DAMAGE ASSESSMENT OF A FULL-SCALE BRIDGE BASED ON THE RESPONSE SURFACE METHOD

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Abstract: As a combination of statistical and mathematical techniques, response surface models have been recently found to be capable of substituting FE models in model updating iterations by using explicit mathematical functions to represent the relationship between the inputs and outputs of a physical system. However, the literature related to this topic is still scarce despite the wide employment of the response surface method in many engineering realms such as chemistry and industry. Due to that, this paper attempts to propose a systematic damage assessment procedure based on the model updating strategy using the response surface method. Instead of the qualitative evaluation traditionally used, here the 2^k factorial design is employed to screen out non-significant updating parameters by quantitative statistical analysis, which considerably improves the screening reliability. Meanwhile, the central composite design is adopted to construct response surface models substituting original FE models during updating. The proposed method is used to detect the damage existing in an experimental full-scale bridge. The results demonstrate the merits of this method in its easy implementation and high computation efficiency, especially for the bridge case.
Keywords: Damage assessment, Model updating, Response surface method, Parameter screening, Bridge

1 INTRODUCTION

The core of structural health monitoring (SHM) is the implementation of a damage assessment process (Farrar and Worden, 2007), in which the model updating strategy has been widely investigated and employed seeking baseline models for specific structures to be updated according to their different damaged states (Friswell and Mottershead, 1995). However, traditional model updating techniques usually require FE models during updating iterations, which results in considerable computational expenses for large structures and thus limits its applications to online SHM systems. Therefore, one may seek other types of models to substitute original FE models for the updating process for the sake of cost-efficiency and an easy connection to the other modules of an online SHM systems. As a result, neural networks have been used for model updating purposes due to their simple physical explanations (Atalla and Inman, 1998). But for neural networks having large dimensions (many input parameters), the numbers of training samples may exponentially increase requiring considerable computational efforts. And the black-box relationships between inputs and outputs also restrain their applications in practice. As an alternative, response surface (RS) models, which is in nature a combination of statistical and mathematical techniques (Myers and Montgomery, 2002), have been recently found to be incorporated into model updating. A response surface model is actually an explicit mathematical function representing the relationship between the inputs and outputs of a physical system, which may highly simplify an updating process. Although the response surface methods have been widely employed in many engineering realms such as chemistry and industry, their applications to damage assessment of civil structures are still scarce in the literature. Using the central composite design (CCD), Guo and Zhang (2004) constructed some high-order polynomial RS models to update an H-shaped structure and they adopted the stiffness quantities and modal frequencies as the input and response properties. The results demonstrated that besides presenting likewise updating accuracy, the proposed method was much more cost-efficient than the traditional FE model updating method. To update a numerical two-span continuous beam, Deng et al (2008) used the uniform design to establish an RS model for updating this beam and they found similar conclusions to those by Guo and Zhang (2004). In another piece of research performed by Cundy (2002), the damage in a simulated mass-spring-damper system and a tested cantilever beam was identified by using the RS models during updating. The research proves the availability of the modal frequencies used as the outputs of the RS models showing some robustness to the experimental variabilities.

Further, in order to identify the damage of more complex structures, this paper attempts to propose a systematic damage assessment procedure based on the RS method. The 2^k factorial design (FD) was employed for parameter screening by means of quantitative statistical analysis and the CCD was used to construct the second-order polynomial models for an experimental full-scale bridge with their modal frequencies being the responses. This study tries to investigate the feasibility and efficiency of RS models in the model updating and the further damage assessment for large-scale civil structures.

2 RESPONSE SURFACE MODELING

For a physical system, an RS model correlates the input parameter vecto ξ to the corresponding response y, which is defined by a mathematical expression:

$$y=f(\xi_1,\xi_2,\cdots,\xi_k)+\varepsilon \quad (1)$$

where f denotes the function mapping ξ to y, k denotes the number of the input parameters and ε represents the variability not considered in f. ε is usually defined as a statistical error having a normal distribution with mean zero. For practical use, ξ can be coded to a dimensionless vector $(x_1,x_2,\cdots,x_k)$ having mean zero and the same standard deviation of ξ.

In engineering applications, second-order polynomial RS models are often employed:

$$y=\beta_0+\sum_{i=1}^{k}\beta_i x_i+\sum_{i=1}^{k}\beta_{ii}x_i^2+\sum_{i<j}\sum_{j=2}^{k}\beta_{ij}x_i x_j+\varepsilon \quad (2)$$

where β denotes the partial regression coefficients obtained through the least squares estimation of ε and the unbiased estimated values of β are represented as b; x_i and $x_i x_j$ give the main effect and the interaction effect between two arbitrary parameters, respectively. Then y has the estimation expression of:

$$\hat{y}=b_0+\sum_{i=1}^{k}b_i x_i+\sum_{i=1}^{k}b_{ii}x_i^2+\sum_{i<j}\sum_{j=2}^{k}b_{ij}x_i x_j \quad (3)$$

The estimated response $\hat{y}$ is namely the RS model for practical use, which also contains some modeling errors.

