Mathematical Morphology in the HSI Colour Space

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Abstract. Mathematical Morphology is a powerful non-linear image analysis techniques based on lattice theory. The definitions of morphological operators need an ordered lattice algebraic structure. In order to apply these operators to the colour images it is required, on one hand the choice of a suitable colour space representation and on the other hand, to establish an order in the colour space providing an ordered lattice algebraic structure. The HSI space represents the colour in terms of physical attributes that separate the achromatic component from the chromatic one and it yields a more intuitive description of the colour properties than the RGB space. The suggested order weighs the hue and the intensity according to the saturation level: it has a lexicographical order in which the intensity has priority if the saturation is high, and the hue has priority if the saturation is low.

1 Introduction

The techniques of Artificial Vision have been developed initially for binary and grayscale images, where the information is codified by 2 and $2^n - 1$, $n \in \mathbb{N}$ levels, respectively. Nevertheless, the colour is an important source of information. For this reason, during the last years these techniques are being developed for colour images. However, at present, both the representation and the treatment of colour images continue to be open problems.

Mathematical Morphology is the natural arena for a rigorous formulation of many problems in image analysis and powerful non-linear techniques including operators for the filtering, texture analysis, shape analysis, edge detection or segmentation. Nevertheless, to define the basic morphological operators, erosion and dilation, it is necessary to define before an order on the space used for processing the images.

For grayscale images, this order comes from the usual order of $\mathbb{R}$. For colour images two problems arise. On one hand, the chromatic space in which the image is processed, and on the other hand, the order that settles down on it.

This paper is structured as follows. First, in Section 2, the basic operators in Morphology, erosion and dilation, are defined in sets and binary images. The natural generalization to grayscale and colour images needs the lattice structure.

In Section 3, in the HSI space a family of orders is suggested that comes defined by a cost function, and makes use of the lexicographical order in the RGB space. In section 4 one of these orders is chosen. It allows to define a lexicographical order with weight in the hue component when the image has high saturation and a with priority in the intensity component when the image has low saturation. Finally, conclusions are included in Section 5.

2 Mathematical Morphology and lattices

Mathematical Morphology is a well-known non-linear technique for the signal processing. It was initiated in the sixties with the works by Matheron [1] and Serra [2] guided by the works on sets by Minkowski. In the eighties, Matheron and Serra proposed the ultimate mathematical formulation of Morphology within the algebraic framework of the lattices [3].

The structuring element in the morphological operations is a finite subset $E \subset \mathbb{Z}^2$ with $(0,0) \in E$. The erosion (resp. dilatation) of $A \subset \mathbb{Z}^2$ by E is defined by the formula $\varepsilon_E(A) = \{x \in \mathbb{Z}^2 \;/\; E_x \subseteq A\} = \bigcap_{s \in E} A_{-s}$ (resp. $\delta_E(A) = \{x \in \mathbb{Z}^2 \,/\, (-E)_x \cap A \neq \emptyset\} = \bigcup_{s \in E} A_s$). Here $A_s = \{x + s \;/\; x \in A\}$. Hence the erosion is an infimum and the dilation is a supremum in the lattice $\mathrm{P}(\mathbb{Z}^2)$ (Parts of $\mathbb{Z}^2$).

A binary image is a map $f : \Omega \subset \mathbb{Z}^2 \to \{0,1\}$ and therefore it is the characteristic function $f = \chi_A$ of A, where $A = \{x \in \mathbb{Z}^2 \;/\; f(x) = 1\}$. Then we can define the erosion and dilation of $f = \chi_A$ by the structuring element $E \subset \mathbb{Z}^2$ as the characteristic functions of $\varepsilon_E(A)$ and $\delta_E(A)$ respectively. Precisely, $\varepsilon_E(f) = \chi_{\varepsilon_E(A)}$ and $\delta_E(f) = \chi_{\delta_E(A)}$. Note that

$$\begin{array}{c} \varepsilon_E(f) = \inf_{s \in E}(f \circ \tau_s) \\ \delta_E(f) = \sup_{s \in E}(f \circ \tau_{-s}) \end{array} \tag{1}$$

where $\tau_s(x) = s + x \quad \forall x \in \mathbb{Z}^2$. Again, the erosion and dilation are an infimun and a supremum respectively, now in binary images lattice. Punctually,

$$\begin{array}{c} \varepsilon_E(f)(x) = \inf_{s \in E}(f(s + x)) \\ \delta_E(f)(x) = \sup_{s \in E}(f(-s + x)) \end{array} \tag{2}$$

A grayscale image is a map $f : \Omega \subset \mathbb{Z}^2 \to \mathbb{Z}$. Since $\mathbb{R}$ is a total order (with its usual order), the grayscale images set is a lattice, which allow us to define the erosion and the dilation of a grayscale image f by the structuring element $E \subset \mathbb{Z}^2$ by (1) and punctually by (2).

