Ruido en Receptores

1- Tipos de ruido

2- Relación señal-ruido (SNR) en receptores

3- Límites del receptor con amplificador

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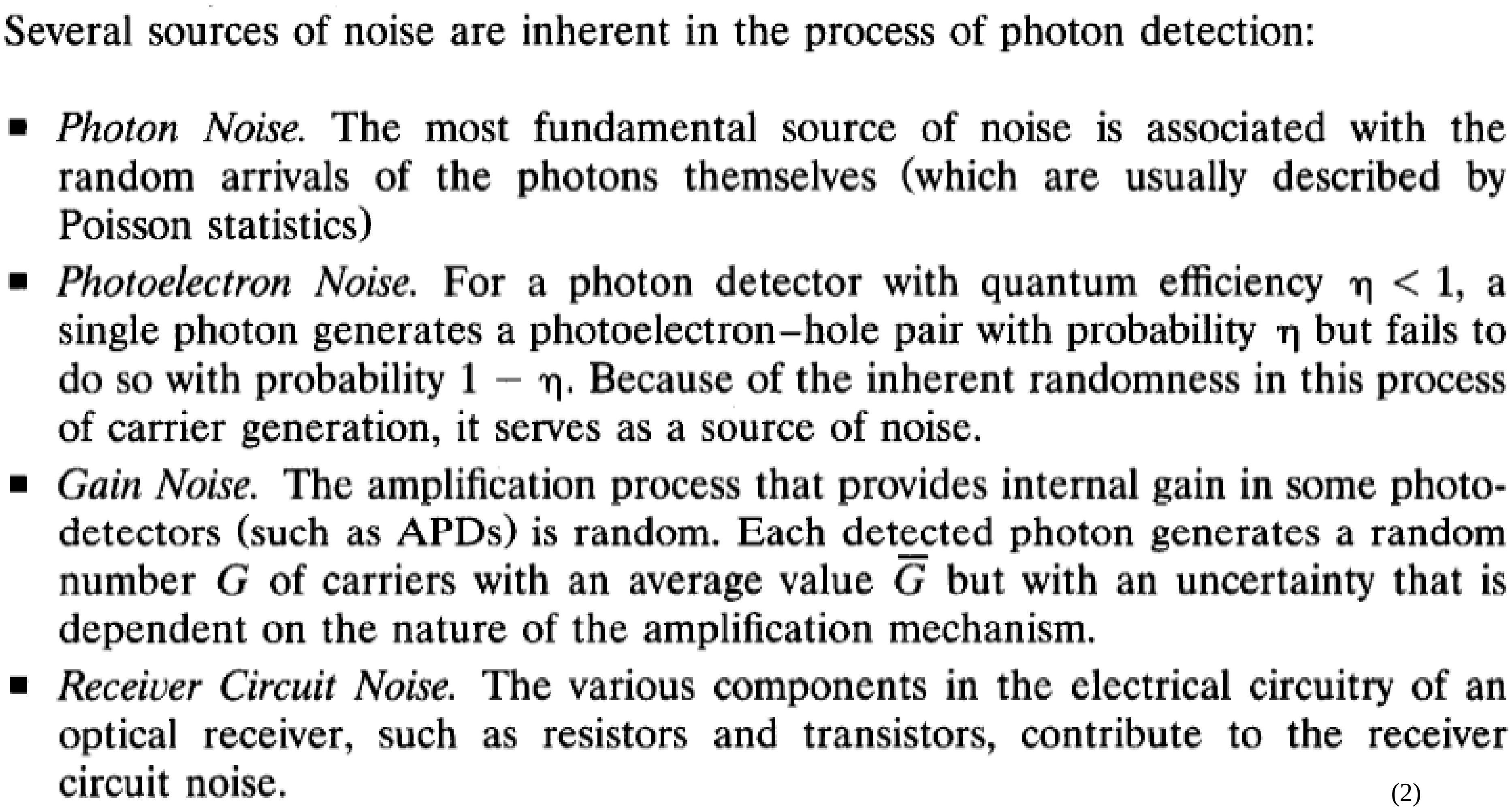
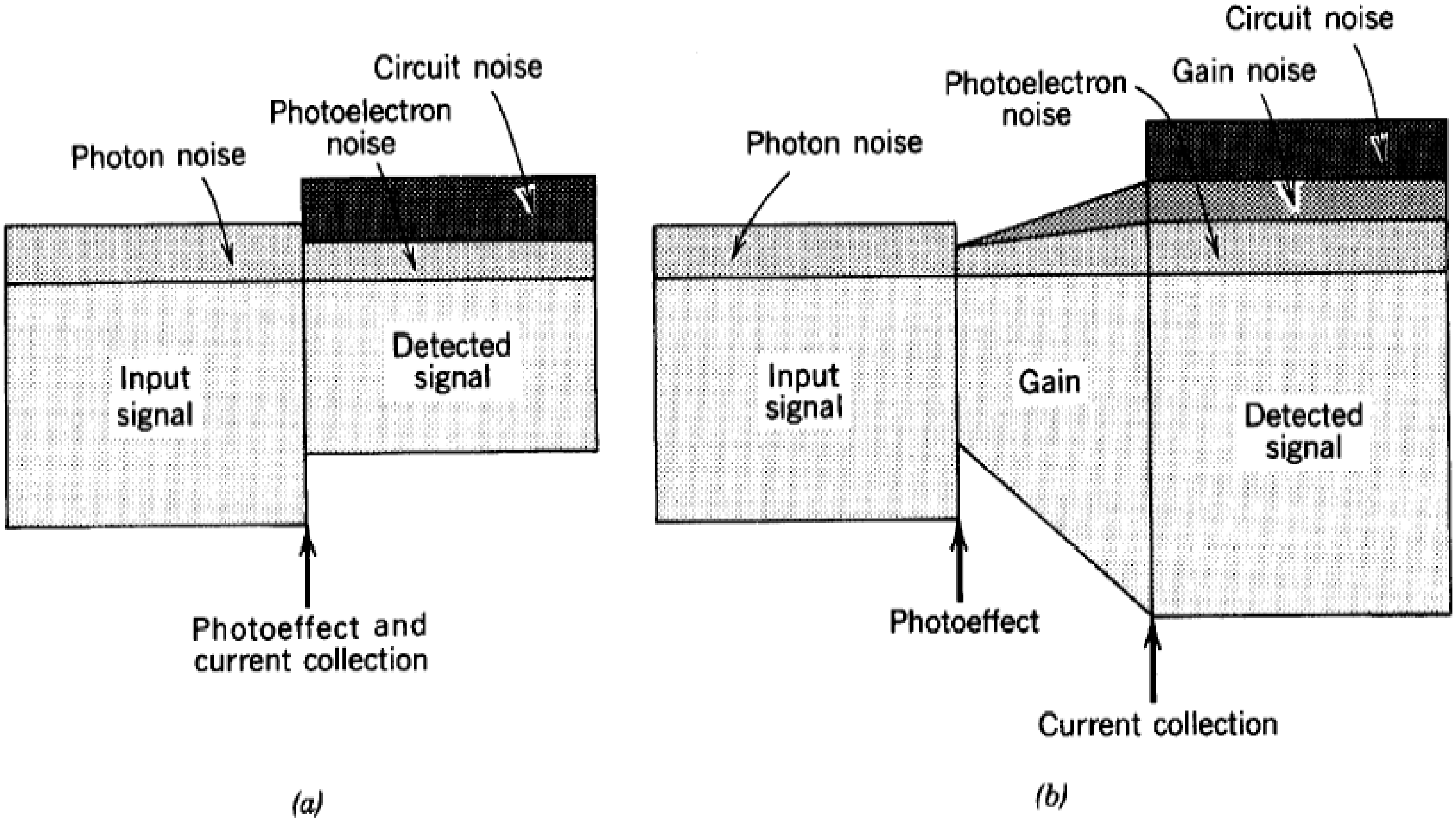
1- Tipos de ruido



