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Three species one-dimensional kinetic model for weakly ionized plasmas

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A three species one-dimensional kinetic model is presented for a spatially homogeneous weakly ionized plasma subjected to the action of a time varying electric field. Planar geometry is assumed, which means that the plasma evolves in the privileged direction of the field. The energy transmitted to the electric charges is channelized to the neutrals thanks to collisions, a mechanism that influences the plasma dynamics. Charge-charge interactions have been designed as a one-dimensional collision term equivalent to the Landau operator used for fully ionized plasmas. Charge-neutral collisions are modelled by a conservative drift-diffusion operator in the Dougherty's form. The resulting set of coupled integro-differential equations is solved with the stable and robust propagator integral method. This semi–analytical method feasibility accounts for non–linear effects without appealing to linearisation or simplifications, providing conservative physically meaningful solutions even for initial or emerging sharp velocity distribution function profiles. It is found that charge-neutral collisions exert a significant effect since a quite different plasma evolution arises if compared to the collisionless limit. In addition, substantial differences in the system motion are found for constant and temperature dependent collision frequencies cases. *Published by AIP Publishing*. [http://dx.doi.org/10.1063/1.4953901]

I. INTRODUCTION, MOTIVATION, AND CONTEXT

Weakly ionized plasmas (WIPs) are ubiquitous in both nature and laboratory, and they configure a topic of continuous research as a multidisciplinary scientific task. With the physical modelling as the key starting point, the comprehension of plasma phenomena involves a great variety of related areas such as numerical computation, the knowledge of chemical reaction processes, and plasma space and industrial applications.[1–3] Quite usually, these physical descriptions are supported by multi-fluid models, theoretically derived from simple kinetic equations by assuming local thermodynamic equilibrium for plasma species described by Maxwellian distribution functions. These models normally do not account for collisional effects among plasma species even though collisions, paradoxically, must have played its role to drive the system to this Maxwellian state. On the other hand, hybrid kinetic-fluid models are frequently used, where the role played by collisional processes is also generally neglected assuming a kinetic collision-free model. On the other hand, if collisional terms are included in a kinetic description, they can be drastically simplified by including simple collision operators taken from similar properly established problems for plasmas or neutral gases kinetic theories, as the so-called BGK term.[3–5] However, these approaches, although justified in many cases if the system is described within certain time and spatial scales, can lead to misleading results. In this sense, special mention should be devoted to the fact that some usual collision operators are established to yield the system to a Maxwellian state, a fact sometimes used as a strong argument to theoretically warrant the model. In contrast, the plasma could naturally evolve to a quite different steady state, as it can be seen in laboratory striated filamentary plasmas or in those ones observed in some astrophysical contexts with suprathermal populations, as in the solar wind.[6,7] In many cases, the steady plasma holds anisotropic temperatures with plasma species distribution functions having tails satisfying a velocity power-law behaviour.[6] Therefore, kinetic models involving collisional effects are required to accounting for the intrinsic mechanisms able to drive the plasma to possible non-Maxwellian steady states. This fact could also reveal some hidden mechanisms responsible of local momentum and heat transfers. The very hard task of thoroughly establishing a well-posed kinetic collisional model sometimes justifies the aforementioned approaches, but at the same time, the search of new models should be stimulated. Consequently, kinetic collisional models are substantial for a better understanding of the basic dynamics leading to important consequences, apart from the departure from Maxwellian equilibrium that could also modify fluid models through transport coefficients computation. An interesting and clarifying discussion about the importance and relevance of accounting for kinetic collisional effects to describe some fundamental plasma features can be found in Ref. 7.

Typical cold plasmas are weakly ionized, i.e., the ratio between electrons or ions and neutrals densities lies in the range of $(10^{-8}\text{–}10^{-4})$. These plasmas cannot be always completely described in the non–collisional regime under the approximation of an immobile high density neutral population. It is quite usual to disregard the charge-neutral interaction for a local kinetic analysis due to the small collision cross-section and the null action of electric fields on the neutrals, but the collective behaviour of the weakly ionized plasma may strongly depend on the charge-neutral interaction.

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Energy transfer processes give rise to a significant impact on the electrons and ions dynamics which are not precisely described by the sole action of an external electric field included in a collisionless Vlasov kinetic description.[8–10] These interactions can be significant because the energy transferred to the charged species by the electric field can be channelled to the neutrals due to collisional processes, modifying the distribution functions with respect to the typical Maxwellian.

In this work, we present a self-consistent non-linear kinetic collisional model for multi-species weakly ionized plasma (WIP) description. In particular, we focus attention to plasma scenarios relatively large enough to consider that a privileged direction of velocity space marks the system evolution. A physical realisation of this model could be related, for instance, to the description of a plasma between two planar metal walls, where an electric field exists.[11,12] In fact, this work is set in the context of a plasma plume description for electric propulsion thrusters. The presence of neutralizing grids and planar emitting cathode, by a time varying electric field, makes the system to evolve accomplishing the hypotheses of a planar space velocity geometry. However, the present approach in not restricted to these laboratory plasmas and a natural extension of the model can be performed in other contexts. For instance, the analysis of the interaction between plasma and emissive or collecting walls is a problem studied under the fluid description[13,14] and seldom dealt under a kinetic point of view,[12,15–17] but proposing pre–defined velocity distribution functions in the Maxwellian form. In this important issue, many works do not account for collisions or other effects that may become relevant for the plasma dynamics, although some approaches such as the one presented in Ref. 18 offers Boltzmann and Vlasov related models to derive the distribution functions of the plasma species. Other methods can be employed to obtain the distribution function, as the one presented in Ref. 19, which is slightly related to the numerical method proposed in this work, but where a split between convective and collisional dynamics is still required, which may introduce non–physical behaviours, as oscillations or artificial viscosities, to obtain a stable solution.

