A model for the economic evaluation of master city plans: a pilot study of Västerås, Sweden

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The paper introduces the framework, problems addressed, objective function, types of variables and so on for a model designed to facilitate the economic evaluation of master city plans. The model presented here has been used in a pilot study of the city of Västerås, Sweden. It consists of three main parts, data, results and method. Some conclusions are drawn.

Key words: mathematical model, economics, city planning

Introduction

Master city plans

A master city plan is a set of decisions made by the city authorities to adapt the city structure efficiently to changes that they do not control. The city planners must face several important planning problems in trying to determine a plan that is capable of efficiently adapting the structure of the city to changes that are beyond their control. Some of these problems are interdependence, irreversibility, durability, uncertainty and externalities.

Interdependence

The various decisions included in a master city plan cannot be determined independently of each other. For example, the minimum cost solution for a plan in which most of the housing is in the form of concentrated multi-family units calls for district heating and transportation by public means. At the opposite extreme, a plan calling for single-family houses in sprawling residential areas would be likely to include electrical heating and commuting by car.[1] Therefore one set of solutions, a consistent master city plan, should be compared with another set of solutions.

Irreversibility, durability and uncertainty

Most of the resources used in city building represent 'sunken costs'. The irreversibility and durability of city structures will limit future options. Of course, it is possible to demolish old urban structures and to build new ones or to convert existing urban structures. However, this usually involves considerable extra cost. Therefore, potential savings can be realized if these characteristics are adequately taken into account when drafting master city plans.

Uncertainty about the future also creates problems for the city planners, since they do not know for certain how the demand for a particular city facility will develop during the period under consideration. Thus they have to supply durable facilities (i.e. facilities that are indivisible over time) under conditions of uncertainty.[1–3]

Objective of the model and theoretical framework

The model presented here provides a method for evaluating alternative master city plans according to economic criteria. In principle all essential effects, that is, all the costs and benefits that the decisions of a master city plan imply should be included in such an evaluation. However, the present model considers only the costs for residences, heating, transportation and schools and is thus an example of cost-effectiveness analysis, rather than of cost-benefit analysis. But, given two master city plans with different costs and different benefits, it is possible to use this model to calculate how much greater the benefits of the more costly master city plan must be for that plan to be preferred to the less costly one.

Another important use for the model would be in making improvements in a given master city plan by repetitive use of the model in a feedback procedure in which marginal changes are made each time.[4] The theoretical framework of the model is drawn from the field of welfare economics. In the model developed here the market mechanism, in the form of economic incentives for individuals, has been used to simulate their behaviour.

However, this model is a planning model, because there are many circumstances where sole reliance on the workings of a decentralized market economy does not lead to situations that are efficient from the point of view of society as a whole. Public intervention in markets may improve the allocation of resources.[5]

Several large-scale models have been criticized for their 'black-box' character. In particular, the lack of a basis of economic theory has been pointed out as an important explanation of their failure.[6] The present model, however, is explicitly based on fundamental economic theory. Economic incentives are modelled to simulate the behaviour of individuals and economic decision parameters are explicitly introduced into the model besides an objective function for economic evaluation: minimization of total costs.

Outline of the model

The model presented in this paper is the result of successive steps and studies carried out during recent years by the authors. Descriptions of preliminary models have already been published.[1–3] However, until recently, none of these models have been applied to a real city. A pilot study has been carried out using the Swedish city of Västerås. One of the results of this pilot study has been a number of changes to the model to make it reflect reality more accurately. The results of this research are briefly described in this paper. A more complete presentation is to be found in a forthcoming book.[4]

It is convenient to distinguish among the following types of variables in the model:

Variables not controlled by the city authorities.
Some example of these exogenously given (data) variables are employment opportunities in the city over time, fuel prices, etc,

Decision variables, i.e. variables directly controlled by the city planners.
Examples of these variables are the potential number of apartments to be demolished, remodelled or constructed, the number, size and location of schools, locations and capacities of new roads and heating pipes, pricing policies for housing, heating and transportation, etc.
These variables are also given as data.

Result variables, i.e. variables controlled indirectly by changes in the decision variables.
The main results are total costs for the different master city plans. Other results variables of interest include the assignment of the workers to different working place centres, the distribution of commuting among the different modes of commuting, traffic congestion on the various routes, the distribution of land rents, etc.

Three main parts of the model may be distinguished: data, method and results; see *Figure 1*. The first part consists of the exogenously given variables and parameters. The second comprises all the computational procedures used to obtain the results. These computations can in turn be divided into two steps. In the first step all relevant consequences for different city activities included in the model of a given master city plan are simulated. Then the simulated consequences are evaluated economically, i.e. total costs are computed. The results, i.e. the endogenous variables of interest for the city planners are obtained from the data and the computations just described.