2.1 2^k Factorial Designs

In order to avoid the experiential evaluation of parameter significance traditionally used, one may introduce the 2^k FD to such evaluation based on statistical analysis. Parameter x_i may or may not be kept according to its contribution to the total model variance (Myers and Montgomery, 2002). Two coded levels (± 1) is used to represent the lower and upper bounds of x_i in the design space. And design points spreading over the design space are defined as responses computed by the different level combinations of all input parameters through FE analysis. A total of 2^k design points is required for a 2^k FD and some additional center points (level zero) may also be considered to evaluate the curvature of the middle region of the design space. It should be mentioned that in most cases, design points are numerically generated taking into account experimental costs and limitations. For example, it's not practical of artificially introducing damage into a bridge for the sake of obtaining different damage scenarios (design points).

2.2 Central Composite Designs

As illustrated in Fig. 1, a CCD can be derived from a 2^k FD by including some "axial" points (level $\pm\alpha$, $\alpha\geqslant 1$), which actually extend the bounds of a design space. In general, the design space prefers "rotatability" giving a stable distribution of the scaled prediction variance. However, when there are many parameters, the operability and rationality of the design space might collapse due to a large α (e.g. mass of 0 is not logical for a beam). In this case, a practical CCD with smaller α and some rotatability is desired. For a CCD design, the number of design points required is 2^k+2k+n_c having $2k$ axial points and n_c center points generally set as 5 or 6.

A CCD establishes a second-order polynomial RS model suitable for structural simulation. And a detailed description of the 2^k FD and CCD is given by Myers and Montgomery (2002).

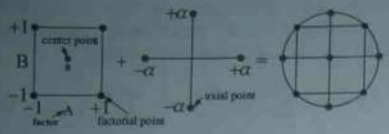

Figure 1. Two-factor central composite design

3 DAMAGE ASSESSMENT PROCEDURE

In this study, the systematic damage assessment procedure comprises four sequential steps of: *feature selection*, which chooses the input and response features; *parameter screening*, which discovers input parameters with low significance; *primary modeling and updating*, which establishes a baseline model to the intact experimental model; *reference-state modeling*, which serves the final damage assessment.

3.1 Feature Selection

In general, the material and geometric properties are chosen as input features (parameters). For response features, time or frequency domain features may be the option. In this study, the stiffness property of section inertias was chosen for the inputs and modal frequencies were for the responses.

3.2 Parameter Screening

By using the 2^k FD, a quantitative, instead of qualitative, evaluation of input parameters can be achieved by using statistical analysis. Parameters are kept or screened out according to their contributions to the total model variance. Here the analysis of variance (ANOVA) is employed and the parameter significance is evaluated by its statistical F-test value:

$$F_A = \frac{SS_A/d_A}{SS_E/d_E} \sim F_{0.05,d_A,d_E} \qquad (4)$$

in which F_A and $F_{0.05,d_A,d_E}$ denote the F-test value of parameter A and the criterion value, respectively; SS_A and SS_E denote the sums of squares with respect to A and the residual (error), respectively; d_A and d_E are the degrees of freedom of A and the error. A is significant to a specific response feature if $F_A > F_{0.05,d_A,d_E}$. Otherwise it will be neglected. The interaction effects of two arbitrary parameters can also be analyzed by this means when necessary.

3.3 Primary Modeling and Updating

In the proposed method, a primary RS model is firstly constructed and then updated to the baseline model having similar responses to those from the intact experimental model, as described below.

3.3.1 Primary modeling

Firstly bounds of each input parameter should be defined to form the bounds of the design space, e.g. ±30% changes to the initial value. Then by the design of experiement (DoE), design points are numerically generated using Ansys® (Ansys, 2003) and the primary RS model is subsequently constructed by the program Design Expert® (Design Expert, 2005).