Here, special attention is therefore paid to constructing a one dimensional velocity space collision operator in the convection-diffusion form, accounting for elastic collisional effects among all plasma species. The collision term is able to address how momentum and energy are transfer among plasma species, in presence of an electric field. A three species (neutrals, ions, and electrons) 1D kinetic model is proposed to study the dynamics of a weakly ionized plasma when spatially homogeneous densities are assumed. The collision term responsible of the interaction among charged particles is modeled by a non-linear Fokker-Planck term that resembles the properties of the Landau collision operator appearing in fully ionized plasmas. This term is constructed upon the basis of considering average Coulombian dry friction as a dynamical force, which is in connection with a non–linear diffusion coefficient, responsible of stochastic fluctuations. Otherwise, the contribution of collisional effects with neutrals is described by a Dougherty operator based upon the assumption of the extremely high density neutral population remains distributed close to a Maxwellian distribution function. The resulting operator is coherent with the planar geometry imposed by the existence of a preferred direction. This geometry is, in practice, a natural constraint implicitly included in the collision term itself, which shapes the plasma evolution. This non-linear collision operator obeys conservative properties to feasibly extract information about particles energization processes that cannot be explained by collisionless operators.

Aside this contribution, another fundamental subject is carefully dealt in this work, which is the problem of solving the resulting highly non–linear equations by an appropriate numerical computational method. This is another arduous task for any non-linear model that sometimes leads, again, to further additional simplification or linearisation procedures that could produce misleading results. Some standard solution methods can result in non–physical or inaccurate descriptions for those systems involving sharp phase space profiles of initial or boundary conditions. Among these conditions, the existence of localized charge structures in a plasma, as in plasma-wall interaction, or peaked velocity distribution functions, as those describing particle beams injection, can produce spurious numerical instabilities and physicalness inaccuracies. The capture of the possible nonlocal effects of this local discontinuities or sharp velocity space structures is of capital importance for a better understanding of some plasma phenomena,[7] but this description, however, should not be distorted by inappropriate numerical algorithms.

Hence, the search for an efficient resolution method for the resulting non-linear kinetic equations also motivates this work. To numerically solve these equations, the use is made of the Propagator Integral Method (PIM), a procedure previously applied in several problems of plasma physics.[20–22] This method takes advantages of the fact that an approximate solution working as a propagator, understood as a Green's function, for the distribution can be found. The explicit integral advancing scheme results in a robust semi-analytical methodology suitable for this kind of equations. This method is able to deal with conservation and entropic properties as well as to manage the evolution of sharp and abrupt conditions, such as initial histogram-like distributions, large gradients in velocity space, or piecewise defined drift and diffusion coefficients.[21,22] Hence, the PIM method apprehends the essential nature of smoothing effects of diffusion by supplying a physically meaningful advancing scheme which operates as if the short-time propagator were the unknown analytical one. It is important here to advert another relevant property which justifies the use of the PIM in this work. This method allows to naturally describing all non–linear effects without using linearisation or further simplifications of the collision terms. Another relevant feature of this method related to the actual problem is that the same time step can be used to advance the fast (electrons) and slow species (ions and neutrals) without impacting the convergence or stability of the scheme. The time step simultaneously valid for several time scale dynamics can be relatively large, up to a tenth of the characteristic relaxation time of the fastest species. In addition, no boundary conditions in velocity space are required, even in the numerical domain, which can be extended up to the size leading to vanishingly small propagator, analytically defined for all velocity values. The results may be relevant for further analysis to kinetically describing some typical problems in WIP contexts as, for example, the description of plasma sheaths in front infinite flat emissive walls,[11,13] where the merging of peaked species velocity values may contribute to effective collisional effects usually neglected.

This paper is structured as follows. In Sec. II, the whole kinetic model is presented, a derivation of the charge-charge collision term is provided. The charge-neutral collision contribution is justified and established also in the same section. Sec. III contains the benchmark problems used to test conservative and operational properties of the model. The explored cases are displayed in Subsections III A–III C: conservation properties of one species system with only self–collisions, influence of collisions in WIP, and evolution of the three species system under the action of an abruptly time varying electric field. Finally, the conclusions of this work are summarized. The Appendix with the dimensionless problem as a sort review of the applied semi-analytical method is also included.

II. THREE SPECIES KINETIC MODEL

In plasmas where the ratio between charge and neutrals number densities is small, the dynamics may depend on the charge-neutral interaction. This interaction becomes important due to the high density of neutrals, which receives part of the energy transmitted to the charged species by the electric field. Moreover, the interplay of several transfer phenomena, governed by quite disparate time scales, demands a kinetic description where collisions have to be properly accounted. In this section, it is derived a set of collision terms having the form of the so-called Fokker–Planck form. In these terms, dynamic friction and diffusion in velocity space are self–consistently included. The plasma is assumed to remain spatially homogeneous, which means that the dynamics is ultimately controlled by an external uniform electric field, which strongly dominates the plasma motion in the privileged direction. This suggests that a one–dimensional velocity space kinetic model suffices to analyse the motion of any distribution function along the preferred direction. This planar geometry, a constraint in practice, is thus prescribed by the field.

Although in weakly ionized plasmas collisions between electrons and neutrals play the fundamental role, all collision terms are included here. These contributions are constructed under the binary collision approximation but with peculiar descriptions for both dynamical friction and diffusion processes for each type of interaction. Hence, each resulting kinetic equation describes the time evolution of the velocity distribution function $f_\gamma(v,t)$, where $\gamma = 0, i, e$ (neutrals, ions, and electrons, respectively). Therefore, the presented kinetic model consists of a system of three non–linear coupled 1D convection-diffusion equations. The collision operators are constructed to satisfy the standards of conservative properties accounting for elastic collisions. Inelastic collision terms, as ionization or recombination processes could be included in a straightforward manner as non–homogeneous source-sink terms.[20,21] The kinetic equation for a time evolving distribution function for a charged species $\alpha = e, i$ has the form

$$\frac{\partial f_\alpha}{\partial t} + \frac{q_\alpha E(t)}{m_\alpha}\frac{\partial f_\alpha}{\partial v} = \sum_\gamma \left(\frac{\partial f_\alpha}{\partial t}\right)_{\alpha\gamma}, \tag{1}$$

where m_α and q_α are the mass and charge of the species α, respectively, and $E(t)$ is the electric field. The same form also stands for neutrals by setting $q_\alpha = 0$. The term $(\partial f_\alpha/\partial t)_{\alpha\gamma}$ symbolizes the rate of change of the distribution function due to collisional effects between species of kinds α and γ. Each of these exchanges are modelled by a drift-diffusion operator, similar to the usual Fokker–Planck collisional one, as

$$\left(\frac{\partial f_\alpha}{\partial t}\right)_{\alpha\gamma} = -\frac{\partial}{\partial v}\left\{A_{\alpha\gamma} - \frac{\partial}{\partial v}D_{\alpha\gamma}\right\}f_\alpha. \tag{2}$$

