

Automation in Dynamic Analysis and Generative Design of Prestressed Concrete Railway Bridge Infrastructures

Khuong Le Nguyen¹, Thong M. Pham³, Khanh Nguyen⁴, Saeed Banihashemi^{5*}

¹ Department of Civil Engineering, University of Transport Technology, Hanoi, 100000, Vietnam;
khuongln@utt.edu.vn

² Faculty of Arts and Design, University of Canberra, 11 Kirinari St, Bruce ACT 2617
khuong.lenguyen@canberra.edu.au

³ UniSA STEM, University of South Australia, Mawson Lakes, SA 5095, Australia Email:
thong.pham@unisa.edu.au

⁴ Escuela Técnica Superior de Ingeniería Aeronáutica y del Espacio, Universidad Politécnica de Madrid, 3 Pza. Cardenal Cisneros, Madrid, 28040, Spain, khanhnguyen.gia@upm.es

⁵ School of Built Environment, University of Technology Sydney, Australia.
Saeed.Banihashemi@uts.edu.au

*Corresponding author

Abstract

This study presents an innovative method for the dynamic analysis and generative design of high-speed ballasted railway bridges subjected to High-Speed Locomotive Multiple Articulated (HSLM-A) train loads. Compliant with Eurocode standards, a comprehensive database of over 4 million data points was generated, including maximum vertical displacement and acceleration data for more than 10,000 bridges affected by ten HSLM-A models at speeds ranging from 150 to 350 km/h. The key contribution of this research lies in a novel surrogate model that incorporates semantic search and advanced decoding techniques, significantly enhancing the calculation time and accuracy of dynamic behaviour predictions for single-span high-speed railway bridges. The performance of the developed model was verified through case studies on existing 30m and 50m span bridges, evidenced by an R^2 value of 0.999, highlighting the model's precision and rapid prediction capabilities. Additionally, the research introduces a cutting-edge framework for optimising the cross-sectional geometry of prestressed concrete railway bridges. A case study was then conducted for a typical box girder bridge to identify 25 feasible solutions better than the original design in terms of mass per unit length. This research showcases the synergy between advanced technology and structural optimisation, and it opens new avenues for future studies in this field.

Keywords: Semantic Surrogate Modelling; High-Speed Railway Bridge Dynamics; Generative Design Optimisation; Structural Performance Prediction; Advanced Decoding Techniques.

33 **1 Introduction**

34 High-speed railway (HSR) systems represent a notable advancement in transportation, providing fast,
35 environmentally friendly, and efficient travel options. The increasing global deployment of these
36 systems has made the design and operation of high-speed railway bridges extremely important. Unlike
37 slower, heavier freight trains that primarily induce quasi-static effects, high-speed passenger trains can
38 potentially cause resonance when the frequency of the loading, determined by the train speed and axle
39 spacing, approaches the natural frequency of the bridge. As new train designs with varying axle
40 configurations are introduced and operate at increasingly high speeds, the likelihood of matching these
41 frequencies increases. The complexity of these dynamic interactions, coupled with the need to design
42 for future train developments, makes the analysis and design of high-speed railway bridges particularly
43 challenging [1–5].

44 In addressing these challenges, simply supported prestressed concrete box girders have risen as a
45 favourite structural solution. Renowned for their structural efficiency, these girders effectively resist
46 dynamic forces and vibrations of high-speed train transit, offering great torsional stiffness and strength
47 vital for maintaining bridge stability at high velocities [6–9]. Moreover, the incorporation of prestressing
48 techniques significantly mitigates tension, enabling the construction of longer spans, thereby enhancing
49 the durability and longevity of the structures [10–13].

50 Due to the many advantages of these structures, extensive research has delved into the dynamic
51 performance of simple span railway bridge girders, using a multi-dimensional approach associated with
52 various factors such as resonance, material properties, and friction [14–19]. More specifically, the
53 authors [20–22] examined the resonance characteristics of bridges under high-speed train passage,
54 revealing the critical need to differentiate between dominated train frequencies and bridge natural
55 frequencies to mitigate resonance risks. They also demonstrated that axial stiffness between simple
56 girders could significantly reduce vibrations near resonance conditions. Other researchers [23–27]
57 focused on the interaction between the "train-bridge" system and bridge deck resonant vibrations,
58 highlighting the importance of dynamic phenomena control at the wheel-rail contact level.

59 In the design process, modal analysis and time history analysis are often required to determine the
60 dynamic behaviours such as displacements, accelerations, and bending moments of the bridge subjected
61 to fast-moving train. Eurocode 1 [28] considers various high-speed load models (e.g., HSLM-A) for
62 dynamic analyses of all railway bridges in the network. When new high-speed trains are designed with
63 dynamic signatures that fall within the envelope of the HSLM-A models, additional dynamic checks of
64 the bridges may not be required, as the bridges are already designed for this envelope of dynamic effects.
65 However, for trains with characteristics outside this envelope, or as train technology evolves beyond the
66 parameters considered in the original HSLM-A development, additional dynamic analysis may be
67 necessary [29,30]. This approach ensures that bridges are prepared for future train designs, making the
68 design process more efficient and the bridges safer and more reliable.

69 However, a critical challenge remains: incorporating dynamic analysis into a design framework,

70 especially different constraints and requirements are essential for comparing design alternatives [31,32].
71 To address this, various researchers have explored ways to optimise bridge designs for high-speed trains.
72 For example, Cho *et al.* [33] proposed a method to determine the optimal span length of railway bridges,
73 focusing on minimising dynamic responses to high-speed trains. Similarly, Song and Lee [34] advanced
74 this field by introducing sophisticated techniques for assessing the interaction between bridges and high-
75 speed vehicles. Futher, Zhai *et al.* [35] highlighted the progress in train-bridge interaction models and
76 underscored the critical role of accurate track modeling. These studies collectively enhance our
77 understanding of how to design bridges that can efficiently support high-speed rail systems while
78 ensuring safety and longevity.

79 Despite these advances, opportunities remain for further innovation in bridge design optimization. While
80 significant advances have been made through probabilistic approaches by Allahvirdizadeh *et al.* [36]
81 and surrogate-assisted reliability-based design optimization [37], the integration of exhaustive dynamic
82 analysis with cross-sectional optimization using semantic search and generative design techniques
83 remains to be fully explored. In recent years, emerging paradigms such as large language models (LLMs)
84 and neuro-symbolic models have demonstrated promising capabilities in related fields. For instance,
85 Wong *et al.* [38] and Dikmen *et al.* [39] have shown that LLMs, when adapted with construction-specific
86 knowledge, can enhance tasks like risk identification and damage detection in infrastructure projects.
87 These advances offer valuable insights that could extend to the dynamic analysis and design optimization
88 of prestressed concrete railway bridges. Similarly, Onchiş *et al.* [40] employed neuro-symbolic models
89 for damage detection in structural components, providing a new direction for improving the reliability
90 and explainability of computational models used in infrastructure maintenance. These studies, published
91 in *Computers in Industry*, underscore the potential of combining data-driven techniques with traditional
92 simulation-based methods to address complex design challenges.

93 To address these challenges, this research introduces an innovative approach combining a surrogate
94 model with a generative design framework. The surrogate model accelerates the dynamic analysis by
95 accurately predicting the performance of bridge designs without extensive simulations, while the
96 generative design framework efficiently explores a broad spectrum of design alternatives to identify
97 optimal cross-sectional configurations.

98 Key highlights of this research include:

- 99 • Comprehensive Database Development: Generating over 4 million data points covering
100 displacement and acceleration data for more than 10,000 bridges, particularly focusing on High-
101 Speed Locomotive Multiple Articulated (HSLM-A) trains.
- 102 • Surrogate Modelling for Efficient Analysis: The novel surrogate model, derived from this
103 extensive database, significantly reduces the time required for dynamic analysis.
- 104 • Practical Application via KD-Railway Tool [41]: Integrating the surrogate model into the KD-
105 Railway tool bridges the gap between research and application.
- 106 • Emphasis on Generative Design: This study adopts generative design in optimising the cross-

107 sectional design of bridges, utilising automation for enhanced design efficiency.
108 • Adherence to Standards: The design framework is aligned with Eurocode standards to increase
109 its applicability in practice.

110 The findings of this research are summarised in a video presentation, which can be viewed in this
111 [Youtube video](#).

112 **2 Methodology**

113 **2.1 Research Workflow Overview**

114 The workflow on the dynamic analysis and generative design of prestressed concrete railway bridges is
115 illustrated in Figure 1 and consists of four distinct stages:

116 ***Stage 1: Data Generation***

117 The first stage involves generating the highest possible vertical acceleration and displacement data of
118 10,000 bridges subjected to loading of 10 HSLM-A train models. All bridges were designed in
119 accordance with Eurocode 1 [28] and analysed using KD-Railway [41], an in-house software tool
120 developed by the authors. Built on the open-source Cast3M finite element analysis solver [42], KD-
121 Railway specializes in dynamic analysis of railway bridges and automates several critical processes
122 including moving load function generation and envelope diagram plotting. The tool's parallel computing
123 capabilities and automated processes significantly enhance computational efficiency while maintaining
124 accuracy comparable to commercial software, as validated through multiple case studies. Using this
125 validated tool, a total of 4 million data points were generated.

126 ***Stage 2: Surrogate Model Development and Validation***

127 Using the generated database, a surrogate model was developed to predict the envelope curves for the
128 maximum acceleration and displacement of railway bridges under loading induced by HSLM-A train
129 model. Subsequently, this surrogate model was employed to forecast the dynamic characteristics of two
130 actual bridges that were in the initial set of 10,000 bridges of Stage 1. The predicted values were
131 compared to those obtained using the Finite Element Method (FEM) in order to assess the reliability and
132 accuracy of the model.

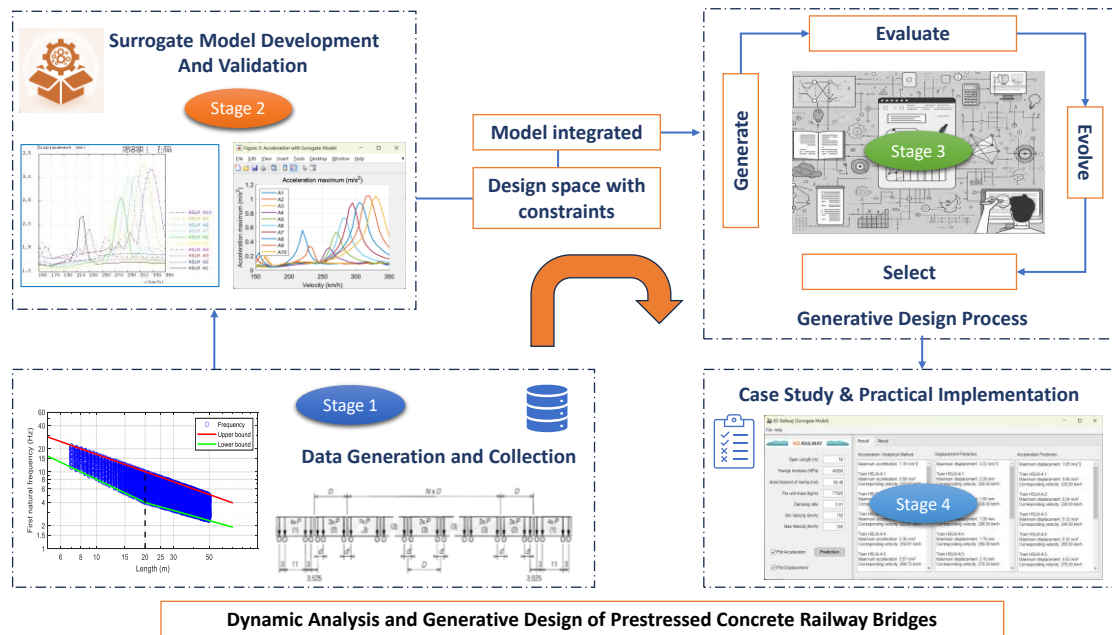
133 ***Stage 3: Generative Design Process***

134 Based on the trained models, this step presented a design approach that generate an optimal cross-section
135 design for simply-supported prestressed concrete box girder bridges while meeting multiple constraints
136 and goals.

137 ***Stage 4: Case Study and Practical Implementation***

138 The surrogate model was incorporated in a revised edition of the KD-Railway software and made
139 available on GitHub. The proposed framework was utilised to produce 25 viable and optimal alternatives
140 for the cross-section of a 30-m span simply-supported prestressed concrete box girder bridge. This phase

141 aimed to showcase the practicality of the surrogate model and the generative design process.



142

143

Figure 1. Research workflow

144 This study focuses exclusively on single-span simple supported beams without skew or complex
145 geometry. This choice is deliberate and aligns with the prevalence of such structures in high-speed
146 railway systems, as noted in the introduction. The methodology developed here is tailored for the
147 preliminary design process, where simplicity and computational efficiency are paramount. While more
148 complex bridge types exist, they typically require detailed design considerations beyond the scope of
149 this preliminary design optimization framework.

150 2.2 Data Generation

151 2.2.1 Input parameters for dynamic analysis

152 The dynamic behaviour of high-speed ballasted railway bridges depends on several key factors,
153 including the properties of the bridge's cross-section, span length, boundary conditions, material
154 characteristics, and the moving load train models.

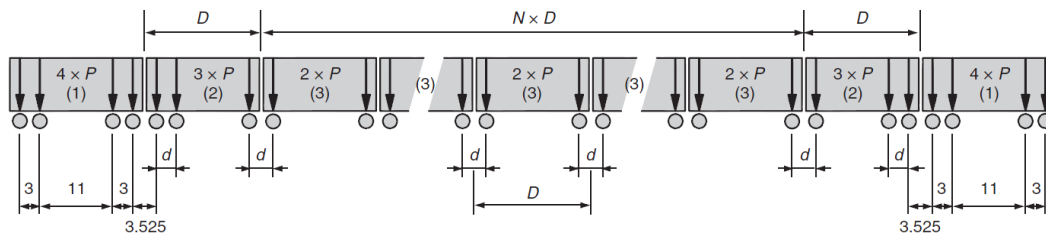
155 According to Eurocode 1 [28], the major input parameter for dynamic analysis is the initial natural
156 frequency of the bridge. To improve the authenticity of the produced data, the frequency of each bridge
157 was randomly generated within a range bounded by the upper and lower limits given in Eurocode 1.
158 Furthermore, the bridge's mass, including its own weight and the weight of all its superstructural
159 elements, is another crucial factor. The mass is quantified per unit length of the bridge and is determined
160 following the guidelines provided by ERRI [43]. This assists engineers in determining suitable values
161 for dynamic analyses.

162 Regarding train loads, Eurocode 1 [28] recommends a suite of standard load models that mimic the
163 effects of various high-speed train types. Among these, the High-Speed Load Model (HSLM) is a key
164 standard for representing the loads from articulated, conventional, and high-speed passenger trains. This

165 study focuses on bridges with simple support spanning from 7 to 50 meters. While this span range was
 166 used for database generation to ensure the surrogate model captures a wide spectrum of dynamic
 167 behaviors, it's important to note that the model's underlying parameters (mass, stiffness, frequency) are
 168 generic and not tied to any specific cross-sectional type.

169 The HSLM-A model has been consistently employed for all 10,000 bridges. The specific parameters for
 170 the HSLM load models are summarised in Table 1.

171 **Table 1. Universal Train HSLM A1-A10 Parameters [28]**



Universal train	Number of intermediate coaches N	Coach length D (m)	Bogie axle spacing d (m)	Point force P (kN)
A1	18	18	2.0	170
A2	17	19	3.5	200
A3	16	20	2.0	180
A4	15	21	3.0	190
A5	14	22	2.0	170
A6	13	23	2.0	180
A7	13	24	2.0	190
A8	12	25	2.5	190
A9	11	26	2.0	210
A10	11	27	2.0	210

172 Finally, the damping factor is another pivotal parameter within dynamic analysis. This study sets the
 173 damping factor at 1%, as commonly adopted for prestressed concrete railway bridges [43].

174 2.2.2 Frequency Considerations and HSLM-A Model Selection

175 In high-speed railway bridge dynamics, multiple frequency sources influence the structural response,
 176 including travelling frequencies from periodic axle loads, natural frequencies of the bridge and train, and
 177 frequencies arising from rail irregularities. Research by Fryba [44] and Yang et al. [45] has consistently
 178 demonstrated that the travelling frequency, determined by the spacing of train axles and bogies, is the
 179 most significant contributor to bridge dynamics. This finding is reflected in design standards such as
 180 Eurocode EN 1991-2:2003, which prioritizes travelling frequency in resonant speed calculations.