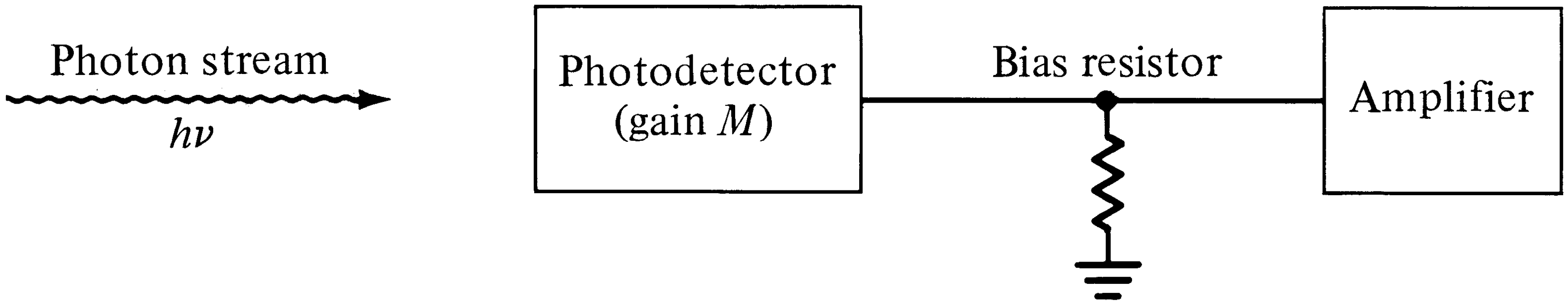
Several sources of noise are inherent in the process of photon detection:

- *Photon Noise.* The most fundamental source of noise is associated with the random arrivals of the photons themselves (which are usually described by Poisson statistics)
- *Photoelectron Noise.* For a photon detector with quantum efficiency $\eta < 1$, a single photon generates a photoelectron–hole pair with probability η but fails to do so with probability $1 - \eta$. Because of the inherent randomness in this process of carrier generation, it serves as a source of noise.
- *Gain Noise.* The amplification process that provides internal gain in some photodetectors (such as APDs) is random. Each detected photon generates a random number G of carriers with an average value $\overline{G}$ but with an uncertainty that is dependent on the nature of the amplification mechanism.
- *Receiver Circuit Noise.* The various components in the electrical circuitry of an optical receiver, such as resistors and transistors, contribute to the receiver circuit noise.

(2)



Signal and various noise sources for (*a*) a photodetector without gain (e.g., a *p-i-n* photodiode) and (*b*) a photodetector with gain (e.g., an APD). (2)



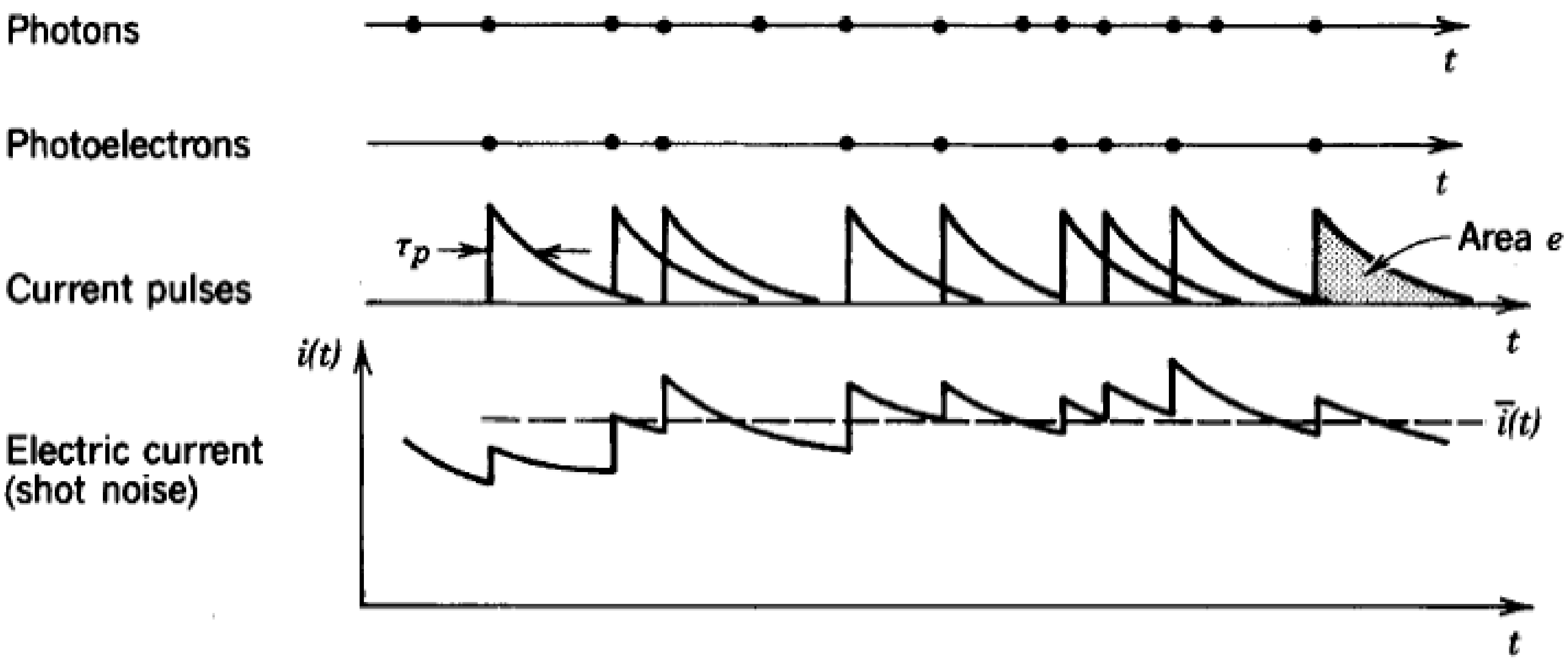
- Photon detection quantum noise (Poisson fluctuation)

