda}_p\} \quad (42)$$

It is evident that phasor $\hat{\underline{U}}_{tp}$ has been explicitly eliminated in both relations of (42) if functions a_{3p} , a_{4p} and $\underline{a}_{12p}$ are evaluated from (32) to (35) and (41).

A. Nonlinear Equations and Their Solution

The relations of (42) lead to the following nonlinear equations for interval p :

$$\Delta F_{a,p} = a_{3p} - \text{Im}\{\underline{a}_{12p} \cdot \underline{\lambda}_p\} = 0 \quad (43)$$

$$\Delta F_{b,p} = a_{4p} - \text{Im}\{\underline{\nu} \cdot \underline{a}_{12p} \cdot \underline{\lambda}_p\} = 0 \quad (44)$$

A system of 12 mismatch equations or residues, based on (43) and (44), must be built, where basic angles α_p and δ_p are treated as variables or unknowns. An iterative algorithm, such as the Newton method, is used, whose form can be written as:

$$\begin{bmatrix} \Delta F_{a,p} \\ \Delta F_{b,p} \end{bmatrix} = - \sum_{q=1}^{n_p} \begin{bmatrix} \frac{\partial \Delta F_{a,p}}{\partial \alpha_q} & \frac{\partial \Delta F_{a,p}}{\partial \delta_q} \\ \frac{\partial \Delta F_{b,p}}{\partial \alpha_q} & \frac{\partial \Delta F_{b,p}}{\partial \delta_q} \end{bmatrix} \begin{bmatrix} \Delta \alpha_q \\ \Delta \delta_q \end{bmatrix} \quad (45)$$

The application of this algorithm requires to obtain the partial derivatives of the residues with respect to angles α_p and δ_p . The analytical expressions of these derivatives are relatively simple due to the compact form of this procedure, and they are given in Appendix A. On the other hand, the structure of the corresponding Jacobian Matrix is relative sparse as shown in (46).

$$\begin{bmatrix} x & x & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ x & x & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & x & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & x & x & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x & x & x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & x & x & x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x & x & x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x & x & x & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x & x & x & 0 \\ x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x & x \\ x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & x & x \end{bmatrix} \quad (46)$$

The nonzero elements of (46) are denoted by symbol x . For instance, the first two rows of (46) correspond to the terms of the first interval ($p = 1$), and they depend on angles α_1 , δ_1 and α_2 . The next two rows of (46) correspond to the terms of the second interval ($p = 2$), and they depend on angles α_2 , δ_2 and α_3 . It is evident from this scheme that the coupling among the first and second interval is due to the participation of angle α_2 in both intervals. The initial values for basic angles α_p and δ_p are obtained from the ideal behaviour of the converter [27]: balanced and sinusoidal ac voltages; infinite capacitive dc smoothing ($B_c = \infty$) and $R_s = R_t = 0$.

244 B. Determination of the Transient Component

245 Once the iterative process is performed, the final values of
246 basic angles α_p and δ_p are known. With these results, the final
247 values of functions $a_{1p}(\alpha_p)$, $a_{2p}(\alpha_p)$ and $\underline{a}_{12p}$ are obtained
248 from (32), (33) and (41) respectively. After that, and taking into
249 account (36) and (41), phasor $\hat{\underline{U}}_{tp}$ is determined from:

$$\hat{\underline{U}}_{tp} = \underline{a}_{12p} \cdot \varepsilon^{-\nu\alpha_p} \quad (47)$$

250 and voltage u_{tp} of (17) can be expressed by:

$$u_{tp}(\tau) = \frac{1}{j2} (\hat{\underline{U}}_{tp} \varepsilon^{\nu\tau} - \hat{\underline{U}}_{tp}^* \varepsilon^{\nu^*\tau}) \quad (48)$$

251 Voltage u_{sp} is known before performing the iterative process,
252 and according to (9) and (15), it can be written as:

$$u_{sp}(\tau) = \frac{1}{j2} \sum_{m=0}^{n_h} (\underline{c}_m \cdot \hat{\underline{E}}_{p,m} \varepsilon^{jm\tau} - \underline{c}_m^* \cdot \hat{\underline{E}}_{p,m}^* \varepsilon^{-jm\tau}) \quad (49)$$

253 The expressions of (48) and (49) are the basis for calculating
254 the harmonic currents, as shown in Appendix B.

255 IV. DC AND AC HARMONICS

256 The goal of this section is to provide analytical expressions
257 for harmonic components of the ac currents and of the dc mag-
258 nitudes (voltage and current). The dc components are also eval-
259 uated. To do that, harmonic currents $\hat{\underline{I}}_{dk}$ of dc current $i_d(\tau)$ are
260 firstly evaluated from:

$$\hat{\underline{I}}_{dk} = \frac{j}{\pi \cdot \rho_k} \int_{\alpha_1}^{\alpha_1+2\pi} i_d \cdot \varepsilon^{-jk\tau} d\tau = \sum_{p=1}^{n_p} \hat{\underline{I}}_{dk,p} \quad (50)$$

261 where $\rho_k = 1$ for $k \neq 0$ and $\rho_k = 2$ for $k = 0$. Term $\hat{\underline{I}}_{dk,p}$ is
262 the harmonic contribution of current i_d in subinterval s_{px} . The
263 analytical expression of this contribution is given in Appendix B.
264 Once dc harmonic currents are evaluated, dc harmonic voltages
265 $\hat{\underline{U}}_{dk}$ are obtained from:

$$\hat{\underline{U}}_{dk} = \frac{\hat{\underline{I}}_{dk}}{G_d + jkB_c} \quad (51)$$

266 AC harmonic currents can be easily evaluated from harmonic
267 components $\hat{\underline{I}}_{dk,p}$ taking into account that currents $i_a(\tau)$ and
268 $i_c(\tau)$ satisfy the following conditions:

$$i_a = \begin{cases} +i_d & (p = 1, 2) \\ -i_d & (p = 4, 5) \end{cases}; i_c = \begin{cases} +i_d & (p = 5, 6) \\ -i_d & (p = 2, 3) \end{cases} \quad (52)$$