3.3.2 Model adequacy checking

The adequacy of an RS model has to be checked according to three criterions having values within [0,1] (Myers and Montgomery, 2002). The first one refers to R^2 measuring the amount of variation around the mean explained by the model:

$$R^2 = 1 - \frac{SS_E}{SS_T} \qquad (5)$$

where SS_T denotes the total sum of squares of the model. R^2 is desired to be close to 1.0 but this does not always imply an adequate model because including a non-significant parameter will also increase R^2. Due to it, an adjusted criterion R^2_{adj} should also be taken into account since it often decreases with the augmentation of non-significant parameters:

$$R^2_{adj} = 1 - \frac{SS_E/d_E}{SS_T/d_T} = 1 - \frac{d_T}{d_E}\left(1 - R^2\right) \qquad (6)$$

where d_T denotes the total degree of freedom of the model. If R^2 and R^2_{adj} are both close to 1.0 and they are slightly different, a model can be deemed fitting well the design points. Meanwhile, an adequate model must process satisfactory predicion ability to unseen data. Hence, the third criterion R^2_{pred} measuring the amount of variation in new data is also used for checking:

$$R_{\mathrm{pred}}^2 = 1 - \frac{\mathrm{PRESS}}{SS_{\mathrm{T}}} \qquad (7)$$

where PRESS denotes the predicted residual error sum of squares, which answers how the model fits the design points in the design space. In practice, a difference between R_{adj}^2 and R_{pred}^2 within 0.2 is acceptable.

3.3.3 Reference parameter updating

The primary model is then updated to find the baseline model of the intact experimental model. Here a multiobjective optimization algorithm is adopted for updating:

$$\min_{x,\gamma} \gamma, \quad \begin{cases} \mathrm{F}(x) - \omega\gamma \leqslant goal \\ lb \leqslant x \leqslant ub \end{cases} \qquad (8)$$

where $\mathrm{F}(x) = abs\left(\left(f_{\mathrm{RSM}} - f_{\mathrm{exp}}\right)/f_{\mathrm{exp}}\right)$ is the dimensionless objective function with f_{RSM} and f_{exp} representing the physical quantities such as frequencies predicted by the RS model and measured by the experiments, respectively; γ is a slack variable used as a dummy argument to minimize $\mathrm{F}(x)$ and the weight ω controls the relative under-attainment or over-attainment of the objectives; *goal* is a set of values that the objectives try to attain; *lb* and *ub* are the lower and upper bounds of *x*.

3.4 Reference-state Modeling and Damage Assessment

With the baseline model, the reference-state model can be constructed and then used for damage assessment purposes.

3.4.1 Reference-state modeling

In this part, the objective structure is divided into some substructures, each of which has an individual parameter. The initial value of each parameter is given by the baseline model and naccording to specific damage situation, bounds for each parameter should be redefined and then the reference-state model is constructed using the same procedure in the primary modeling.

3.4.2 Damage identification

Based on the experimental data from the damaged structure, the reference-state model is updated to identify the damage represented by changes in the input parameters. The same multiobjective optimization algorithm introduced above is used here to minimize the discrepancies of physical quantities between the reference-state model and the damaged experimental model.

4 THE I-40 BRIDGE

For a further validation of the proposed method on experimental structures, the I-40 bridge was adopted (Farrar *et al.*, 1996). This bridge comprised twin spans and one of them with 3 continuous spans (Figure 2) was designedly tested for damage identification purposes. The web and the flange of the north plate girder at substructure 5 was damaged by torch cuts to different damage severities. Force vibration tests were applied to the intact and damaged bridge and the experimental frequencies of the First 3 flexural modes were 2.48, 3.50 and 4.08Hz of the intact bridge and 2.30Hz, 3.49Hz and 3.99 Hz for the damaged bridge.

A two-dimensional beam model was used to simulate the bridge, as illustrated in Fig. 2. Sixty-one elements were generated to simulate the equivalent girder and 10 elements were used for each equivalent pier. According to the real boundary conditions, the pier bottoms were restrained from any movement and the connections between the girders and the pier tops were set to be rotatable. Meanwhile, since the real bridge had the thin and thick (substructures S3 and S7) girders, two sets of beam cross-sectional properties were developed having identical the Young's modulus of 200 GPa. The initial equivalent values of the section areas and inertias were A1=0.5685 m^2, I1=0.512 m^4 (thin girder) and A2=0.6436 m^2, I2=0.812 m^4 (thick girder), where considerable errors might exist due to some simplification and measurement errors resulting in an incorrect model.

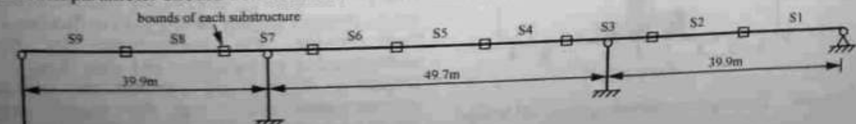

Figure 2. Schematic diagram of the I-40 bridge

4.1 Parameter Screening

These four parameters, A1, I1 and A2, I2, were firstly screened to check their significance and the 3 flexural frequencies were used as the responses for RS modeling. The piers were not considered for updating because they were not sensitive to the frequency changes due to the rotatable connections between the plate girders and the pier tops.