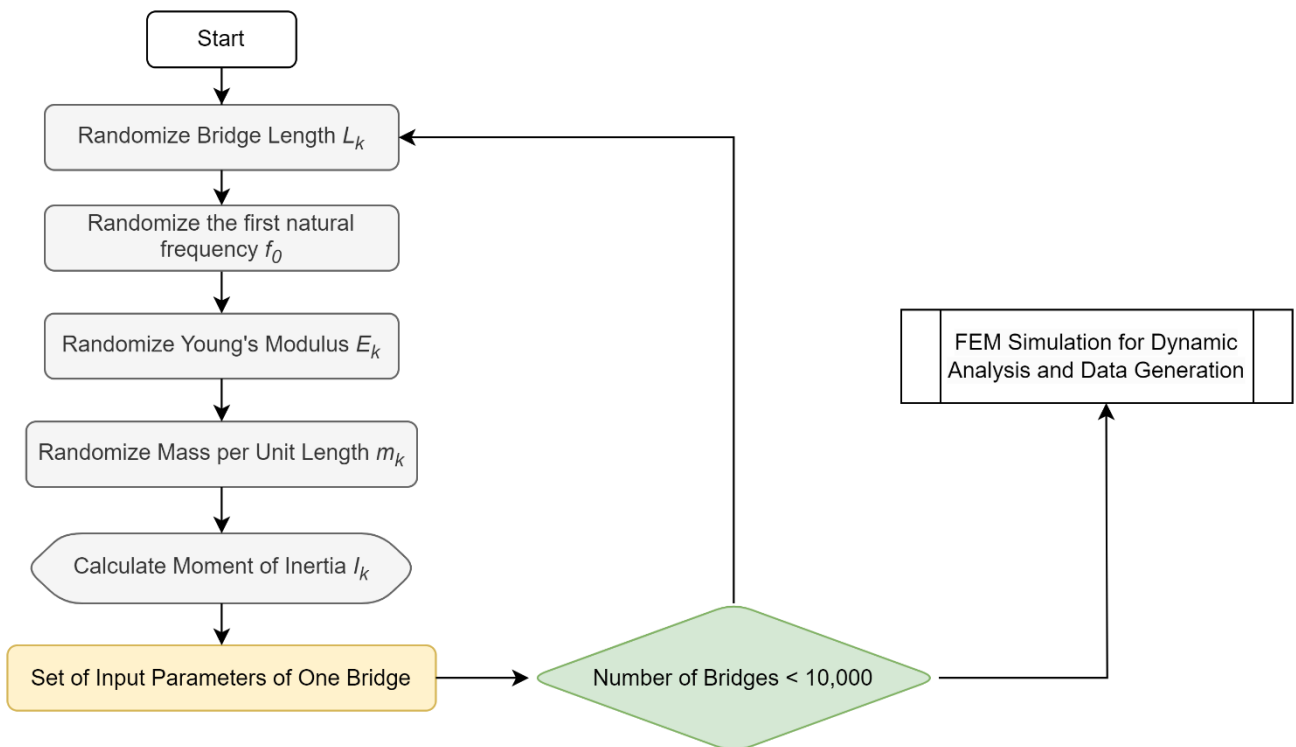
181 This study employs the HSLM-A model, which, while a simplification, effectively captures this crucial
 182 travelling frequency component. Although it does not account for train-bridge interaction or rail
 183 irregularities, studies by Majka and Hartnett [46] and Domenech et al. [47] have shown that rail
 184 irregularities, while significant for lateral dynamics and train response, have a comparatively minor
 185 influence on vertical bridge acceleration and dynamic amplification. The effects of rail irregularities are

186 indirectly addressed in Eurocode EN 1991-2:2003 through a coefficient $\phi''/2$ in the dynamic
 187 amplification factor. However, recent research conducted by Patrick *et al.* [48] indicates that while this
 188 coefficient works well for displacement predictions, it may be less conservative for acceleration
 189 responses when using simplified moving load approaches rather than full train-track-bridge interaction
 190 models.

191 The HSLM-A model's focus on moving loads offers an optimal balance between accuracy and
 192 computational efficiency for vertical dynamics analysis. This approach aligns with current best practices
 193 in bridge engineering and is particularly suitable for the study's emphasis on vertical dynamic analysis
 194 and design optimization. While more detailed models would be necessary for studies on lateral
 195 dynamics, train safety, or passenger comfort, the HSLM-A model provides an appropriate level of detail
 196 for the purposes of this research.

197 2.2.3 Randomness selection of input parameters

198 The input parameters considered for this analysis include first natural frequency of the bridge (f_0),
 199 Young's modulus (E_k) in GPa, moment of inertia (I_k) in m^4 , span length (L_k) in meters, and mass per unit
 200 length (m_k) in kg/m.



201
 202 **Figure 2. Flowchart of the Input and Output Generation Processes for 10,000 Railway Bridges**

203 Figure 2 outlines the process for generating randomised data:

- 204 1. The length of the bridge (L_k) is randomised between [7, 50] meters using a uniform distribution.
 205 2. Given L_k , the first natural frequency (f_0) was randomly generated within its bounds as per
 206 Eurocode 1.

- 207 3. Young's modulus (E_k) was randomised within the typical range for prestressed concrete, [30, 40]
208 GPa.
- 209 4. The mass per unit length (m_k) was also randomised, considering the span length and
210 corresponding guidelines to ensure a realistic distribution of bridge masses.
- 211 5. Finally, the moment of inertia (I_k) is calculated using the values obtained from the previous steps.

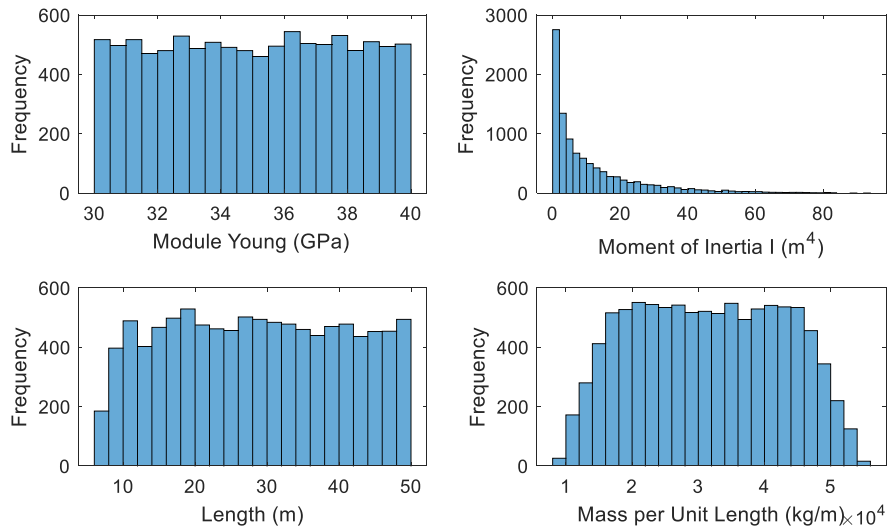
212 **2.2.4 Data generated using KD-Railway source code**

213 The data generation of 10,000 bridges was conducted through an automated process, leveraging the
214 capabilities of the KD-Railway software [41]. To thoroughly analyse the characteristics of each bridge,
215 a diverse range of train velocities ranging from 150 to 350 km/h, with increments of 5 km/h, was chosen.
216 As a result, a total of 41 simulations was conducted for each train model. Therefore, to adhere to
217 Eurocode rules, 10 distinct HSLM-A train models necessitates a substantial quantity of 410 simulations.
218 A total of more than 4 million simulations were carried out, with each simulation yielding the maximum
219 displacement and maximum acceleration of the respective bridge when subjected to a single train
220 crossing at a specific speed.

221 The pivotal steps in the automated calculation process are outlined as follows (Figure 2):

- 222 1. Division into small groups: The bridges were batch-processed in smaller groups of 1,000 to
223 improve computing performance. This method enabled parallel computations, resulting in a
224 substantial optimisation of computational resource utilisation.
- 225 2. Initialization Loop: The computation sequence was designed with an initialisation loop, where
226 each iteration corresponded to a bridge indexed from 1 to 10,000.
- 227 3. Bridge Construction and Load Application: In each iteration, the bridge model was constructed
228 with the specified input parameters—length, elastic modulus, moment of inertia, and mass per
229 unit length. The loadings were 10 different HSLM-A train models.
- 230 4. Result Recording: Upon completing the dynamic analysis for each bridge, the Cast3M software
231 automatically compiled and recorded the results, including the maximum displacement and
232 acceleration values as functions of train speed, were saved into individual .csv files for further
233 processing and analysis.

234 The computation, conducted on a personal laptop with moderate specifications (All-in-One Desktop
235 Computer, 11th Gen Intel Core i7-1165G7), spanned over four days. More than 4 million data points for
236 maximum vertical acceleration and displacement were recorded. Figure 3 displays histograms for the
237 input features: Young's modulus, moment of inertia, length, and mass per unit length, offering a visual
238 representation of the distribution of these critical bridge parameters. The histograms demonstrate that
239 the input parameters are distributed across their respective engineering-relevant ranges (e.g., Young's
240 modulus from 30-40 GPa) with sufficient sampling density to capture the variety of possible bridge
241 configurations, avoiding bias toward particular values.



242

243

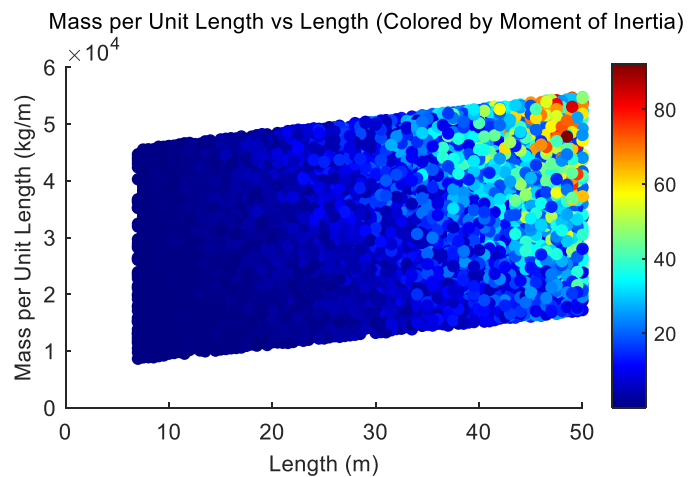
Figure 3. Histogram of Input Features: Module Young, Moment of Inertia, Length, Mass per Unit Length

244

Figure 4 illustrates the relationship between mass per unit length and length, revealing how these variables are distributed across these bridges. The colour intensity corresponds to the density of data points, providing insight into common structural properties within the dataset.

245

246



247

248

Figure 4. Mass per Unit Length vs Length Relationship, with Color Scale Indicating Moment of Inertia (m⁴)

249

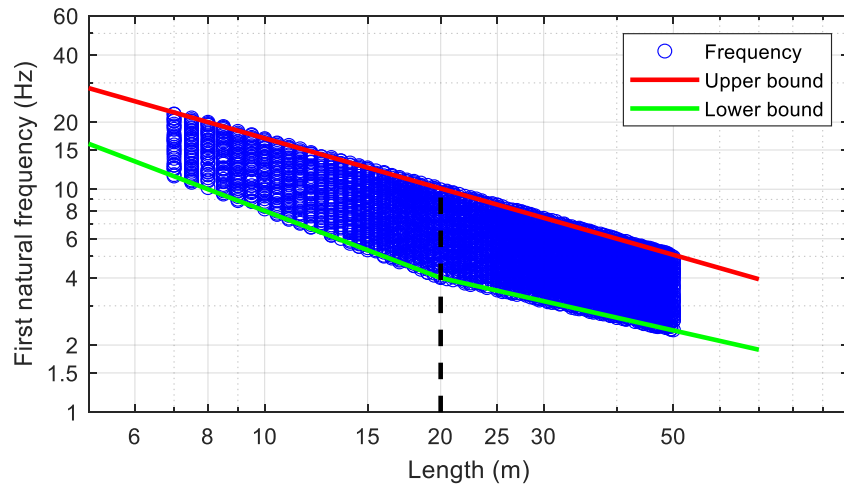
Figure 5 summarises the first natural frequency distribution of the 10,000 bridges plotted in logarithmic terms against length (m), with upper and lower bounds delineated according to Eurocode 1 [28]. This graph highlights the natural frequency range expected for typical bridge designs and showcases the bounds within which most bridges' natural frequencies lie, ensuring compliance with Eurocode guidelines for dynamic performance.

250

251

252

253



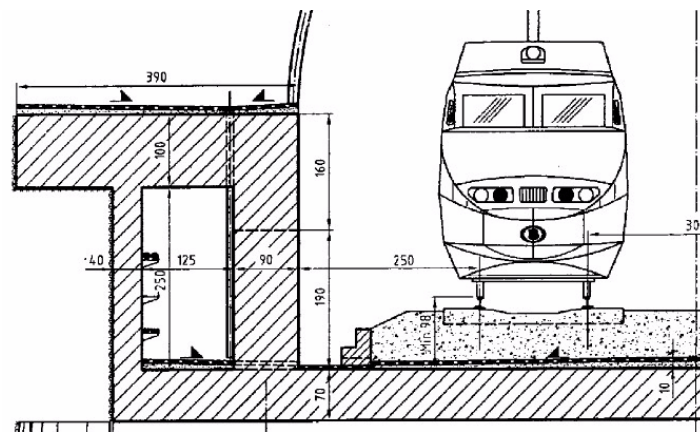
254

255 **Figure 5. First Natural Frequency Distribution of 10,000 Bridges Plotted in Logarithmic Terms Versus Length (m)**
 256 **With Upper Bound and Lower Bound According to Eurocode 1 [28]**

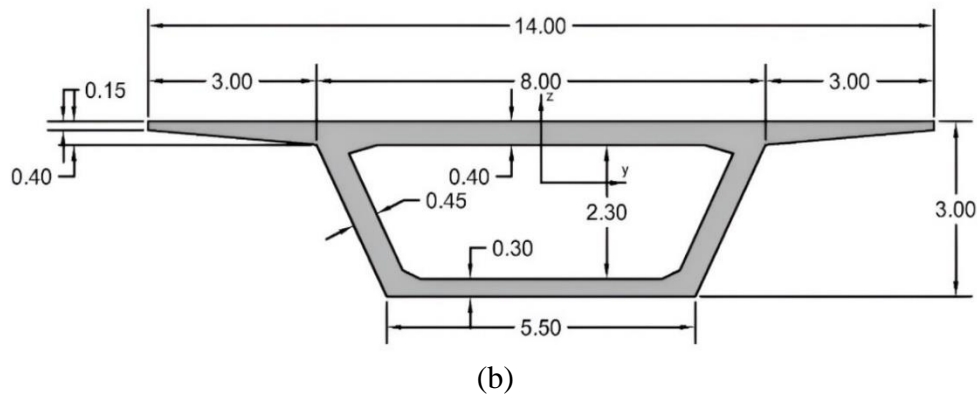
257 **2.3 Data for Validation and Application Process**

258 The developed surrogate model was validated by examining its predictive performances against the
 259 dynamic behaviours of two real bridges that were not included in the generated data for model training.
 260 The initial focus is on the Antoing Bridge near the French-Belgian border, on the high-speed railway
 261 network connecting Paris with Brussels. This multi-span structure features simply-supported prestressed
 262 concrete (PC) girders with U-shaped sections of 50 meters. The bridge's dynamic properties were
 263 obtained through extensive real-world measurements by Xia *et al.*[16]. The second bridge is a 30-meter
 264 span, simply-supported box girder bridge designed for the UK's High Speed 2 (HS2) project [49].

265 **Error! Reference source not found.** illustrates a typical overpass configuration within the two selected
 266 bridges.