269 The remaining current i_b is obtained from $i_b = -i_a - i_c$.
270 Taking into account (50) and (52), the harmonic components of
271 the ac currents are given by:

$$\begin{aligned} \hat{\underline{I}}_{a,k} &= \hat{\underline{I}}_{dk,1} + \hat{\underline{I}}_{dk,2} - \hat{\underline{I}}_{dk,4} - \hat{\underline{I}}_{dk,5} \\ \hat{\underline{I}}_{c,k} &= \hat{\underline{I}}_{dk,5} + \hat{\underline{I}}_{dk,6} - \hat{\underline{I}}_{dk,2} - \hat{\underline{I}}_{dk,3} \\ \hat{\underline{I}}_{b,k} &= -\hat{\underline{I}}_{a,k} - \hat{\underline{I}}_{c,k} = \hat{\underline{I}}_{dk,3} + \hat{\underline{I}}_{dk,4} - \hat{\underline{I}}_{dk,1} - \hat{\underline{I}}_{dk,6} \end{aligned} \quad (53)$$

272 The dc components are derived from (3), (50), (51), and (53)
273 when $k = 0$.

V. BOUNDARY OF THE DCM

274

275 The operation of the DCM means that no more than two
276 diodes are in conduction. For instance, diodes 1 and 6 are in
277 conduction in subinterval s_{1x} ($p = 1$) and the next diode to
278 be fired, namely diode 2, presents an anode-cathode voltage
279 v_2 which satisfies the condition $v_2(\tau) < 0$ for $\tau \in s_{1x}$, and
280 hence $v_2(\delta_1) < 0$ for the operation under DCM. The condi-
281 tion $v_2(\delta_1) = 0$ is used in [15] to identify the boundary of the
282 DCM when an ideal scheme is assumed: balanced and sinu-
283 soidal ac voltages, infinite capacitive dc smoothing ($B_c = \infty$),
284 and $R_s = R_t = 0$. The scheme considered in this paper is much
285 more complex than the ideal scheme of [15]. Hence, instead
286 of identifying the boundary of the DCM, a verification of the
287 DCM is proposed in this paper which is based on checking
288 the conditions $v_2(\delta_1) < 0$ for diode 2, $v_3(\delta_2) < 0$ for diode 3
289 etc. In accordance with Fig. 1 and Table I, $u_b - u_c = e_3$ and
290 voltage $v_2(\delta_1)$ of diode 2 at the end of subinterval s_{1x} is given
291 by: $v_2(\delta_1) = R_s \cdot i_d(\delta_1) + X_s \cdot i'_d(\delta_1) + e_3(\delta_1)$. The voltages of
292 the remaining diodes present a similar form. In general, these
293 voltages can be expressed by:

$$v_{p+1}(\delta_p) = R_s \cdot i_d(\delta_p) + X_s \cdot i'_d(\delta_p) + e_{p+2}(\delta_p) \quad (54)$$

294 where $p = 1$ to 6, $v_7(\delta_6) = v_1(\delta_6)$, $e_7(\delta_5) = e_1(\delta_5)$ and $e_8(\delta_6)$
295 $= e_2(\delta_6)$. Taking into account (11), terms $i_d(\delta_p)$ and $i'_d(\delta_p)$ are
296 obtained from:

$$\begin{aligned} i_d(\delta_p) &= G_d \cdot u_d(\delta_p) + B_c \cdot u'_d(\delta_p) \\ i'_d(\delta_p) &= G_d \cdot u'_d(\delta_p) + B_c \cdot u''_d(\delta_p) \end{aligned} \quad (55)$$

297 According to (12), term $u''_d(\delta_p)$ can be calculated from $u_d(\delta_p)$
298 and $u'_d(\delta_p)$ by:

$$u''_d(\delta_p) = f_p(\delta_p) - 2\xi \cdot u'_d(\delta_p) - k_o^2 \cdot u_d(\delta_p) \quad (56)$$

299 Taking into account (14)–(19), voltage $u_d(\delta_p)$ and its deriva-
300 tive are estimated from:

$$\begin{aligned} u_d(\delta_p) &= u_{sp}(\delta_p) + u_{tp}(\delta_p) \\ u'_d(\delta_p) &= u'_{sp}(\delta_p) + u'_{tp}(\delta_p) \end{aligned} \quad (57)$$

301 In summary, once the iterative process is performed, the volt-
302 age $v_{p+1}(\delta_p)$ is calculated for each diode using (54)–(57). The
303 verification of the DCM requires that $v_{p+1}(\delta_p) < 0$ for all the
304 diodes. Otherwise we obtain a no coherent result. In this case, a
305 lower conductance G_d should be selected in order to satisfy the
306 condition $v_{p+1}(\delta_p) < 0$ for all the diodes.

307 A. Intermediate Conduction Mode (ICM)

308 Each section of model 1b (DCM) and model 1a (CCM) is
309 defined by two basic angles and it presents two subintervals:
310 conduction and discontinuity subintervals for DCM, commuta-
311 tion and conduction subintervals for CCM [6]. The transition
312 from DCM to CCM is not direct since an intermediate conduc-
313 tion mode (ICM) exists among DCM and CCM. Each section
314 with ICM is defined by four basic angles and it presents four
315 subintervals: conduction, commutation, conduction and discon-
316 tinuity subintervals.

ICM can be viewed as a variant of CCM in where a small discontinuity subinterval appears within the primitive full conduction subinterval of the ac currents (see Fig. 7 in [15]). ICM can be viewed too as a variant of DCM in where the dc current wave form contains, at the end of the primitive conduction subinterval, a commutation subinterval and a new conduction subinterval. For instance, and according to Table I, section s_1 ($p = 1$) presents the following four subintervals: conduction (1, 6), commutation (1, 2, 6), conduction (1,2) and discontinuity (no diodes).