- Bulk dark current
- Surface leakage current
- Statistical gain fluctuation (for avalanche photodiodes)

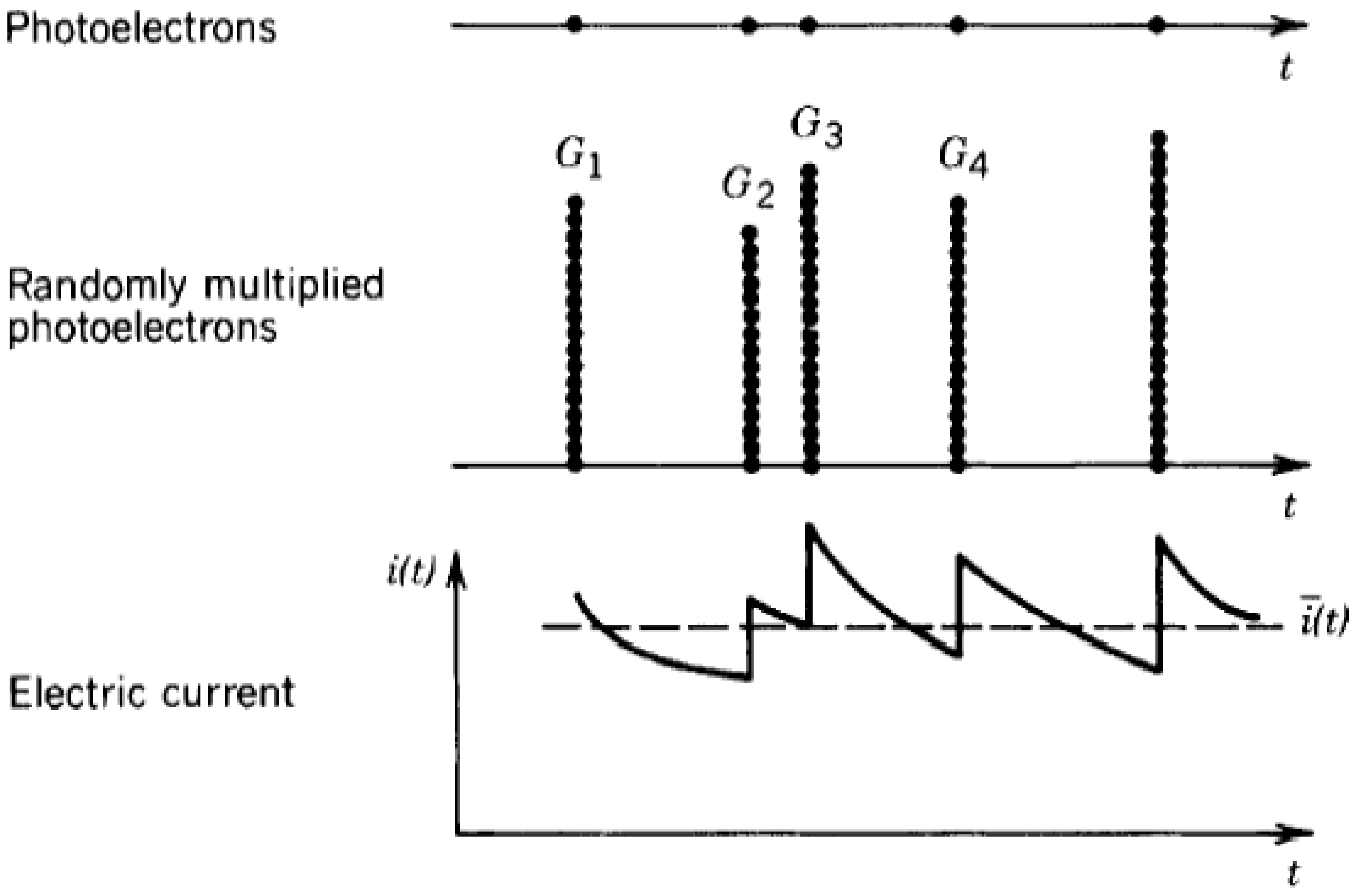
- Thermal noise

- Amplifier noise

(2)


Photons
t
Photoelectrons
t
Current pulses
τ_p
Area e
t
$i(t)$
Electric current
(shot noise)
$\bar{i}(t)$
t

(2)


Photoelectrons
t
G1
G2
G3
G4
Randomly multiplied photoelectrons
t
i(t)
ī(t)
Electric current
t

(2)

Ruido Shot (PIN)

Naturaleza discreta de los fotones detectados → (disparo, granalla, lluvia, gravilla)

Densidad espectral de ruido constante → $\underbrace{S_s(f)}_{\text{Densidad espectral potencia}} = q\bar{I}$ (blanco → todo el espectro)

$$\boxed{\sigma_{i_S}^{\ 2} = 2q\bar{I}\underbrace{\Delta f}_{\text{Ancho de banda eléctrico del receptor}}} \quad \left[\mathrm{A}^2\right]$$

$$\underbrace{\bar{I}}_{\text{Corriente media continua}} = \Re(P_{in} + P_{BG}) + I_d \approx \underbrace{\Re P_{in}}_{I_{ph}} + I_d = I_{ph} + I_d$$

Señal contínua → $\boxed{P_{señal} = R_L I_{ph}^{\ 2} = R_L(\Re P_{in})^2}$ $[\mathrm{W}]$

Ruido shot → $\boxed{P_{\substack{ruido\\shot}} = R_L \sigma_{i_S}^{\ 2} = R_L 2q\bar{I}\Delta f}$ $[\mathrm{W}]$

Ruido Shot (APD)