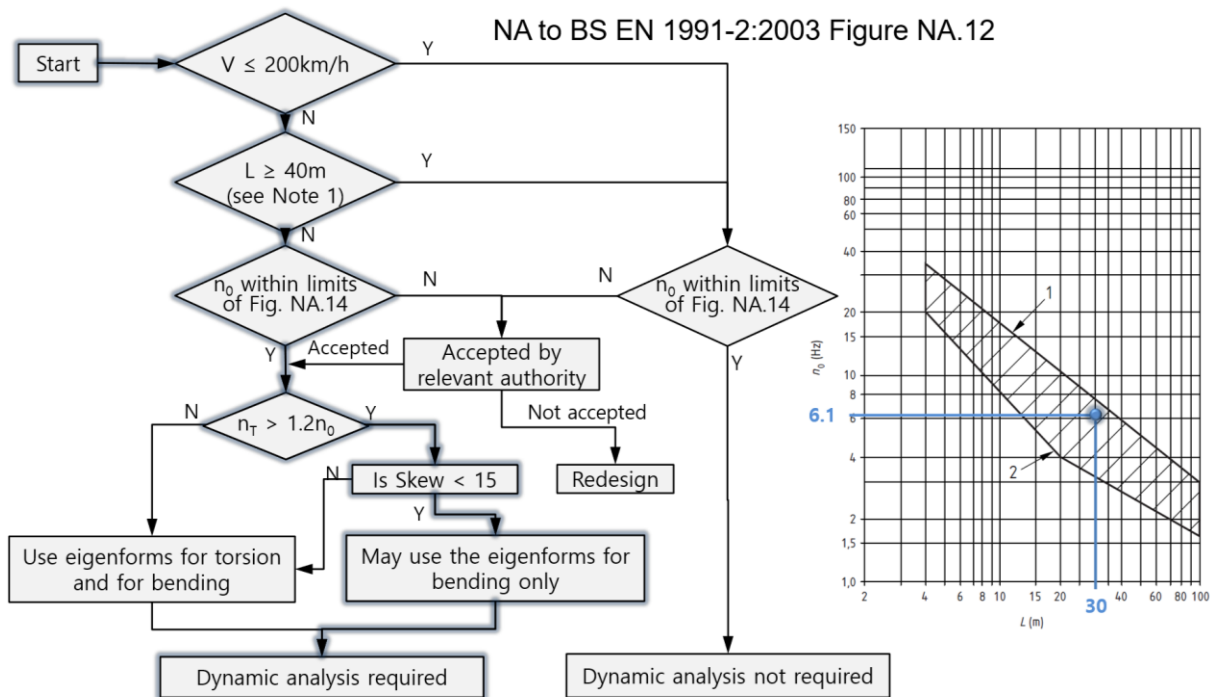


(a)



267 **Figure 6. Cross-sectional configurations of the validation case studies: (a) Antioing Bridge - a prestressed concrete**
 268 **(PC) girder with U-shaped sections [16]; (b) Typical simply supported box girder bridge from UK's High Speed 2**
 269 **(HS2) project [49]**

270 For the latter bridge type—prestressed concrete box girders, we examined the design protocol used by
 271 HS2 for dynamic analysis requirements. This strategic protocol, as implemented by Pere Alfaras,
 272 Principal Bridge Engineer at Arcadis UK [49], follows the assessment framework defined in BS EN
 273 1991-2:2003 [28] (Figure 7), which dictates when dynamic analysis is required for railway bridges.



274 **Figure 7. Process to evaluate whether the bridge requires a dynamic analysis**
 275

276 As part of validating our model with the HS2 bridge case study, we examined its first natural frequency
 277 of 6.1 Hz at a 30m span length against this Eurocode framework. For the HS2 bridge case, its frequency
 278 falls within the acceptable bounds, as shown in the right-hand graph. While its span length is less than
 279 40m, dynamic analysis is required due to operational speeds exceeding 200 km/h. By validating our
 280 surrogate model against a real bridge that requires dynamic analysis according to Eurocode standards,
 281 we demonstrate its practical value as an efficient alternative to traditional analysis methods while

282 maintaining compliance with code requirements. The accurate prediction of dynamic responses for the
283 HS2 bridge (detailed in Section 3.1.2) confirms our model's reliability for cases where Eurocode
284 mandates dynamic analysis.

285 **2.4 Surrogate Model Development**

286 **2.4.1 Indexing and Semantic Search**

287 The bridges were divided into groups, each stored in a separate file (*.mat). This division is semantically
288 meaningful, as it organises bridges into manageable sets for faster retrieval. When querying data for a
289 specific bridge, instead of traditional index identification, a semantic search query is processed. The
290 input values (descriptors of the bridge) are analysed semantically to compute a query vector, which is
291 then used to navigate through the semantic index to identify the most contextually relevant .mat file(s).

292 Semantic search in this context is about finding the most relevant dataset for a given bridge based on a
293 semantic understanding of the query, rather than a direct lookup. This involves interpreting the query's
294 semantic context — in this case, identifying which group of data (group) a specific bridge belongs to.
295 The semantic indexing is reflected in how the data is organised and accessed.

296 This approach offers several computational benefits over traditional methods:

- 297 1. Improved efficiency: Search times are significantly reduced compared to conventional database
298 queries, as the system quickly narrows down the relevant data from the extensive 4-million-point
299 database.
- 300 2. Enhanced accuracy: By understanding the context of bridge parameters, the search retrieves more
301 relevant data points, potentially leading to improved prediction accuracy.
- 302 3. Scalability: As the database grows, the semantic search maintains its efficiency, unlike traditional
303 methods that might slow down with larger datasets.

304 The implementation involves converting bridge characteristics into high-dimensional semantic vectors
305 (embeddings) that capture their engineering relationships. These embeddings are then indexed using an
306 efficient nearest-neighbor search algorithm. When analysing a new bridge, its parameters are similarly
307 embedded, and the system rapidly identifies the most relevant existing data points.

308 **2.4.2 Value Decoding and Estimation**

309 The decoding process translates raw data into meaningful parameters that can be used to estimate the
310 maximum acceleration (A_{\max}), which involves several mathematical operations:

311 Data reshape:

$$312 \quad D_k = \text{reshape}(D(k - (g - 1) \times 500, :), 410, 8)$$

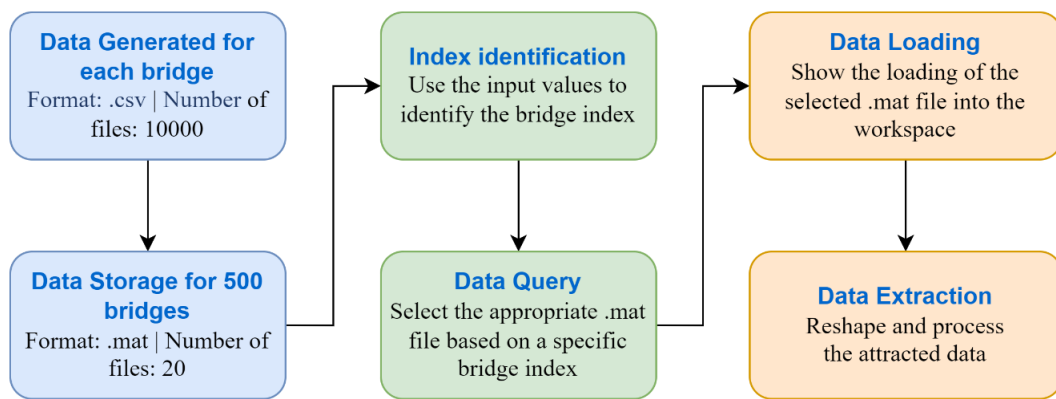
313 where D represents the original data matrix, k is the unique identifier for a specific bridge, g is the group
314 to which the bridge belongs. The reshape function then reorganises this row into a new matrix with
315 dimensions 410 by 8.

316 The factor variable scales the raw acceleration data based on the bridge's characteristics and comparison
 317 to standard parameters (E_0 , I_0).

318
$$f = \frac{D_k(3,2) \times D_k(3,3)}{E_0 \times I_0}$$

319 D_k is the matrix derived from the original data set, specific to a certain bridge, where $D_k(i, j)$ correspond
 320 to specific properties of interest (e.g., material properties, geometric dimensions, etc.). E_0 and I_0 are
 321 constants representing the standard values of elasticity and moment of inertia, respectively, against
 322 which the bridge data is being normalised.

323 The workflow depicted in Figure 8 illustrates a sophisticated approach for developing surrogate models
 324 to estimate the maximum displacement and acceleration of prestressed concrete railway bridges.



325

326

Figure 8. Surrogate Model Development and Data Manipulation Workflow

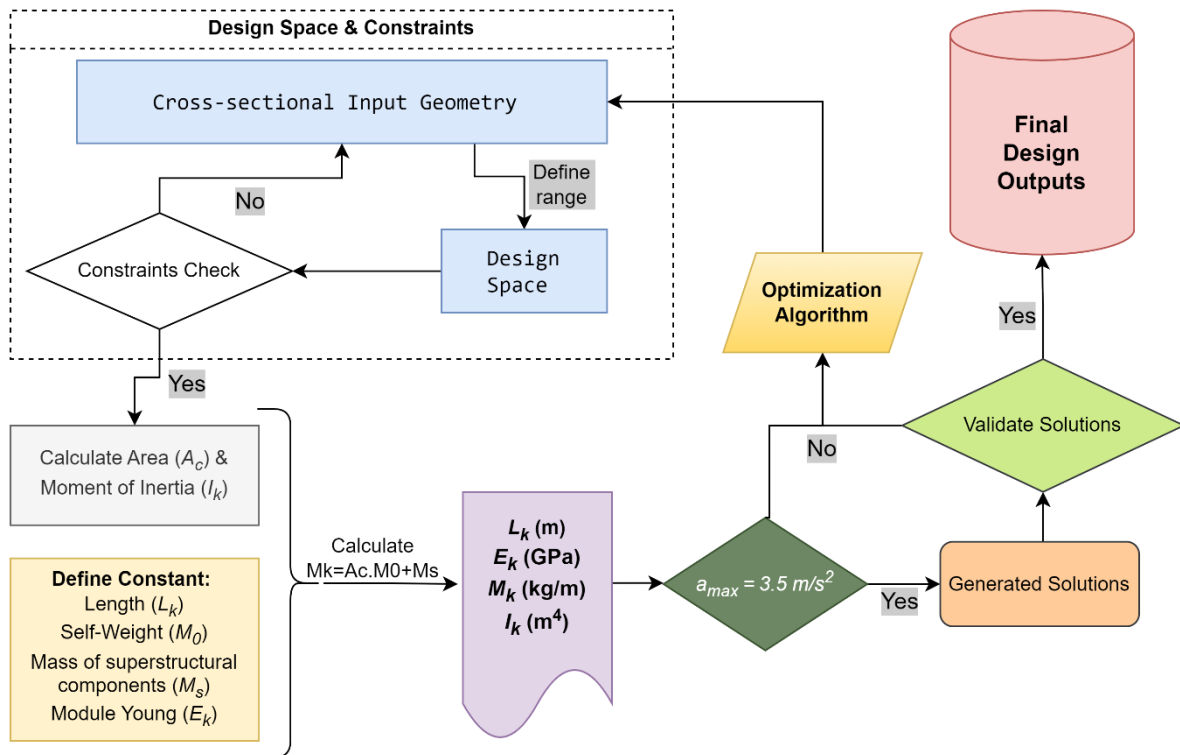
327 The surrogate model employs semantic search with nearest neighbor analysis to predict dynamic
 328 responses for new bridge configurations. When a query with new bridge parameters is received, the
 329 model first normalizes these parameters using the same scaling factors applied to the training data. It
 330 then computes a semantic similarity metric based on both geometric properties (span length, cross-
 331 sectional dimensions) and physical parameters (mass, stiffness, natural frequency). The K nearest
 332 neighbors ($K=5$) from our database are identified using this metric. Rather than selecting a single closest
 333 match, the model performs weighted interpolation between these neighbors, with weights inversely
 334 proportional to their semantic distance from the query point. This interpolation considers the physical
 335 constraints of bridge dynamics to ensure predictions remain within realistic bounds. For example, if a
 336 bridge's parameters fall between two known cases with maximum accelerations of 2.1 m/s^2 and 2.4 m/s^2
 337 respectively, the model will predict an intermediate value based on the relative similarity to each case
 338 while respecting the underlying physical relationships. The model's accuracy is evidenced by its high R^2
 339 values (0.999) when validated against FEM results for real bridge cases.

340 2.5 Generative Design Process for Cross-Sectional Optimisation

341 The research yielded a proposed generative design approach as a significant outcome. It skilfully
 342 combines automated data processing with human's expertise evaluation. The generative design initiative

343 aims to comply with the vertical acceleration criterion set by the Eurocode for ballasted track bridges,
 344 which restricts acceleration to 3.5 m/s^2 . This criterion is established to prevent ballast destabilization,
 345 which can compromise the track's structural integrity and stability at high speeds. The objective of the
 346 method is to ensure that the vertical acceleration remains below this limit by minimising the bridge's
 347 weight and ensuring the design's reliability through human intervention based on expertise.

348 Figure 9 outlines the proposed generative design framework for cross-sectional optimisation, combining
 349 computational efficiency with professional insights.



350

351

Figure 9. Flowchart of the proposed Generative Design Process for Cross-Sectional Optimisation

352 **Definition of Input Variables with Constraints:** The procedure begins by defining an initial design
 353 space that includes the bridge's dimensions, elastic modulus (E), precise cross-section geometry, and
 354 mass per unit length. Every input variable is allocated a value that allows for generating a wide range of
 355 output combinations while staying within the limits of realistic practicality. For instance, the web
 356 thickness must be at least 30 cm and should not exceed 60 cm.

357 **Initialisation of NSGA-II Algorithm:** The optimisation process commences with the initialisation of
 358 the NSGA-II algorithm [50]. This multi-objective optimisation algorithm begins with a randomly
 359 generated initial population of potential cross-sectional designs, each representing a possible solution
 360 within the defined design space.

361 **Optimisation Loop:** Central to the generative design process is the iterative optimisation loop, wherein
 362 each generation of design solutions is rigorously evaluated and evolved. Within this loop, the following
 363 steps are iterated:

- 364 • **Calculate Vertical Acceleration:** The vertical acceleration for each design is computed using the

- 365 surrogate model.
- 366 • Apply Genetic Operators: Genetic operators—selection, crossover, and mutation—are applied
 - 367 to the population of designs to explore the design space and generate new design variants.
 - 368 • Fitness Evaluation: The fitness of each design is evaluated based on its vertical acceleration and
 - 369 mass efficiency. Designs that yield lower accelerations and reduced mass are favoured, aligning
 - 370 with performance and cost-effectiveness optimisation objectives.
 - 371 • Convergence Check: The process checks for convergence, determining whether the design
 - 372 solutions have reached a state of optimality as defined by the algorithm's stopping criteria.

373 **Optimal Design Selection:** The possible alternatives that meet the maximum acceleration limitation
374 (e.g., 3.5 m/s²) will be chosen from the generated solutions for the following phase, which requires
375 human experience.

376 **Design Analysis and Selection for Validation:** The final phase involves a detailed analysis of the
377 selected designs from the previous step. Each solution must be analysed based on project requirements,
378 considering cost, structural performance, site execution, and environmental impact. The designs that
379 show the most potential and meet the specific criteria of the project are chosen for additional analysis
380 and empirical validation.

381 **2.6 Limitations and Assumptions of the Generative Design Process**

382 While the proposed generative design framework aims to optimize the cross-sectional geometry of
383 prestressed concrete railway bridges, it is important to acknowledge certain limitations and assumptions:

- 384 1. Shear deformability: The current model does not account for shear deformability effects, which
385 can be significant in short-span bridges or bridges with low height-to-span ratios.
- 386 2. Transverse flexibility: The optimization process focuses primarily on longitudinal behavior and
387 does not explicitly consider transverse flexibility, which may affect the distribution of stresses
388 in wide bridge decks.
- 389 3. Torsional effects: While torsional resistance capacity is mentioned as a key effect, the current
390 optimization framework does not explicitly model or optimize for torsional behavior.

391 While this study demonstrates the framework's application through a box girder bridge case study, the
392 surrogate model's underlying parameters (mass, stiffness, frequency) are generic and could be applied
393 to optimize various bridge deck types appropriate for different span ranges. Future work could extend
394 this framework to include cross-section type selection as part of the optimization process.

395 An additional limitation of the current methodology relates to the load application model. Our
396 calculations implement moving concentrated loads without considering their distribution through the
397 ballast and sleepers. In reality, train axle loads applied at rail level are distributed through the track
398 structure before reaching the bridge deck, an effect that is particularly significant for short-span bridges.
399 While more refined approaches, such as the triangular distribution (25%-50%-25%) proposed by ERRI
400 Report D 214/RP 9 [43], can provide more realistic results, our simplified approach yields conservative
401 predictions of dynamic responses. This conservatism provides an additional safety margin in our design
402 optimizations, though it may lead to slightly over-designed structures, particularly for shorter spans.
403 Future refinements of the methodology could incorporate these load distribution effects to achieve more

404 economical designs while maintaining safety requirements.