The parameters $A_{\alpha\gamma} = A_{\alpha\gamma}(f_\alpha, f_\gamma, v, t)$ and $D_{\alpha\gamma} = D_{\alpha\gamma}(f_\alpha, f_\gamma, v, t)$ are referred to the non–linear convection and diffusion coefficients, respectively. As a first stage, the charge-charge collision term is constructed for the one-dimension velocity case by taking as a reference the complete well-known Landau collisional operator[23,24] in an spatially homogeneous plasma. In such a system, the planar geometry leads to plasma species distribution functions depending only on the velocity component v lying in the privileged direction established by the electric field. Since a test particle of mass m_α and velocity v is scattered by particles of velocity v' with distribution function $f_\beta(v', t)$, a small change of the particle velocity Δv in the preferred direction may be considered as a result of an average Coulombian interaction force among charges. Thence, the dynamical frictional effect experienced by m_α can be phenomenologically modelled as an effective force of uniform intensity opposite to the relative velocity $v - v'$. This contribution can be understood as a Coulomb's like law for dry friction, a case also studied in the theoretical frame of Brownian motion.[25] The cumulative effect of many interactions gives rise to the drift coefficient $A_{\alpha\beta}$ which is related to the change of the expectation value of Δv per unit of time $(\langle \Delta v\rangle/\Delta t)$ as

$$A_{\alpha\beta} = \frac{\langle \Delta v\rangle}{\Delta t} = -\mu_{\alpha\beta}\left(1 + \frac{m_\alpha}{m_\beta}\right)\int_{-\infty}^{\infty} \mathrm{sgn}(v - v')f'_\beta \,\mathrm{d}v'. \tag{3}$$

Here, $\mathrm{sgn}(\cdot)$ is the sign function with $\mathrm{sgn}(0) = 0$, primes over a distribution function mean $f'_\gamma = f_\gamma(v', t)$ and $\mu_{\alpha\beta}$ is a parameter related to plasma properties. Besides the friction force, the test charge is subjected to random fluctuating forces of stochastic nature, responsible for the f_α spreading in velocity space. This diffusive behaviour is computed by the coefficient $D_{\alpha\beta}$, related to the average value of $(\Delta v)^2/2$ per unit of time $(\langle(\Delta v)^2\rangle/2\Delta t)$. Assuming that Δv is of order $v - v'$ meanwhile $\Delta v/\Delta t$ is proportional to the friction term

$\mathrm{sgn}(v - v')$, a diffusion coefficient proportional to the average value of $(v - v')\mathrm{sgn}(v - v') = |v - v'|$ is proposed

$$D_{\alpha\beta} = \frac{\langle(\Delta v)^2\rangle}{2\Delta t} = \mu_{\alpha\beta}\int_{-\infty}^{\infty} |v - v'| f'_\beta \mathrm{d}v', \tag{4}$$

assuming that $\int |v| f_\beta \mathrm{d}v$ remains finite for any v. With this selection, the resulting 1D collisional term for charge–charge interaction behaves as the complete plasma physics Fokker–Planck–Landau operator, in fact, it can be easily checked that it provides a well–posed conservative collision operator for a one dimensional plasma. Moreover, similar differential relations for the drift and diffusion coefficients fulfilled by the Fokker–Planck–Landau operator are also satisfied. In particular, making use of the relation $\partial_v \mathrm{sgn}(v - v') = 2\delta(v - v')$ the properties

$$A_{\alpha\beta} = -\left(1 + \frac{m_\alpha}{m_\beta}\right)\frac{\partial}{\partial v} D_{\alpha\beta}, \tag{5}$$

$$\frac{\partial}{\partial v} A_{\alpha\beta} = -2\mu_{\alpha\beta}\left(1 + \frac{m_\alpha}{m_\beta}\right) f_\beta, \tag{6}$$

are obtained. These are equivalent to the relations held by the divergences of the diffusion tensor and the drift vector for the Fokker–Planck–Landau operator.[3,21] The parameter $\mu_{\alpha\beta}$ does not alter the conservation properties of the operator, and its value has been deduced from the equivalent one appearing in the complete Landau operator for fully ionized plasmas. Particularly, $\mu_{\alpha\beta} = 4\pi\lambda_{\alpha\beta} q_\alpha^2 q_\beta^2 / m_\alpha^2 V_{th_\alpha}^2$ which is a parameter related to collision frequencies and energy and momentum rate transfers. Thus, $\lambda_{\alpha\beta}$ is the Coulomb logarithm, $V_{th_\beta} = \sqrt{kT_\beta / m_\beta}$ is the thermal velocity of the species β, and k is the Boltzmann constant. The conservation of norm, momentum, and energy of the whole system is satisfied since the complementary collision parameter verifies $\mu_{\beta\alpha} = \mu_{\alpha\beta}(m_\alpha / m_\beta)^2$.