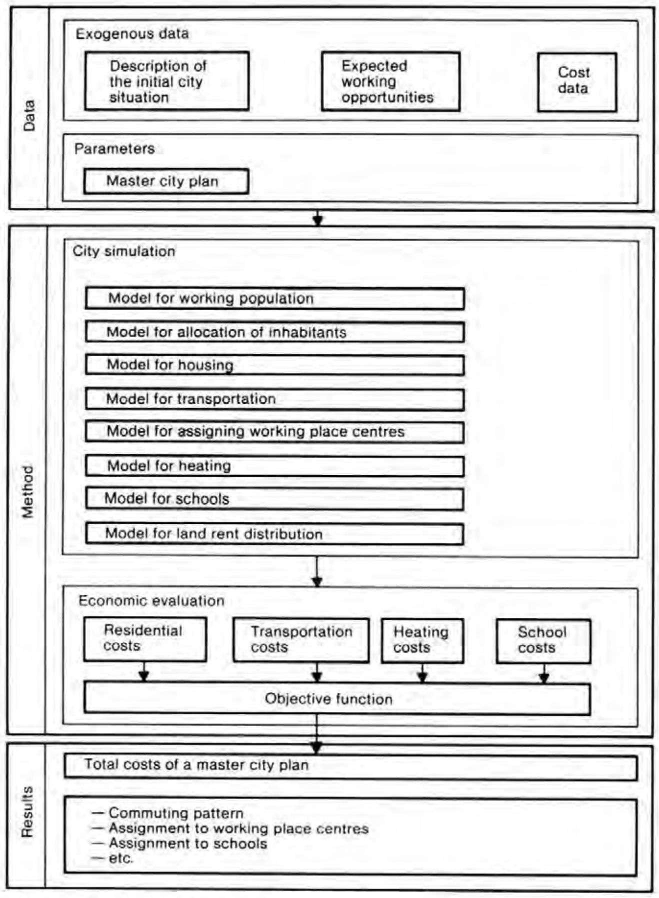

Figure 1 Outline of model

In a perfect competitive market an efficient equilibrium solution is automatically reached through the tâtonnement process. However, where the development of cities is concerned, these processes do not lead to an efficient solution owing to the various characteristics discussed in this section of the paper. Therefore, it may be possible to use an iterative procedure carried out by a computer to find solutions that are an improvement over some given preliminary plans.

Pilot study of Västerås

As has already been mentioned, the model described here has been applied to Västerås, Sweden. Västerås fulfills several conditions that were considered important for a pilot study. First, the city is expected to expand for some time in the future. Second, it is medium in size, with a population of 120,000, and so not too complex to be used as a test case. Third, the data required by the model are available from the city planning office. Fourth, the city planners of Västerås have been willing to invest sufficient resources in the form of time and effort in collaborating with the authors to adapt the model to realistically reflect the city. Finally, several different master city plans were supplied by the city planners for the purposes of this evaluation.

Data

The first part of the model consists of the exogenously given variables and the decision parameters. These input data can be divided into the following four categories:

(a) Data required to describe the city at the initial point of time.
(b) Cost data relevant to housing, heating, schools and transportation.
(c) Data on the expected limits for working population and working place centres through time.
(d) Data used to define the master city plans.

These groups of data will be described in the same order as they were presented above.

Description of existing city

The total area of the city of Västerås is divided into about 400 blocks or nodes. These nodes are assumed to be homogeneous in some respects, e.g. with regard to types of residences, year of construction, income level of the inhabitants, etc. The following data are provided for each node: the coordinates of the centre of gravity of the node (x, y); the main way land is used (for residences, working places, open space, parking places, etc.); the intensity of land use (neighbourhood area per apartment); number of storeys; year of construction; number of apartments and rooms; total population and working population; number of school children; and average disposable income per household. Ten different working place centres are explicitly considered. The total number of potential employment opportunities is given for each such centre.

The existing transportation system, with its routes for buses and cars, is modelled using the concept of transportation nodes. A link is defined by a pair of such nodes. Consecutive links are connected to form routes from residence nodes to working place centres as well as the congestion per unit of time along the routes.

All of the values listed above are known for each node of the city for the initial point of time.

Cost data

The cost data include values for the following kinds of items:

The real rate of interest.
The expected economic lifetime and building and maintenance costs for residences, schools, heating pipes, roads, etc.
Costs for conversion of one type of facility into another, including such items as demolition costs for old residences.
Costs for the operation of vehicles including depreciation, maintenance, fuel, etc.
Velocities for each mode of commuting.
Times for commuting.
Costs for oil and distribution losses for heating.
Amount of space demanded for the different facilities.