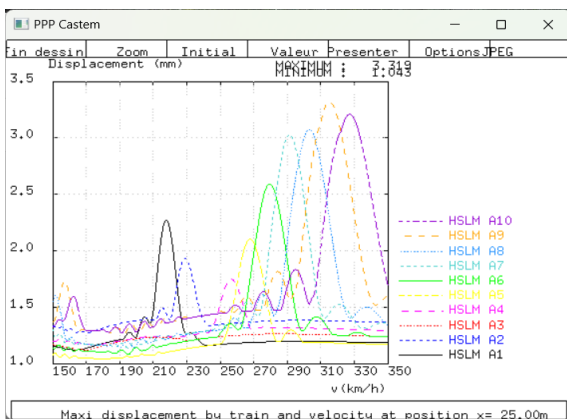
405 3 Surrogate model validation and implementation

406 3.1 Validation of the Surrogate Model

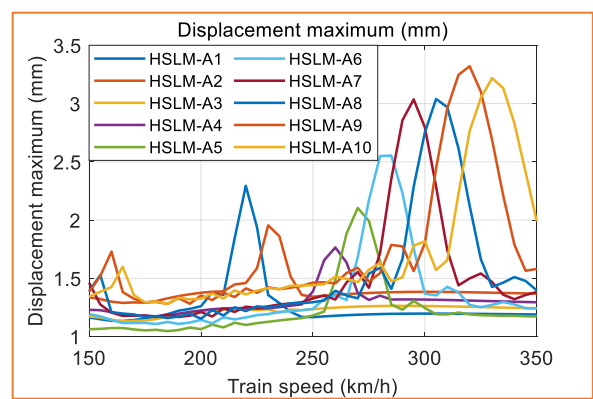
407 The surrogate model's validation is an essential step to verify its accuracy and reliability in simulating
408 the dynamic behaviour of railway bridges. This section details a methodical comparison between the
409 surrogate model's predictions and FEM results for two actual bridges: Antoing Bridge and one from the
410 High Speed 2 (HS2) railway line. The focus is on two critical dynamic response parameters: maximum
411 displacement and acceleration, evaluated over a spectrum of train speeds using HSLM-A train models.

412 3.1.1 Antoing Bridge

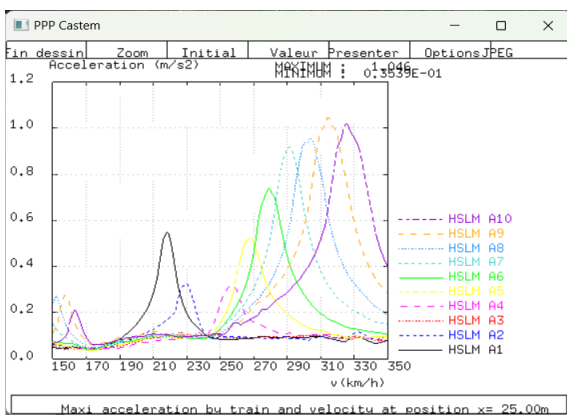
413 Dynamic analyses were performed on Antoing Bridge to assess the developed surrogate model's
414 predictions against FEM-derived maximum displacement and acceleration responses. Figure 10 presents
415 a detailed comparison of the maximum displacement and acceleration experienced by the Antoing
416 Bridge as a function of train speed, analysed using 10 different HSLM-A train models. The figure
417 comprises two graphs, one depicting displacement and the other acceleration, with each graph displaying
418 a series of peaks corresponding to the different train models.



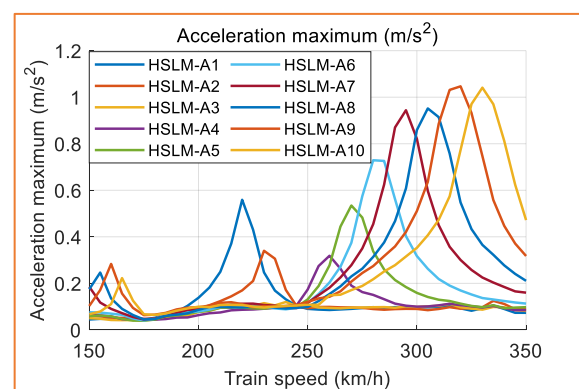
(a) Displacement results by KD-Railway



(b) Displacement results by surrogate model



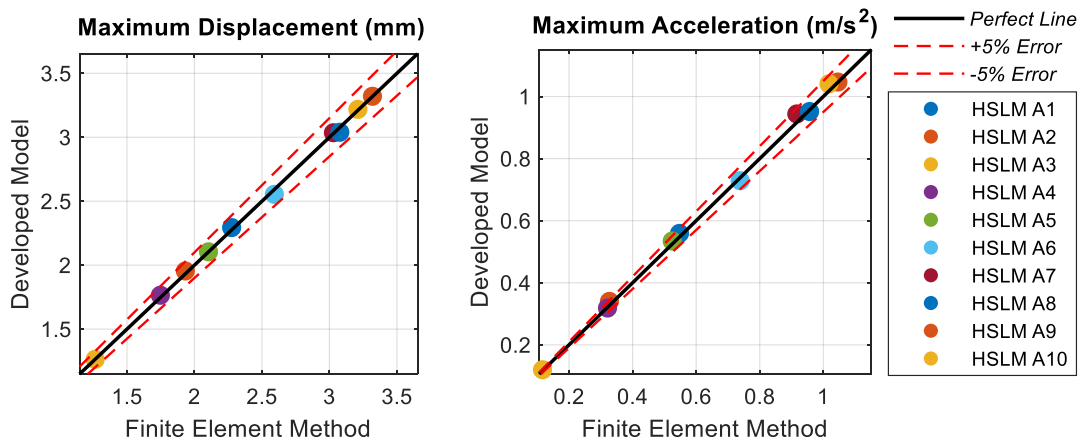
(c) Acceleration results by KD-Railway



(d) Acceleration results by surrogate model

419 **Figure 10. Antoing Bridge - Comparison of Maximum Displacement and Maximum Acceleration using KD-**
 420 **Railway-based Cast3M and developed model**

421 Figure 11 offers a quantitative validation using a scatter plot of the maximum displacement and
 422 acceleration as predicted by the surrogate model against the FEM results for various HSLM-A train
 423 models. The left graph illustrates the comparison for maximum displacement (in mm), while the right
 424 graph does the same for maximum acceleration (in m/s^2). Both graphs include a "Perfect Line" which
 425 represents an ideal 1:1 correlation between the surrogate model and FEM results. Most data points are
 426 closely aligned along the Perfect Line, falling within a narrow band that represents a $\pm 5\%$ error margin
 427 for both maximum displacement and acceleration responses.

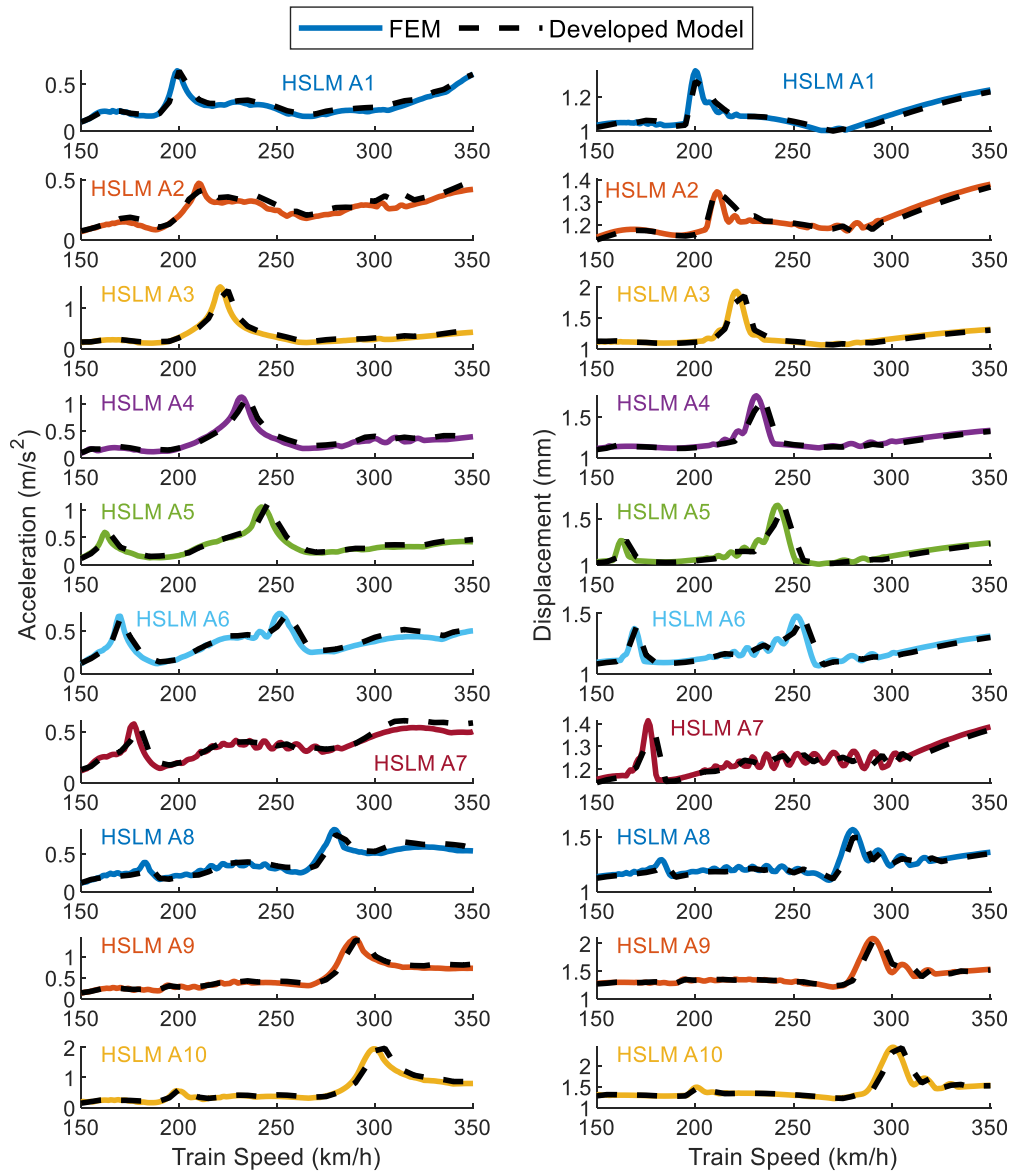


428
 429 **Figure 11. Comparison of Maximum Displacement and Maximum Acceleration at the point at the middle span of**
 430 **the Antoing Bridge subjected to 10 conventional train models using FEM and developed model**

431 **3.1.2 HS2's bridge**

432 The validation process extended to the HS2 Bridge. Similarly, comparative dynamic analyses were
 433 conducted using both the KD-Railway-based Cast3M software and the developed surrogate model.
 434 Figure 12 presents plots that compare a range of train speeds for different HSLM-A models, illustrating
 435 the surrogate model's fidelity in reproducing the dynamic responses. These responses are consistent with
 436 those from the FEM, even across varying train speeds and models, indicating the model's robustness.

437 Figure 13 directly compares the maximum displacement and acceleration of each bridge subjected to 10
 438 train models. The data points, when plotted against the Perfect Line, primarily lie within the $\pm 5\%$ error
 439 bars, indicating a strong connection between the surrogate model and FEM results. The uniformity
 440 observed in several train models highlights the surrogate model's capacity to accurately represent a
 441 diverse range of dynamic reactions.

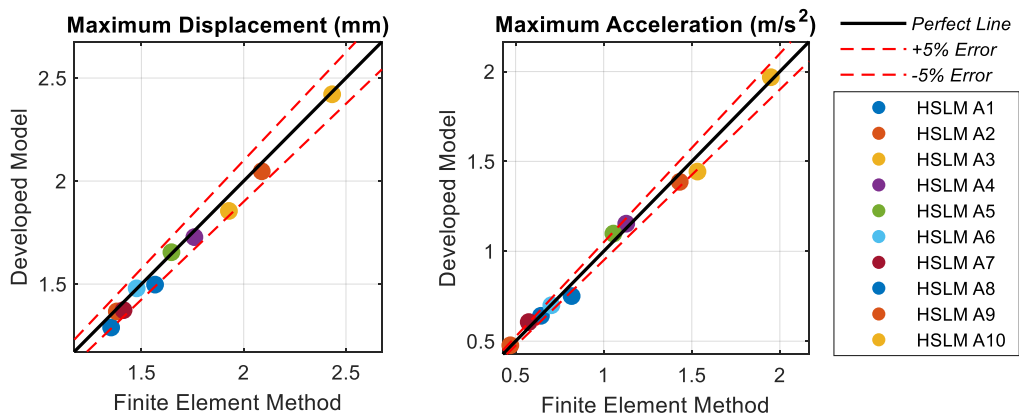


442

443

444

Figure 12. HS2 bridge - Comparison of Maximum Displacement and Maximum Acceleration using KD-Railway-based Cast3M and developed model



445

446

447

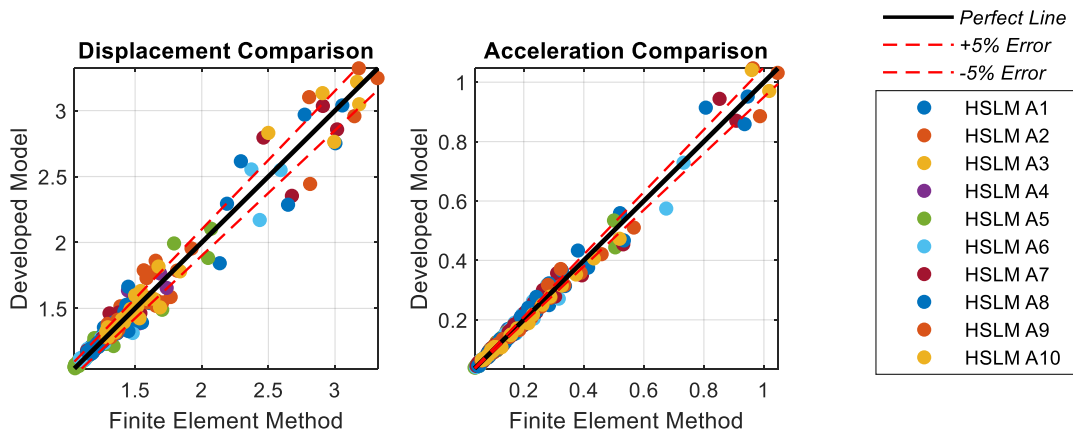
448

Figure 13. Comparison of Maximum Displacement and Maximum Acceleration at the point at the middle span of the HS2's Bridge subjected to 10 conventional train models using FEM and developed model

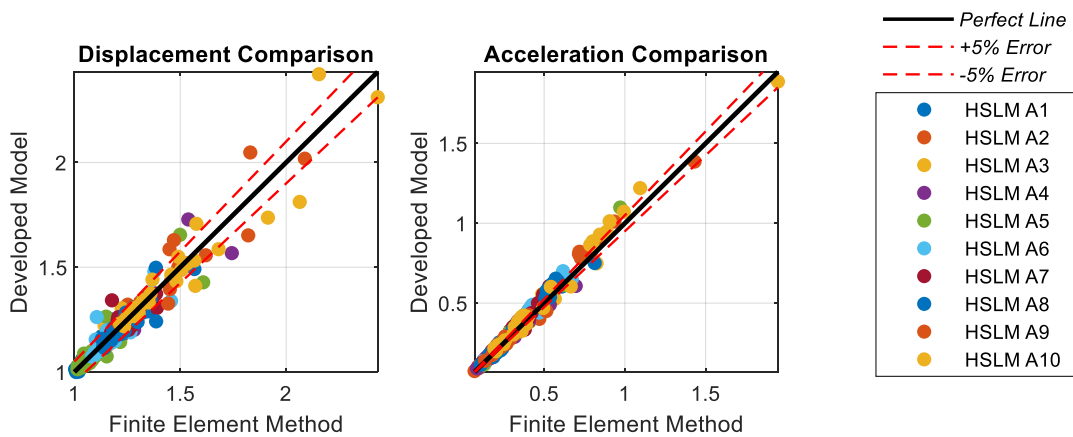
449 **3.2 Enhanced Validation and Future Development Directions**

450 In the validation phase, only the singular maximum displacement or acceleration values were compared
 451 for each bridge subjected to train models moving within the speed range of [150, 350] km/h. However,
 452 to derive a more detailed dynamic profile, this section will adopt a more comprehensive validation
 453 approach by comparing the spectrums results. Specifically, 41 discrete data points representing the
 454 maximum displacement and acceleration corresponding to each incremental speed within the speed
 455 range, with a step of 5 km/h, will be used. In total, 410 values of displacement and 410 values of
 456 acceleration will be compared for each bridge.

457 The comparison between the values predicted by the surrogate model and those obtained via FEM is
 458 illustrated in Figure 14. Notably, the surrogate model exhibited a commendable predictive accuracy for
 459 acceleration, with an error margin under 5%.