Along the theoretical lines established to describe the interaction between charges, the charge-neutral and neutral-neutral contributions to the collisional rates are also constructed in the form of conservative drift-diffusion operators. The effects of collisions with neutral particles should be, again, described by both dynamical friction and velocity space diffusion in a self–consistent way as a whole. Due to the fact that in the mixture, the neutrals distribution does not deviate drastically from a Maxwellian, a multi-species Dougherty collision operator[26,27] in the Fokker–Planck form is proposed. The drift and diffusion coefficients are implicitly defined in the final expression

$$\left(\frac{\partial f_\gamma}{\partial t}\right)_{\gamma 0} = -\frac{\nu_{\gamma 0}}{n_0}\frac{\partial}{\partial v}\left\{-\int_{-\infty}^{\infty}(v - v') f'_0 \mathrm{d}v' - \frac{1}{n_\gamma}\frac{\partial}{\partial v}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{(v'' - v')^2}{2} f'_0 f''_\gamma \mathrm{d}v' \,\mathrm{d}v''\right\} f_\gamma, \tag{7}$$

for the rate of change of f_γ due to the interaction of species γ with neutrals. Now, the parameter $\nu_{\gamma 0} = n_0 \sigma_{\gamma 0} V_{th_\gamma}$ is related to typical frequencies of each collisional process between species γ and neutrals.[28] Hence, $\sigma_{\gamma 0}$ is a cross-section measure for charge-neutral and neutral-neutral interaction. It is important, however, to mention here that under a microscopic point of view the real collision frequency is velocity dependent, as in the case of charge-charge interaction. In view of Eq. (7), it is clear that the convective contribution stands for an average viscosity force, which is the result of the cumulative effects of a microscopic friction force proportional to the relative velocity $v - v'$, between a test particle with velocity v and a neutral particle of velocity v'. The ensemble of neutrals conforms a medium characterized by a velocity distribution $f_0(v', t)$, close to a Maxwellian distribution, with density n_0 much higher than n_e and n_i. This fact justifies why the charges, and neutrals as well, experience a viscous friction force stated by Eq. (7). On the other hand, the diffusion coefficients only depend on macroscopic magnitudes due to the double integral over primed variables stated above. The drift and diffusion coefficients included in Eq. (7) ensure energy, momentum, and norm conservation thanks to the reciprocity relations $\nu_{0\gamma} = \nu_{\gamma 0} m_\gamma / m_0$. Thence, this operator is also fully conservative and it leads to a Maxwellian distribution function when it acts as a self–collision term for a one species system in absence of external forces and inelastic terms. The characteristic evolution frequencies of this Dougherty operator do no differ so much from the corresponding ones for a complete Fokker–Planck–Landau operator, as it was analysed in Ref. 24.

As for Eq. (1) with $q_\alpha = 0$, the velocity distribution function for the neutrals obeys a convection-diffusion equation of the form

$$\frac{\partial f_0}{\partial t} = \sum_\gamma \left(\frac{\partial f_0}{\partial t}\right)_{0\gamma}, \tag{8}$$

which accounts for the energy transferred from the charged species to neutrals. The collision terms follow Eq. (7) with the indexes γ and 0 interchanged. It can be seen that although the neutrals do not directly experience the action of electromagnetic forces, the velocity distribution f_0 may evolve due to collisional effects with charged species. As a consequence, the neutrals acquire a small amount of the electrical energy transferred to the charges.[29] This means that all the species feel directly or indirectly the perturbation induced by an electric field. It is important to mention that the collision parameters $\mu_{\alpha\beta}$ and $\nu_{\gamma 0}$ depend on the temperature through the thermal velocity, the cross-sections and the Coulomb logarithm, introducing a new source of non–linearities. In the numerical solution, these parameters must be computed at each time step to take into account the different exchanges among the particles when their temperatures vary.

III. NUMERICAL RESULTS

In this section, three benchmark problems are solved using the PIM to test the established kinetic model. The procedure to apply this method can be found in previous works,[20–22,29,30] and it is not largely presented here, although a short review is presented in the Appendix for a self–consistent reading of this work. To this propose, only self–collisions are first reported to study how the terms described in Section II behave individually, i.e., when they act alone on a distribution function. To analyze the effect of collisions, a comparison between the evolution of a non–collisional plasma and the three species model is also shown. Second, the complete kinetic model presented in Sec. II is computed with a time variable electric field. In addition, a comparison between the previous case with constant and temperature dependent collision parameters $\mu_{\alpha\beta}$ and $\nu_{\gamma 0}$ is performed.

A. One species conservative collision term

The collision terms described in this paper are analytically conservative, preserving in time the system number density, momentum and energy. To properly include these terms in more exhaustive simulations, the numerical solution of simple dimensionless problems only accounting for self–collisions should conserve these macroscopic variables. Therefore, for the charge-charge self-interaction, i.e., if $\alpha = \beta$, the collision term reads

$$\left(\frac{\partial f}{\partial t}\right)_{\alpha\alpha} = -\frac{\partial}{\partial v}\left\{-2\int_{-\infty}^{\infty}\operatorname{sgn}(v-v')f'\mathrm{d}v' - \frac{\partial}{\partial v}\int_{-\infty}^{\infty}|v-v'|f'\mathrm{d}v'\right\}f, \tag{9}$$

where the parameter $\mu_{\alpha\alpha}$ has been set to 1. The initial condition is a Maxwellian distribution function with dimensionless density, mass and temperature equal to unity. The main results of this test problem are shown in Fig. 1. When this collision term acts alone, the solution evolves to a distribution function with power-law tails[20] (Fig. 1(a)), but preserving the initial temperature, density, and mean velocity (Fig. 1(b)). The moments of the distribution function are computed through the relations

$$\begin{pmatrix} N \\ U \\ T \end{pmatrix} = \int_{-\infty}^{\infty}\begin{pmatrix} 1 \\ v \\ \frac{1}{2}(v-u)^2 \end{pmatrix} f(v,t)\,\mathrm{d}\,v, \tag{10}$$

where $u = U/N$. If the difference of these macroscopic quantities respect to the initial condition are analysed (Fig. 1(c)), a very small difference in the numerical results appears. The order of these errors ($\sim 10^{-14}$) is small enough to consider the numerical solution unperturbed, even for a large number of iterations.