Los pares e$^-$-h$^+$ secundarios se generan en tiempos aleatorios

→ ruido shot añadido

$$\underbrace{\bar{I}_{APD}}_{\text{Corriente media continua}} = M(\Re(P_{in}+P_{BG})+I_d) \approx M\,\underbrace{\Re P_{in}}_{I_{ph}} + MI_d$$

Llamando $\underbrace{\bar{I}}_{M=1} = \Re P_{in} + I_d = I_{ph} + I_d \rightarrow \bar{I}_{APD} = M\,\bar{I}$

$$\boxed{\sigma_{i_S}{}^2 = 2q\boxed{M^2\ \underbrace{F_{APD}}_{\text{Excess noise factor}}}\bar{I}\Delta f} \qquad \left[\mathrm{A}^2\right]$$

$$F_{APD}(M) \approx M^x \quad \left[0 \le x \le 1\right]\begin{cases} Si \rightarrow 0,3 < x < 0,5 \\ Ge, III\text{-}V \rightarrow 0,7 < x < 1 \end{cases}$$

$$\boxed{M=1 \quad F_{APD}=1 \quad \rightarrow PIN}$$

Señal contínua → $\boxed{P_{señal} = R_L M^2 I_{ph}{}^2 = R_L(M\Re P_{in})^2}$

Ruido Shot → $\boxed{P_{\substack{ruido\\shot}} = R_L\sigma_{i_S}{}^2 = R_L 2qM^2F_{APD}\bar{I}\Delta f}$

Ruido Térmico (Johnson noise, Nyquist noise)

$T \neq 0 \rightarrow$ movimiento de e$^-$ $\rightarrow$ fluctuaciones aleatorias de corriente $\rightarrow$

$\rightarrow$ ruido térmico

Relación con el cuerpo negro $\rightarrow f < 1THz \rightarrow \underbrace{S_T(f)}_{\text{Densidad espectral potencia}} = \dfrac{2k_BT}{R_L}$ $\quad T\left[{}^{\text{o}}K\right]$

$$\boxed{\sigma_{i_T}{}^2 = \frac{4k_BT}{R_L} \underbrace{\Delta f}_{\substack{\text{Ancho de banda}\\ \text{eléctrico}\\ \text{del receptor}}}} \quad \left[\text{A}^2\right]$$

$$\boxed{P_{\substack{ruido\\ térmico}} = R_L\sigma_{i_T}{}^2 = 4k_BT\Delta f} \quad \left[\text{W}\right]$$

Ruido del Amplificador

Hay dos modelos

1) Con figura de ruido del amplificador

-Ruido Shot de componentes activos

-Ruido térmico de los componentes resistivos

Existe aunque $T = 0^{\circ} K,$ y no haya señal de entrada

-G (Ganancia del amplificador)

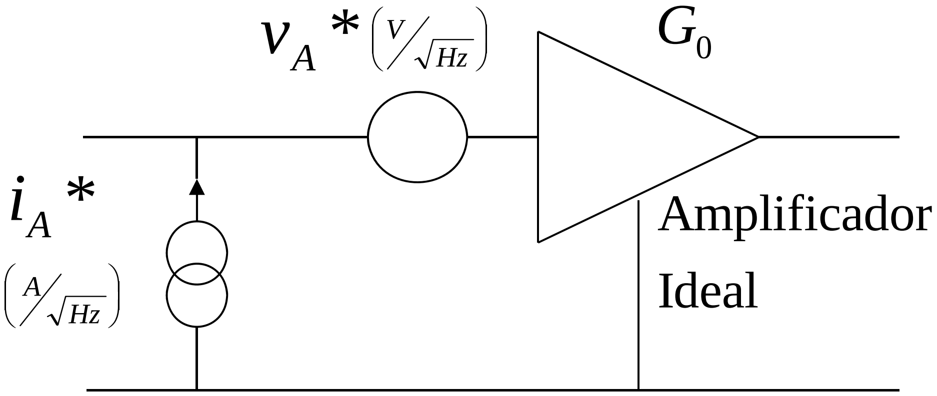
$$\boxed{\sigma_{i_A}{}^2 = \frac{4Gk_B \boxed{T_A}}{R_L} \Delta f}$$

$$\boxed{T + T_A = \underbrace{T_e}_{\text{Temperatura equivalente}} = \underbrace{F_n}_{\text{Figura de ruido del amplificador}} T} \qquad F_n = 1 + \frac{T_A}{T}$$

2) Con tensión y corriente de ruido del amplificador

$* \rightarrow$ Por unidad de ancho de banda (densidad espectral)

Modelo de amplificador real



$$\left(\sigma_{i_S}{}^*\right)^2 = 2qM^2F_{APD}\overline{I}$$

$$\left(\sigma_{i_T}{}^*\right)^2 = \frac{4k_BT}{R_L}$$

$$\left(i_T *\right)^2 = \left(\sigma_{i_S}{}^*\right)^2 + \left(\sigma_{i_T}{}^*\right)^2 + \left(i_A *\right)^2 = 2qM^2F_{APD}\overline{I} + \frac{4k_BT}{R_L} + \left(i_A *\right)^2$$

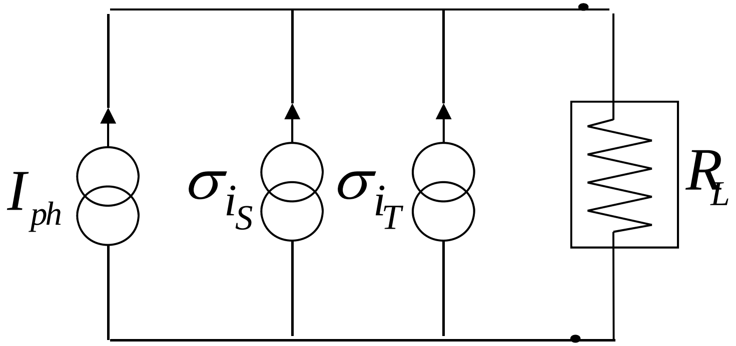
2- Relación señal-ruido (SNR) en receptores

(I_{ph} contínua)

1) Modelo 1

1-a) Receptor sin amplificador

$$SNR=\frac{S}{N}=\frac{M^2R_LI_{ph}^2}{R_L\sigma_{i_S}^2+R_L\sigma_{i_T}^2}=\frac{M^2\Re^2\boxed{P_{in}^2}}{2qM^2F_{APD}(\Re\boxed{P_{in}}+I_d)\Delta f+\frac{4k_BT}{R_L}\Delta f}$$