For the extension of the basic morphological operations to colour images, we need a lattice structure in colour images set. A colour image can be represented by a map $f : \Omega \subset \mathbb{Z}^2 \to C$, where $C \subset \mathbb{R}^3$ is a colour space. If C has a lattice structure with the order $\leq_C$, then the colour images set is a lattice with the order

$$f \leq g \;\Leftrightarrow\; f(x) \leq_C g(x) \quad \forall x \in \Omega \quad \forall f, g : \Omega \subset \mathbb{Z}^2 \to C \tag{3}$$

This allows to define the erosion and dilation of a colour image $f : \Omega \subset \mathbb{Z}^2 \to C$ by a structuring element $E \subset \mathbb{Z}^2$ by (1) and punctually by (2), where the infimum and the supremum are calculated with the order $\leq_C$ in the colour space.

For binary and grayscale images, in (2) infimum and supremum are minimum and maximum respectively. For colour images, if the colour space has a total order, we also have minimum and maximum. However, if the colour space is partially ordered, infimum and supremum do not have to be minimum nor maximum. This could originate fake colours, i.e., colours that were not in the original image.

Eroding binary images is the same as diminishing white objets and dilating them makes white objets bigger. For grayscale images, eroding is the same as darkening them and dilating is clarifying them. Nevertheless, for colour images erosion and dilation do not have this univocal meaning: they depend on the order relation on the chromatic space. We must select this order depending on the image features or the type of image processing task.

3 Order in the HSI colour space

In the RGB space, colours are specified as (R, G, B) which give the amount of each red, green and blue primary stimulus in the colour. The transformation from RGB to hue, saturation and brightness coordinates is simply a transformation from a cartesian coordinates system to a cylindrical coordinates system. The achromatic axis is formed by all the achromatic points ($R = G = B$). The perpendicular plane to the achromatic axis, and intersecting it at the origin, called chromatic plane, contains all the colour information. The hue and saturation coordinates are determined within the achromatic plane. Hanbury and Serra [4], [5], [6] adopt a family of HSI spaces using norms in $\mathbb{R}^3$, and proving the independence between chromatic and achromatic components. The equations of transformation between RGB and HSI using the max-min semi-norm are given by

$$\begin{cases} I = 0.213R + 0.715G + 0.072B \\ S = \max(R, G, B) - \min(R, G, B) \\ \begin{cases} \theta = \arccos\left(\dfrac{2R - G - B}{2(R^2 + G^2 + B^2 - (RG + RB + GB))^{\frac{1}{2}}} \right) \\ H = \begin{cases} 2\pi - \theta & \text{si } B > G \\ \theta & \text{si } B \leq G \end{cases} \end{cases} \end{cases} \tag{4}$$

where I, S and H are the intensity, the saturation and the hue respectively.

All different orders defined in colour spaces are based on other previous orders defined in every component of colour. In the specific case of HSI space, there are two linear components, saturation and intensity; therefore, it is possible to work with the usual order of the real numbers. However, the hue is an angle value, $H \in [0, 2\pi)$, and the unit circle neither has relevant order nor dominant position. In mathematical terms, we cannot construct a lattice on the unit circle if we do not assign it to an arbitrary origin.

Peters [7] and Hanbury [8] fix a reference hue, H_{ref} and establish an order. This reference hue is chosen as the minimum value. The hue circle is ordered by the distance:

$$d(H, H_{ref}) = \begin{cases} |H - H_{ref}| & \text{if } |H - H_{ref}| \leq \pi \\ 2\pi - |H - H_{ref}| & \text{if } |H - H_{ref}| > \pi \end{cases} \tag{5}$$

From this distance we obtain an order for the hue:

$$H_1 \leq_{H_{ref}} H_2 \Leftrightarrow \begin{cases} d(H_1, H_{ref}) < d(H_2, H_{ref}) \\ \text{or} \\ d(H_1, H_{ref}) = d(H_2, H_{ref}) \text{ y } H_1 \leq H_2 \end{cases} \tag{6}$$

It should be observed that the natural order in the linear components, saturation and intensity, agrees with the intuitive order. But there are different orders for hue component depending on the value H_{ref}, so that the intuitive idea of smaller or bigger point disappears. For this reason some rare results can be obtained when an angular component plays an important role in the order defined on the chromatic space.