The same procedure is applied for the collision term of Eq. (7). If only collisions between the neutral particles are accounted ($\gamma = 0$), the mutual collision term becomes

$$\left(\frac{\partial f}{\partial t}\right)_{00} = -\frac{\partial}{\partial v}\left\{-\int_{-\infty}^{\infty}(v-v')f'\mathrm{d}v' \times -\frac{\partial}{\partial v}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{(v''-v')^2}{2}f'f''\mathrm{d}v'\mathrm{d}v''\right\}f, \tag{11}$$

where, again, the collision frequency ν_{00}, density, mass, and temperature equal 1. Figures 2(a) and 2(b) show how the initial Maxwellian distribution remains constant in practice, since only small numerical errors (presented in Fig. 2(c)) appear. These tests evidence that the PIM is conservative and only numerical errors related to the machine's precision and the mesh finiteness appear in the simulations.

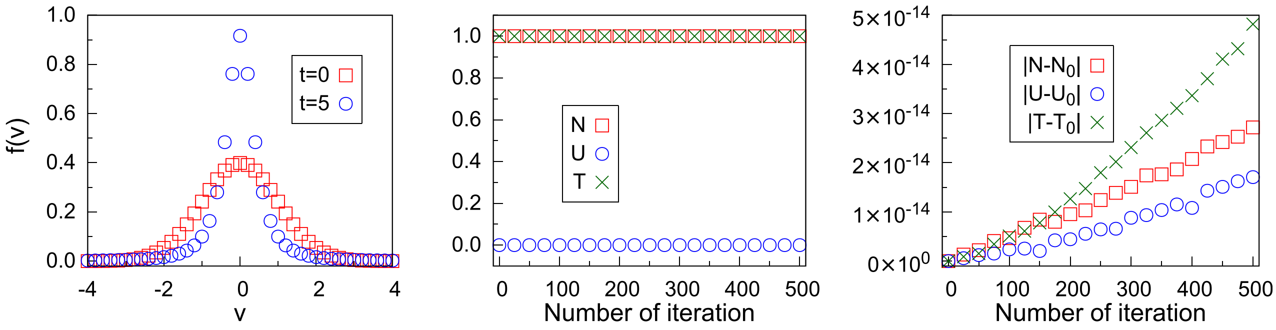
B. Influence of collisions

To test the effect and relevance of the energy and momentum transfers due to collisions in a weakly ionized plasma, a comparison between a non–collisional and the three species kinetic model is presented here. The system of equations that describe the evolution of a non–collisional plasma derives from Eqs. (1) and (8) without collision terms. Thus, the Vlasov equations under space homogeneity are

$$\frac{\partial f_e}{\partial t} - \frac{|q_e|E}{m_e}\frac{\partial f_e}{\partial v} = 0, \quad \frac{\partial f_i}{\partial t} + \frac{|q_i|E}{m_i}\frac{\partial f_i}{\partial v} = 0. \tag{12}$$

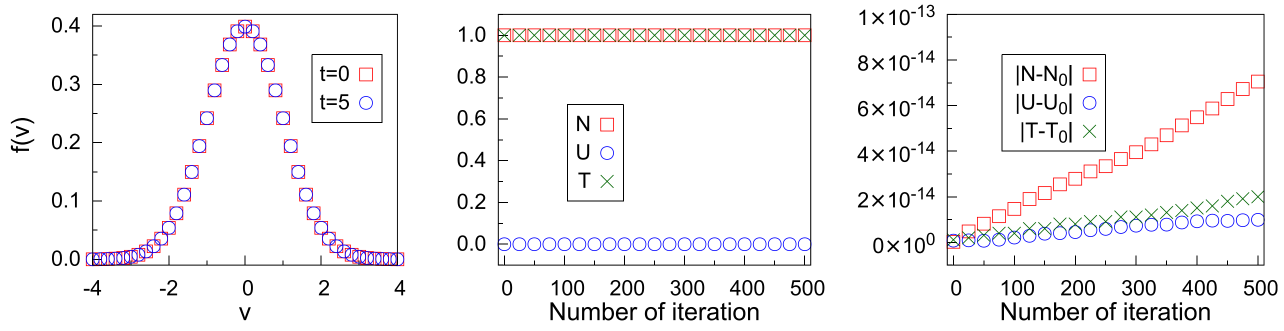
The fluid velocities for the charged species can be analytically calculated, if the electric field is given as a time varying function, as

$$u_\alpha(t) = \mp\frac{|q_\alpha|}{m_\alpha}\int_{t_0}^{t}E(t')\mathrm{d}t' + u_\alpha(t_0), \tag{13}$$



(a) (b) (c)

FIG. 1. Solution for the charge-charge self–interaction. The time step is $\tau = 0.01$. (a) Distribution function for initial condition (red squares) and at the end of the simulation (blue circles). (b) Macroscopic moments time evolution. (c) Errors, with respect to the initial conditions.



(a) (b) (c)

FIG. 2. Solution for the neutral-neutral interaction problem, with $\tau = 0.01$. (a) Distribution function for initial condition (red squares) and at the end of the simulation (blue circles). (b) Macroscopic moments time evolution. (c) Errors, with respect to the initial conditions.

where t_0 is the initial time, sign (–) corresponds to electrons and (+) to ions. The species norm and temperature do not change for this non–collisional homogeneous plasma. Neither the macroscopic momentum nor the distribution function of neutrals change ($\partial f_0/\partial t = 0$) because no interaction with the electric field or with any other species appear. To check the influence of collisions, a simple test-case is solved for the electric field

$$E(t) = \begin{cases} E_0 & \text{if } t < 2.5\,\text{ms} \\ -E_0 & \text{if } t \geq 2.5\,\text{ms}, \end{cases} \tag{14}$$

where $E_0 = 0.1\,\text{V/m}$, which is within the order of magnitude of the signals used for plasma diagnostic in the laboratory plasma of Ref. 14. In order to emulate this experimental Argon plasma, the following parameters are employed with Maxwellian distribution functions as initial conditions

$$\begin{aligned} &m_0 = m_e + m_i; \quad T_0 = 0.03 \times T_e; \quad n_0 = 10^{13}\,\text{cm}^{-3} \\ &m_i = 72819.6 \times m_e; \quad T_i = 0.05 \times T_e; \quad n_i = r_i \times n_0 \\ &T_e = 1\,\text{eV}; \quad n_e = r_i \times n_0, \end{aligned} \tag{15}$$