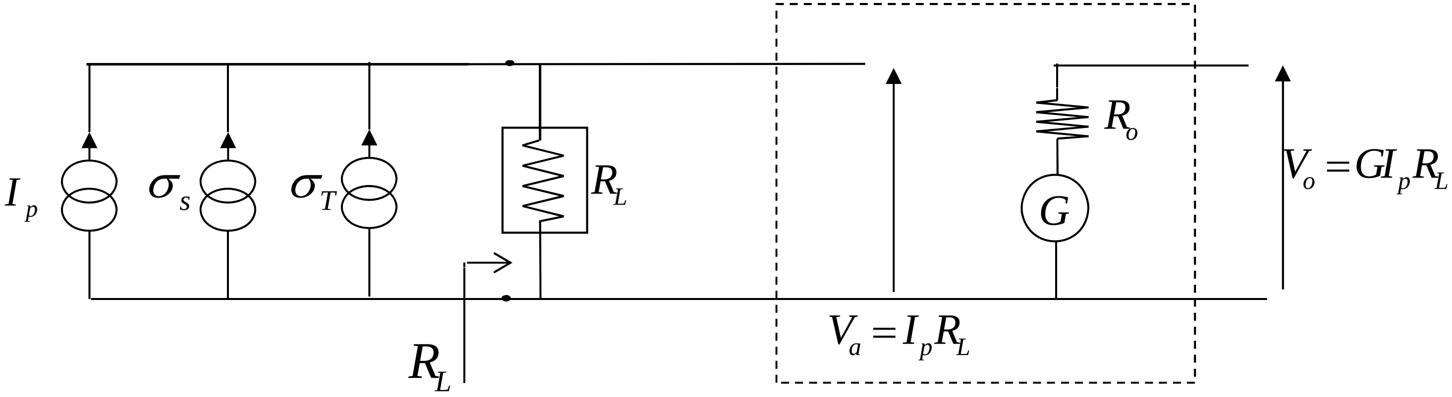


1-b) Receptor con un amplificador (alta impedancia) (HZ)

$$SNR = \frac{S}{N} = \frac{GM^2 R_L I_{ph}^2}{GR_L\sigma_{i_S}^2 + GR_L\sigma_{i_T}^2 + R_L\sigma_{i_A}^2} = \frac{M^2 I_{ph}^2}{\sigma_{i_S}^2+\sigma_{i_T}^2+\frac{1}{G}\sigma_{i_A}^2} =$$

$$= \frac{M^2 I_{ph}^2}{2qM^2F_{APD}(I_{ph}+I_d)\Delta f + \underbrace{\frac{4k_BT}{R_L}+\frac{4k_BT_A}{R_L}}_{\frac{4k_BT}{R_L}\boxed{F_n}}\Delta f} =$$

$$= \frac{M^2\Re^2\boxed{P_{in}^2}}{2qM^2F_{APD}(\Re\boxed{P_{in}}+I_d)\Delta f + \frac{4k_BF_nT}{R_L}\Delta f}$$

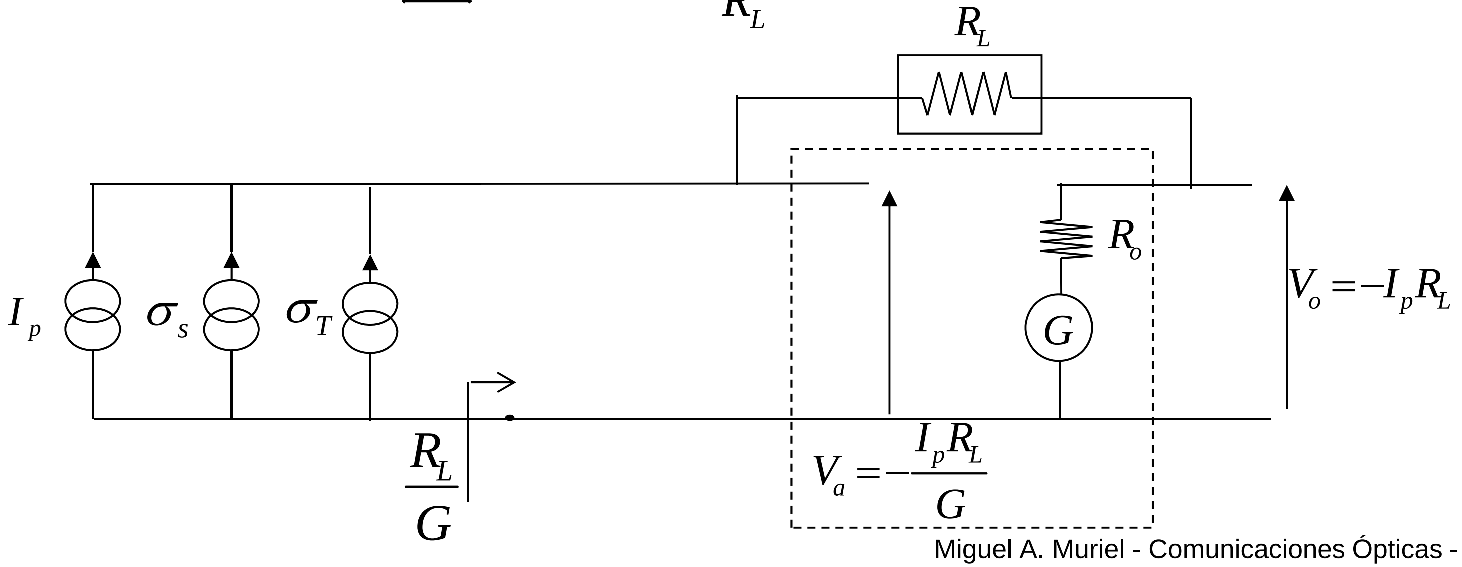


1-c) Receptor con un amplificador (transimpedancia) (TZ)

$$SNR=\frac{S}{N}=\frac{M^2 R_L I_{ph}{}^2}{G(\frac{R_L}{G})\sigma_{i_S}{}^2+G(\frac{R_L}{G})\sigma_{i_T}{}^2+(\frac{R_L}{G})\sigma_{i_A}{}^2}=\frac{M^2 I_{ph}{}^2}{\sigma_{i_S}{}^2+\sigma_{i_T}{}^2+\frac{1}{G}\sigma_{i_A}{}^2}=$$

$$=\frac{M^2 I_{ph}{}^2}{2qM^2F_{APD}(I_{ph}+I_d)\Delta f+\underbrace{\frac{4k_BT}{R_L}+\frac{4k_BT_A}{R_L}}_{\frac{4k_BT}{R_L}\boxed{F_n}}\Delta f}=$$

$$=\frac{M^2\Re^2\boxed{P_{in}{}^2}}{2qM^2F_{APD}(\Re\boxed{P_{in}}+I_d)\Delta f+\frac{4k_BF_nT}{R_L}\Delta f}$$