Expected limits for working population paths and working place centres

This set of data includes the most probable values as well as the upper and lower limits for employment opportunities for all times from the initial point of time to the time horizon (20 years in the pilot study). The values used for Västerås are shown in *Figure 3*, which also shows values for the potential distribution of the working opportunities among the working place centres.

Data defining the master city plans

These data are preliminary decisions made by the city planners regarding modifications related to residences, the heating system, the transportation system and schools. The following types of data are necessary:

The potential number and type of apartments to be constructed in various city areas at different points of time.
The potential number of apartments to be demolished in various city areas at different points of time.
Changes in number, sizes and locations of working place centres.
Changes in number, sizes and locations of schools.
Changes in layout of transportation system and in bus routes.
Changes in layout of heating system.

Several different master city plans for Västerås have been studied. Only two of them will be discussed here. These are an alternative for which a majority of the the new residences will be built in the satellites and outskirts of the city (alternative B) and the polar case in which urban renewal provides most of the new residences (alternative D). The data are potential values for the number of apartments and their distribution over the various city areas and are shown in *Table 1*.

Some of the data used to define the plans are potential starting values. They may be changed by the model in accordance with a simulated path for the working population.

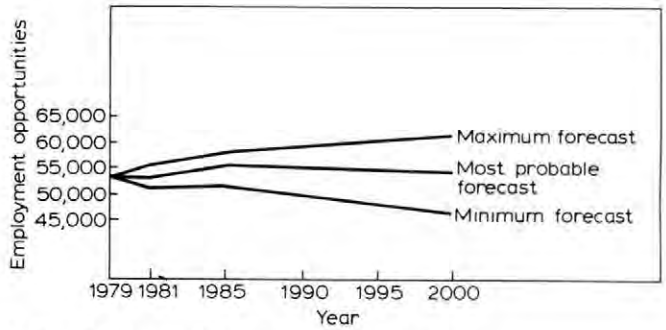

Figure 2 Forecasted employment opportunities

Table 1 Two master city plans for Västerås. Number of apartments

Master city plan	B	D
Type of housing	$F = 25\%$ $S = 75\%$	$F = 75\%$ $S = 25\%$
Outskirts (dense area including Hökåsen and Tillberga)	2 700	3 300
Satellites	5 000	700
Urban renewal in inner city	3 300	10 000
Gross production	10 000	14 000
Demolished apartments	3 300	6 300

F = apartments in multi-family housing
S = apartments in single-family housing

Method

General features

As was pointed out earlier the method used in the model involves two main aspects: simulation and evaluation. The first employs a set of submodels that are strongly interdependent in the sense that the results of one submodel are used as inputs for another. This means that the program involves a great deal of iteration. The data used by a given

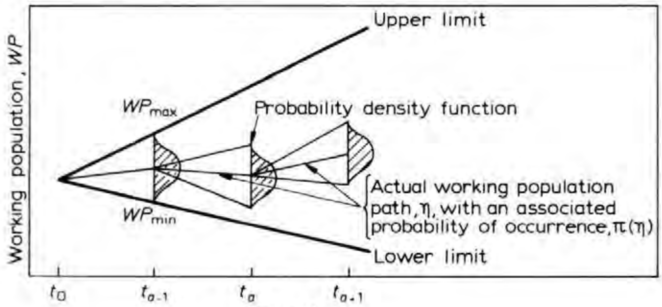

Figure 3 Simulation of working population paths

submodel in one iteration are the results from other submodels in the previous iteration. The iterations will continue until certain equilibrium conditions are met for all of the submodels. In broad terms, equilibrium is when the results obtained in two consecutive iterations are approximately equal. Then the results can be evaluated economically. The various submodels used will be described briefly in the following paragraphs.

Model for working population

This model is solved in three main stages:

(a) Simulation of the working possibilities (η) for the time span of the study and the probability of occurrence $\pi(\eta)$. The path and its associated probability $\pi(\eta)$ are obtained for each intersection point of time t_a by assuming a given probability distribution between the upper and lower bounds of the working population possibilities (see *Figure 3*). The path is represented by the following set of values:

$$\eta \equiv [WP(t_1), WP(t_2), \ldots, WP(t_a)] \quad (1)$$

(b) Allocation of the working possibilities $WP(t_a)$ among the different working place centres using an exogeneously given rule.
(c) The ratios s are given for each node of the city and each intersection time, and:

$$s = \frac{\text{working population}}{\text{total population}}$$

The total population over time can be calculated using these ratios.