Fix a (cost) function $c : HSI \to \mathbb{R}$, and consider the following order:

$$(H_1, S_1, I_1) \leq^c_{H_{ref}} (H_2, S_2, I_2)$$

$$\Leftrightarrow \begin{cases} c(H_1, S_1, I_1) < c(H_2, S_2, I_2) \text{ (a)} \\ \text{or} \\ c(H_1, S_1, I_1) = c(H_2, S_2, I_2) \text{ and } I_1 < I_2 \text{ (b)} \\ \text{or} \\ c(H_1, S_1, I_1) = c(H_2, S_2, I_2) \text{ and } I_1 = I_2 \text{ and } H_1 <_{H_{ref}} H_2 \text{ (c)} \\ \text{or} \\ c(H_1, S_1, I_1) = c(H_2, S_2, I_2) \text{ and } I_1 = I_2 \text{ and } H_1 = H_2 \text{ and } S_1 \leq S_2 \text{ (d)} \end{cases} \tag{7}$$

Case a) of (7) determines the order almost everywhere. Only for pairs of points over the same surface $c(H, S, I) = c$ the order must be decided via the HSI lexicographical order with priority I, H, S. Certainly, RGB space is the most employed colour space in images acquisition. In addition, if we want to work using the order defined by (7), it requires a high computational cost for calculating the equations of a change of coordinates between the spaces RGB and HSI. However, we can avoid to calculate inverse transform equations if the lexicographical order given by (7) is considered on RGB space.

4 Erosion and dilation in the *HSI* colour space

The lexicographical orders are total orders with priority of components. In real images, the intensity is the attribute that offers greater definition of scenes, therefore the priority of lexicographic order I, H, S offers good visual results. If the image has a high saturation, it is mainly determined by the hue, so we set the hue as first position in the lexicographical order. [8], [9], [10], [11], [12], [13].

A new order is suggested for HSI space defined by (7), where the cost function is a function of the hue for high saturation level case and a function of the intensity for low saturation.

Fixed a hue reference value H_{ref}, we defined the normalized hue value by the formula

$$h = \frac{d(H, H_{ref})}{\pi} \in [0, 1] \tag{8}$$

The above cost function is defined by

$$c(H, S, I) = a(S)h + (1 - a(S))I \tag{9}$$

where $a : [0, 1] \rightarrow [0, 1]$ is increasing with $a(0) = 0$ and $a(1) = 1$. Initially, it is possible to choose $a(S) = S$ [14], so

$$c(H, S, I) = Sh + (1 - S)I \tag{10}$$

and the order is expressed by

$$(H_1, S_1, I_1) \leq_{H_{ref}} (H_2, S_2, I_2)$$

$$\Leftrightarrow \begin{cases} (1 - S_1)I_1 + S_1 h_1 < (1 - S_2)I_2 + S_2 h_2 \\ \text{or} \\ (1 - S_1)I_1 + S_1 h_1 = (1 - S_2)I_2 + S_2 h_2 \text{ and } R_1 < R_2 \\ \text{or} \\ (1 - S_1)I_1 + S_1 h_1 = (1 - S_2)I_2 + S_2 h_2 \text{ and } R_1 = R_2 \text{ and } G_1 < G_2 \\ \text{or} \\ (1 - S_1)I_1 + S_1 h_1 = (1 - S_2)I_2 + S_2 h_2 \text{ and } R_1 = R_2 \text{ and } G_1 = G_2 \\ \qquad \text{and } B_1 \leq B_2 \end{cases} \tag{11}$$

where(R, G, B) and (H, S, I) are the components of a point in the RGB and HSI spaces respectively.

We remark that:

- Saturation component S and its complementary value $1 - S$ are weights of the normalized hue and the intensity respectively. To establish the order relation it must be taken into account that the hue component has bigger weight when the saturation is high, whereas the intensity has bigger weight when the saturation is low.
- If the image has a high saturation level, then the fixed reference hue value plays an important role. For example, if the predominant colour is red and we select $H_{ref} = 0$, then eroding (resp. dilating) is the same that increasing (resp. decreasing) the size of saturate red objects.

The image Miro (368×271), used by Hanbury [11], [8] has low saturation (Fig. 1). However, the image Colours (249×245) has medium-high saturation (Fig. 2). These images are used to testing the goodness of the order above suggested. For the image Miro, the order works by intensity level and for the image Colours, the order works by hue level.