where m_γ, T_γ, and n_γ are the mass, temperature, and density of the species γ, respectively; and the ionization ratio (r_i) is 10^{-6}. In addition, the fluid velocities are zero ($u_\gamma = 0$) and single charge ions are assumed ($Z = 1$). The cross-sections for the charge-neutral and neutral-neutral interactions are constant, particularly $\sigma_{00} = \sigma_{i0} = 10^{-14}\,\text{cm}^{-2}$, $\sigma_{e0} = 10^{-16}\,\text{cm}^{-2}$ although these parameters should change with the species energy.[2] The remaining parameters required to compute $\mu_{\alpha\beta}$ and $\nu_{\gamma 0}$ are extracted from the literature.[2,28] To properly depict the evolution of the system, collision parameters $\mu_{\alpha\beta}$ and $\nu_{\gamma 0}$ are updated at each time step in the simulation. It can be seen in Eq. (15) that very disparate time scales appear naturally in the problem due to the huge ratios of temperature, density, and mass among the species. Classical numerical methods may become unstable, leading to non–physical solutions when these ratios or non–linearities appear in the numerical problem, a fact that poses no difficulty for the PIM.

In Fig. 3, the mass fluxes ($j_\gamma = m_\gamma n_\gamma u_\gamma$) for the non–collisional plasma (Fig. 3(a)) and the model described in this work (Fig. 3(b)) are presented. Complete different dynamics appears when collisions are taken into account. In one hand, dynamics without collisions follows Eq. (13) for the charged species and no change for the neutrals. On the other hand, when collisions are accounted, electron mass flux reaches a fast maximum and then remains constant until the electric field changes ($t = 2.5\,\text{ms}$). Ions and neutrals also reach a steady velocity, but a longer time is required due to their larger mass. One interesting result is that neutrals are accelerated even when this species does not directly feel the electric field. This evolution is a result of the energy exchanged to the neutrals by the charged species, which implies that the charge-neutral interaction is important to study the dynamics of WIP. Even when the total energy introduced to the system is the same in both, collisional and non–collisional cases, this energy is redistributed in a different way. Mass fluxes reached for charged species are orders of magnitude below the obtained for the non–collisional case. This occurs because when collisions are introduced, part of the energy transferred by the electric field is finally channelled to the neutrals, which act as a background that slowdown the charged species.

It is interesting to note that the different time scales introduced by the disparate referred ratios are irrelevant to the non–collisional model, where the mass fluxes for electrons and ions evolve symmetrically. If collisions are accounted, each species shows quite more realistic different dynamics. This means that, in weakly ionized plasmas, collisions should not be neglected for large time scales. Also, the distribution function evolution of the neutrals is conditioned by the exchange of energy and momentum with the charged species.

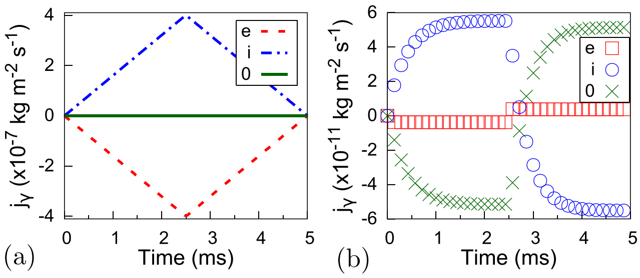


FIG. 3. Mass fluxes j_γ for the three species in cases (a) non-collisional and (b) 1D kinetic collisional plasma.

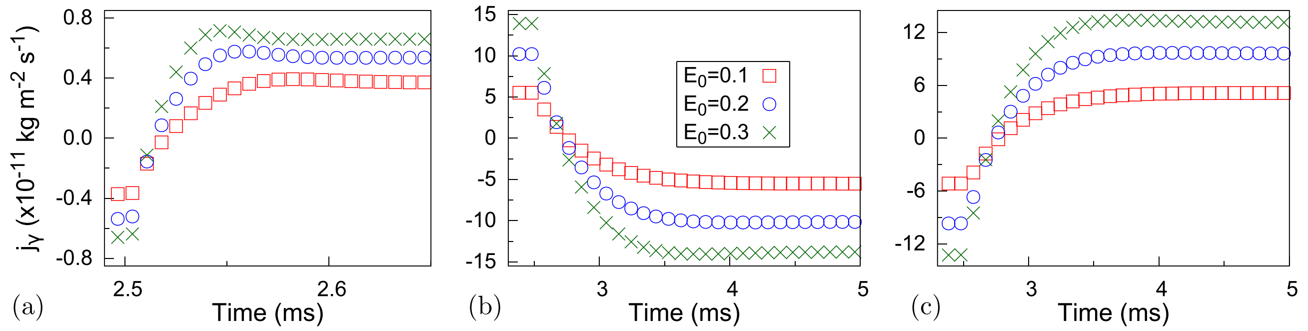


FIG. 4. Mass fluxes for (a) electrons, (b) ions, and (c) neutrals. Electrons have a fast relaxation time due to the high thermal velocity. Meanwhile, heavy and cold species require more time to recover a constant velocity.

C. Three species evolution under an abrupt electric field

Finally, to stress the role played by collisions and the non-linearities, the previous case is extended to describe a system forced by different electric field magnitudes, as hose used in plasma diagnosis experiments.[14] Here, the whole three species kinetic model is tested with the electric field of Eq. (14) for different values of E_0 showing a comparison between the evolution of the system with constant and temperature dependent collision parameters. In Fig. 4, a magnified view of the mass fluxes when the electric field changes ($t = 2.5\,\mathrm{ms}$) for $E_0 = 0.1, 0.2, \mathrm{and}\, 0.3\,\mathrm{V/m}$ is shown. Thanks to these results, nondistorted by numerical discretization, different dynamics for fast (electrons) and slow (ions and neutrals) species can be appreciated. First of all, electrons only require fractions of millisecond to recover a constant velocity. On the other hand, ions and neutrals naturally require several milliseconds to reach a steady state. These different behaviours are directly related to the exchange rate of energy with neutrals.