Model for allocation of inhabitants

In general the population of a node depends on the populations of the other nodes. The nodal populations are usually obtained by a minimization procedure. The main purpose of this model is to simulate a reasonable allocation of the population over the city area. Therefore, strong interdependence among these nodal populations has been ensured by the use of the following function:

$$P_n = A\bar{\Omega}_n \exp[b \cdot lr_n] \quad (2)$$

where:

P_n = population at node n
$\bar{\Omega}_n$ = neighbourhood area at node n per unit of land area
A and b are two parameters
lr_n = land rent value at node n. (See section on model for land rent distribution)

Equation (2) is an extension of the empirical function given by Clark.[7]

There the population density is simply given as a function of the radius to the city centre. Muth[8] suggested that the population density be calculated as a function of the individual commuting costs. This was done in some of our previous work.[2,3] The final step has been to make the density distribution a function of the land rent, as has been done here. See also Mills.[9]

Figure 4 provides an illustration of equation (2). There the slope character of the parameter b can be observed. The value of A is obtained from an equilibrium condition for the city: the total city population must have residences within the city limits.

The model for housing

Once the population has been allocated to the city nodes, it may be necessary to construct new residences. There are three main steps in determining this.

(a) Determination of the average size of household at each node (nf_n). The average size of household for the city as a whole is known from existing statistics. Using exogenously given rules the size of the households at each node and at each intersection time can be determined.
(b) Determination of the number of apartments at each node (na_n). The following simple formula is used for new residences:

$$na_n = \frac{\text{population}}{\text{household size}} = \frac{P_n}{nf_n} \quad (3)$$

(c) Determination of the number of storeys at each node (α_{1n}). To find these values the concept of neighbourhood area $\bar{\Omega}$ has been introduced. (The subscript n will be dropped in the remainder of this paragraph for the sake of convenience.)

The neighbourhood area is the fraction of the total area of a node required to build residences, including open space, local roads and the like. *Figure 5* illustrates this concept. The proportion between the neighbourhood area and the total area is called the exploitation factor ($\bar{e}$) and is included in the exogenous data. On the basis of *Figure 5* the following relationship can be seen to hold:

$$\bar{\Omega} = \bar{e} \cdot \Delta x \, \Delta y \quad (4)$$

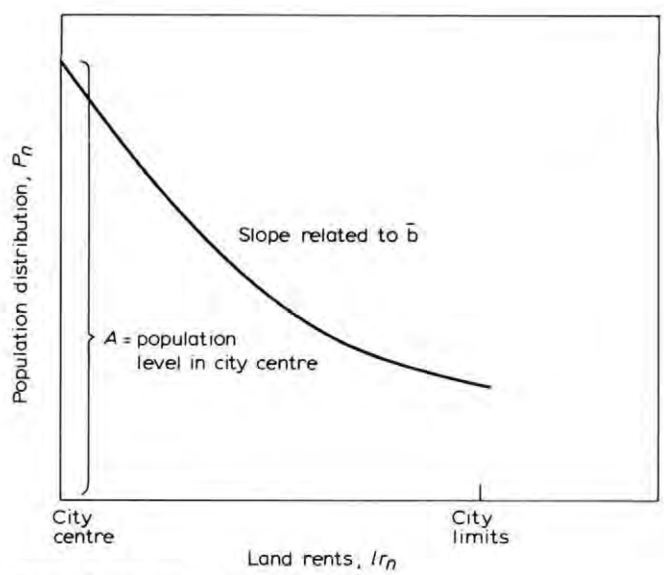

Figure 4 Population density function

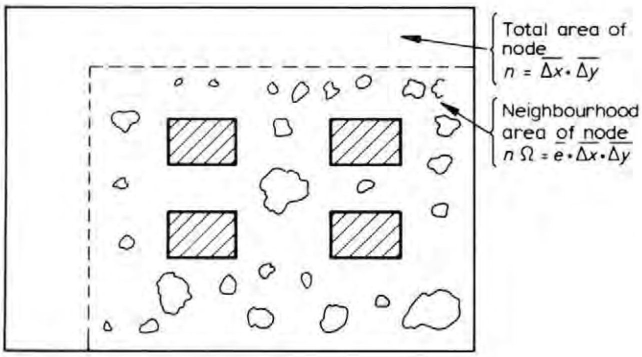

Figure 5 Neighbourhood area

Table 2 Relation between neighbourhood area per apartment and number of storeys

Storeys	1.5	4.0	7.0	10.0
Apartment (m²)	700	350	160	150

The neighbourhood area required per apartment can be expressed as a function of the number of storeys, i.e.:

$$\bar{\omega} = \bar{\omega}(\alpha_1)$$

For Västerås the relationship shown in *Table 2* has been used.
On the basis of the concepts introduced above, the number of storeys at each node can be computed as follows:

$$\bar{\Omega} \geqslant na \cdot \omega(\alpha_1) \tag{5}$$

$$\bar{\alpha}_{1\,\min} \leqslant \alpha_1 \leqslant \bar{\alpha}_{1\,\max} \tag{6}$$