460
 461 (a) Antaoing Bridge



462
 463 (b) HS2's bridge

464 **Figure 14. Comparison of envelop displacements and acceleration of two bridges subjected to 10 conventional train**
 465 **models using FEM and developed model**

466 The performance metrics of the predictions are summarised in Table 2, with the surrogate model
 467 achieving an R^2 of 0.99 and RMSE below 0.045 for the maximum acceleration and displacement values.
 468 When evaluating the complete set of data points for the envelope curves (410 values), the predictive

469 performance remains strong, with R^2 of 0.96 and 0.93 for displacement predictions for the Antoing and
 470 HS2 bridges, respectively. Acceleration predictions are notably accurate, with R^2 of 0.99 and 0.97 for
 471 the Antoing and HS2 bridges, respectively.

472 **Table 2. Prediction performance metrics on envelop and maximum acceleration and displacements for Antoing and**
 473 **HS2 bridges**

		R-squared	RMSE	MAE
Antoing Bridge	Envelop Displacement	0.964	0.02	0.011
	Maximum Displacement	0.999	0.02	0.015
	Envelop Acceleration	0.987	0.075	0.037
	Maximum Acceleration	0.998	0.013	0.011
HS2 Bridge	Envelop Displacement	0.926	0.045	0.026
	Maximum Displacement	0.984	0.043	0.035
	Envelop Acceleration	0.971	0.036	0.028
	Maximum Acceleration	0.991	0.043	0.033

474
 475 Building on the current findings, there are several promising directions for advancing the research and
 476 develop other surrogate models. First, increasing the detail in the dataset could refine the model's
 477 accuracy. If the speed interval is narrowed from 5 km/h to 2 km/h or even 1 km/h, more data could be
 478 created. This data will provide more insights of how bridges behave under different speeds. For instance,
 479 with a 2 km/h speed interval, the model would generate 101 discrete outputs for vertical displacements
 480 for a single train model, considering each bridge within the speed range from 150 to 350 km/h.

481 Furthermore, expanding the range of speeds could also improve its usefulness. If the upper speed limit
 482 is increased up to 450 km/h, the model would be better suited for future high-speed rail developments.
 483 The same methodology can be applied to a wider variety of bridge designs, such as continuous, short-
 484 span, and skew bridges.

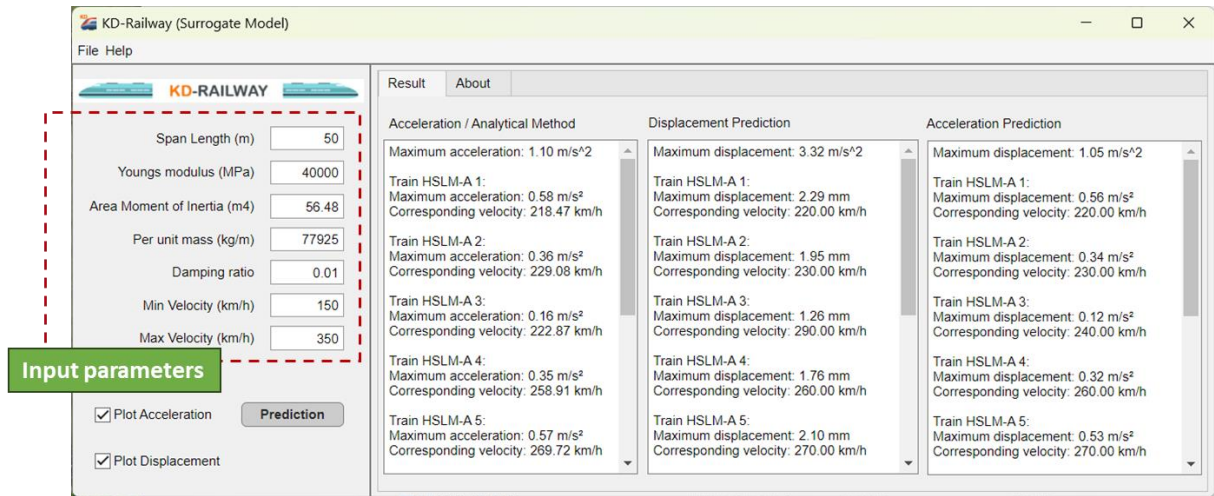
485 Future research could also enhance the current methodology by incorporating more sophisticated
 486 modeling approaches. Particularly, the inclusion of detailed train-track-bridge interaction models and
 487 improved representation of rail irregularity effects could enhance acceleration prediction accuracy.
 488 While the current HSLM-A model with the $\phi''/2$ coefficient provides a practical approach aligned with
 489 Eurocode standards, more comprehensive modeling of track irregularities could further improve the
 490 surrogate model's predictive capabilities, especially for acceleration responses. This enhancement would
 491 be particularly valuable for cases where detailed acceleration analysis is critical for design decisions.

492 Finally, a wide range of damping ratios could be considered rather than a fixed value since different
 493 bridges react differently. A more flexible approach to damping could give us more precise results,
 494 making the model even more reliable for predicting bridge behaviour.

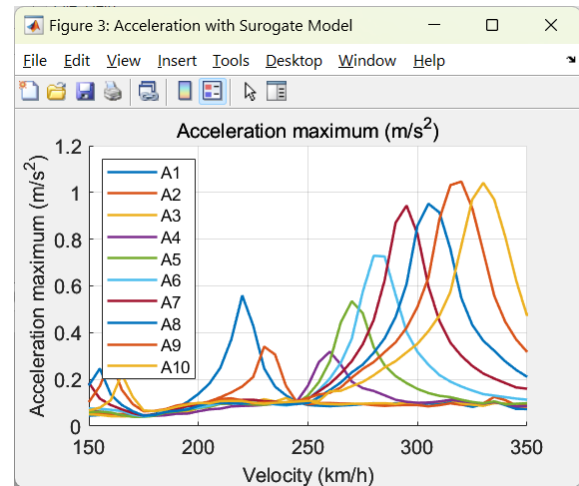
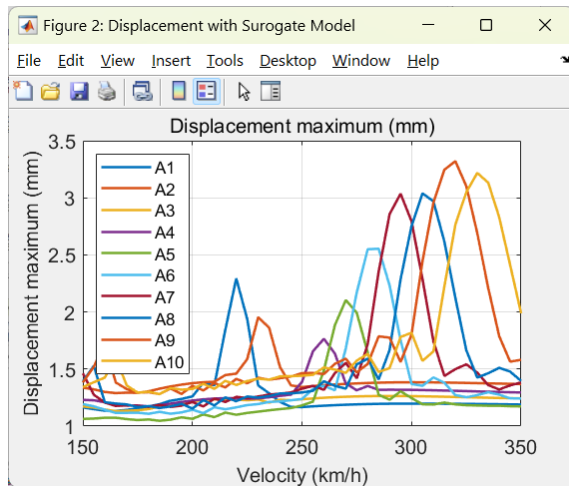
495 **3.3 Practical Implementation: Fast KD-Railway for Rapid Dynamic Analysis**

496 The surrogate model's performance across different HSLM-A train models and speeds indicates its
 497 potential as a practical tool in engineering analysis and design of railway bridges, providing a
 498 computationally efficient alternative to traditional FEM without compromising accuracy. In the context

499 of rapid dynamic analysis, the surrogate model has been integrated into the KD-Railway software
 500 recently developed by authors [41], serving as an autonomous module that obviates the necessity for the
 501 Cast3M open-source code. This integration furnishes users with an alternative for independent operation.



502



503

504 **Figure 15. Fast KD-Railway User Interface and Dynamic Results of Antong Bridge Using Developed Fast KD-**
 505 **Railway Tool**

506 The interface of the Fast KD-Railway tool is presented in Figure 15. The surrogate model's predictive
 507 functionality, specifically for maximum vertical acceleration and maximum vertical displacement, has
 508 been integrated into the updated version, which is composed of an area for defining the input parameters
 509 and an area for result display:

- 510 • **Input Parameter Definition Area:** This interface segment allows input of the bridge's parameters,
 511 setting the stage for the subsequent analysis.
- 512 • **Selection Area:** Users can switch between graphical and textual result formats, offering
 513 flexibility in result presentation.
- 514 • **Text-based Result Display Area:** This section presents a tabulated view of dynamic response
 515 metrics, listing the maximum accelerations experienced by the bridge under HSLM-A train
 516 traversals, along with the corresponding speeds, all formatted for ease of data extraction and
 517 report integration.

- 518 • Image-based Result Display Area: By activating the "Plot Results" function, users can
519 graphically represent the results, facilitating a more intuitive analysis and interpretation of the
520 data.

521 The Fast KD-Railway software represents a significant advancement in the field of dynamic analysis of
522 bridges by offering a marked reduction in model development time compared to FEM simulation tools
523 such as Midas, Abaqus, or Robot Structural Analysis. Traditional FEM software necessitates a full
524 working day to construct a model capable of dynamic analysis. Moreover, these conventional methods
525 require extensive manual data interpretation whenever input parameters are varied, contributing to the
526 protracted timeframes for model adjustment and analysis [41].

527 A comparative analysis of computational efficiency reveals significant improvements in calculation time
528 across different methods for dynamic analysis of railway bridges. This comparison is particularly
529 relevant when considering the comprehensive evaluation required by Eurocode standards, which
530 necessitates 1510 simulations across 10 HSLM-A train models and speed ranges from 100 to 400 km/h.
531 Traditional Finite Element Method (FEM) analysis, using software like ANSYS, requires approximately
532 22 minutes for a single bridge analysis. The KD Railway software, developed as an intermediate
533 improvement, substantially reduces this simulation time to about 20 seconds per analysis. However, the
534 proposed surrogate model further diminishes the calculation time to approximately 0.1-0.5 seconds per
535 analysis. This represents a remarkable reduction in computation time compared to traditional FEM
536 methods, from 22 minutes to as low as 0.1 seconds. Such near-instantaneous computation enables
537 extensive simulations and iterative design processes that were previously impractical due to time
538 constraints. Crucially, this significant gain in efficiency does not compromise accuracy. The surrogate
539 model's precision has been rigorously verified against real-world structures, including the Antoining
540 bridge (please see the section 4), demonstrating its reliability for practical applications in railway bridge
541 design and analysis.

542 **4 Case study: Cross-Sectional Generative Design**

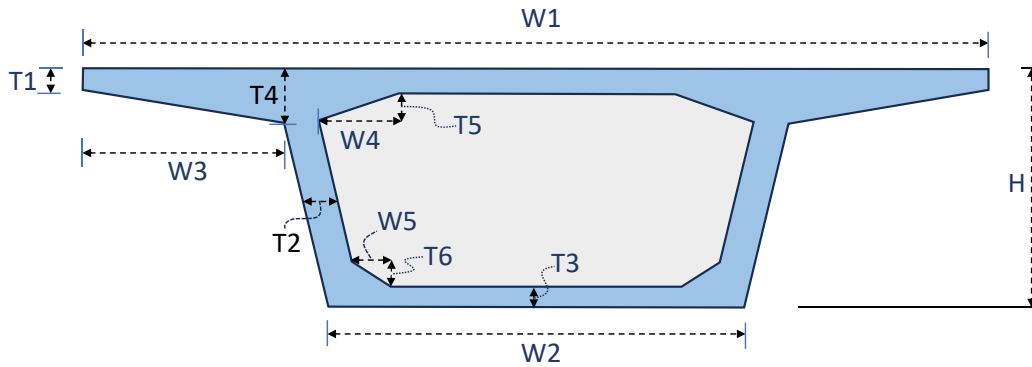
543 **4.1 Problem description**

544 In this section, the generative design framework described in Section 2.5 will be used to refine design
545 parameters for the cross-sectional geometry of a standard simply-supported box girder bridge deck
546 (Figure 16), ensuring efficient and effective solutions. The design process considers two primary
547 conditions affecting the bridge deck's cross-section: constructive and resistant conditions. Constructive
548 conditions depend on the builder's experience, site conditions, and available resources, making them
549 challenging to assess due to their variability. Resistant conditions, on the other hand, are clearer and
550 include several key effects worth emphasising:

- 551 • Bending resistance capacity depends on the deck's height (H) and the areas of the top and bottom
552 slab.
- 553 • Transverse flexibility and shear deformability: induce a distortion of the box, leading to an

554 inequality in the distribution of longitudinal bending stresses.

- 555 • Torsion resistance capacity: depends on the thickness of the box walls ($T2$) and the area enclosed
- 556 by them.
- 557 • Resistance capacity of the upper slab against a direct load: significant transverse bending may
- 558 occur and this depends on the separation between webs and the length of the external cantilevers
- 559 ($W3$).



560

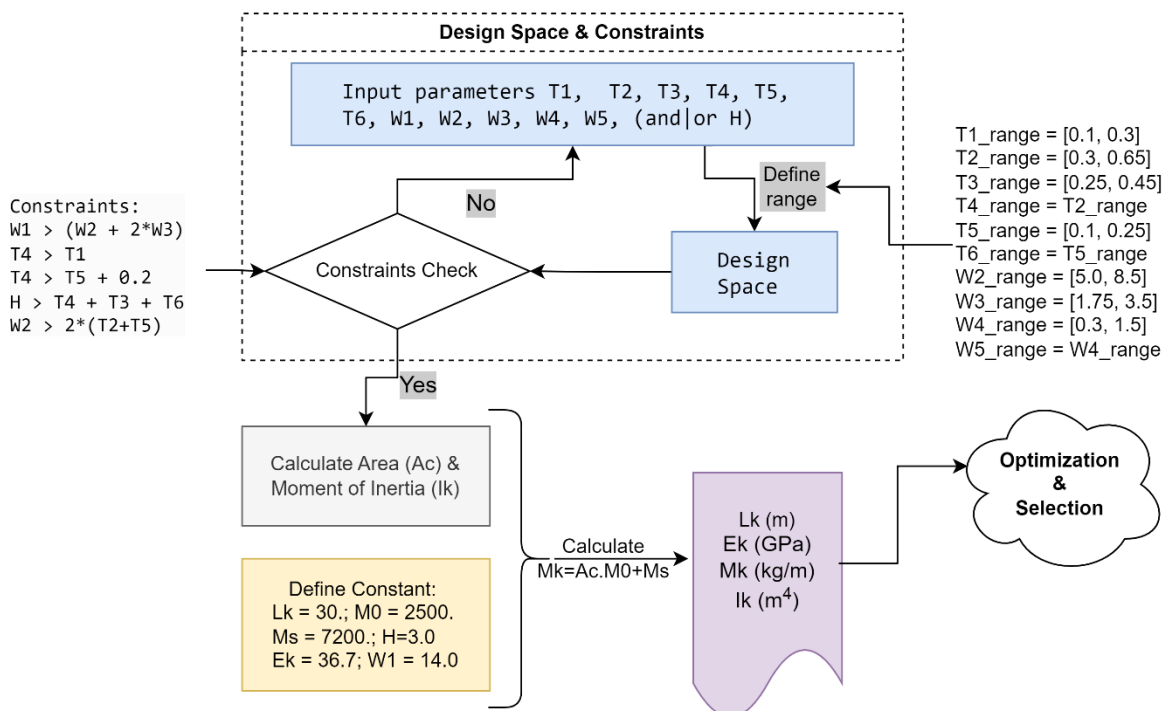
561 **Figure 16. Cross-Sectional Dimensions and Parameters of a Structural Bridge Component**

562 In this sense, eleven parameters of the bridge deck's cross-section can be selected as cross-sectional

563 design input that will be optimised through the generative design framework. The range and constraints

564 established for each parameter, the used constants, and the meticulously structured process are

565 represented in Figure 17.



566

567 **Figure 17. Cross Section, Design Space and Constraints of the Generative Design Process**

568 The process consists of:

- 569
- 570 • **Design Space & Constraints:** The design space is initially characterised by specifying input
571 parameters (T1-T6, W1-W5, H), which collectively define the box girder bridge's geometry.
572 Constraints are methodically imposed to maintain design feasibility and uphold structural
573 integrity. These include proportional relations among widths and thicknesses, such as W1 being
574 mandated to exceed the combined widths of W2 and W3, as well as stipulating T4 as superior to
575 T1. Additionally, a hierarchical constraint system ensures that T4 surpasses T5 by a quantifiable
576 margin and that other constraints regarding H and W2 are met.
 - 577 • **Constraints Verification:** Subsequently, a verification step assesses the alignment of design
578 parameters with the imposed constraints. Non-compliance prompts a re-evaluation, necessitating
579 an iterative refinement of parameters to achieve a viable design configuration.
 - 580 • **Structural Property Computation:** Upon satisfying all constraints, the process progresses to
581 calculate essential structural properties—area (A_c) and moment of inertia (I_k)—integral to
582 assessing the dynamic response of the bridge.
 - 583 • **Definition of Constants:** In parallel, constants are delineated: bridge span (L_k), concrete density
584 (M_0), ballast mass per unit length (M_s), Young's modulus of concrete (E_k), and bridge width (W1).
585 These constants serve as foundational benchmarks for the ensuing optimisation.
 - 586 • **Optimisation & Selection:** The optimisation phase iteratively manipulates the design
587 parameters within the established space to discern the optimal combination that satisfies
588 performance criteria, such as minimising weight in keeping structural efficiency.