The operating range of ICM is very small [5] in comparison with the operating range of DCM or CCM, and hence, its relevance and impact are more reduced. In order to verify this statement, the ideal scheme of [15] is considered in where μ is the overlap angle. The boundary DCM-ICM is obtained when $\mu = 0$, while $\mu = \mu_0 = 3.18^\circ$ defines the boundary CCM-ICM, namely, only a difference of 3.18° for the overlap angle exists for the ICM limits. The difference between limits is low for the dc voltage: $U_d(\mu = 0) = 1.0086U_d(\mu = \mu_0)$, while it is higher for the load: $G_d(\mu = \mu_0) = 1.444G_d(\mu = 0)$, but in any case this last difference is very moderate if it is compared with the difference between CCM limits: $G_d(\mu = 60^\circ) = 54.61G_d(\mu = \mu_0)$. These calculations confirm that most practical cases can be treated with model 1b (DCM) and model 1a (CCM).

Finally, if the coherence checking of model 1b is ignored, model 1b can be used in the operating range of ICM with acceptable errors for the first harmonics. This question is treated in Section VI-A. In any case, it is evident that a DAP with a detailed representation of the ICM would require a higher complexity than model 1b of this paper and model 1a described in [6], and hence, future developments are needed in this area.

VI. RESULTS

The system proposed in [3], [6] and [28] is also here used. It consists of a six pulse diode bridge of 400 V, 20 kVA, 50 Hz with the following parameters:

$$R_s = 0.04 \Omega; L_s = 0.51 \text{ mH}; C = 1.284 \text{ mF}; \\ R_t = 0.1 \Omega; L_t = 0.61 \text{ mH};$$

The accuracy of the proposed DAP is checked against PSCAD/EMTDC, using a simulation time of 0.8 s (the first 40 cycles). The steady-state solution with PSCAD is detected by no apparent changes in the harmonic results. The ideal model for the diode is selected in PSCAD.

A. Test 1: Balanced Operation

The phase to neutral ac voltages only present fundamental component with a positive sequence of $400/\sqrt{3}$ V, 0° . Two points of operation are defined. In the first one, a resistance of $R_d = 70 \Omega$ is used. With it, a clearly discontinuous conduction mode is obtained, as shown in Fig. 4. The convergence properties of the DAP are indicated in Table II and dc harmonic components are shown in Table III. A very good agreement with PSCAD is achieved as shown in Fig. 4 and Table III.

Using the method described in Section V, the boundary DCM-ICM in test 1 is reached with $R_d = 64.82 \Omega$. DC harmonic components for this operating point are given in Table IV, and

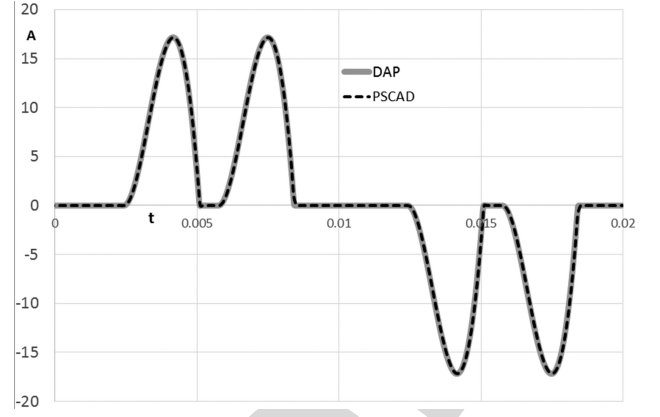


Fig. 4. Test 1: waveform for current i_a with $R_d = 70 \Omega$.

TABLE II
TEST 1: CONVERGENCE PROPERTIES WITH $R_d = 70 \Omega$

Iter	$\alpha_1 (^\circ)$	$\delta_1 (^\circ)$
0	43.67824	92.77824
1	43.61132	91.54128
2	43.58835	91.51915
3	43.58835	91.51925
4	43.58835	91.51925

also a very good agreement with PSCAD is obtained as shown in Table IV.

A third case is considered with $R_d = 41 \Omega$ in where the coherence checking of model 1b is ignored. Under this load, the converter operates clearly under ICM and very close to CCM, namely the worst case for model 1b without coherence checking. In spite of that, model 1b and PSCAD present similar waveforms for the ac current as shown in Fig. 5. Of course, model 1b is unable to reproduce the small waveform, similar to a triangle and marked in Fig. 5, in where three diodes are in conduction (from zero to the peak value of this triangle). The harmonics of current i_a are shown in Table V for this case in where the results from the DAP are indicated on the left and the results from PSCAD on the right in bracket. The discrepancies are due to the no coherent operation of model 1b when $R_d = 41 \Omega$. However, these discrepancies are relative low for the amplitudes and for the arguments of the first harmonics. A less critical case (for instance: $R_d = 50 \Omega$) provides lower discrepancies since it is more close to DCM.