The 2^k FD was employed for screening using $2^4+5=21$ design points including 5 center points. The bounds of each parameter were set to be ±20% change with respect to the initial values. The screening model has the model adequacy indices R^2 , R^2_{adj} and R^2_{pred} all greater than 0.999 indicating satisfactory modeling precision. It can be seen from Fig. 3 hat A1 and I1 highly affect the frequency changes in a contrary tendency from the lower to higher modes. On the contrary, I2 has a little influence only on the last two frequencies and A2 shows no effects on the frequency feature. The observation discovers the significant parameters, A1, I1 and I2, to be updated subsequently.

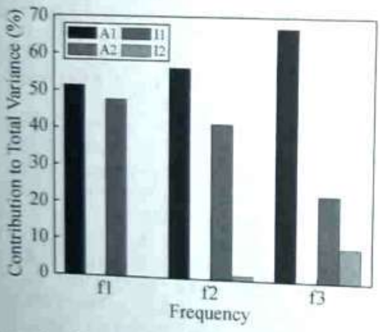

Figure 3. Parameter screening of the I-40 bridge

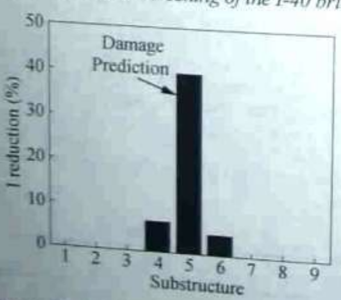

Figure 4. Damage assessment of the I-40 bridge

Table 1. Input parameter bounds of the primary RS model of the I-40 bridge

	$-\alpha$	-1	0	+1	$+\alpha$
A1	0.864	1.0	1.2	1.4	1.536
I1	0.464	0.6	0.8	1.0	1.136
I2	0.464	0.6	0.8	1.0	1.136

4.2 Primary Modeling and Updating

For the initial FE model of the intact bridge, the discrepancies between the numerical and experimental frequencies were 14.41%, 25.35% and 37.90%. A CCD was used to construct the primary RS model and 20 design points including 6 center points ($2^3+2*3+6=20$) were numerically generated with the bounds of each parameter given by Table 1 with $\alpha=1.68$. Then the primary model (a second-order polynomial model without considering the interaction terms is adopted for this case study) was updated by means of the multiobjective optimization algorithm. The frequency discrepancies of the updated model decreases to -4.43%, 1.71% and 2.78% with the average MAC value of 0.9982. The updated three parameters turn to be A1=0.7712 m^2, I1=0.512 m^4 and I2=0.377 m^4.

4.3 Reference-state Modeling and Damage Assessment of the I-40 Bridge

The entire bridge girder was divided into 9 substructures from S1 to S9 (Fig. 2) each of them comprising some beam elements. The damage was assumed to be the stiffness reduction caued by the loss of the section inertias, I1 and I2. A CCD having a practical α of 1.73 was used to construct the reference-state RS model using 70 design points. The bounds of the design space were defined as $0.727I_0$, $0.8I_0$, I_0 and $1.073I_0$ for $-\alpha$, -1, +1 and $+\alpha$, respectively. I_0 represents the updated values of I1 and I2 obtained from the primary updating. Then the reference-state model was constructed having three model adequacy indices all greater than 0.975. It shoud be noted that the reference-state model inherits the modeling errors from the primary model where the error of the first frequency is more than 4% resulting in some perturbation to the damage predictions. Thus one may expect to eliminate such perturbation by refining the reference-state model through slightly tuning the coefficients *b*. Such tuning was implemented by the multiobjective optimization and after that, the frequency discrepancies between the refined reference-state model and the experimental model were very close to zero, which enhances the accuracy of the damage predictions.

Subsequently, based on the experimental data from the damaged bridge, the damage at substructure S5 was well identified by the multiobjective updating using the refined reference-state model, as shown in Fig. 4. After updating, the average frequency error between the RS and experimental models is 3.07% with the average MAC value of 0.9957, which proves a satisfactory damage assessment performance of the proposed method in the real and large-scale engineering structure.

5 CONCLUSIONS

A systematic damage assessment procedure based on the RS method is presented in this paper. By using the 2^k FD, the significance of updating parameters can be quantitatively evaluated based on statistical analysis. At the same time, second-order polynomial RS models constructed by the CCD can be used to represent the dynamic behaviors of a physical system and thus be employed for damage assessment purposes. The proposed method was verified against an experimental full-scale bridge, where its feasibilty and effiency in damage assessment has been well proved.

ACKNOWLEDGEMENTS

This research is supported by the Ministry of Education and Science of Spain (Project: BIA2007-67790) and by the Science & Technology Development Foundation of Fuzhou University (Project:2010-XQ-26).

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