Equation (5) is the neighbourhood area condition and equation (6) the physical condition. If α_1 turns out to be greater than $\bar{\alpha}_{1\,\max}$, only $\bar{\alpha}_{1\,\max}$ storeys may be built. The remainder of the people assigned to that node must be reassigned to another node. If α_1 turns out to be less than $\bar{\alpha}_{1\,\min}$, $\bar{\alpha}_{1\,\min}$ storeys must be built anyway, but in a smaller neighbourhood area $\bar{\Omega}_a$. The remaining neighbourhood area, $\bar{\Omega}_{a+1} = \bar{\Omega}\bar{\Omega}_a$ is then available for use at a later time.

Model for transportation

The only transportation considered explicitly in the model is commuting between the residential nodes and the working place centres. There are four possible modes of commuting: walking, bicycling, by car and by bus.

The transportation layout is given as data for the city at the initial point of time as well as for the different intersection points of time for the study. All possible routes for commuting to the working place centres are also given as data.

Each worker living at node n and working at working place centre p will choose among the possible routes and modes of commuting the combination of route and mode that minimizes his individual commuting costs:

$$c_{bj} = \min_r c^r_{bj}(n,p) \tag{7}$$

and

$$c_b = \min_j c_{bj} \tag{8}$$

where the $c^r_{bj}(n,p)$ are the commuting costs from node n to working place centre p using route r and mode j.

These costs include such mode dependent costs as time, gasoline, maintenance, depreciation, parking fees, bus fares and the like. The costs of traffic congestion are determined endogenously by the model. The city planners will receive information on the congestion on all the road links used. This information may provide clues as to desirable revisions of the road building plans.[10]

Model for assessing working place centres

Finding the most efficient choice of working place centre for each residential node is an example of the assignment problem. To attain a 'reasonable' solution to this problem it is assumed that the so-called gravity rule holds.

The main idea of the gravity rule is to define an 'attraction value' AV_{np} between residential node n and working place centre p for each node and centre. The following rule is used:

$$AV_{np} = \frac{WP_n \cdot \overline{WP}^p}{c_b(n,p)} \tag{9}$$

where WP_n is number of workers living at node n, $\overline{WP}^p$ is number of jobs at centre p, $c_b(n,p)$ is cost for commuting from node n to centre p.

The attraction values of equation (9) are then used to assign workers to working place centres by means of the following conditions:

(a) A working place centre p is preferred to p' by the workers living at residential node n if AV_{np} is less than $AV_{np'}$.
(b) Workers living at node n are more likely to be assigned to working centre p than workers living at node n' if AV_{np} is less than $AV_{n'p}$.
(c) For all nodes n with workers assigned to working place centre p the following condition must hold:

$$\sum_n WP_n \leqslant \overline{WP}^p \tag{10}$$

Model for heating

This model is similar to the transportation model. The mode of heating chosen by the residents living at node n is the one that minimizes individual heating costs:

$$c_h = \min_j c_{hj} \tag{11}$$

where c_{hj} are the individual heating costs corresponding to heating mode j.

In the case of Västerås the heating model has been oversimplified since only district heating is allowed.

Model for schools

This model can be said to be a simplified combination of the models for transportation and assignment to working place centres. These have been described earlier in the paper.

The model for schools can be divided into the following three main computational steps:

(a) Distribution of the total number of school children among the different residential nodes. This number is given as input data for the initial point of time as well as for the full time span of the study. The distribution of children among the schools is known for the initial point of time.

Some specific rules are than applied to the initial distribution to determine the development of the distribution of the children among the schools for the full time span.

(b) Assignment of the school children to the schools is done by minimizing commuting costs. This means that both the school and mode of transportation are determined for each school child.

(c) The following constraints must be satisfied:

(1) Only walking and busing are permitted as means of transportation for school children.

(2) A maximum distance for commuting is specified. For Västerås the maximum distance for walking is 2 km and for busing 5 km.

(3) The number of children attending a given school must be no greater than the total capacity of the school, which is given as input data.