B. Test 2: Unbalanced Operation with Negative Sequence Voltage at Fundamental Frequency

A negative sequence voltage of $6/\sqrt{3}$ V, 120° , at fundamental frequency, is added to the ac voltages considered in Test 1. Under this new condition, the boundary for the DCM is obtained when $R_d = 116.3 \Omega$. A value of $R_d = 117 \Omega$ is chosen for Test 2. Convergence properties are shown in Table VI and the final values for the main angles are indicated in Table VII. These angles exhibit a clear asymmetry due to the unbalanced operation. It leads, as shown in Tables VIII and IX, to a considerable

TABLE III
TEST 1: DC HARMONIC COMPONENTS WITH $R_d = 70 \Omega$

k	DAP				PSCAD (Time step $10 \mu s$)			
	$\hat{I}_{dk}(A)$	$\phi_{dk}(^\circ)$	$\hat{U}_{dk}(V)$	$\theta_{dk}(^\circ)$	$\hat{I}_{dk}(A)$	$\phi_{dk}(^\circ)$	$\hat{U}_{dk}(V)$	$\theta_{dk}(^\circ)$
0	7.75986	90	543.19026	90	7.76106	90	543.18972	90.00000
6	9.09465	14.92679	3.75762	-74.73502	9.09445	14.92749	3.75749	-74.73241
12	1.36776	-121.3781	0.28256	148.79104	1.36767	-121.38004	0.28258	148.79026
18	0.79427	58.34962	0.10939	-31.53765	0.79408	58.34907	0.10939	-31.54377
24	0.47139	-136.118	0.04869	133.96652	0.47119	-136.11197	0.04869	133.96048
30	0.27538	31.78962	0.02276	-58.14274	0.27522	31.80515	0.02276	-58.15461
36	0.17377	-153.1386	0.01197	116.91772	0.17363	-153.11923	0.01197	116.87244
42	0.1308	23.71833	0.00772	-66.23336	0.13059	23.74394	0.00775	-66.41348
48	0.10787	-164.8511	0.00557	105.1912	0.10770	-164.83808	0.00557	105.15920

TABLE IV
TEST 1: DC HARMONIC COMPONENTS WITH $R_d = 64.82 \Omega$

k	DAP				PSCAD (Time step $10 \mu s$)			
	$\hat{I}_{dk}(A)$	$\phi_{dk}(^\circ)$	$\hat{U}_{dk}(V)$	$\theta_{dk}(^\circ)$	$\hat{I}_{dk}(A)$	$\phi_{dk}(^\circ)$	$\hat{U}_{dk}(V)$	$\theta_{dk}(^\circ)$
0	8.36609	90.00000	542.29002	90.00000	8.36733	90.00000	542.28601	90.00000
6	9.60288	13.34235	3.96759	-76.29244	9.60380	13.33981	3.96795	-76.29437
12	1.39815	-130.90805	0.28884	139.27455	1.39822	-130.92765	0.28886	139.25641
18	0.79165	48.36727	0.10903	-41.51100	0.79158	48.34530	0.10903	-41.53212
24	0.48558	-148.15330	0.05016	121.93801	0.48552	-148.17736	0.05016	121.91589

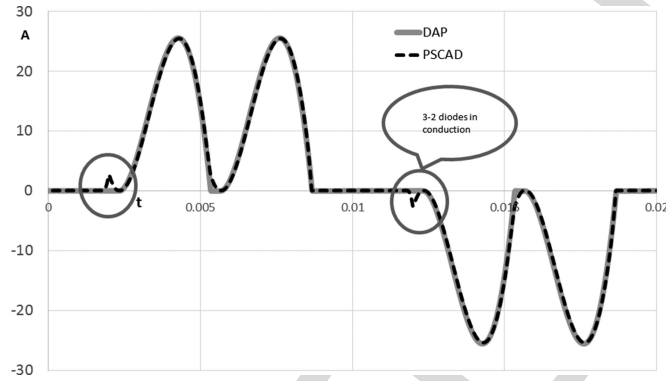


Fig. 5. Test 1: waveform for current i_a with $R_d = 41 \Omega$ in ICM.

TABLE V
TEST 1: AC HARMONIC CURRENTS WITH $R_d = 41 \Omega$ IN ICM
(TIME STEP $10 \mu s$)

k	$\hat{I}_{a,k}(A)$	$\phi_{a,k}(^\circ)$	k	$\hat{I}_{a,k}(A)$	$\phi_{a,k}(^\circ)$
1	14.827 (14.805)	-13.802 (-13.783)	17	0.567 (0.546)	104.388 (112.974)
5	9.416 (9.214)	108.655 (106.545)	19	0.470 (0.488)	-160.491 (178.818)
7	5.664 (5.693)	-104.744 (250.902)	23	0.340 (0.336)	-109.133 (266.285)
11	1.179 (1.111)	-52.758 (-50.074)	25	0.260 (0.271)	-9.071 (-39.449)
13	1.157 (1.183)	47.247 (35.941)	29	0.224 (0.230)	34.335 (57.558)

TABLE VI
TEST 2: CONVERGENCE PROPERTIES

Iter	$\alpha_1(^\circ)$	$\delta_1(^\circ)$
0	44.85869	88.15869
1	44.98046	87.38156
2	45.17384	86.45016
3	45.10902	86.49631
4	45.11187	86.49272
5	45.11187	86.49273
6	45.11187	86.49273

TABLE VII
TEST 2: FINAL VALUES FOR THE MAIN ANGLES

p	1	2	3	4	5	6
α_p	45.11187	110.93964	162.81625	225.11187	290.93964	342.81625
δ_p	86.49273	140.05373	212.18947	266.49273	320.05373	392.18947
γ_p	41.38086	29.11409	49.37322	41.38086	29.11409	49.37322

agreement with PSCAD is achieved as shown in Tables VIII and IX. 404 405

C. Test 3: Unbalanced Operation With a Second Harmonic Voltage of Positive Sequence 406 407

A second harmonic voltage of $0.8/\sqrt{3} V$, 0° , of positive sequence, is added to the ac voltages considered in Test 2. Under this new condition, the boundary for the DCM is obtained when $R_d = 121.49 \Omega$. A value of $R_d = 121.5 \Omega$ is chosen for Test 3. Convergence properties are shown in Table X. A full harmonic spectrum on the dc and ac sides can be seen as shown in Tables XI and XII. Significant levels of dc component are 408 409 410 411 412 413 414

level of non characteristic even harmonics on the dc side; to a significant level of negative sequence current at fundamental frequency; to the generation of triplen harmonics and to the generation of non characteristic sequences of the characteristic harmonics on the ac side. All odd harmonics of Table IX present quantities of positive (+) and negative (-) sequence. A very good