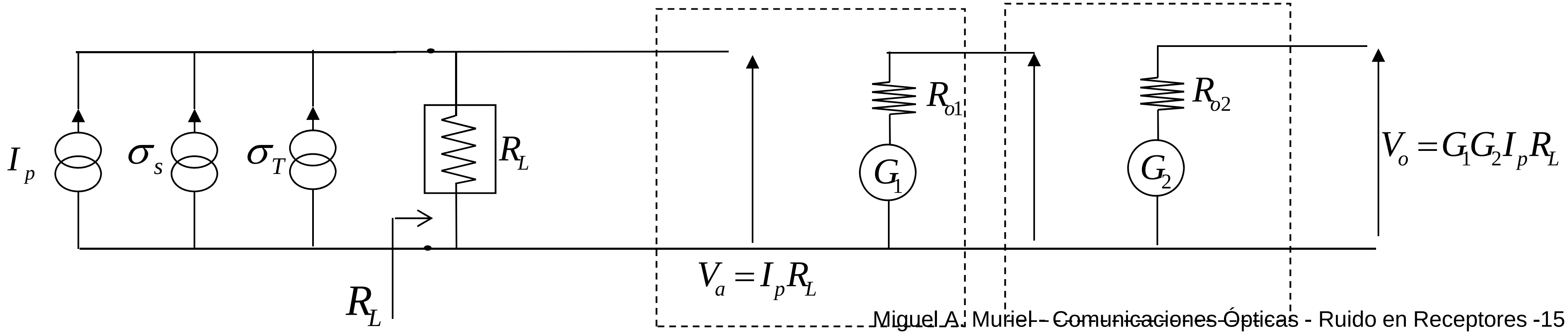
1-d) Receptor con dos amplificadores (alta impedancia) (HZ)

$$SNR=\frac{S}{N}=\frac{G_1G_2M^2R_LI_{ph}^{\ 2}}{G_1G_2R_L\sigma_{i_S}^{\ 2}+G_1G_2R_L(\frac{4k_BT}{R_L}\Delta f)+G_1G_2R_L(\frac{4k_BT_{A1}}{R_L}\Delta f)+G_2R_L(\frac{4k_BT_{A2}}{R_L}\Delta f)}=$$

$$=\frac{M^2I_{ph}^{\ 2}}{\sigma_{i_S}^{\ 2}+(\frac{4k_BT}{R_L}\Delta f)+(\frac{4k_BT_{A1}}{R_L}\Delta f)+\frac{1}{G_1}R_L(\frac{4k_BT_{A2}}{R_L}\Delta f)}=$$

$$=\frac{M^2I_p^{\ 2}}{2qM^2F_{APD}(I_{ph}+I_d)\Delta f+\frac{4k_BT}{R_L}(T+T_{A1}+\frac{T_{A2}}{G_1})\Delta f}=\frac{I_p^{\ 2}}{2qM^2F_{APD}(I_{ph}+I_d)\Delta f+\frac{4k_BT}{R_L}F_{n1}\Delta f+\frac{4k_B}{R_L}\frac{T_{A2}}{G_1}\Delta f}$$

$$=\frac{M^2\Re^2\boxed{P_{in}^{\ 2}}}{\underbrace{\underbrace{2qM^2F_{APD}(\Re\boxed{P_{in}}+I_d)\Delta f+\frac{4k_BT}{R_L}F_{n1}\Delta f}_{\text{1 amplificador}}+\frac{4k_B}{R_L}\frac{T_{A2}}{\boxed{G_1}}\Delta f}_{\text{2 amplificadores}}}$$



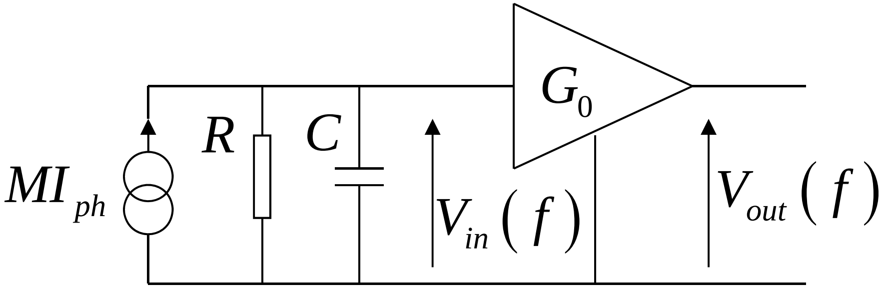
2) Modelo 2

2-a) Amplificador de tensión (HZ)

Cálculo de la señal

$$C = C_{photodiode} + C_{amplifier}$$

$$R = R_{photodiode} \| R_{amplifier}$$



Sin ecualizar →

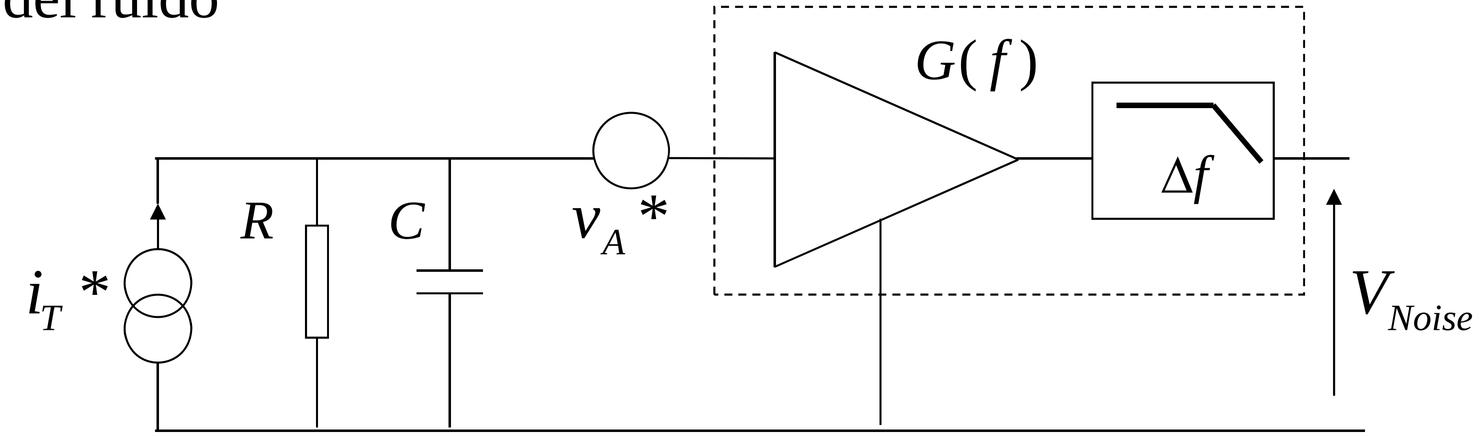
$$V_{in}(f) = MI_{ph}(R\|C) = MI_{ph}\left(\frac{1}{\frac{1}{R} + j2\pi fC}\right) = MI_{ph}\left(\frac{R}{1 + j2\pi fRC}\right)$$