As it was indicated in Eq. (15), the relation $V_{th_e} \gg V_{th_i}$ between electron and ion thermal velocities holds. At the same time, the charge-neutral collision frequency $\nu_{\gamma 0}$ is directly proportional to the thermal velocity, so that, the rate of electron-neutral exchange is higher than the ion-neutral one. This results in a faster relaxation time, even when $\sigma_{e0} < \sigma_{i0}$. Here, relaxation time refers to the time required by one species to recover a constant velocity after the electric field changes its magnitude. Besides, if increasingly higher values of E_0 are applied, substantial differences in the overshoots and relaxation times appear, especially for electrons. The overshoot in the mass flux of electrons (Fig. 4(a)) grows with the electric field, but electrons recover quickly the constant velocity. This means that a higher temperature for the charged species is reached, which results in a higher charge-neutral collision frequency. It can be seen in Fig. 5, how temperature increases with the electric field. As it was explained above, the temperature modifies the collision frequencies through the thermal velocity, resulting in more encounters with neutrals. As a consequence, the value of electric field influences not only the velocity of the charges but also their temperature, which ends up modifying the collision parameters and the dynamics of the whole system. On the contrary, when the electric field changes, temperature decreases until the electrons reach again a constant velocity, which produces a lower electron-neutral interaction effects. This allows electrons to reach a fast velocity, until temperature increases again and the excess of energy is finally transferred to neutrals. This behaviour can only be obtained if the plasma is modeled in a self–consistently way.

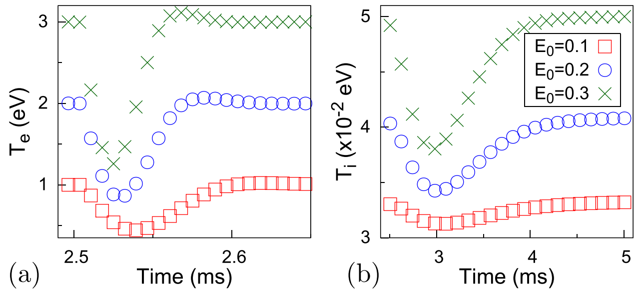


FIG. 5. Temperatures for (a) electrons and (b) ions. The species recover faster a steady state when high electric field is applied.

To test the influence of the non–constant collision parameters, the same test cases are performed keeping the initial values of these parameters through all the simulations. The mass flux of electrons is studied in Fig. 6 for different values of E_0 for both cases. A difference in the relaxation times can be appreciated due to the use of constant collision frequencies. When collision parameters depend on the species temperature, an overshoot in the electrons mass flux appear due to the decrement in temperature. With constant collision frequencies, no change or overshoot in the dynamics of the species appear because, even at low temperature, electrons interact with neutrals at a higher rate than the one corresponding to their temperature.

These results indicate that keeping collision parameters as their initial (or any other constant) value could produce inaccurate results when species temperatures change substantially during the simulations, which can occur, for example, when abrupt changes in the electric field appear. Regarding the question if the obtained solutions show a physical time evolution, the system entropy S has been also analysed. With the Boltzmann entropy computed as

$$S(t) = -k \sum_{\gamma} \int f_{\gamma}(v,t) \log f_{\gamma}(v,t) \mathrm{d}v. \tag{16}$$

The $S(t)$ time derivative, shown in Fig. 7, is obtained by a simple first order central numerical derivative scheme. As it

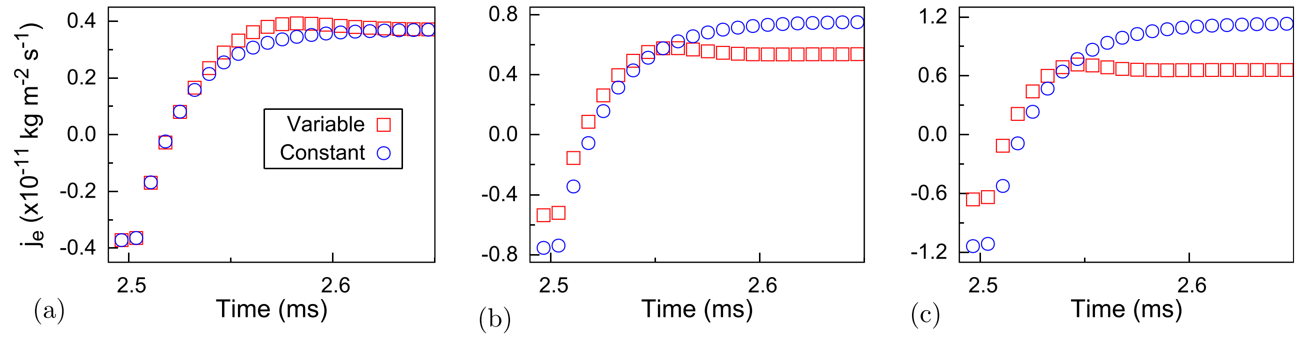


FIG. 6. Comparison of the electron mass flux with variable and constant collision parameters. Very different dynamics appear for both problems and no overshoot raises with constant collision parameters. Differences in the evolution of the slow species are also found (a) $E_0 = 0.1$ V/m, (b) $E_0 = 0.2$ V/m, (c) $E_0 = 0.3$ V/m.

can be seen, this value is always positive, even when an abrupt change in the electric field occurs and different evolution rates appear. This fact essentially means that the numerical solution agrees with the expected thermodynamical evolution of increasing entropy.