TABLE VIII
TEST 2: DC HARMONIC COMPONENTS

k	DAP				PSCAD (Time step $1 \mu s$)			
	$\hat{I}_{dk} (A)$	$\phi_{dk} (^{\circ})$	$\hat{U}_{dk} (V)$	$\theta_{dk} (^{\circ})$	$\hat{I}_{dk} (A)$	$\phi_{dk} (^{\circ})$	$\hat{U}_{dk} (V)$	$\theta_{dk} (^{\circ})$
0	4.69642	90.00000	549.48075	90.00000	4.70096	90.00000	549.48073	90.00000
2	4.46089	39.87167	5.52907	-49.52135	4.45792	39.86732	5.52906	-49.52138
4	3.20248	63.75408	1.98475	-25.94242	3.20344	63.80122	1.98475	-25.94240
6	5.72400	21.69209	2.36500	-68.10558	5.72554	21.68634	2.36500	-68.10549
8	1.87167	-56.17520	0.57999	-146.02345	1.87071	-56.10017	0.57999	-146.02337
10	0.37627	-61.38765	0.09328	-151.26625	0.37897	-61.35962	0.09328	-151.26479
12	0.83921	-75.89204	0.17337	-165.79088	0.83934	-76.07417	0.17337	-165.79012
14	0.81307	129.52176	0.14397	39.60847	0.81234	129.60626	0.14397	39.60883
16	0.23758	119.38362	0.03681	29.45950	0.24059	119.30939	0.03681	29.46439
18	0.27148	86.38100	0.03739	-3.55155	0.27287	85.82876	0.03739	-3.54644
20	0.42138	-27.99462	0.05223	-117.93392	0.42113	-28.04941	0.05223	-117.93187
22	0.14546	-125.58524	0.01639	144.46994	0.14702	-124.48907	0.01639	144.48178
24	0.11019	-138.14783	0.01138	131.90276	0.11280	-138.67891	0.01138	131.91699

TABLE IX
TEST 2: AC HARMONIC SEQUENCE CURRENTS

k	DAP				PSCAD (Time step $1 \mu s$)			
	$\hat{I}_{k+} (A)$	$\phi_{k+} (^{\circ})$	$\hat{I}_{k-} (A)$	$\phi_{k-} (^{\circ})$	$\hat{I}_{k+} (A)$	$\phi_{k+} (^{\circ})$	$\hat{I}_{k-} (A)$	$\phi_{k-} (^{\circ})$
1	5.35232	-11.10762	2.64792	142.89774	5.35195	-11.12448	2.64799	142.88667
3	2.42630	-63.55436	2.13546	166.57343	2.42651	-63.60203	2.13526	166.51317
5	1.53198	-39.47313	3.86558	123.56321	1.53170	-39.56775	3.86551	123.47863
7	2.72659	-80.60730	1.36155	53.99653	2.72671	-80.72611	1.36190	53.87275
9	0.83895	-169.96274	0.37105	68.58471	0.83933	189.87440	0.37074	68.42754
11	0.17986	152.65820	0.78022	38.46301	0.17962	152.39908	0.78053	38.27491
13	0.29571	158.41147	0.48269	-113.67144	0.29600	158.20937	0.48307	246.10527
15	0.44527	14.76059	0.18799	-131.37728	0.44565	14.50033	0.18775	228.28799
17	0.08133	4.24814	0.19491	-161.18517	0.08105	3.81108	0.19504	198.58954
19	0.11167	-29.09047	0.27430	91.54894	0.11181	-29.30115	0.27469	91.21312
21	0.22206	-147.79659	0.08382	-13.33553	0.22243	211.84089	0.08403	-13.91303
23	0.07344	119.18350	0.06266	-20.91137	0.07363	118.51819	0.06248	-21.12156

TABLE X
TEST 3: CONVERGENCE PROPERTIES

It	1	2	3	4	5	6
0 α	44.99118	104.99118	164.99118	224.99118	284.99118	344.99118
0 δ	87.89118	147.89118	207.89118	267.89118	327.89118	387.89118
1 α	45.09627	107.66448	163.50356	225.06258	287.37569	343.31879
1 δ	86.94984	144.64077	210.9351	267.36291	325.27137	391.14875
2 α	45.3224	111.0962	162.9055	225.16597	290.34069	342.4241
2 δ	85.76235	139.70785	212.0829	266.76397	321.52514	392.53322
3 α	45.27363	111.65587	163.26849	225.08954	290.71904	342.70788
3 δ	85.7659	138.35184	211.67179	266.84448	320.79185	392.19117
4 α	45.27594	111.67286	163.27733	225.09359	290.7262	342.71312
4 δ	85.76305	138.25189	211.65761	266.83924	320.76709	392.18224
5 α	45.27594	111.67288	163.27731	225.0936	290.7262	342.71312
5 δ	85.76305	138.25173	211.65764	266.83924	320.76715	392.18225

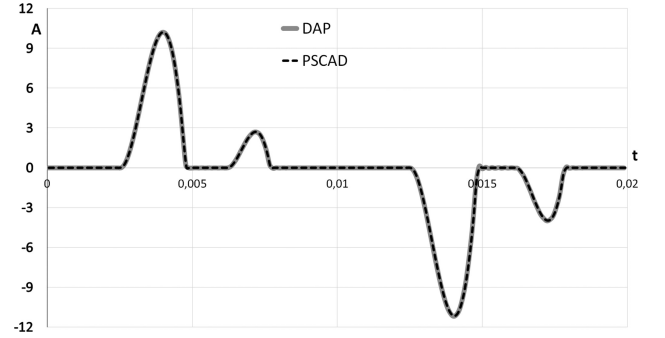


Fig. 6. Test 3: waveform for current i_a with $R_d = 121.49 \Omega$.

present in the ac currents ($I_{a,0} = -0.16 A$, $I_{b,0} = -0.072 A$ and $I_{c,0} = 0.23 A$) and non characteristic harmonics are present on the dc side (Table XI) (mainly third harmonic) and on the ac side (Table XII) (mainly second harmonic). A very good agreement with PSCAD is achieved as shown in Fig. 6 and Tables XI and XII.