$$\rightarrow V_{out}(f) = G_0 V_{in}(f) = G_0 \frac{RMI_{ph}}{1 + j2\pi fCR} \rightarrow \boxed{\Delta f = \frac{1}{2\pi RC}} \leftarrow \text{Muy baja}$$

Con ecualización →

$$G(f) = G_0\left(1 + j2\pi fCR\right) \rightarrow \boxed{V_{out}(f) = G_0 RMI_{ph}}$$

Cálculo del ruido



$$\left(i_T\,*\right)^2=\left(\sigma_{i_S}{}^{*}\right)^2+\left(\sigma_{i_T}{}^{*}\right)^2+\left(i_A\,*\right)^2=2qM^2F_{APD}\overline{I}+\frac{4k_BT}{R}+\left(i_A\,*\right)^2$$

$$\left|V_{Noise}\right|^2=\int_0^{\Delta f}\left|G(f)\right|^2(v_A*)^2\,df+\int_0^{\Delta f}\frac{\left|G(f)\right|^2R^2(i_T*)^2}{\left|1+j2\pi fCR\right|^2}df=$$

$$=G_0{}^2\int_0^{\Delta f}\left[\left|1+4\pi^2f^2R^2C^2\right|(v_A*)^2+R^2(i_T*)^2\right]df=$$

$$=G_0{}^2\left(\left(1+\left(\frac{4\pi^2}{3}\right)R^2C^2\Delta f^2\right)(v_A*)^2+R^2(i_T*)^2\right)\Delta f$$

$$\frac{S}{N}=\left(\frac{V_{out}}{\left|V_{Noise}\right|}\right)^2=$$

$$=\frac{(G_0RMI_{ph})^2}{G_0{}^2\left((v_A*)^2\left(1+\frac{4\pi^2}{3}\left(\Delta f\right)^2R^2C^2\right)+2qR^2\left(I_{ph}+I_d\right)M^2F_{APD}+4k_BTR+R^2(i_A*)^2\right)\Delta f}$$

$$=\frac{I_{ph}{}^2}{\left(\frac{(v_A*)^2}{M^2}\left(\underbrace{\frac{1}{R^2}}_{a}+\underbrace{\frac{4\pi^2}{3}\left(\Delta f\right)^2C^2}_{b}\right)+\underbrace{2q\left(I_{ph}+I_d\right)F_{APD}}_{c}+\underbrace{\frac{4k_BT}{M^2R}}_{d}+\underbrace{\frac{(i_A*)^2}{M^2}}_{e}\right)\Delta f}$$

M ↑→ S/N ↑ hasta que (c) se hace significativo → Valor óptimo de M

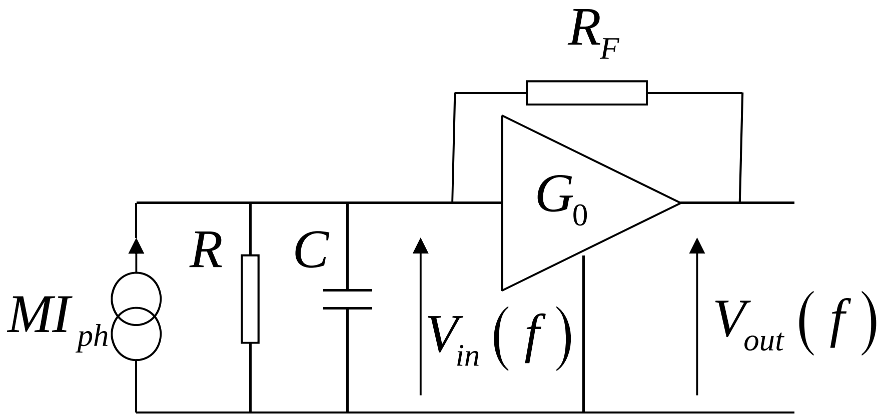
R ↑→ S/N ↑ *m*ientras que (a) y (d) sean significativos

Δf ↑→ el término predominante es (b)

2-b) Amplificador de transimpedancia (TZ)

$$C = C_{photodiode} + C_{amplifier}$$

$$R = R_{photodiode} \, \| \, R_{amplifier}$$



$$V_{in}(f) = \frac{V_{out}(f)}{G_0}$$

$$MI_{ph} + \frac{V_{out}(f) - V_{in}(f)}{R_F} = V_{in}(f)\left(\frac{1}{R} + j2\pi fC\right) \rightarrow$$

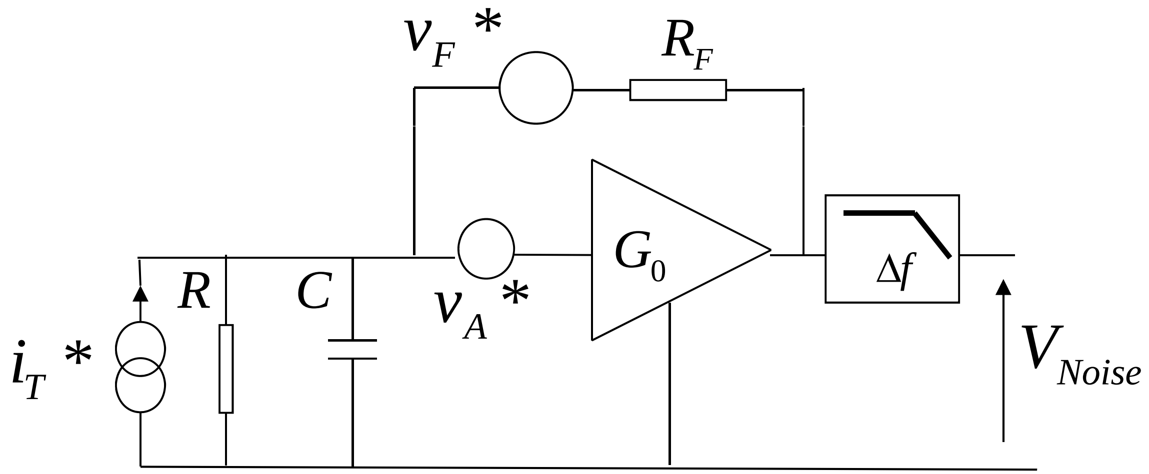
$$MI_{ph} = V_{out}(f)\left(-\frac{1}{R_F} + \frac{1}{G_0 R_F} + \frac{1}{G_0 R} + \frac{j2\pi fC}{G_0}\right) \approx V_{out}(f)\left(-\frac{1}{R_F} + \frac{j2\pi fC}{G_0}\right) \rightarrow$$