Model for land rent distribution

The land rent distribution is a set of shadow values for land that give an indication of values of one particular allocation of the inhabitants over the city area with respect to the locations of the relevant city activities. The following condition is used for an individual in a given income class dwelling at a given node:

$$\text{apartment rent} + \text{heating costs} + \text{commuting costs} = \text{constant} \qquad (12)$$

where the constant is independent of the node of residence.

This equation corresponds to that of Mohring[11] and is illustrated in *Figure 6*. Equation (12) represents the indifference of an individual of a given income class with regard to node of residence when only his own costs for housing, heating and commuting are taken into consideration.

The expressions used to evaluate the individual's commuting and heating costs have already been presented in earlier paragraphs. The apartment rent for an individual is determined by the following formula:

$$ar = \left(\frac{lr}{\alpha_1} + bc\right)\alpha \qquad (13)$$

where:

lr is land rent per unit of area per unit of time
α_1 is number of storeys
bc corresponds to building costs (construction and maintenance) per unit of area per unit of time and is a function of number of storeys and size of apartment
α is habitable space demanded per person. It is assumed in the model that it is a function of income and price:[12]

$$\alpha = h(ic)^{\theta_1}(ar)^{\theta_2} \nless \alpha_{min}{}^* \qquad (14)$$

where:

h is a constant
ic is income class assumed for individual
θ_1 and θ_2 are given elasticity coefficients. For Västerås these have been assumed to be 1 and −0.4, respectively.

* This demand function can be deduced from a utility function of Cobb-Douglas type

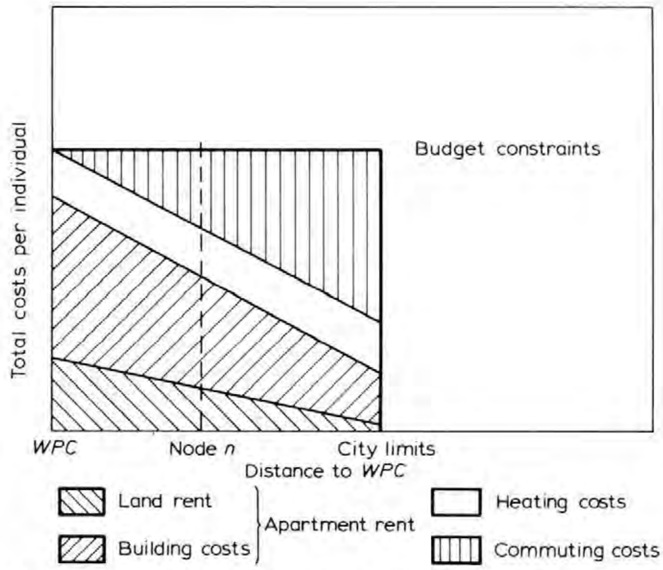

Figure 6 Equilibrium condition for determination of land rent distribution

The value of the constant in equation (12) can be obtained for each income class by applying this equation to the land at the city limits. The land rent at the city limits corresponds to the value of the land when used for agriculture and should be quite low. For Västerås it was assumed to be zero. The city limits are determined by the set of nodes for which the land rent is a minimum, i.e.:

$$lr = \text{minimum} \qquad (15)$$

Therefore, using equation (12), the city limits can also be obtained as the set of nodes for which:

$$\frac{\alpha_1}{\alpha}(bc \cdot \alpha + \text{commuting costs} + \text{heating costs}) = \text{minimum} \qquad (16)$$

When the value of the constant in equation (12) has been determined, the land rent for each income class at each node can be obtained using (12) again at this node.

The calculated land rent distribution reflects the scarcity of land in urban uses. That is, land is more expensive near the city centre than at the city limits. Therefore when residences are built in a city node n, the land costs per unit of time can be evaluated using the following formula (use of the subscript n is resumed):

$$\frac{\alpha_n \cdot P_n}{\alpha_{1n}} \cdot lr_n \qquad (17)$$

Expression (17) is consistent with (13).

Model for economic evaluation

The models described above are used to simulate the various city activities. They are solved iteratively, and once convergence to an equilibrium solution has been achieved, the model for economic evaluation can be used.