A time step of $10 \mu s$ (instead of $1 \mu s$) provides also acceptable results for the harmonic analysis of test 3 via PSCAD and its use increases the simulation speed. A CPU time of 8 s is required

to reach the steady-state solution via PSCAD with a time step of $10 \mu s$. However, the CPU time is less than 0.016 s when the DAP is implemented in C, and running in Windows, namely, the simulation speed is 500 times higher with the proposed DAP.

D. Discussion and Application

If an identical representation of the diode is used for model 1b and PSCAD, the harmonic analysis by means of model 1b (with coherence checking) is more accurate than by means of PSCAD since the accuracy with PSCAD depends on the time step to

TABLE XI
TEST 3: DC HARMONIC COMPONENTS

k	DAP				PSCAD (Time step 1 μ s)			
	$\hat{I}_{dk}$ (A)	ϕ_{dk} ($^\circ$)	$\hat{U}_{dk}$ (V)	θ_{dk} ($^\circ$)	$\hat{I}_{dk}$ (A)	ϕ_{dk} ($^\circ$)	$\hat{U}_{dk}$ (V)	θ_{dk} ($^\circ$)
0	4.52583	90.00000	549.84293	90.00000	4.52764	90.00000	549.84291	90.00000
1	0.40163	120.34274	0.99545	31.51172	0.40087	120.76210	0.99545	31.51169
2	4.33075	39.44878	5.36779	-49.96667	4.33164	39.46018	5.36779	-49.96667
3	0.26502	44.68297	0.21899	-44.92732	0.26564	44.79794	0.21899	-44.92785
4	3.11460	64.95581	1.93028	-24.75190	3.11389	64.93827	1.93028	-24.75190
5	0.26451	-21.14909	0.13115	-110.91526	0.26184	-20.97510	0.13114	-110.91561
6	5.55443	22.14077	2.29494	-67.66438	5.55430	22.15250	2.29494	-67.66432
7	0.20319	57.90538	0.07196	-31.92760	0.20605	57.70042	0.07196	-31.92531
8	1.86259	-54.98148	0.57718	-144.83534	1.86382	-54.99418	0.57718	-144.83526
9	0.09537	-121.98594	0.02627	148.14397	0.09682	-121.01324	0.02627	148.14211
10	0.37023	-54.37746	0.09178	-144.26054	0.36909	-54.42606	0.09178	-144.26019
11	0.07755	-160.65120	0.01748	109.45508	0.07615	-159.62847	0.01748	109.45859
12	0.83870	-74.05487	0.17326	-163.95744	0.83946	-74.00497	0.17327	-163.95708
13	0.04808	-22.36811	0.00917	-112.27818	0.04669	-25.44275	0.00917	-112.28483
14	0.78264	132.99241	0.13859	43.07592	0.78401	132.98244	0.13859	43.07634
15	0.08688	95.20446	0.01436	5.28240	0.08966	95.99331	0.01436	5.28663

TABLE XII
TEST 3: AC HARMONIC SEQUENCE CURRENTS

k	DAP				PSCAD (Time step 1 μ s)			
	I_{k+} (A)	ϕ_{k+} ($^\circ$)	I_{k-} (A)	ϕ_{k-} ($^\circ$)	I_{k+} (A)	ϕ_{k+} ($^\circ$)	I_{k-} (A)	ϕ_{k-} ($^\circ$)
1	5.15881	-11.03714	2.56702	142.35052	5.15857	-11.05410	2.56680	142.33441
2	0.22323	19.06813	0.17136	150.44203	0.22344	19.03852	0.17184	150.32943
3	2.36052	-63.84074	2.07235	167.61794	2.36037	-63.88955	2.07224	167.56774
4	0.13001	-62.54149	0.17712	84.64989	0.13035	-62.78287	0.17708	84.47330
5	1.49477	-38.07669	3.74267	123.92894	1.49477	-38.16141	3.74271	123.84311
6	0.12770	-128.54481	0.14170	157.59673	0.12752	231.20343	0.14177	157.44079
7	2.65328	-80.06695	1.34707	54.67088	2.65347	-80.18881	1.34717	54.55660
8	0.09441	-41.74618	0.05425	4.58904	0.09442	-41.95762	0.05457	4.08130
9	0.83844	-168.00835	0.37225	72.78936	0.83862	191.84595	0.37235	72.62637
10	0.05988	118.04875	0.05374	-42.08478	0.06024	117.61448	0.05342	-42.59650
11	0.16388	159.25617	0.77455	39.73591	0.16400	159.06373	0.77481	39.53244
12	0.03803	80.71940	0.03380	88.26153	0.03771	79.98212	0.03377	87.91614
13	0.29526	161.19303	0.46506	-109.93115	0.29551	160.94557	0.46523	249.86461
14	0.02523	-136.02886	0.05661	-151.27920	0.02521	223.50874	0.05679	208.12821
15	0.43006	18.16551	0.17977	-129.19351	0.43027	17.92328	0.17994	230.51389

estimate the switching instants and to calculate the harmonic components via FFT. Both limitations do not exist in model 1b since the basic angles are determined with very high resolution and the harmonic components are estimated using analytical expressions.

It is important to note that the harmonic analysis with the proposed DAP of model 1b is several hundred times faster than with the conventional time domain simulation via PSCAD. This interesting property makes very attractive the integration of model 1b as nonlinear load into the iterative harmonic analysis (IHA) or into the harmonic power flow (HPF) [29]–[33] in where model 1b must be solved several times with different ac harmonic voltages sources within the iterative process. The linear network of the IHA or of the HPF can present any complexity, and hence filters, connected on the ac side as it is indicated in [8], can be directly included into the linear network. Moreover, an easy and direct integration of model 1b into the IHA or into the HPF is possible if the harmonic interaction converter-linear network is solved by means of the Gauss algorithm.