588 In the optimisation framework detailed within this study, we intentionally omitted factors like transverse
589 flexibility, shear deformability, and torsion resistance capacity. This decision was rooted in the desire to
590 simplify the optimisation process, acknowledging that while these elements are crucial in specific
591 scenarios, their inclusion would unduly complicate the methodology without markedly advancing the
592 study's core aims. We concentrated on key parameters significantly influencing the bridge's longitudinal
593 dynamic behaviour during the initial design phase.

594 This generative design process is exemplified through two separate case studies, one involving ten
595 variables and the other eleven, demonstrating its versatility and adaptability across different scenarios.

596 • **Case Study with 10 Variables:**

597 In the first case study, ten variables (T1, T2, T3, T4, T5, T6, W2, W3, W4, W5) were used to
598 define the cross-sectional parameters of the box girder bridge. The bridge deck's height (H) is
599 considered constant and takes a value of 3.0 m ($L/10$). The absence of height parameter H in this
600 scenario means that the design optimisation was focused on the planar cross-sectional properties
601 without considering the variable height dimension.

602 • **Case Study with 11 Variables:** The second case study extended the design space to include an
603 additional variable, height (H), thereby expanding the generative design process to consider the
604 vertical dimension of the bridge's cross-section. This additional variable introduces a new layer
605 of complexity, as height influences the structural behaviour in terms of overall stiffness.

606 Incorporating H allows for a more holistic optimisation, factoring in the effects of the bridge's
607 elevation profile on dynamic performance and stability.

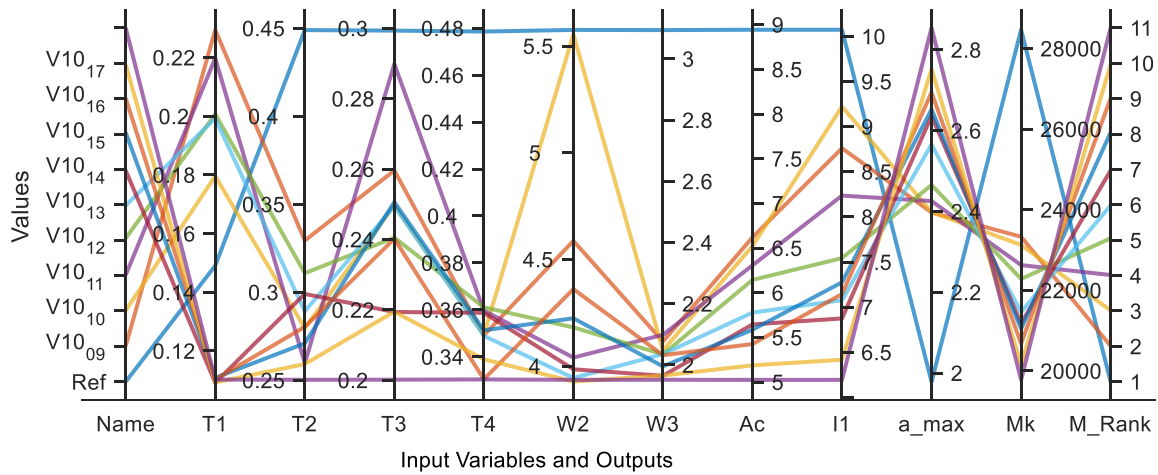
608 **4.2 Generative design solutions**

609 The generative design process yielded a variety of feasible solutions for each case study analysed. For
610 instance, Figure 18 presents eleven distinct solutions derived from a case study involving ten variables.
611 These solutions significantly diverge from the initial parameters specified in the reference section,
612 predominantly showcasing a reduction in nearly all variable values. Such a reduction aligns with the
613 optimisation objective of minimising the mass per unit length by reducing the cross-sectional area.
614 Importantly, all proposed solutions comply with the Eurocode requirement that the bridge deck's
615 maximum acceleration does not exceed 3.5 m/s^2 . Nonetheless, as a direct consequence of the reduced
616 mass per unit length, the acceleration in each solution is naturally higher compared to the reference
617 scenario.

618 To further elucidate the results, it is noteworthy that the thickness parameters (T1-T4) and width
619 parameters (W2-W3) consistently show lower values across all optimized solutions compared to the
620 reference design. This trend indicates that the generative design algorithm effectively explores the design
621 space to find more efficient cross-sectional geometries. The cross-sectional area (A_c) and moment of
622 inertia (I1) also decrease accordingly, reflecting the overall reduction in material usage.

623 Solution $V10_{17}$, for example, achieves the lowest mass per unit length (Mk) while still maintaining
624 structural integrity and meeting the acceleration criterion. This solution represents a significant
625 optimization, with a mass reduction of approximately 30% compared to the reference design. However,
626 it is crucial to note that this mass reduction comes with a trade-off, as the maximum acceleration (a_{max})
627 for this solution approaches the 3.5 m/s^2 limit.

628 Interestingly, the solutions exhibit a range of trade-offs between mass reduction and dynamic
629 performance. Solutions $V10_{09}$ to $V10_{16}$ offer varying degrees of mass reduction with corresponding
630 increases in maximum acceleration, allowing designers to choose the most appropriate balance for
631 specific project requirements. This spectrum of options demonstrates the power of the generative design
632 approach in providing multiple viable alternatives, each with its own set of advantages in terms of
633 material efficiency and dynamic behavior.



634

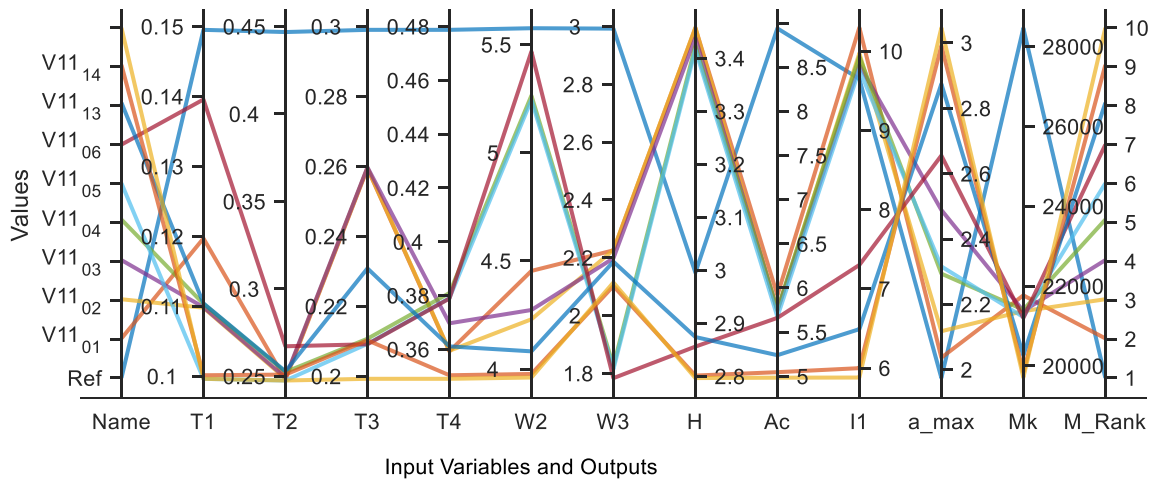
635

Figure 18. Feasible Solutions for the Generative Design Process with 10 Input Variables

636

In the second case study, as depicted in Figure 19, fifteen viable solutions were identified, with the deck height (H) being included as a design variable. Of these solutions, six feature a deck height exceeding the reference model, while the remaining nine solutions exhibit a reduced height, as detailed in Table 2. Consistent with findings from the case study involving ten variables, a reduction in the values of the other ten variables was also observed. This approach aligns with the objective of minimising mass per unit length, reflecting the optimisation trends observed in the preceding analysis.

641



642

643

Figure 19. Feasible Solutions for the Generative Design Process with 11 Input Variables

644

Upon closer examination of Figure 19, several notable patterns emerge. The inclusion of deck height (H) as a variable introduces an interesting dynamic to the optimization process. Solutions $V11_{01}$ to $V11_{05}$ showcase increased deck heights compared to the reference model, ranging from approximately 3.4m to 3.46m. This increase in height allows for a more significant reduction in other dimensional parameters, particularly the thickness variables ($T1$ - $T4$) and width variables ($W2$ - $W3$).

648

649

Interestingly, the solutions with increased deck height tend to achieve lower mass per unit length (M_k) while maintaining relatively low maximum accelerations (a_{max}). Solutions $V11_{06}$ to $V11_{15}$ demonstrate

650

651 that viable designs can also be achieved with reduced deck heights, some as low as 2.64m. These
 652 solutions generally show less dramatic reductions in other variables, indicating a trade-off between
 653 height reduction and the need to maintain adequate cross-sectional properties for structural integrity.

654 The moment of inertia (I_I) values vary significantly across the solutions, reflecting the complex interplay
 655 between deck height and other cross-sectional dimensions. Solutions with higher deck heights tend to
 656 maintain higher I_I values, contributing to their improved dynamic performance despite reduced mass.

657 **Table 3. Selected solutions for the Generative Design process with 10 and 11 variables**

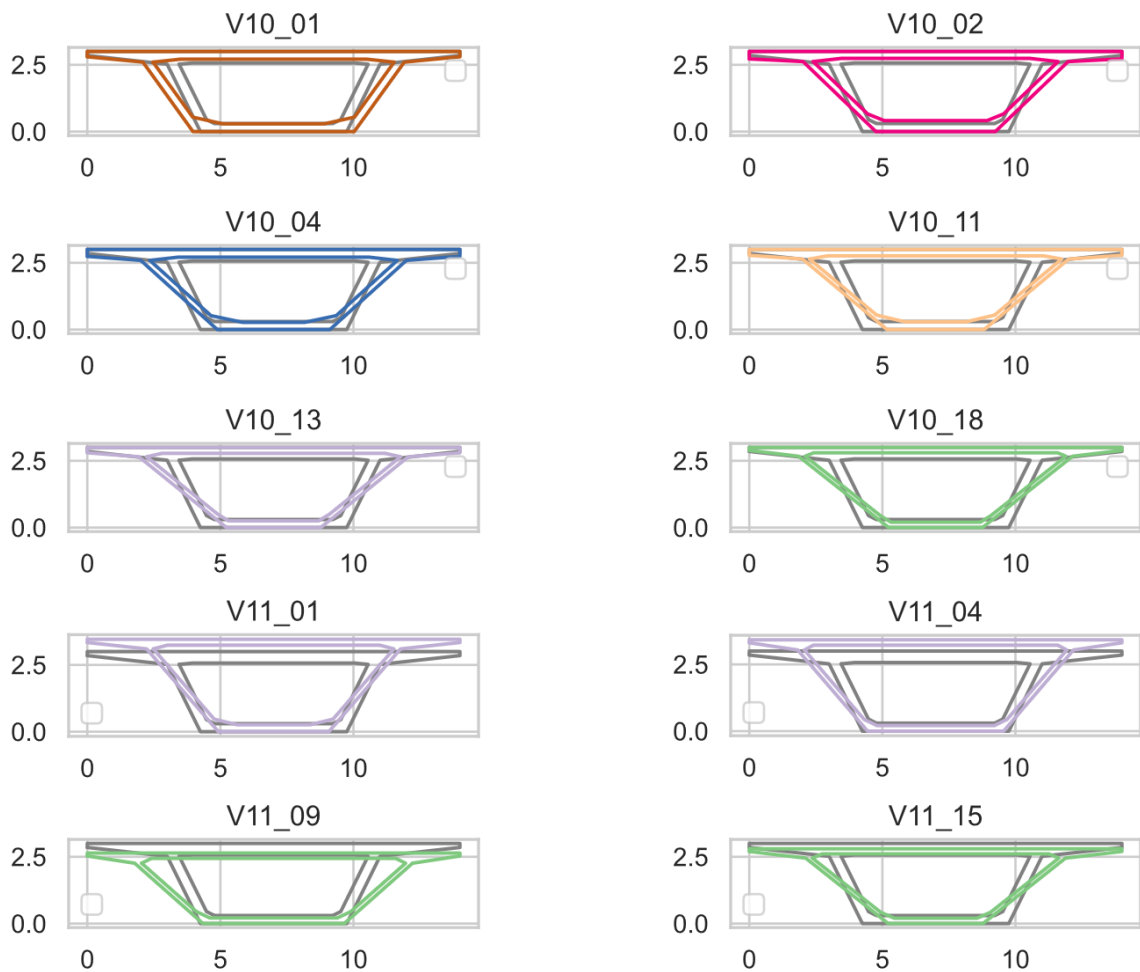
Name	T1	T2	T3	T4	T5	T6	W2	W3	W4	W5	H	Area	I _I	a_max	M _k
Reference	0.15	0.45	0.30	0.48	0.05	0.15	5.50	3.00	0.50	0.30	3	8.96	10.09	1.98	29566
Validated design for Optimisation process with 10 variables															
V10_01	0.20	0.38	0.29	0.40	0.11	0.25	6.06	2.09	1.03	1.13	3	8.08	11.11	2.09	27398
V10_02	0.28	0.38	0.41	0.38	0.12	0.25	4.49	2.02	1.06	0.62	3	7.93	10.63	2.12	27031
V10_03	0.17	0.33	0.36	0.42	0.11	0.25	4.26	1.97	1.23	0.67	3	7.76	10.00	2.20	26597
V10_04	0.26	0.34	0.27	0.41	0.12	0.25	4.23	2.01	1.06	1.19	3	7.44	9.18	2.22	25807
V10_05	0.26	0.33	0.26	0.40	0.12	0.25	4.19	1.98	0.84	1.19	3	7.19	9.01	2.26	25176
V10_06	0.14	0.32	0.37	0.35	0.12	0.25	4.65	1.95	0.96	1.35	3	7.04	10.08	2.30	24790
V10_07	0.17	0.34	0.26	0.40	0.12	0.25	4.09	1.89	0.66	0.50	3	6.93	8.40	2.37	24528
V10_08	0.17	0.35	0.39	0.34	0.13	0.25	4.28	2.01	0.85	0.41	3	6.75	9.41	2.40	24084
V10_09	0.23	0.33	0.26	0.35	0.12	0.25	4.31	1.98	0.59	1.15	3	6.63	8.77	2.45	23772
V10_10	0.18	0.28	0.25	0.35	0.11	0.25	5.47	1.95	0.71	0.98	3	6.53	9.23	2.45	23534
V10_11	0.22	0.26	0.29	0.36	0.12	0.25	3.65	2.00	0.68	1.06	3	6.31	8.24	2.48	22965
V10_12	0.20	0.31	0.24	0.36	0.13	0.25	3.81	1.93	0.80	0.61	3	6.13	7.53	2.52	22535
V10_13	0.20	0.29	0.25	0.35	0.13	0.25	3.54	1.94	0.58	0.41	3	5.79	7.09	2.64	21677
V10_14	0.11	0.30	0.22	0.36	0.13	0.24	3.59	1.87	0.39	0.48	3	5.66	6.89	2.72	21362
V10_15	0.11	0.27	0.25	0.35	0.13	0.24	3.86	1.89	0.51	0.47	3	5.57	7.26	2.73	21137
V10_16	0.11	0.28	0.24	0.33	0.13	0.24	4.03	1.93	0.33	0.39	3	5.42	7.14	2.78	20747
V10_17	0.11	0.26	0.22	0.34	0.13	0.24	3.52	1.87	0.33	0.41	3	5.20	6.43	2.85	20211
V10_18	0.11	0.25	0.20	0.33	0.13	0.24	3.52	1.85	0.32	0.39	3	5.02	6.19	2.96	19740
Validated design for Optimisation process with 11 variables															
V11_01	0.12	0.25	0.26	0.36	0.14	0.21	4.15	2.23	0.53	0.86	3.46	5.93	10.89	2.04	22019
V11_02	0.11	0.25	0.26	0.36	0.15	0.21	3.88	2.23	0.45	0.96	3.46	5.75	10.48	2.12	21566
V11_03	0.11	0.25	0.26	0.37	0.17	0.21	3.93	2.20	0.44	0.72	3.44	5.75	10.29	2.49	21563
V11_04	0.11	0.25	0.21	0.38	0.18	0.23	5.11	1.82	0.34	0.48	3.42	5.74	10.4	2.29	21550
V11_05	0.10	0.25	0.21	0.38	0.18	0.23	5.10	1.82	0.34	0.48	3.42	5.69	10.2	2.32	21415
V11_06	0.14	0.27	0.21	0.38	0.18	0.24	5.37	1.79	0.39	0.57	2.86	5.68	7.15	2.66	21395
V11_07	0.11	0.25	0.26	0.35	0.16	0.21	3.88	2.18	0.41	0.68	3.45	5.66	10.17	2.52	21360
V11_08	0.17	0.27	0.21	0.37	0.15	0.22	3.83	2.11	0.41	0.54	2.79	5.55	5.88	2.69	21085
V11_09	0.11	0.26	0.21	0.38	0.18	0.24	5.31	1.78	0.37	0.51	2.64	5.4	5.81	2.73	20712
V11_10	0.11	0.25	0.26	0.36	0.16	0.22	3.65	2.06	0.44	0.80	2.72	5.34	5.77	2.75	20555
V11_11	0.10	0.25	0.20	0.38	0.18	0.24	5.10	1.77	0.33	0.49	2.77	5.32	6.21	2.81	20506
V11_12	0.10	0.25	0.20	0.38	0.18	0.24	5.04	1.78	0.32	0.50	2.64	5.24	5.53	2.97	20309
V11_13	0.11	0.25	0.23	0.36	0.15	0.22	3.69	2.18	0.38	0.52	2.87	5.22	6.09	2.87	20256
V11_14	0.10	0.25	0.21	0.35	0.16	0.22	3.57	2.10	0.37	0.54	2.80	5.04	5.49	2.99	19810
V11_15	0.10	0.25	0.20	0.35	0.16	0.22	3.56	2.12	0.37	0.51	2.80	5.00	5.37	3.05	19710

659 **4.3 Discussion**

660 In this study, a wide range of design options by setting the parameters were selected—like the width
661 (W2) between 5 and 8.5 meters, and the thicknesses (T3 and T2) from 0.25 to 0.45 meters and 0.3 to
662 0.65 meters, respectively—beyond the usual limits seen in everyday bridge construction. This approach
663 allowed to thoroughly investigate various design possibilities, pushing the limits of what's commonly
664 done in bridge design.