The following cost items are included in the economic evaluation:

Costs for land	TC^A
Costs for residences	TC^B
Costs for the transportation system	TC^C
Costs for heating	TC^D

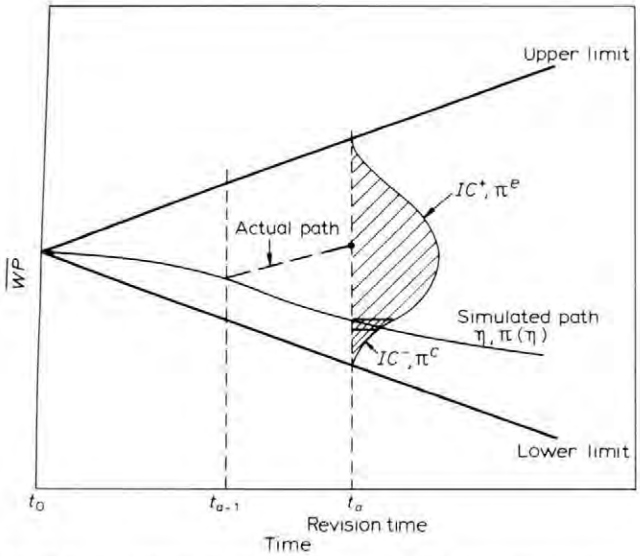

Figure 7 Simulation of working opportunities for expansion and contraction cases

Costs for schools	TC^E
Revision costs	TC^F

All of these costs are computed for each intersection point of time, t_a. Revision costs are illustrated in *Figure 7*.

At each time t_a it is possible to review the situation, comparing the simulated path for working possibilities with the path that has actually occurred up to that point of time. There may of course be deviations in either direction, that is there may be expansion or contraction relative to the simulated path. The differences in costs for these two cases can be computed and designated IC^+ and IC^-, respectively. The probabilities associated with the two cases are π^e and π^c, respectively. The revision cost is then formulated as follows:

$$\mu_1 IC^+\pi^e + \mu_2 IC^-\pi^c \qquad (18)$$

where μ_1 and μ_2 are two risk aversion factors. These factors will usually have the value 1.

For each period of time the total costs are calculated as follows:

$$TC(t_a) = TC^A + TC^B + TC^C + TC^D + TC^E + TC^F \qquad (19)$$

Various planning alternatives can be compared with each other by the basis of an objective function OF. The one used here is defined according to the following considerations. The total costs incurred along the simulated path η, designated U, are:

$$U = \int_0^T TC(t_a)\,\mathrm{d}t_a \equiv \mu_\eta \qquad (20)$$

U is a random variable that depends on the working population path and the probability of its occurrence is the probability of η, i.e. $\pi(\eta)$. The mean value and variance of U can be defined as follows:[13]

$$\text{Mean value} = \bar{U} = E[U] \qquad (21)$$

$$\text{Variance} = \sigma^2 = E[(U - \bar{U})^2] \qquad (22)$$

The objective function is then expressed as follows:

$$OF = \bar{U} + \mu_3\sigma^2 \qquad (23)$$

where μ_3 is a risk aversion factor.

This objective function permits the various master city plans to be compared.

Results

The main results of this study are total costs for different master city plans. Examples of such results will be presented in the next section. In addition, many types of complementary results can be obtained from the model. Examples of preliminary results of this type are presented later.

Total costs

The results presented here are preliminary. Results corresponding to only a single specific path for the working population are presented. The same path has been used for each of the alternatives studied to facilitate comparison. The costs for two different alternatives, a satellite alternative (B) and an urban renewal alternative (D) are shown in *Table 3*. The costs are given in 1979 values.

The following observations can be made on the basis of *Table 3*. As expected, the urban renewal alternative is more expensive than the satellite alternative, costing SEK 519 million more. Land costs are somewhat greater in the urban renewal alternative, because of the higher land rents in the inner city. Also, an additional 3000 apartments in the inner city must be demolished and replaced in the urban renewal alternative as compared to the satellite alternative. Owing to this, costs for residences are SEK 624 million more in alternative D.

On the other hand, the other costs are lower in the urban renewal alternative. The reasons for this are:

Fewer new roads are necessary in alternative D; the existing road system in the inner city can be used more intensively. Commuting costs are lower since the average commuting distance is shorter.

A larger, more efficient boiler can be used for the district heating system and fewer new heating pipes need be installed.

The capacity of the existing schools of the inner city can be used to a greater extent. Fewer new schools need be built.

The difference in commuting costs between the two plans is small and not significant. The reasons for this may be the existence of two counterbalancing forces: the average commuting distance is greater in the satellite alternative, but the congestion is relatively greater in the urban renewal alternative. The commuting patterns for the two alternatives are similar.