The average modeling of [9] has been recently proposed for the analysis of the first characteristic harmonics with DCM, CCM-1 and CCM-2, and balanced operation. This procedure is about three times faster than the conventional time domain simulation [9]. Moreover, the analysis of noncharacteristic harmonics, due to nonideal conditions, have not been considered in [9]. However, the average modeling of [9] allows to simulate the dynamic response of the converter, for instance, the transient response from DCM to CCM due to a sudden change in the load R_d . Models 1b and 1a provide the state state response in DCM and CCM-1 operation with non noncharacteristic harmonics. However, the transient response from DCM to CCM-1 is not possible with models 1b and 1a since they are only oriented to the harmonic analysis under steady-state operation.

VII. CONCLUSION

Following the method proposed in [6], a DAP for calculating ac and dc harmonic components of a diode rectifier under non-ideal conditions is presented. Discontinuous conduction mode

(DCM) is considered in this paper, and a procedure for detecting the boundary of DCM is also proposed.

A set of twelve analytical non linear equations allows an easy calculation of switching angles, which become the only variables to be updated in this DAP. Once they are determined, characteristic and non characteristic harmonic components can be easily estimated by a set of compact analytical expressions.

The switching angles are calculated by a full Newton formulation, and the accuracy of this DAP has been tested by comparison with time domain simulation via PSCAD-EMTDC. The steady-state solution with PSCAD requires a considerable computer effort while the simulation speed with the proposed DAP is very high. That is a clear advantage in order to include this DAP in harmonic power flows. This DAP has been implemented in the low-level programming language C, and included in the EDF ExpertEC program as a plug in.

The procedure for determining the boundary of DCM; the accuracy of this DAP under nonideal conditions; the systematic form of defining and solving the nonlinear equations and the set of analytical expressions for harmonic calculations are additions to previous DAPs.

APPENDIX A

TERMS OF THE JACOBIAN MATRIX

According to (36), the derivatives of term $\underline{\lambda}_p$ are given by:

$$\frac{\partial \underline{\lambda}_p}{\partial \alpha_p} = -\frac{\partial \underline{\lambda}_p}{\partial \delta_p} = -\underline{\nu} \cdot \underline{\lambda}_p \quad (58)$$

Functions a_{1p} , a_{2p} and $\underline{a}_{12p}$ only depend on angle α_p , and they are defined in (32), (33) and (41). Hence, the corresponding derivatives are:

$$\frac{\partial a_{1p}}{\partial \alpha_p} = \sum_{m=0}^{n_h} \text{Im} \left\{ jm \cdot (1 - \underline{c}_m) \cdot \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm \alpha_p} \right\} \quad (59)$$

$$\frac{\partial a_{2p}}{\partial \alpha_p} = - \sum_{m=0}^{n_h} \text{Im} \left\{ jm \cdot (\sigma + jm \cdot \underline{c}_m) \cdot \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm \alpha_p} \right\} \quad (60)$$

$$\frac{\partial \underline{a}_{12p}}{\partial \alpha_p} = \frac{1}{k_n} \left(\frac{\partial a_{2p}}{\partial \alpha_p} - \underline{\nu}^* \frac{\partial a_{1p}}{\partial \alpha_p} \right) \quad (61)$$

Functions a_{3p} and a_{4p} only depend on angles α_{p+1} and δ_p , and they are defined in (34) and (35). Hence, the corresponding derivatives are:

$$\frac{\partial a_{3p}}{\partial \alpha_{p+1}} = \sum_{m=0}^{n_h} \text{Im} \left\{ (\sigma + jm) \hat{\underline{E}}_{p+1,m} \cdot \varepsilon^{(\sigma + jm) \alpha_{p+1} - \sigma \delta_p} \right\} \quad (62)$$

$$\frac{\partial a_{4p}}{\partial \alpha_{p+1}} = - \sum_{m=0}^{n_h} \text{Im} \left\{ \sigma (\sigma + jm) \hat{\underline{E}}_{p+1,m} \cdot \varepsilon^{(\sigma + jm) \alpha_{p+1} - \sigma \delta_p} \right\} \quad (63)$$

$$\frac{\partial a_{3p}}{\partial \delta_p} = - \sum_{m=0}^{n_h} \text{Im} \left\{ \sigma \hat{\underline{E}}_{p+1,m} \cdot \varepsilon^{(\sigma + jm) \alpha_{p+1} - \sigma \delta_p} + jm \underline{c}_m \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm \delta_p} \right\} \quad (64)$$

$$\frac{\partial a_{4p}}{\partial \delta_p} = \sum_{m=0}^{n_h} \text{Im} \left\{ \sigma^2 \hat{\underline{E}}_{p+1,m} \cdot \varepsilon^{(\sigma + jm) \alpha_{p+1} - \sigma \delta_p} + m^2 \underline{c}_m \hat{\underline{E}}_{p,m} \cdot \varepsilon^{jm \delta_p} \right\} \quad (65)$$