665 However, expanding the range of these parameters means it might come up with designs that work well
666 in theory but might not be practical due to various reasons. As a result, expert review becomes crucial
667 during this stage.

668 Looking more closely at the case study (Figure 20), the bridge deck design is looked at from two
669 perspectives, e.g., constructivity and resistance. From a construction point of view, some designs suggest
670 very thin parts (about 0.25 meters thick), which does not comply with the construction constraints about
671 minimal thickness, e.g., 4-5 times thicker than the prestressing sheaths. On the other hand, from a
672 structural standpoint, designs with steeply angled or very thin parts present significant challenges. These
673 designs might not be strong enough to resist certain types of damage, like shear, which is a concern for
674 the overall durability of the bridge.



675 **Figure 20. Representative Geometry Comparison of Selected Solutions Relative to the Reference, Coloured by Mass**

676

per Unit Length

677 Furthermore, this study's primary emphasis on developing a surrogate model and introducing a
678 generative design process meant that a detailed examination of displacement limitations was not
679 included in our initial analysis. It's crucial to understand that the process of checking for displacement
680 limitations entails a thorough assessment of both static and dynamic loads affecting the bridge structure.
681 This checking ensures that the total displacement remains within the safety and serviceability thresholds
682 set by applicable standards and codes. While this evaluation is essential for the practical implementation
683 of bridge designs, it constitutes a separate phase of analysis that could be smoothly incorporated into our
684 research framework in subsequent developments. Integrating this step will guarantee that the innovative
685 designs produced through our surrogate model and generative design methodology not only demonstrate
686 structural creativity but also comply with all relevant engineering and safety standards.

687 **5 Conclusion**

688 This study introduces a significant step forward in optimising high-speed railway bridge design. It
689 highlights the crucial role of surrogate modelling, generative design, and optimisation within this
690 domain. The compilation of an extensive database, comprising over 4 million data points for dynamic
691 analysis, adheres to Eurocode standards and serves as a critical resource for investigating the dynamic
692 behaviours of over 10,000 high-speed, single-span prestressed concrete railway bridges subjected to
693 HSLM-A train loads.

694 The surrogate model, developed as the core of this research, featuring semantic search and advanced
695 decoding techniques, markedly improves the efficiency and accuracy of dynamic behaviour predictions.
696 This model has been rigorously validated across diverse bridge structures, demonstrating its broad
697 applicability and robustness. Antoining Bridge and the HS2 project's bridge, representing both traditional
698 and contemporary construction techniques, served as benchmarks for this validation. This facilitated a
699 meticulous comparison of empirical data with the model's predictions, affirming the model's precision
700 (evidenced by an R^2 value of 0.999) and its relevance to modern design practices. Through this dual
701 analysis, the study not only verifies the model's accuracy but also illustrates its versatility in addressing
702 various engineering challenges.

703 This study further introduces a novel methodology aimed at refining the cross-sectional geometry of
704 railway bridges. Employing this methodology in a targeted case study yielded multiple designs that
705 outperformed the original design in reducing mass per unit length. This outcome effectively
706 demonstrates the methodology's capability to enhance the efficiency of structural design, underscoring
707 its significant potential for optimising bridge construction practices.

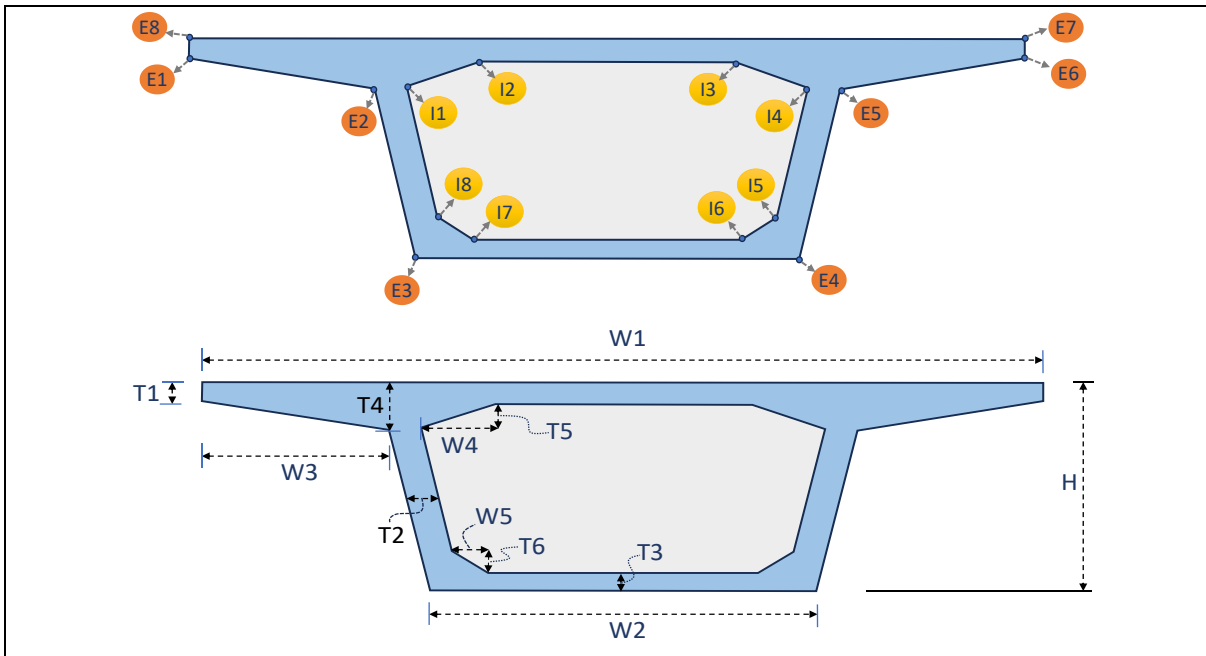
708 Looking ahead, this study proposes several avenues for further research. These include expanding the
709 velocity range within the database to enhance the model's applicability and accuracy and refining the
710 selection process for bridge models to streamline data generation. Additionally, leveraging the amassed
711 database to explore advanced machine-learning techniques could pave the way for innovative analytical

712 tools. Such tools could transform into accessible web applications, extending the impact of this research
 713 within the computational bridge engineering field and beyond. A complete process could be developed
 714 by incorporating comprehensive checks for displacement limitations to ensure that the designs represent
 715 structural innovation and meet all necessary standards for safety, serviceability, and compliance with
 716 engineering codes.

717 In essence, this research builds a bridge between advanced technology and structural optimisation,
 718 offering new perspectives for future studies in bridge design and analysis. Its contributions are poised to
 719 shape the future landscape of computational bridge engineering, emphasising efficiency, accuracy, and
 720 the integration of cutting-edge technology in the design and analysis of modern infrastructure.

721 Annexe

722 **Dimensions of the section and their corresponding coordinates in the calculation of moment of inertia.**

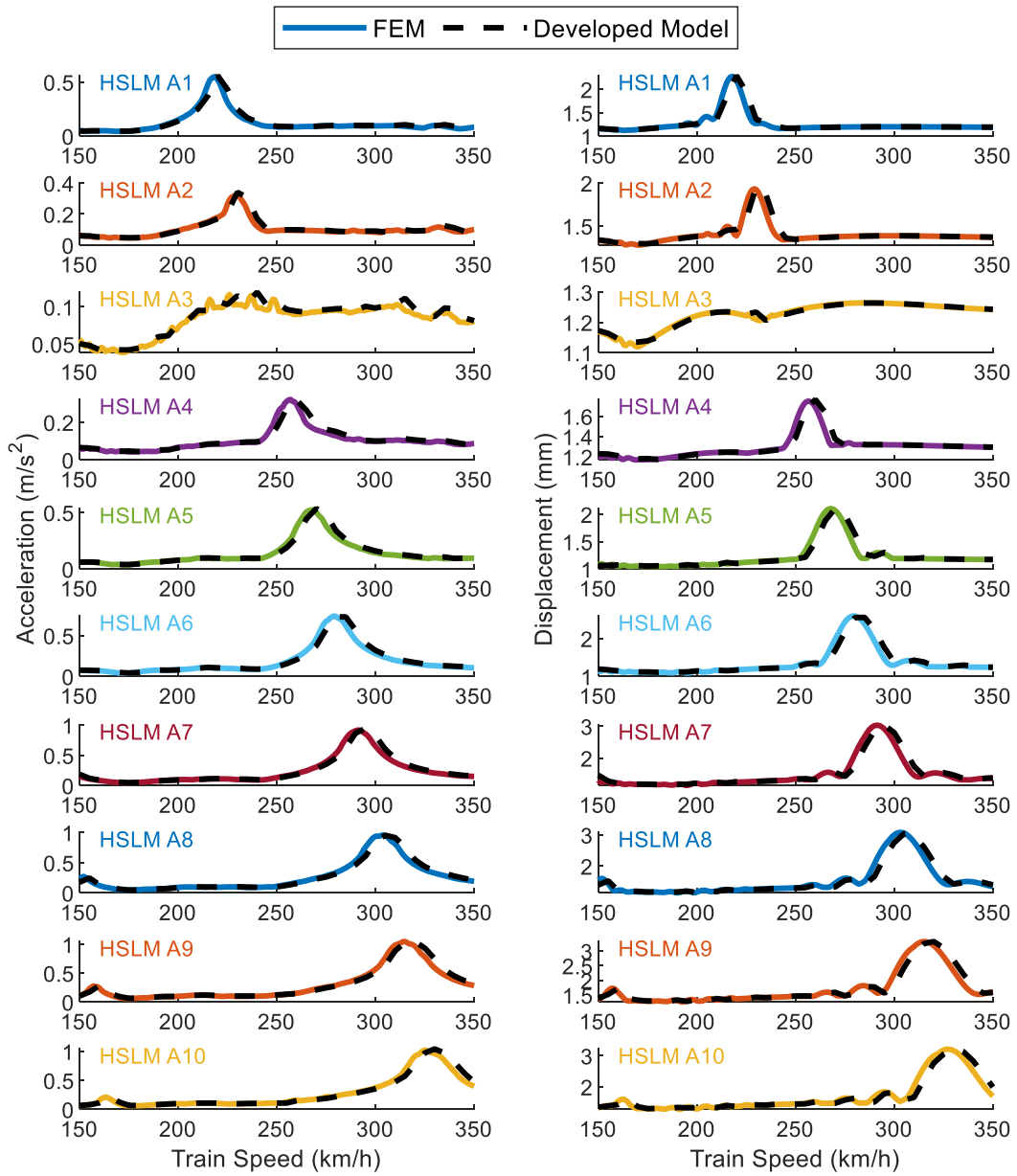


Points	X	Y	Points	X	Y
E1	0	H-T1	I1	W3+T2	H-T4
E2	W3	H-T4	I2	W3+T2+W4	H-T4+T5
E3	(W1-W2)/2	0	I3	W1-W3-T2-W4	H-T4+T5
E4	(W1+W2)/2	0	I4	W1+W3-T2	H-T4
E5	W1-W3	H-T4	I5	W1-[(W1-W2)/2-(T3+T6)*tan(alpha)+T2]	T3+T6
E6	W1	H-T1	I6	X(I5)-W5	T3
E7	W1	H	I7	W1-X(I5)+W5	T3
E8	0	H	I8	W1-X(I5)	T3+T6

723 where: $\alpha = \arctan([(W1-W2)/2-W3]/(H-T4))$

724

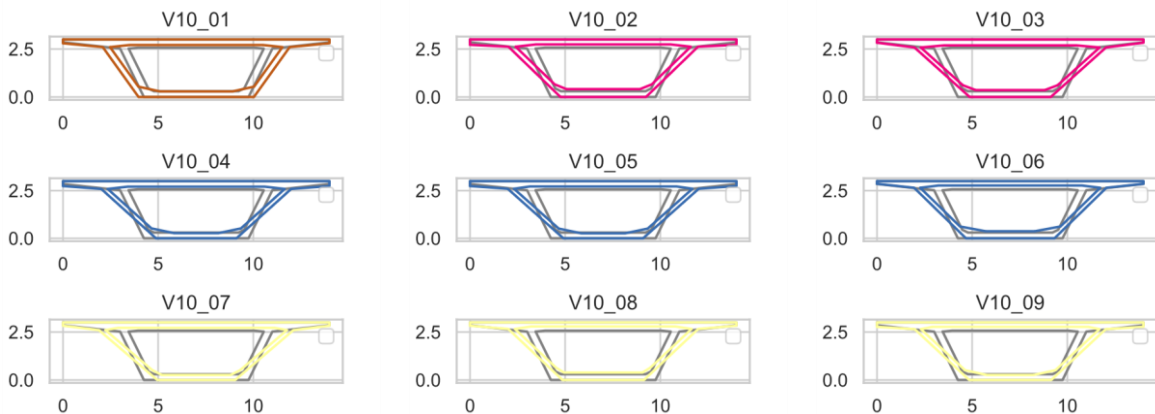
725 **Comparison of envelope curves for displacement and acceleration derived using the Finite**
 726 **Element Method (FEM) with the proposed model**