Inspection of the results show where it may be possible to improve a planning alterntive by making marginal changes. For example, the urban renewal alternative may be

Table 3 Total costs for alternatives B and D

Group of cost items	Satellite alternative (B)	Urban renewal alternative (D)	Difference (D – B)
Costs of land	17	40	+23
Costs for residences	1049	1673	+624
Costs for roads	100	54	–46
Commuting costs	1144	1118	–26
Heating costs	1465	1432	–33
School costs	209	186	–23
Total costs	3984	4503	+519

All costs are expressed in millions of Swedish Crowns (SEK)

Table 4 Observed and simulated commuting patterns in Västerås

Mode of commuting	Observed commuting pattern for 1975*	Simulated commuting pattern for 1980–81
Walking	6 079	4 521
Bicycling	9 583	10 716
By car	25 055	26 147
By bus	11 553	11 745

*Source: Bertil Claesson, Road and Traffic Office in Västerås

improved by decreasing the number of apartments to be demolished, thus reducing the costs for residences significantly.

Some complementary results

The results presented below mainly concern the simulation of different activities in the city.

It is important to calibrate the model so that the initial siuation as modelled corresponds to the actual initial situation of the city. This means that the various parameters of the model must be given values so that the values obtained from the model for the initial year are close to the observed values. *Table 4* shows the observed and simulated commuting patterns in Västerås as an example. As can be seen, the correspondence is quite good.

Several results for the various activities of the city can be obtained from the model for the various points of time. Some of them are:

Figure 8 Localization of schools. Initial time

Population and working population.
Distribution of population density.
Distribution of income classes throughout city.
Number of workers assigned to various working place centres.
Number of school children assigned to various schools.
Number of apartments, type of housing, size of habitable area per person and number of storeys in various nodes.
Pattern of commuting.
Congestion pattern.
Distribution of land rents.

Some of these results are shown in graphical form in *Figures 8–10. Figure 8* shows the location of the 43 schools

Figure 9 Assignment of residential nodes to schools. Alternative B. Period 1995–2000

in Västerås at the initial point of time. Several of these schools were closed during the period of the study due to physical obsolescence according to the data assumptions used. *Figures 9* and *10* show the locations of the schools remaining as well as for the new schools for alternatives B and D, respectively, for the period 1995-2000. Also the assignment of the school children from different residential nodes to the schools according to the rule of least individual school commuting costs is presented. In the satellite of Dingtuna there is a new school. (no. 46) in alternative B during this period (see *Figure 9*). However, in alternative D the old school is closed and no new one built in this satellite. Also, a maximum busing distance of 5 km is assumed in this computer run. Therefore, in this plan the

Figure 10 Assignment of residential nodes to schools. Alternative D, Period 1995-2000

school children of Dingtuna have no possibility of attending school at all (see *Figure 10*). In this way these maps can be used to illustrate the consequences of different plans and may give clues for changes in the plans that can be worthwhile in an iterative evaluation procedure.

Conclusions

Owing to the strong degree of interdependency among the various sections of a city and to irreversibilities, it would seem that the municipal government has to develop some sort of master city planning if the problems of city planning are to be solved efficiently. This means that the government needs some method to evaluate alternative master city plans as to their costs and benefits. The model presented in this paper is an attempt to create such a tool. Although only the cost side is explicitly dealt with, it may be a valuable tool in the decision process in that it enables the city planners to determine just how great the benefits of a more costly alternative should be to make that alternative preferable to a less costly alternative with lesser benefits.

This model is presently being applied o the city of Västerås, Sweden, in a pilot study. Application of the model to two master city plans for Västerås that are polar in a sense – one 'satellite' alternative and an 'urban renewal' alternative – has given seemingly reasonable preliminary results.

However, it is obvious that there are numerous ways in which this first version of the model can be improved. For example, alternative modes of heating, such as electric heating and individual oil heating, can be introduced. The water and sewage system could also be included in the model. Items on the benefit side may also be included, even though the problems of estimation involved are considerably more difficult.

Another possible area of improvement would be the determination of the optimal number of nodes or zones necessary for an accurate simulation of the city activities. Perhaps the number of nodes used for Västerås has been unnecessarily large. The input data could be arranged in a way that is easier for the user to understand and change. Steps in this direction have already been taken.

The models can also be divided into its submodels so that the planning offices for the various sectors can better 'control' the setting up of 'their' sector plans. This may facilitate the improvement of the plans under consideration using a common data bank.

It is also possible to refine the submodels to make them more sophisticated. However, it is necessary to realize that there is a conflict between the users' requirements for greater realism in the modelling and their simultaneous demand that the model be simplified so that it is easier to understand and manage. A model for the evaluation of master city plans necessarily treats different city activities simultaneously. Drastic simplifications, that might be too simple for the individual sector models separately, are necessary in such a model. Thus, finding the optimum trade-off between realism and simplicity where master city plans are concerned is a delicate matter. At present, the planners in Västerås are particularly desirous of a simpler model.

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