IV. CONCLUSIONS

In this work, a one-dimensional kinetic model has been presented to describe the evolution of three species spatially homogeneous weakly ionized plasma. The importance of searching for kinetic collision term has been stressed by showing some results. These test practical cases are related to measurement processes in plasma diagnosis devices that use electric potential signals. A planar geometry induced by the existence of the preferred direction of a strong electric field has been assumed to design all the 1D velocity-space elastic collision terms. The proposed charge-charge collision contribution has been established as an average of a dry friction phenomenological force acting on a test particle. The resulting collisional term behaves as the one-dimensional equivalent Fokker-Planck-Landau operator. On the other hand, a non–linear Dougherty operator with viscous–like drift force has been adopted for the collisional terms involving neutrals, because this species, highly predominant in the plasma, remains close to a Maxwellian state in a WIP. The semi-analytical propagator integral method is employed to solve the coupled equations using the same time step for fast and slow species dynamics. This advancing scheme provides a conservative evolution of the system regardless non–linearities or sharp conditions involved in the drift-diffusion parameters. It is found that if usual linearisation or simplification procedures are applied, e.g., constant collision frequencies, some dynamics could remain hidden, such as the overshoots that appear for fast changes in the electric field. According to the properties of the collision operators, time evolving velocity distributions functions are obtained, independently of the species mass, density, and temperature ratios. An entropic consistent evolution of the system is also obtained for all values of the applied electric field, even though this external force changes abruptly with respect to time. Furthermore, these drastic changes in the electric field do not affect the consistency of the numerical method, which produces a smooth transition in the time depending macroscopic variables.

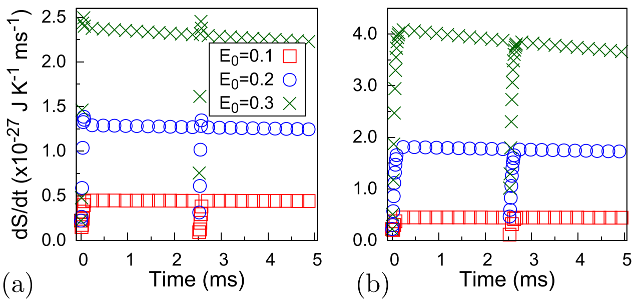


FIG. 7. Time entropy derivative for different electric fields with (a) variable (temperature dependent collision parameters) and (b) constant collision parameters.

We can conclude that the resolution of WIP from a kinetic point of view in a self–consistent way provides a physical transient solution for all the species that could be extremely difficult to obtain with other solvers. In particular, we stress that collisional effects, especially charge-neutral interactions, can exert a significant influence in the dynamics of weakly ionized plasmas. Thus, the high density of neutrals produces a viscous-like effect on the charged species. This effect transfers part of the energy transmitted by the electric field to the charges to the neutral population. We remark that such an effect is impossible to be detected with a non–collisional plasma kinetic approximation. Additionally, the effect of the temperature on the collision parameters influences the dynamics of the system.

It is expected that further exploration of the 1D kinetic collision model would explain significant departures from Maxwellian equilibrium and local energization processes on plasma species found in laboratory and natural plasmas. The present approach can be applied to a better understanding of results obtained by diagnosis procedures based upon the use of fast sweep signals, a topic that will be dealt in a forthcoming work.

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APPENDIX: DIMENSIONLESS NUMERICAL SIMULATION

For the dimensionless problem presented in Sec. III A, 10 000 points are used with a grid that goes from –250 to 250 dimensionless velocities in the charge-charge case and from –25 to 25 in the neutral-neutral problem. The long grid for the charge-charge problem is required due to the behaviour of the tails, which require more space to decline up to a value unable to perturb the problem. The time step for both problems is $\tau = 0.01$. To numerically solve the three species kinetic model presented in Sec. II, the following characteristic parameters are employed:

$$\begin{aligned}&n_c = n_0;\quad m_c = m_e;\quad q_c = q_e;\quad T_c = T_e;\\ &\sigma_c = \sigma_{e0} = 10^{-16}\,\mathrm{cm}^{-2};\quad v_c = V_{th_e};\\ &\mu_c = \sigma_c v_c;\quad \nu_c = n_c\mu_c;\quad t_c = \nu_c^{-1};\quad E_c = 1\,\mathrm{V/m}.\end{aligned}\tag{A1}$$

The choice of the electron-neutral collision as a characteristic parameter provides a good compromise between the high density of neutral particles and high temperature of electrons. The collision parameters are now rescaled with respect to the characteristic values $\tilde{\mu}_{\alpha\beta} = \mu_{\alpha\beta}/\mu_c$ and $\tilde{\nu}_{\gamma 0} = \nu_{\gamma 0}/\nu_c$ where $\tilde{}$ represent dimensionless variables. The remaining microscopic and macroscopic variables are also rescaled in the same way. To advance the distribution functions in time, a simple integral scheme[22,29,30] in the absence of source-sink terms is used. This scheme essentially can be cast into

$$f_\gamma(v, t+\tau) = \int_{-\infty}^{\infty} P_{\tau_\gamma}(v, v') f_\gamma(v', t)\,\mathrm{d}v',\tag{A2}$$

where

$$P_{\tau_\gamma}(v, v') = \frac{1}{\sqrt{4\pi D'_\gamma \tau}} \exp\left(-\frac{\left(v - v' - \tau A'_\gamma\right)^2}{4D'_\gamma \tau}\right)\tag{A3}$$

is a Gaussian probability density distribution, known as a propagator, where A_γ and D_γ are the convection and diffusion parameters that contain all the collision terms coefficients and external forces for the species γ. A prime over the convection-diffusion parameters means they are evaluated at the source variables and τ is the time step. This analytical scheme is numerically integrated by using a simple rectangle rule

$$f^{n+1}_{\gamma_j} = \sum_{j'=0}^{v_{max}} P^n_{\gamma_{jj'}} f^n_{\gamma_{j'}} \Delta v_\gamma,\tag{A4}$$

where j and j' are the array indexes, v_{max} is the maximum number of points, n is the current iteration, and Δv_γ is the grid step for the species γ. The numerical integrals are evaluated from $-L_{v_\gamma}/2$ to $L_{v_\gamma}/2$, where L_{v_γ} is the mesh length in the velocity space for the species γ. For better representation of the distribution function, different grid lengths are used for each species: $L_e = 100$, $L_i = 0.05$, and $L_0 = 0.03$ with 5000 points for each species. The three distribution functions are advanced with the same time step $\tau = 0.01$.

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