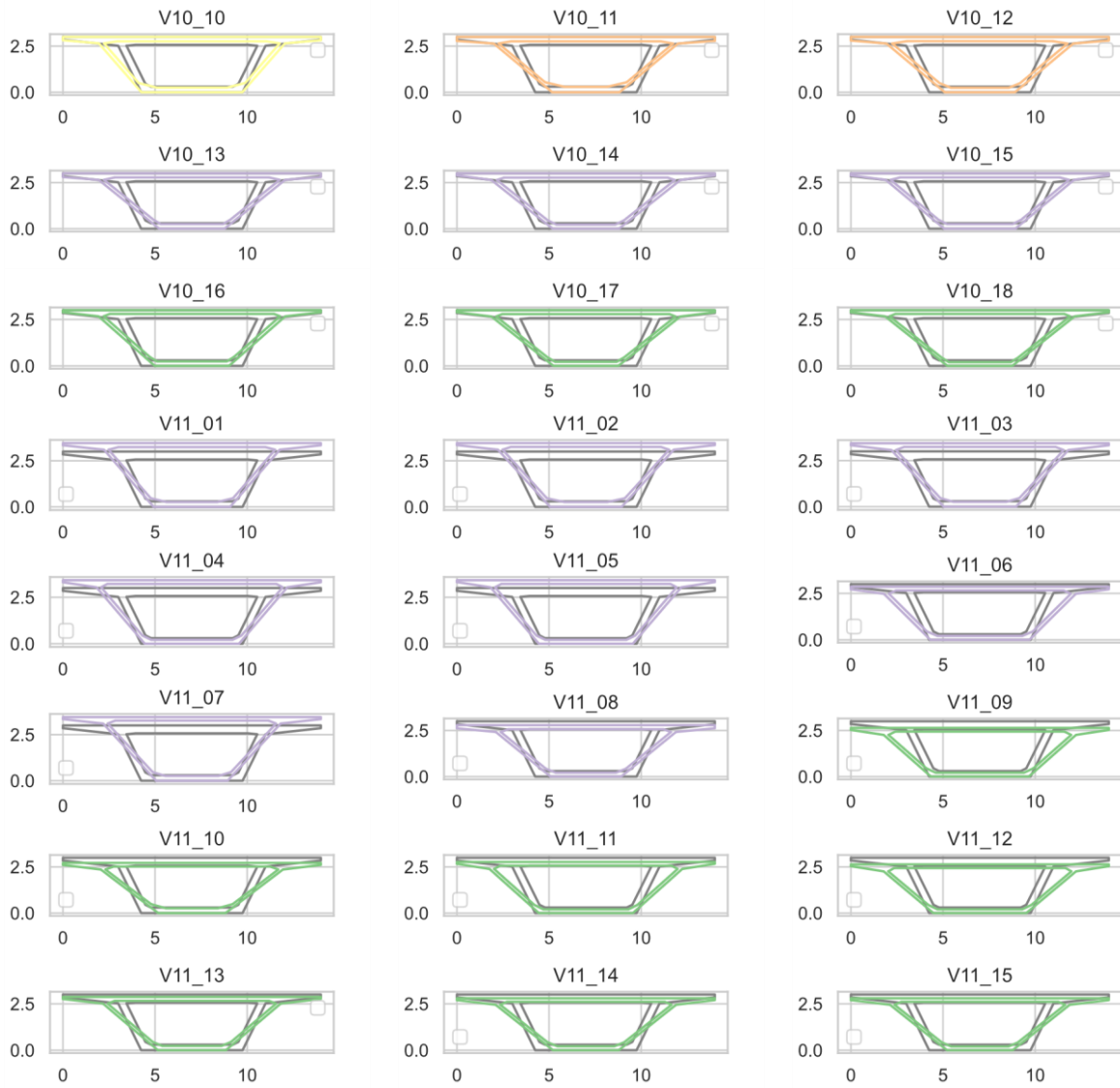


727

728

Figure 21. Antoing Bridge





729 **Figure 22. Geometry Comparison of Selected Solutions Relative to the Reference design, Coloured by Mass per Unit**
 730 **Length**

731

732

733 **References**

- 734 [1] H. Xia, N. Zhang, Dynamic analysis of railway bridge under high-speed trains, *Comput. Struct.* 83
735 (2005) 1891–1901. <https://doi.org/10.1016/j.compstruc.2005.02.014>.
- 736 [2] B. Yan, G.-L. Dai, N. Hu, Recent development of design and construction of short span high-speed
737 railway bridges in China, *Eng. Struct.* 100 (2015) 707–717.
738 <https://doi.org/10.1016/j.engstruct.2015.06.050>.
- 739 [3] X. He, T. Wu, Y. Zou, Y.F. Chen, H. Guo, Z. Yu, Recent developments of high-speed railway
740 bridges in China, *Struct. Infrastruct. Eng.* 13 (2017) 1584–1595.
741 <https://doi.org/10.1080/15732479.2017.1304429>.
- 742 [4] Y. Yang, J. Yau, LC. Hsu, Vibration of simple beams due to trains moving at high speeds, *Eng*
743 *Struct* 19 (1997) 936.
- 744 [5] M. Fawad, M. Salamak, Q. Chen, M. Uscilowski, K. Koris, M. Jasinski, P. Lazinski, D. Piotrowski,
745 Development of immersive bridge digital twin platform to facilitate bridge damage assessment and
746 asset model updates, *Comput. Ind.* 164 (2025) 104189.
747 <https://doi.org/10.1016/j.compind.2024.104189>.
- 748 [6] S. Ju, HT. Lin, Resonance characteristics of high-speed trains passing simply supported bridges, *J*
749 *Sound Vib* 267 (2003) 1127.
- 750 [7] J. Hansen, K. Hanna, M.K. Tadros, Simplified transverse post-tensioning construction and
751 maintenance of adjacent box girders, *PCI J.* 57 (2012) 64–79.
752 <https://doi.org/10.15554/pcij.03012012.64.79>.
- 753 [8] A. EI-Remaily, M.K. Tadros, T. Yamane, G. Krause, Transverse Design of Adjacent Precast
754 Prestressed Concrete Box Girder Bridges, *PCI J.* 41 (1996) 96–113.
755 <https://doi.org/10.15554/pcij.07011996.96.113>.
- 756 [9] L.-H. Tran, T.-M. Duong, B. Claudet, K. Le-Nguyen, A. Nordborg, F. Schmidt, Comparative
757 analysis of beam models for vertical rail vibrations under dynamic forces, *Eur. J. Mech. - ASolids*
758 110 (2025) 105497. <https://doi.org/10.1016/j.euromechsol.2024.105497>.
- 759 [10] J. Lall, S. Alampalli, E.F. DiCocco, Performance of Full-Depth Shear Keys in Adjacent Prestressed
760 Box Beam Bridges, *PCI J.* 43 (1998) 72–79. <https://doi.org/10.15554/pcij.03011998.72.79>.
- 761 [11] X. Zhang, W. Zhai, Z. Chen, Characteristic and mechanism of structural acoustic radiation for box
762 girder bridge in urban rail transit, *Sci Total Env.* 627 (2018) 1303.
- 763 [12] J. Su, X. Ma, B. Chen, K. Sennah, Full-scale bending test and parametric study on a 30-m span
764 prestressed ultra-high performance concrete box girder, *Adv. Struct. Eng.* 23 (2020) 1276–1289.
765 <https://doi.org/10.1177/1369433219894244>.
- 766 [13] A. Fam, H. Honickman, Built-up hybrid composite box girders fabricated and tested in flexure,
767 *Eng. Struct.* 32 (2010) 1028–1037. <https://doi.org/10.1016/j.engstruct.2009.12.029>.
- 768 [14] G. Dai, M. Su, Y.F. Chen, Design and Construction of Simple Beam Bridges for High-Speed Rails
769 in China: Standardization and Industrialization, *Balt. J. Road Bridge Eng.* 11 (2016) 274–282.
770 <https://doi.org/10.3846/bjrbe.2016.32>.
- 771 [15] Y. Yang, JD. Yau, Resonance of high-speed trains moving over a series of simple or continuous
772 beams with non-ballasted tracks, *Eng Struct* 143 (2017) 295.
- 773 [16] H. Xia, G. De Roeck, N. Zhang, Experimental analysis of a high-speed railway bridge under Thalys
774 trains, *J Sound Vib* 268 (2003) 103.
- 775 [17] Y. Cao, H. Xia, Z. Li, A semi-analytical/FEM model for predicting ground vibrations induced by
776 high-speed train through continuous girder bridge, *J Mech Sci Technol* 26 (2012) 2485.

- 777 [18] N. Khoshamadi, S. Banihashemi, M. Poshdar, H. Abbasianjahromi, A. Tabadkani, A. Hajirasouli,
778 Parametric and generative mechanisms for infrastructure projects, *Autom. Constr.* 154 (2023)
779 104968. <https://doi.org/10.1016/j.autcon.2023.104968>.
- 780 [19] N.K. Kedia, R. Wangtawesap, C. Ngamkhanong, Dynamics responses of railway bridges
781 influenced by flooded ballasted tracks subjected to high-speed trains, *Transp. Eng.* 20 (2025)
782 100334. <https://doi.org/10.1016/j.treng.2025.100334>.
- 783 [20] M. Martínez-Rodrigo, J. Lavado, P. Museros, Dynamic performance of existing high-speed railway
784 bridges under resonant conditions retrofitted with fluid viscous dampers, *Eng Struct* 32 (2010) 808.
- 785 [21] G. Gu, Resonance in long-span railway bridges carrying TGV trains, *Comput Struct* 152 (2015)
786 185.
- 787 [22] José M. Goicolea, Resonant effects in short span high speed railway bridges: modelling and design
788 issues, (2011).
- 789 [23] L. Diachenko, A. Benin, A. Diachenko, Research of interaction of the “train – bridge” system with
790 bridge deck resonant vibrations, *MATEC Web Conf.* 239 (2018) 05002.
791 <https://doi.org/10.1051/mateconf/201823905002>.
- 792 [24] G. Bianchi, C. Fanelli, F. Freddi, F. Giuliani, A. La Placa, Systematic review railway infrastructure
793 monitoring: From classic techniques to predictive maintenance, *Adv. Mech. Eng.* 17 (2025)
794 16878132241285631. <https://doi.org/10.1177/16878132241285631>.
- 795 [25] T. Arvidsson, R. Karoumi, Train–bridge interaction – a review and discussion of key model
796 parameters, *Int J Rail Transp* 2 (2014) 147.
- 797 [26] T. Arvidsson, R. Karoumi, Train–bridge interaction—a review and discussion of key model
798 parameters, *Int. J. Rail Transp.* 2 (2014) 147–186.
- 799 [27] T. Arvidsson, R. Karoumi, Train–bridge interaction – a review and discussion of key model
800 parameters, *Int. J. Rail Transp.* 2 (2014) 147–186. <https://doi.org/10.1080/23248378.2014.897790>.
- 801 [28] British Standards Institution, Eurocode 1: Actions on structures - Part 2: Traffic loads on bridges
802 (BS EN 1991-2:2003), (2003).
- 803 [29] C. Blanquart, M. Koning, The local economic impacts of high-speed railways: theories and facts,
804 *Eur. Transp. Res. Rev.* 9 (2017) 1–14. <https://doi.org/10.1007/s12544-017-0233-0>.
- 805 [30] M. Reiterer, A. Firus, A. Vorwagner, G. Lombaert, J. Schneider, A.M. Kohl, Railway bridge
806 dynamics: development of a new high-speed train load model for dynamic analyses of train
807 crossing, in: *IABSE Congr. Ghent 2021*, 2021: pp. 1633–1642.
808 <https://doi.org/10.2749/ghent.2021.1633>.
- 809 [31] K. Le Nguyen, Application of XGBoost Model for Predicting the Dynamic Response of High-
810 Speed Railway Bridges, in: C. Ha-Minh, A.M. Tang, T.Q. Bui, X.H. Vu, D.V.K. Huynh (Eds.),
811 *CIGOS 2021 Emerg. Technol. Appl. Green Infrastruct.*, Springer, Singapore, 2022: pp. 1765–1773.
812 https://doi.org/10.1007/978-981-16-7160-9_178.
- 813 [32] R. Allahvirdizadeh, A. Andersson, R. Karoumi, Probabilistic Dynamic Design Curves Optimized
814 for High-Speed Reinforced Concrete Railway Bridges Using First-Order Reliability Method, *Int.*
815 *J. Struct. Stab. Dyn.* (2024) 2540012. <https://doi.org/10.1142/S0219455425400127>.
- 816 [33] J.-R. Cho, K. Jung, K. Cho, J.-W. Kwark, Y.J. Kim, B.-S. Kim, Determination of the optimal span
817 length for a railway bridge crossed by various types of high-speed trains, *Proc. Inst. Mech. Eng.*
818 *Part F J. Rail Rapid Transit* 230 (2016) 531–543. <https://doi.org/10.1177/0954409714549083>.
- 819 [34] T. Cho, M.-K. Song, D.H. Lee, Reliability analysis for the uncertainties in vehicle and high-speed
820 railway bridge system based on an improved response surface method for nonlinear limit states,
821 *Nonlinear Dyn.* 59 (2010) 1–17. <https://doi.org/10.1007/s11071-009-9521-0>.

- 822 [35] W. Zhai, Z. Han, Z. Chen, L. Ling, S. Zhu, Train–track–bridge dynamic interaction: a state-of-the-
823 art review, *Veh. Syst. Dyn.* 57 (2019) 984–1027.
- 824 [36] R. Allahvirdizadeh, A. Andersson, R. Karoumi, Probabilistic Dynamic Design Curves Optimized
825 for High-Speed Reinforced Concrete Railway Bridges Using First-Order Reliability Method, *Int.*
826 *J. Struct. Stab. Dyn.* (2024) 2540012. <https://doi.org/10.1142/S0219455425400127>.
- 827 [37] R. Allahvirdizadeh, A. Andersson, R. Karoumi, Improved dynamic design method of ballasted
828 high-speed railway bridges using surrogate-assisted reliability-based design optimization of
829 dependent variables, *Reliab. Eng. Syst. Saf.* 238 (2023) 109406.
830 <https://doi.org/10.1016/j.res.2023.109406>.
- 831 [38] S. Wong, C. Zheng, X. Su, Y. Tang, Construction contract risk identification based on knowledge-
832 augmented language models, *Comput. Ind.* 157–158 (2024) 104082.
833 <https://doi.org/10.1016/j.compind.2024.104082>.
- 834 [39] I. Dikmen, G. Eken, H. Erol, M.T. Birgonul, Automated construction contract analysis for risk and
835 responsibility assessment using natural language processing and machine learning, *Comput. Ind.*
836 166 (2025) 104251. <https://doi.org/10.1016/j.compind.2025.104251>.
- 837 [40] D.M. Onchis, G.-R. Gillich, E. Hoge, C. Tufisi, Neuro-symbolic model for cantilever beams
838 damage detection, *Comput. Ind.* 151 (2023) 103991.
839 <https://doi.org/10.1016/j.compind.2023.103991>.
- 840 [41] L.-N. Khuong, N. Van Dang, KD-Railway 1.0 – A structural dynamics software for high-speed rail
841 bridge based on open source Cast3m platform, *J. Sci. Transp. Technol.* (2022).
- 842 [42] E. Le Fichoux, *Présentation Et Utilisation De Cast3m*, (2011). <http://www-cast3m.cea.fr/>.
- 843 [43] ERRI Specialists’ Committee D214, Rail Bridges for Speeds > 200km/h, Final Report: Part A
844 Synthesis of the Results of D214 Research, (1999).
- 845 [44] L. Frýba, *Dynamics of Railway Bridges*, T. Telford, 1996.
- 846 [45] Y. Yang, C.W. Lin, Vehicle-bridge interaction dynamics and potential applications, *J Sound Vib*
847 284 (2005) 205.
- 848 [46] M. Majka, M. Hartnett, Dynamic response of bridges to moving trains: A study on effects of
849 random track irregularities and bridge skewness, *Comput. Struct.* 87 (2009) 1233–1252.
850 <https://doi.org/10.1016/j.compstruc.2008.12.004>.
- 851 [47] A. Doménech, P. Museros, M.D. Martínez-Rodrigo, Influence of the vehicle model on the
852 prediction of the maximum bending response of simply-supported bridges under high-speed
853 railway traffic, *Eng. Struct.* 72 (2014) 123–139. <https://doi.org/10.1016/j.engstruct.2014.04.037>.
- 854 [48] P. Salcher, C. Adam, A. Kuisle, A Stochastic View on the Effect of Random Rail Irregularities on
855 Railway Bridge Vibrations, *Struct. Infrastruct. Eng.* 15 (2019) 1649–1664.
856 <https://doi.org/10.1080/15732479.2019.1640748>.
- 857 [49] A. Pere, Dynamic Analysis of High Speed 2 Rail: HS2 Project, (n.d.).
858 [https://www.midasbridge.com/en/blog/casestudy/dynamic-analysis-of-high-speed-rail-insights-](https://www.midasbridge.com/en/blog/casestudy/dynamic-analysis-of-high-speed-rail-insights-from-the-hs2-project)
859 [from-the-hs2-project](https://www.midasbridge.com/en/blog/casestudy/dynamic-analysis-of-high-speed-rail-insights-from-the-hs2-project) (accessed December 10, 2023).
- 860 [50] K. Deb, S. Agrawal, A. Pratap, T. Meyarivan, A Fast Elitist Non-dominated Sorting Genetic
861 Algorithm for Multi-objective Optimization: NSGA-II, in: M. Schoenauer, K. Deb, G. Rudolph,
862 X. Yao, E. Lutton, J.J. Merelo, H.-P. Schwefel (Eds.), *Parallel Probl. Solving Nat. PPSN VI*,
863 Springer Berlin Heidelberg, Berlin, Heidelberg, 2000: pp. 849–858. [https://doi.org/10.1007/3-540-](https://doi.org/10.1007/3-540-45356-3_83)
864 [45356-3_83](https://doi.org/10.1007/3-540-45356-3_83).