$$V_{out}(f) = -\frac{G_0 R_F MI_{ph}}{G_0 - j2\pi f R_F C} = -\frac{R_F MI_{ph}}{1 - j2\pi f \frac{R_F}{G_0} C}$$

$\rightarrow \boxed{\Delta f = \frac{G_0}{2\pi R_F C}}$ → No se necesita ecualización si $f < \Delta f$

Si se ecualiza → $V_{out}(f) = -\frac{R_F MI_{ph}}{1 - j2\pi f \frac{R_F}{G_0} C}\left(1 - j2\pi f \frac{R_F}{G_0} C\right) = -R_F MI_{ph}$

Ganancia de transimpedancia → R_F



$$\left(i_T*\right)^2=\left(\sigma_{i_S}{}^{*}\right)^2+\left(\sigma_{i_T}{}^{*}\right)^2+\left(i_A*\right)^2=2qM^2F_{APD}\overline{I}+\frac{4k_BT}{R}+\left(i_A*\right)^2$$

$$v^{*}{}_{F}=\sqrt{4kTR_F}$$

$$\frac{S}{N}=\frac{I_{ph}{}^{2}}{\left(\frac{(v_A*)^2}{M^2}\left(\left(\frac{1}{R}+\frac{1}{R_F}\right)^2+\frac{4\pi^2}{3}\left(\Delta f\right)^2C^2\right)+2q\left(I_{ph}+I_d\right)F_{APD}+\frac{4k_BT}{M^2}\left(\frac{1}{R}+\frac{1}{R_F}\right)+\frac{(i_A*)^2}{M^2}\right)\Delta f}$$

$$\underbrace{\frac{1}{R}}_{HZ}\rightarrow\underbrace{\frac{1}{R}+\frac{1}{R_F}}_{TZ}$$

Ganancia óptima del APD

$$SNR_{\text{Máximo}} \rightarrow \frac{d\left(SNR\right)}{dM} = 0$$

$$M_{\text{óptimo}} \neq \Delta f$$

$$\text{Si} \rightarrow M_{\text{opt}} \approx 100$$

$$\text{InGaAs} \rightarrow M_{\text{opt}} \approx 10$$

3- Límites del receptor con amplificador

Modelo-1

1) Limite ruido térmico $(\sigma_{i_T}{}^2 \gg \sigma_{i_S}{}^2)$

$$\lim_{(\sigma_{i_T}{}^2 \gg \sigma_{i_S}{}^2)} \frac{S}{N} = \lim_{(\sigma_{i_T}{}^2 \gg \sigma_{i_S}{}^2)} \frac{M^2 \Re^2 \boxed{P_{in}{}^2}}{2qM^2 F_{APD}(\Re \boxed{P_{in}} + I_d)\Delta f + \frac{4k_B F_n T}{R_L}\Delta f}$$

$$SNR = \frac{I_{ph}{}^2}{\sigma_{i_T}{}^2} = \frac{I_{ph}{}^2}{\frac{4k_B T}{R_L} F_n \Delta f} = \underbrace{\frac{R_L \Re^2 \boxed{P_{in}{}^2}}{4k_B T F_n \Delta f}}_{\text{PIN}} M^2$$

La SNR aumenta con el cuadrado de la potencia incidente

Es lo habitual en fotodiodos PIN

Cuantificación del ruido térmico → Potencia equivalente de ruido ($\underbrace{NEP}_{\text{Noise Equivalent Power}} = \frac{P_{in}}{\sqrt{\Delta f}}$)

→ Potencia mínima necesaria, por unidad de raiz de ancho de banda eléctrico, para tener $\frac{S}{N} = 1$

$$\rightarrow \frac{S}{N} = \frac{R_L \Re^2}{4k_B T F_n} \underbrace{\frac{{P_{in(mímima)}}^2}{\Delta f}}_{(NEP)^2} = 1 \quad \rightarrow \quad NEP = \frac{P_{in(mínima)}}{\sqrt{\Delta f}} = \frac{2}{\Re}\sqrt{\frac{k_B T F_n}{R_L}} = 2\frac{h\nu}{q\eta}\sqrt{\frac{k_B T F_n}{R_L}}$$

Valores típicos de NEP → $1\text{-}10 \left[pW / \sqrt{Hz} \right]$

$$P_{in} = \frac{S}{N}(NEP\sqrt{\Delta f})$$

-También se usa:

Detectividad (Detectivity) → $D = \frac{1}{NEP}$

Detectividad específica → $D^* = D\sqrt{Area}$

2) <u>Limite ruido shot</u> $(\sigma_{i_S}{}^2 \gg \sigma_{i_T}{}^2) \quad (I_d = 0)$

$$\lim_{(\sigma_{i_S}{}^2 \gg \sigma_{i_T}{}^2)} \frac{S}{N} = \lim_{(\sigma_{i_S}{}^2 \gg \sigma_{i_T}{}^2)} \frac{M^2 \Re^2 \boxed{P_{in}{}^2}}{2qM^2 F_{APD}(\Re \boxed{P_{in}} + I_d)\Delta f + \dfrac{4k_B F_n T}{R_L}\Delta f}$$

$$SNR = \frac{S}{N} = \frac{I_{ph}{}^2}{\sigma_{i_S}{}^2} = \frac{\Re P_{in}}{2qF_{APD}\Delta f} = \underbrace{\frac{\eta_R \boxed{P_{in}}}{2h\nu\Delta f}}_{\text{PIN}} \frac{1}{F_{APD}}$$

La SNR aumenta proporcionalmente a la potencia incidente

Es lo habitual en los forodiodos APD

(1) Agrawal, "Fiber-Optic Communication Systems", 3rd Ed., Wiley, 2002

(2) Saleh and Teich, "Fundamentals of Photonics", 1st Ed., Wiley, 2007