Residues $\triangle F_{a,p}$ and $\triangle F_{b,p}$ depend on angles α_p , δ_p and α_{p+1} , and they are defined in (43) and (44). Hence, the corresponding terms of the Jacobian matrix present the following expressions:

$$\frac{\partial \triangle F_{a,p}}{\partial \alpha_p} = -\text{Im} \left\{ \frac{\partial \underline{a}_{12p}}{\partial \alpha_p} \underline{\lambda}_p + \underline{a}_{12p} \frac{\partial \underline{\lambda}_p}{\partial \alpha_p} \right\} \quad (66)$$

$$\frac{\partial \triangle F_{b,p}}{\partial \alpha_p} = -\text{Im} \left\{ \underline{\nu} \frac{\partial \underline{a}_{12p}}{\partial \alpha_p} \underline{\lambda}_p + \underline{\nu} \cdot \underline{a}_{12p} \frac{\partial \underline{\lambda}_p}{\partial \alpha_p} \right\} \quad (67)$$

$$\frac{\partial \triangle F_{a,p}}{\partial \delta_p} = \frac{\partial a_{3p}}{\partial \delta_p} - \text{Im} \left\{ \underline{a}_{12p} \cdot \frac{\partial \underline{\lambda}_p}{\partial \delta_p} \right\} \quad (68)$$

$$\frac{\partial \triangle F_{b,p}}{\partial \delta_p} = \frac{\partial a_{4p}}{\partial \delta_p} - \text{Im} \left\{ \underline{\nu} \cdot \underline{a}_{12p} \cdot \frac{\partial \underline{\lambda}_p}{\partial \delta_p} \right\} \quad (69)$$

$$\frac{\partial \triangle F_{a,p}}{\partial \alpha_{p+1}} = \frac{\partial a_{3p}}{\partial \alpha_{p+1}}; \quad \frac{\partial \triangle F_{b,p}}{\partial \alpha_{p+1}} = \frac{\partial a_{4p}}{\partial \alpha_{p+1}} \quad (70)$$

We can observe that the terms of the Jacobian matrix present simple analytical expressions.

APPENDIX B

HARMONIC CURRENTS

In accordance with (11) and (14), current $i_d(\tau)$ can be obtained from voltage $u_d(\tau)$, and this voltage can be expressed in function of components u_{tp} and u_{sp} . Hence, current i_d can be also split into two components i_{tp} and i_{sp} , so that:

$$i_d = i_{tp} + i_{sp}; \quad \begin{cases} i_{tp} = G_d \cdot u_{tp} + B_c \cdot u'_{tp} \\ i_{sp} = G_d \cdot u_{sp} + B_c \cdot u'_{sp} \end{cases} \quad (71)$$

Taking into account (48) and (49), components i_{tp} and i_{sp} can be written as:

$$i_{tp}(\tau) = \frac{1}{j2} (\underline{d}_t \cdot \hat{\underline{U}}_{tp} \varepsilon^{\underline{\nu} \tau} - \underline{d}_t^* \cdot \hat{\underline{U}}_{tp}^* \varepsilon^{\underline{\nu}^* \tau}) \quad (72)$$

$$i_{sp}(\tau) = \frac{1}{j2} \sum_{m=0}^{n_h} (\underline{d}_m \cdot \hat{\underline{E}}_{p,m} \varepsilon^{jm \tau} - \underline{d}_m^* \cdot \hat{\underline{E}}_{p,m}^* \varepsilon^{-jm \tau}) \quad (73)$$

$$\underline{d}_t = G_d + \underline{\nu} B_c; \quad \underline{d}_m = \underline{c}_m (G_d + jm \cdot B_c) \quad (74)$$

Harmonic contributions of terms i_{tp} and i_{sp} are defined as:

$$\hat{\underline{I}}_{dk,tp} = \frac{j}{\pi \rho_k} \int_{\alpha_p}^{\delta_p} i_{tp} \cdot \varepsilon^{-jk \tau} d\tau; \quad \hat{\underline{I}}_{dk,sp} = \frac{j}{\pi \rho_k} \int_{\alpha_p}^{\delta_p} i_{sp} \cdot \varepsilon^{-jk \tau} d\tau \quad (75)$$

$$\hat{\underline{I}}_{dk,p} = \hat{\underline{I}}_{dk,tp} + \hat{\underline{I}}_{dk,sp}; \quad \rho_k = \begin{cases} 1 & \text{for } k \neq 0 \\ 2 & \text{for } k = 0 \end{cases} \quad (76)$$

Taking into account (72) and (73), the harmonic contributions present the following compact form:

$$\hat{\underline{I}}_{dk,tp} = \underline{\beta}'_{k,tp} \cdot \underline{d}_t \cdot \hat{\underline{U}}_{tp} + \underline{\beta}''_{k,tp} \cdot \underline{d}_t^* \cdot \hat{\underline{U}}_{tp}^* \quad (77)$$

$$\hat{\underline{I}}_{dk,sp} = \sum_{m=0}^{n_h} (\underline{\beta}'_{km,sp} \cdot \underline{d}_m \cdot \hat{\underline{E}}_{p,m} + \underline{\beta}''_{km,sp} \cdot \underline{d}_m^* \cdot \hat{\underline{E}}_{p,m}^*) \quad (78)$$

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$$\underline{\beta}'_{k,tp} = \frac{\varepsilon^{(\underline{\nu}-jk)\delta_p} - \varepsilon^{(\underline{\nu}-jk)\alpha_p}}{2\pi\rho_k(\underline{\nu}-jk)} \quad (79)$$

$$\underline{\beta}''_{k,tp} = -\frac{\varepsilon^{(\underline{\nu}^*-jk)\delta_p} - \varepsilon^{(\underline{\nu}^*-jk)\alpha_p}}{2\pi\rho_k(\underline{\nu}^*-jk)} \quad (80)$$

$$\underline{\beta}'_{km,sp} = j\frac{\varepsilon^{-j(k-m)\delta_p} - \varepsilon^{-j(k-m)\alpha_p}}{2\pi\rho_k(k-m)} \quad (81)$$

$$\underline{\beta}''_{km,sp} = -j\frac{\varepsilon^{-j(k+m)\delta_p} - \varepsilon^{-j(k+m)\alpha_p}}{2\pi\rho_k(k+m)} \quad (82)$$

$$\underline{\beta}'_{kk,sp} = \frac{\delta_p - \alpha_p}{2\pi\rho_k} \quad \text{for } m = k \quad (83)$$

$$\underline{\beta}''_{00,sp} = -\underline{\beta}'_{00,sp} \quad \text{for } m = k = 0. \quad (84)$$

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