Fig. 3 shows the erosion and dilation of image Miro by a disk of width 4 with the order (11) with $H_{ref} = 0^o$ in a) and b) and with $H_{ref} = \pi$ in c) and d). At Fig. 1 c) we can see: the red and yellow shaded regions are areas of high saturation; the blue and green shaded body and the blue and green coloured spots over background have medium saturation; the white background, with green-gray coloured spots and dark border of imagen and dark spots have low saturation values.

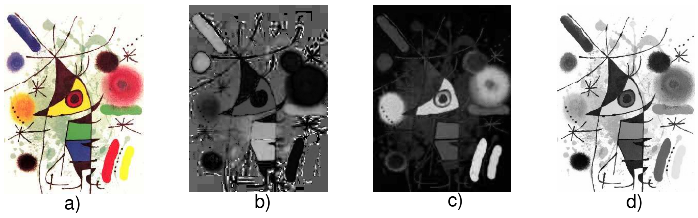

Fig. 1. a) Image Miro b) Distance hue with $H_{ref} = 0^o$, c) Saturation d) Intensity.

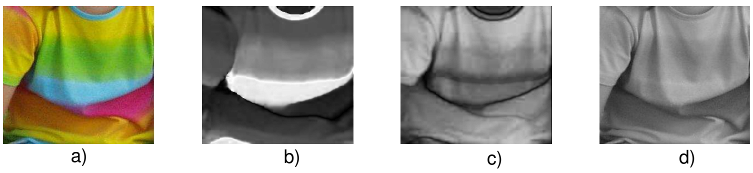

Fig. 2. a) Image Colours b) Distance hue with $H_{ref} = 0^o$, c) Saturation d) Intensity.

Noting that with $H_{ref} = 0^o$ the result of erosion increases the size of the image borders and dark spots. In the dilation by a structuring element big enough, these elements with low intensity disappear. Green-gray spots over white background (high intensity), also increase their size after an erosion operation. Finally, the yellow and red coloured spots at the right bottom of the image, over white background, are objects with high saturation that also increase their size. However, yellow and red regions of face with dark border decrease.

Any change of reference hue value give rise to no different behavior of low saturation areas, but we can appreciate some differences at middle-high saturation regions. For example, if we select $H_{ref} = \pi$, close to blue colour, then the red colour has a high hue level. Fig. 3 c) shows at the right bottom of the image that as result of erosion, the edge of the red spot is not enhanced due to green-gray spots over background.

Fig. 4 shows the erosion and the dilation of image Colours by a disk of width 4, with the order (11) with $H_{ref} = 0^o$ in a) and b) and with $H_{ref} = \pi$ in c) and d). At Fig. 2 *c*) we can see that image Colours has middle-high saturation level;

therefore, the order works like an intermediate order between the lexicographical order with intensity as priority component and the lexicographical order with hue as priority component. Fig. 4 is agreed with our intuitive perception: the reference hue value has more influence than in the above case.

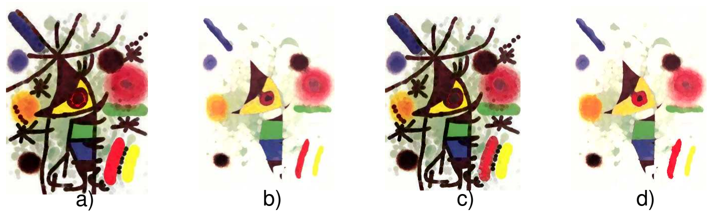

Fig. 3. a) Erosion b) Dilation with $H_{ref} = 0^o$ c) Erosion d) Dilation with $H_{ref} = \pi$ of image Miro.

Fig. 4. a) Erosion c) Dilation with $H_{ref} = 0^o$ c) Erosion d) Dilation with $H_{ref} = \pi$ of image Colours.

5 Conclusions

A new order has been presented in the HSI colour space in the Mathematical Morphology framework for colour images. This order allows to select the saturation level as a weighting factor for the intensity and the hue.

For low saturation regions, the order works in a lexicographical order with intensity as priority component, whereas if the regions have a high saturation value, the order chooses the hue as priority component, since these components are the right ones to determine the image at every case. When saturation is medium, then the order works like an intermediate order between the lexicographical one with the intensity as priority component and the lexicographical one with the hue as priority component.

It is possible to prove that the fixed hue reference value has a high influence on images with medium-large saturation level, whereas this influence is not significant for images with low saturation level. Another advantage of the order is that it allows to reduce the computational cost that involves the work in colour spaces different the RGB space.

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